

Appendix A

The basic equations are:

$$\frac{dX_1}{dt} = -\alpha X_1 X_2 \quad (\text{A1})$$

$$\frac{dX_2}{dt} = \alpha X_1 X_2 - \gamma X_2 \quad (\text{A2})$$

The boundary conditions at $t = 0$ are:

$$X_1(0) = (N - a) \text{ and } X_2(0) = a \quad (\text{A3})$$

Given these boundary conditions, $X_1(t)$ and $X_2(t)$ are related by (see (13) in Methods):

$$X_2(t) = N - a + \frac{N}{R} \log \left(\frac{X_1(t)}{N-a} \right) - X_1(t) \quad (\text{A4})$$

For small t , we can expand the log in (A4) as follows:

$$\log \left(\frac{X_1(t)}{N-a} \right) = \log \left[1 - \left(1 - \frac{X_1(t)}{N-a} \right) \right] \sim - \left(1 - \frac{X_1(t)}{N-a} \right) \quad (\text{A5})$$

which, when substituted into (A4) gives, after some simple algebra,

$$X_2(t) = \frac{R-1}{R} \left[N - X_1(t) \left(1 - \frac{a}{(R-1)N} \right) \right] \sim \frac{R-1}{R} [N - X_1(t)] \quad (\text{A6})$$

Substituting these into (A1), we get the Logistic Equation:

$$\frac{dX_1}{dt} = -\gamma(R-1)X_1(t) \left[1 - \frac{X_1(t)}{N} \right] \quad (\text{A7})$$

whose solution, with the boundary condition (A3) is:

$$X_1(t) = \frac{N}{\left[1 + \frac{a}{N-a} e^{\gamma(R-1)t} \right]} \quad (\text{A8})$$

For small t , we can expand the denominator to get:

$$X_1(t) \sim N \left[1 - \frac{a}{N-a} e^{\gamma(R-1)t} \right], \text{ } t \text{ small} \quad (\text{A9})$$

Substituting into (A6) gives,

$$X_2(t) \sim \frac{(R-1)aN}{R(N-a)} e^{\gamma(R-1)t} \quad (\text{A10})$$

This analysis shows that for small t , $X_1(t)$ decreases exponentially from its initial value of $(N - a)$ and this decrease feeds an exponential increase in $X_2(t)$.

(A10) also shows that the coefficient of t in the exponent of $X_2(t)$ is $\gamma(R - 1)$. (A11)