**Appendix**

**Table S1**: Six simulation scenarios for studying power of testing treatment effect of intervention B using a constant correlation parameter. In the six scenarios, we vary number of clusters in each sequence ($I$), different coefficients of variation for cluster sizes ($cv$), different ICCs ($α$), different baseline event rates (baseline), different designs in Figure 2 (Design), and varying effect sizes (1-effect size), respectively.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Scenario | $$I$$ | $$cv$$ | $$α$$ | baseline | Design (Figure 2) | 1- effect size |
| 1 | 12,18,36,60 | 0.4 | 0.2 | 0.2 | 1B | $ e^{θ\_{2}}=0.7$, $ e^{θ\_{1}}=0.75$  |
| 2 | 18 | (0,1,0.1) | 0.2 | 0.2 | 1B |
| 3 | 18 | 0.4 | (0.01,0.1,0.02) | 0.2 | 1B |
| 4 | 18 | 0.4 | 0.2 | (0.1,0.5,0.1) | 1B |
| 5 | 24 | 0.4 | 0.2 | 0.2 | 2A-2F |
| 6 | 18 | 0.4 | 0.2 | 0.2 | 1B | $e^{θ\_{2}}=\left(0.4,0.8,0.05\right)$, $e^{θ\_{1}}=0.75$  |

**Table S2**: Six simulation scenarios for studying power of testing treatment effect of intervention B using two correlation parameters in the common block exchangeable correlation structure ($α\_{0}, α\_{1}$).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Scenario | $$I$$ | $$cv$$ | $$(α\_{0}, α\_{1}) $$ | baseline | Design (Figure 2) | 1- effect size |
| 1 | 12,18,36,60 | 0.4 | (0.03,0.015) | 0.2 | 1B | $ e^{θ\_{2}}=0.7$, $ e^{θ\_{1}}=0.75$  |
| 2 | 18 | (0,1,0.1) | (0.03,0.015) | 0.2 | 1B |
| 3 | 18 | 0.4 | $α\_{0}$=0.03,0.06,0.1$α\_{1}$=(0.01,$ α\_{0}$,0.01) | 0.2 | 1B |
| 4 | 18 | 0.4 | (0.03,0.015) | (0.1,0.5,0.1) | 1B |
| 5 | 24 | 0.4 | (0.03,0.015) | 0.2 | 2A-2F |
| 6 | 18 | 0.4 | (0.03,0.015) | 0.2 | 1B | $e^{θ\_{2}}=\left(0.4,0.8,0.05\right)$, $e^{θ\_{1}}=0.75$  |

**Table S3:** 19 combinations of the two correlation parameters for simulations using the common block exchangeable correlation structure ($α\_{0}, α\_{1}$).

|  |  |  |
| --- | --- | --- |
| Combination index of correlations |  $$α\_{0}$$ | $$α\_{1}$$ |
| 1 | 0.03 | 0.01 |
| 2 | 0.02 |
| 3 | 0.03 |
| 4 | 0.06 | 0.01 |
| 5 | 0.02 |
| 6 | 0.03 |
| 7 | 0.04 |
| 8 | 0.05 |
| 9 | 0.06 |
| 10 | 0.1 | 0.01 |
| 11 | 0.02 |
| 12 | 0.03 |
| 13 | 0.04 |
| 14 | 0.05 |
| 15 | 0.06 |
| 16 | 0.07 |
| 17 | 0.08 |
| 18 | 0.09 |
| 19 | 0.1 |



**Figure S1 A-F**. The empirical type I error rate for testing the treatment effect of B in six different scenarios in Table 2 using the misspecified exchangeable working correlation structure. The gray band in each plot represents 95% confidence interval of the nominal type I error rate 0.05.



**Figure S2 A-F.** The statistical power for testing non-zero treatment effect of B in six different scenarios in Table S1 using the true working correlation structure. The empirical power of the eight tests is shown with the predicted power calculated from equation (5).