The State Value

Moustafa Ahmed AbdElaal (✉️ mostafamba@yahoo.com)
Alexandria University

Nesrien Mohamed Elmohamady
Alexandria University

Research

Keywords: State value, Ornstein-Uhlenbeck stochastic process, Wealth of Nations, country assets, Evaluating Natural Resource, country's wealth, Assets Pricing.

Posted Date: October 29th, 2020

DOI: https://doi.org/10.21203/rs.3.rs-97507/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License
Abstract
This paper considers the state value using the real option approach. Our model allows adding unlimited number of factors which affects the state value. We adjusted Ornstein-Uhlenbeck stochastic process to be able to consider unlimited number of pricing factors to calculate the state value. We think this model may be useful to evaluate the performance of the government and making decision process via knowing the optimal value and the optimal time for the decision. This dynamic model differs from the traditional pricing model for evaluating the nation’s wealth using the discounted cash flow model (DCF) which does not allows considering the market condition via using the risk-neutral approach. The states value determinants are divided into two classes, determinants and sub-determinants.

Key words: State value; Ornstein-Uhlenbeck stochastic process; Wealth of Nations; country assets; Evaluating Natural Resource; country’s wealth; Assets Pricing.

Introduction
The traditional method for calculating the wealth of nations uses the Discounted Cash Flows (DCF) model to discount the cash flows of the country's assets. Thus, the traditional DCF only considers the physical assets to calculate the wealth of nations. In reality, there are many valuable assets other than the physical assets; these assets such as the human capital and the reputation of the industry of the country. The reputation of country makes the producers of this country ask a premium on their products. Thus, the reputation factor can increase the GDP of the country and the wealth of this country in the same time. On the other hands, some financial components were not been priced in the previous studies such as the net foreign assets.
Fortunately, the World Bank report (2018) considered the human capital and the net foreign assets using the traditional method of evaluating assets. The traditional ways for pricing the assets use the Discounted Cash Flows (DCF) method. The DCF considers the discounted cash follows of the assets and sometimes consider the real probabilities of theses cash follows to estimate the expected discounted cash follows of these assets. Unfortunately, the DCF does not price the external factors affecting the price of these assets; such as the inflation rate and the technology level. Furthermore, the DCF also does not enable us from considering the stochastic movements of the cash flows. Thus, DCF make it is difficult to evaluate the
performance of the government and its ability to manage the returns and the risks of the
country's assets.
This paper solves this problem by pricing the assets of the state (the wealth of the country)
using the risk-neutral approach. We consider the state as a firm and the government as a board
of directory. This look enables us from using the real-option approach to price the state asset
and, therefore, calculate the state value.
We included the coefficients of the non-priced factors into Ornstein-Uhlenbeck model which
enable us to consider the effect of the external factors, such as the inflation rate and the
technology level, in the pricing process. Hence, this paper aims to answer the following
questions. First, how can we measure the state value? Second, how can we consider the
determinants of the state value in the pricing models?
The next part will be divided into five parts. The first reviews the state value sub-
determinants. Second, reviews the perspectives of the state value. Then we represent the
model in the third part. The forth section introduces the state value. Finally, inspects the future
research.
1. The state value sub-determinants:
Many researchers and organizations presented visions of the wealth of nations. For example,
Adam Smith made a paradigm shift by thought the labor as one of the nation’s assets. He says
in his work "The Wealth of Nations":
"The annual labour of every nation is the fund which originally supplies it with all the
necessaries and conveniencies of life which it annually consumes, and which consist always
either in the immediate produce of that labour, or in what is purchased with that produce from
other nations." (Adam Smith, 1776).
Simon Kuznets defined the nation’s wealth as the stock of country wealth (Simon Kuznets,
1938). Michael Porter introduced new types of national wealth by introducing "The
Competitive Advantage of Nations", that the nation has much wealth increasing its
competitiveness, such as education and health...etc. (Michael Porter, 1990). Moreover, Credit
Suisse S.A each year publishes the Global Wealth Report Data book, which includes the net
wealth of countries. The book includes the real estate prices, exchange rates, equity market
prices, liabilities, the adult population, human resources, natural resources, and the capital and
technological advancements.
In valuing the country's assets, most papers take into account only three factors as an influence
factors which lead the assets prices. For example, Schwartz (1997) used the three-factor model
to price the future contracts of the commodity in order to valuing financial and real assets.
These factors are the logarithm of the spot price of the commodity, the convenience yield of
the commodity, and the interest rate.
This paper, will call these factors the "determinants" of the state value. In the reality, these
factors themselves are affected with other factors such as the inflation rate, the country's
reputation, the governance level, the economic geography, and the Infrastructure. We will call
these factors as the "sub-determinants" of the state value. The contribution of this article is considering the sub-determinants in the pricing model.

In the next section, we will display extra factors, sub-determinants, which may affect the state value, considering that the main determinants will be covered within the model. In this paper, the sensitivity of the three factors of the Schwartz (1997) to the previous sub-determinants will be considered by the coefficient $\beta_i$, the following part will study the sub-determinants of the state value:
1.1. The country’s Reputation

The reputation refers to the view, which had been given to a person, product, business, or organization (Inny, 2014). But more precisely, the reputation refers to the aggregated evaluation of past, present and future actions of a company which makes an organization credible and trustworthy (Ch. Fombrun and C. van Riel, 1997). Therefore, the reputation of a country depends on people’s experiences and countries’ views which in turn determine behavior patterns, attitudes, and activities of people and countries concerning a given country. In the dynamic, contemporary, knowledge-based, technology economy era, one of the most important assets of a country’s wealth is the reputation. In this paper, we not only consider the reputation of businesses or other organizations or individuals, but also of countries. The country’s reputation guarantees achieving strategic advantage and continuing development prospects. This advantage will oblige states to try to preserve their international reputation. Where, the countries that violate international law will have a poor reputation, which leads other states to avoid cooperative opportunities with them in the future, reducing their competitiveness position on the global market, and reducing investment. Poor reputation also leads to escaping talented workers, tourists, consumers, scientists, artists, etc. Therefore, one of the most valuable merits, which help the country to build its wealth, is its reputation. Consequently, countries put in every effort to facilitate a positive perception of their country by maintaining its merits and values for an excellent reputation.

Countries can create their good reputation through the quality of its brands, the efficiency of its internal and external policy, the efficiency of attracting foreign investment and entrepreneurs, the efficiency of attracting skilled workers, the efficiency of attracting scientists and students ... and others, its culture and national heritage, the view to its citizen whether they are famous or unknown, and the size of its tourism (S. Anholt, 2007).

The Reputation Institute has been measuring the countries’ reputations since 1999. Indicators of countries’ reputation are calculated based on 16 attributes within three areas: effective government: safe place, ethical country, responsible participant in the global community, progressive social and economic policies, operates efficiently, favorable environment for business, appealing environment: friendly and welcoming, beautiful country, appealing lifestyle, enjoyable country, advanced economy: contributor to global culture, high-quality products & services, well-educated and reliable workforce, well-known brands, values education, technologically advanced. So, we can say that reputation depends on many factors such as natural resources, geography, political system, culture, traditions, and customs.

In this paper we will consider the country’s reputation in pricing the country assets using the time series of the institute reputation index.

1.2. Economic Geography

The economic geography refers to the connection between geography, location, and economics. In this paper we will investigate how the location, geography affect the country’s economic. This leads to the question: “does the economic geography consider one of the country’s wealth sub-determinants?
A country’s location and geography may directly affect sustainable economic development of the state through their effect on agricultural productivity, disease spared, the availability of natural resources (see, Gallup et al., 1999; Collier and Gunning, 1999; Ndulu, 2007), and institutional quality (Gallup et al., 1999; Rodrik et al., 2004). The location and geography may also reduce the country’s transport costs (Limao and Venables, 2001; Amjadi and Yeats, 1995).

Recently, the new economic geography (NEG) literature (Fujita et al, 1999) has highlighted the effect of the geographical location on a country’s prosperity. It was found that the location has a positive effect on market access, which in turn has a positive effect on economic development (Redding and Venables, 2004). Hence, market access positively affects the amount of forging direct investments (FDI) (Amiti and Javorcik, 2008), and the firm-level productivity (Lall, Shalizi and Deichmann, 2004). On the other hands, many studies found a strong positive effect of market access on income levels (Knaap, 2006; Breinlich, 2006; Amiti and Cameron, 2007; Deichmann, Lall, Redding and Venables, 2008).

Consequently, the location and geography may determinate the relative position and the economic importance of any country on the globe vis-à-vis other. In other words, the location and geography affects the importance of the states’ international relations, which plays a central role in economic development. On other words, the geography determinates the country’s ability to reach the global markets that in turn has a positive effect on the country’s level of income.

According to Fujita et al (1999) the economic geography may be measured via two measures; the distance from core economies (continuous variable) and if the country is landlocked or not (dummy variable). In this study, we will consider the economic geography as a continuous variable, while we will let the economic geography as a dummy variable to the future research.

1.3. Economic and social Infrastructure

Typically, the Infrastructure means to the basic structures that support and facilitate economic activities (ADB, 2017). It includes railways, highways, and roads, airports, mass-transit, sanitation, education, health, electricity, gas, water supply, waste treatment, correctional institutions, police, fire service and judiciary, telecommunication by landlines, mobile phones, and internet systems. Some even added the green infrastructure, which aims to enhance a wide range of important goods and ecosystem services, such as clean water and air, food and green spaces in urban areas, while improving quality of life and supporting a green economy.

Many literatures address the importance of efficient infrastructure in promoting sustainable economic development (Arrow and Kurz, 1970; Aschauer, 1989; Calderon and Serven, 2003; Estache, 2006). Infrastructure is important for two reasons. First, it serves the consumption of residents. Second, it is one of the important inputs of private and public sector production. It is one of sub-the determinants of the nation’s wealth that stimulating sustainable economic growth, by prompting the public and private production, reducing the cost of intermediate goods, or through externality effect in reducing poverty and providing a good life.
Furthermore, Green Infrastructure plays a vital role in supporting contemporary development for urban and rural areas. The Green Infrastructure is creating healthier communities, that can lead to improved mental health and general wellbeing and contribute to stress reduction, reducing the local temperature and energy demand, keeping excessive rainfall.

Moreover, other studies concluded the positive relation between the growth's rate of output per capita and the stock of public capital (see, Barro 1990, 1991). Biruk Birhanu Ashenafi, (2017) illustrated that the infrastructure development relates to economic growth whether in the long run or short run. On the other hands, some studies found that the infrastructure increases productivity (Ratner, 1983; Aschauer, 1989a, b). Nevertheless, other studies focus on both of the differential impacts of capital and current components of public spending on economic growth (Devarajan et al., 1996). Therefore, infrastructure is a nub for the development of an economy.

In this study, we will represent the infrastructure using the method of Calderón and Servén (2004). There are three indexes for the infrastructure stock. The first index is the telecommunication sector index, which is calculated with the number of main telephone lines per 1,000 workers. The second index is the power sector index. We can represent the infrastructure stock of the power sector by the electricity generating capacity of the economy or the MW per 1,000 workers. The last index is the transportation sector which may be represented with the length of the road network. The infrastructure may also be measured by the world economic forum index in the global competitiveness report. It is measured by three pillars (transport infrastructure, utility infrastructure, ICT adoption) (WEF, 2019).

1.4. The governance

Although, there are no Consensus exists about the definition of governance, the common among scholars is that governance referred to the process by which governments are selected, controlling, and replaced, and the participation in the political and decision-making process by nongovernmental institutions (Osborne &Gaebler, 1992; Rhodes, 1997; Peters & Pierre, 1998; Agere, 2000; Newman, 2001; Kettl, 2002; Lovan et al., 2004; Hysing, 2009). It is the way in which public institutions get their authority, and exercise it to provide public goods and services and make public policy. The governance introduces a good relationship between people and government, which includes efficiency, honesty, responsiveness, and quality (de Ferranti et al., 2009). Moreover, it refers to the capacity of the government to effectively formulate and applying sound policies, and earns the respect of the residents and the states for the Institutions that govern economic and social activates (Kaufmann et al., 2009). And it’s alternative to traditional tools of governing (Kettl, 2002; Rhodes, 1997).

For decades, international organizations (IOs) such as the IMF, the United Nations, and the World Bank, have called for an application of good governance as a way for developing human resource and sustainable development (Santiso, 2001; Mimicopoulos et al., 2007; United Nations, 2007).
There are many standards to measure the governance, such as the world’s governance indicators WGI. The indicator consists of the next pillars (Kaufmann et al., 2009):

1. Rule of Law: It means the quality of contract enforcement, including the protecting property rights, the functioning and independence of the judiciary, and the police.
2. Government activity: the quality of policy formulations and implementation and the credibility of the government’s commitment to such policies, the degree of its independence from political pressures, the competence of the bureaucracy, and the quality of public and civil service.
3. Voice and Accountability: the civil and human rights, the degree of people participation in selecting the government, besides the freedom of expression.
4. Regulatory quality: It means the ability to formulate and apply sound policies and regulations that authorization and encourages the development of the private sector.
5. Control of Corruption: reduce using the public power for private benefits.
6. Political Instability and Violence: it means the likelihood of terrorism threats, or changes in government.

The United Nations also presented characteristics of good governance practices in the global standard that adopted by governments which receive their aid. This index helps to measure how governments are achieving governance (Mimicopoulos et al., 2007). According to it, there are eight pillars for the good governance; it is participatory, accountable, consensus-oriented, responsive, transparent, efficient and effective, inclusive and equitable, and follows the rule of law (Biswajit Satpathy, et al., 2013).

Good governance affects all country’s aspects in many ways. First, it affects the quality of services which provide to the citizens, and the quality of government work (Mimicopoulos et al., 2007). The absence of good governance may lead to increase corruption costs, lowering productivity, obstacle investments, lowering confidence in public institutions, destroying a small- and medium-sized enterprises, weakening systems of public financial management, reducing investments in health and education, and reduce the economic growth level (Fisman and Svensson, 2007; Olken, 2006). Second, it relates positively to the levels of economic growth by attracting more investments, encouraging higher levels of human capital accumulation, getting higher credit ratings, introducing higher quality public services, using the foreign aid resources in better way, increasing the productivity of government spending, improving technological innovations, and curbing health and education degradation (Easterly et al., 2006; Lewis, 2006; WB, 2006a). Third, governance improved regulatory quality that can lead to promoting economic growth by creating more incentives for the private sector, helping the poor by creating more opportunities for entrepreneurship. It may decrease opportunities for corruption, improving the quality of public services, and getting labor and housing markets better (Djankov et al., 2006; Dollar et al., 2006; Loayza et al., 2006; WB, 2006b; Jalilian et al., 2007). Fourth, by the rule of law judicial independence promotes a stable investment environment that leads to higher levels of investment and growth and thus helps in reducing poverty through efficient legal systems (Easterly et al., 2006; WB, 2006b). Fifth,
international donors such as international organizations like the IMF and the World Bank and countries; are using the governance to determine the eligibility of States for grants and aids (Santiso, 2001; Poluha & Rosendahl, 2002; Mimicopoulos et al., 2007). Therefore, governance is one of the most important elements of the wealth of States, whose absence may hamper the development process.

The governance level could be measured via the several indicators such as The Worldwide Governance Indicators (WGI).

1.5. The efficiency of Markets (capital – goods & services – housing)

Efficient Markets should pursue good governance rules, whether they are capital, workforce, goods, and services, and housing markets that help it promote the growth and development of the economy.

We will discuss the efficiency of market (capital – goods & services – housing) in the next part:

**Goods and services markets:** The efficient of goods and services market may reduce the transaction costs, encourage industrialization, increase the employment rate, promote the aggregate demand and consumption, stimulate the growth of complementary industries, raise the saving rate and productivity, and create the investment climate. Not to mention with the efficiency of markets play a vital role in the integration into the world economy. The efficient Markets is the main rationale behind trade liberalization and both the GATT and the WTO (David Ricardo, 1817). On the other hands, the costs of market failure are less than the costs of state failure. Therefore, the state should focus on increasing the market efficient, because the improvement in market-enhancing governance conditions would promote growth the advanced countries (Kaufmann, Kraay and Mastruzzi, 2005).

**Financial market:** its development will lead to economic development and growth. The financial sector system is standing behind in developing the developed countries than the developing countries (Fisher, 1930; Gurley and Shaw, 1955. Some studies concluded that the better industrialization is the basic factor to increase the stock market growth and enhance the economic growth of the economy (Greenwood and Jovanovic, 1990; and Bencivenga and Smith, 1991; Marques, 2013; Babajide, 2016). Moreover, the efficient financial markets can raise the GDP per capita because it increases savings, investments, and growth rate. The efficient financial markets also improve the allocation of resources and reduce the costs of establishing a sophisticated financial structure (Gurley and Shaw, 1955; Goldsmith, 1969; Shaw, 1973; Boyd and Prescott, 1986; Berthemy and Varoudakis, 1996, Greenwood and Smith, 1997; Aghion et al., 2004). Levine (2001) showed that the stock market liquidity and development prompted economic growth. Therefore, the economy may stagnate when the stock market got crushed (Barro, 2017).

**The housing market:** it is a fundamental component of development. The construction industry has a positive effect on macroeconomics for several reasons. First, the housing sector plays an essential role in rising consumption, where the buying and selling of houses is a significant portion of the total value in an economy (Jonathan McCarthy and Charles Steindel,
The efficient housing market can improve the performance of the other sectors in the economy via raising the overall consumption and the investment capacity of the community. The second reason is that the housing sector plays a pivotal role in the growth of complementary industries, where several sectors of the economy benefit from increased housing. The most beneficiaries would be the construction and home renovation industries, and the durable goods firms. The efficient housing market has a multiplier effects on real estate agents, surveyors, notaries, and bankers; this has multiplier effects throughout the economy. In most cases, the construction and purchase of new houses increase the demand on household durable goods, furniture, household décor, and increase the employment in the local shipping and delivery services sectors; therefore, it can redouble the GDP (NACCA, 2005). Third, the efficient housing market create a profitable investment opportunities which attract the foreign direct investment (FDI). Forth, the housing markets increase the savings rate because it is one of the most important tools for the "forced savings". This is very useful because the saving represents the future consumption, so it is an essential component of the economy. Fifth, because of the above, the housing sector is a motivation to increase the growth rate; this is because the efficient housing market increases growth rate of the small and medium business, labor abilities, and technology (De Soto, 2000).

In this paper, the efficiency of capital market could be measured by the market index (WEF, 2019). The efficiency of product market is measured by the WEF index (WEF, 2019). The efficiency of housing market can be measured by Housing Market Index (HMI).

1.6. Institutions

The efficient of institutions may play a vital role in economic growth, far more than the natural environment (North, 1988; Rodrick et al., 2002; Petrunya and Ivashina, 2010). Although the importance of geography and cultural to economic growth, the efficient of institutions are essential in explaining long-run economic growth (Acemoglu et al., 2005, Weil, 2008). It enhances the sustainable development in the economy (Easterly, 2008; Acemoglu et al., 2005), the investments in physical and human capital, technology, the organization of production, allocation of physical and human resources (Acemoglu et al., 2005, Rodrick, Subramanian & Trebbi 2002), the supplying the factors of the organization of production and technological (Acemoglu, Johnson & Robinson 2001), the research and development (R&D) (North, 1990; Acemoglu, Johnson & Robinson 2005; Weil, 2008), and develops the distribution of resources in the country (Acemoglu et al., 2005).

The institutions comprise of the organizations of protect property rights (North & Thomas, 1973; Acemoglu et al., 2001), those which mobilize savings for investment (Tchouassi 2014), and those that hold officials accountable by the public (Keefer, 2005; Acemoglu & Robinson, 2012).

The institutions variable could be measured by the world economic forum's index in the global competitiveness report. It is measured by eight pillars (security, social capital, checks and balances, public- sector performance, transparency, property rights, corporate governance, future orientation of government) (WEF, 2019).
1.7. Democracy

The political freedom is very substantial to the economic growth (Kormendi and Meguire, 1985; Scully, 1988). There are various arguments to the link between democracy and the quality of governance. Democracy allows the citizen to peacefully and regularly change inefficient and corrupt government administrations, and allows people to keep more efficient, successful regimes. The rulers in the democratic regimes may want to listen to the people’s needs and face their criticism for getting their support in elections. Without a democracy, transparent political institutions and a free press, people can’t control their governments and understand these risks. Thus, Democracy raises the quality of governance in the long run.

The democracy has a positive effect on education spending and widened education coverage (Lindert, 2005). Stasavage (2005) finds that the democracy improves the primary education. Increasing democracy raise the growth of the human capital (Baum and Lake, 2003; Tavares and Wacziarg, 2001; Doucouliagos and Ulubasoglu, 2008).

Several studies found that the democracy causes positive technological change and thus productivity growth (e.g. Przeworski et al., 2000; Halperin, Siegle and Weinstein, 2005; North, 2005; Pinto and Timmons, 2005; Faust, 2007; Alesina and Trebbi, 2008; North, Wallis and Weingast, 2009).

Some studies such as Baum and Lake (2003) used the popular index six-step Guttman scale of openness of executive recruitment to measure democracy. The index puts sex scales for democracy. Thus, the measurement of the democracy needs to use dummy variables which are out of the interest of this paper. Hence, we let the democracy factor for the future research.

1.8. Exchange rate

Foreign exchange rate refers to the value of the money of a country in the currency of another country. It depends on many factors such as economic growth, inflation rate, unemployment rate, deficit and surplus balance of payments ... etc. If the government financed the economic growth from the budget deficit and borrowing, this will have an adverse effect on the future currency, unless it is managed wisely. Reversely, financing through increased investment and exports will have a positive impact on the exchange rate.

The exchange rate can influence positively or negatively the economy of a country (Chen, 2012). It may affect the ability of the state to achieve sustainable development and increasing GDP (Aizenman and Lee, 2010; McLeod and Mileva, 2011; Benigno et al., 2015). Thus, it considers one of the important factors which makes countries more or less vulnerable to crises.

Conversely, Hausmann, R., Pritchett, L. and D. Rodrik (2005) found that the rapid growth accelerations are often correlated with real exchange rate depreciation. It may have negative effects on the economic growth and the trade sector because of relapsing international competitiveness and job losses. Aman, Q., Ullah, I., Khan, M.I., Khan, S., (2013) illustrated that the change in the exchange rate has a positive effect on the economic growth through the increasing export, and enhancing investment. Glüzmenn, P. A., Levy-Yeyati, E. and F.
Sturzenegger (2012) found that the exchange rate leads to increase saving and investment through lowering labor costs and income redistribution, it does not affect the trade sector but leads to greater domestic savings and investment, as well as employment, in developing countries (Ramzi et al., 2012). Nevertheless, there are limitations on the reduction of the exchange rate, because the large over and undervaluation Hamper growth, while modest undervaluation enhances growth (Razin and Collins, 1997; Aguirre and Calderon, 2005).

The exchange rate can be measured by Exchange Rate Index.

2. The perspectives of state value

The traditional approach for evaluation of any asset is the Discounted Cash Flows (DCF). The DCF method is simple and easy to execute. However, the DCF method does not consider the stability of the cash DCF inputs such as the cash follows, the risk attached with these cash follows, and the factors affects cash follows. The cash follows of any asset is affected with many factors; such as the technology progress and the inflation rate. The DCF may consider the inflation and the growth rate by multiplying the discounting part of the DCF formula by the inflation rate or the growth rate. The DCF does not consider the stochastic movement of the inputs of the DCF. To illustrate this point, let's assume that we price a mine. If we use the DCF model for capital budgeting, the mine value will be affected with the changes in the copper price. The DCF model does not consider the shocks in the copper price or the shock in the discounting rate. In other words, DCF model for capital budgeting failed to consider uncertainty in the cash flows of the projects.

The evolution of capital budgeting's algorithms produced real option model, which is one of the most popular models nowadays, especially in the oil companies. The real option gives the decision maker the flexibility in timing the market and chooses the optimal time to do anything. For example, the real option model gives the management the optimal strategy for abandonment or switch. According to the real option model, the price of the underlying asset follows a geometric Brownian motion (GBM) process. This flexibility is not available in the NBV model.

2.1. Evaluating the state assets on the micro-level:

Many researchers attempted to price the assets of the state as separate units. These studies priced stand-alone projects. For example, they considered estimating the fair value of mines and oil futures contracts. The oil futures models are a suitable to both the future contracts and the real option projects.

The contribution of Brennan and Schwartz (1985) is important on the micro-level. Brennan and Schwartz (1985) evaluated the assets when the cash flows determined by future management decisions, not by the current cash flows. Brennan and Schwartz (1985) did not present a comprehensive look for assets of the country. They evaluated only the mining and other natural resource projects. The evaluation of the state value requires pricing all components of the wealth; such as, produced capital and urban land, natural capital, human capital, and net foreign assets (The Changing Wealth of Nations, 2018).
The real option approach has developed and a stream of researches was presented. Recent papers tried to develop the evaluation models by considering new factors in the pricing process. These factors result in a premium on the asset. Cortazar and Schwartz (1997) presented an evaluation model for undeveloped oil field in case of no arbitrage. The researchers continued in presenting richer model to price the real projects. Schwartz (1997) developed a two and three factor models for commodity prices. The models are suited to financial and real option pricing. Schwartz (1997) considered the long run behavior of the assets and the deviation of its prices from the long-run mean. Cortazar, G., Schwartz, E. S., & Casassus, J. (2003) considered a three-factor model to price oil futures contracts. The model can be used in other future contracts and in asset pricing. Zhang and Sun (2018) estimated the ocean wealth of China. Their study calculated the ocean wealth by summing three types of wealth which are: Ocean manufactured capital, Ocean human capital, and Ocean natural capital. Beside the previous components, they considered the two factors affecting the ocean wealth. These factors are the effect of environmental externalities and the population change. Zhang and Sun presented value of the ocean wealth without investigating the volatility and the stochastic movement of the components of the wealth. The presented model depended only on constant parameters. In this paper we treated this issue by assuming that the assets satisfy a Brownian motion. Although the previous models were effective in evaluating the assets of the country and make decisions on it, they are not qualified to represent the performance of the government at all. The earlier models miss the comprehensive look for the country.

We concluded the problems of the micro-level approach:

- The micro-level approach only takes a snapshot for the assets of the country. Such as mines, copper, and other natural resource projects. The micro-level approach also interested in pricing the foreign assets, and the intangible assets. According to this approach, the assets of market value are separately calculated for each asset. Using this approach, we can evaluate the performance only of the management of this asset. The problem now is that the micro-level approach does not give us a comprehensive look for the entire asset of the country. Thus, we cannot evaluate the performance of the government when we use the micro-level approach.

- The micro-level approach does not give the government the ability to make decision regarding to the total asset of the country; such as the changing the interest rate and impose an extra tax.

- Although some studies in the micro-level approach considered the real option approach in evaluating the country's assets, they did not include the effect of many factors on the assets. For example, Brennan and Schwartz (1985) did not consider the effect of the technology shocks on the evaluated assets.

2.2. Evaluating the state value using the macro-level approach:

There was a parallel stream of research, beside the micro-level approach, to evaluate the state assets. These studies considered a comprehensive look for the assets of the country. The
World Bank (2018) presented four components for evaluating the total wealth of the country. These components are natural capital, produced capital, human capital, and net foreign assets. The natural capital includes minerals, agricultural land, comprising energy, protected areas, and forests.

Many economists now are also paying attention to the importance of natural resources as an important country’s assets. Natural resources can be classified into seven types: fisheries, forests and pastureland, and agricultural land and protected areas, water resources, oil, coal, natural gas, and minerals. Even under this classified, there is a sub-classification based on the sort of each asset such as the type of soil, water.

The abundance of natural resources provides water, soil, forests and materials for all most of the aspects of the life; for example, drinking, washing, fish’s wealth, export, tourism, industries and irrigation. These resources may influence all the country’s aspects by its effect on industrial development, tourism and poverty reduction. Therefore, playing a critical role in economic growth and sustaining development. Wright (1990) argued the major cause of America’s technological progress was the abundance of mineral resources. On the other hand, Barbier (1999) found a difference results. Barbier illustrated that many low and middle-income economies with rich resources have low or stagnant growth rates.

The liberalization of world commodity markets helped the countries suffering from poor resources to get its needs.

Some researchers found that the change in the global during the post-wars era may have affected the primary-product export in the developing economies. Therefore, they no longer become the premier source of economic growth compared to the labor-intensive manufactured exports (Findlay, 1996; Findlay and Wellisz 1993). Undoubtedly, natural recourse could not be worth a valuable wealth to the country unless there is a supporting factor, such as technology, industrial, developing markets.

The second component of the total wealth of the country, according to the World Bank (2018), is the produced capital which contains structures, equipment, comprising machinery, and urban land. The human capital contains some intangible assets such as, experience, knowledge, and skills. The net foreign assets are the summation of the equity, debt, foreign direct investment…, etc. Although the World Bank report presented a comprehensive look for the position of the country wealth, the report mainly depended on the DCF method. The report assumed that the assets’ values are stationary. The report did not consider the stochastic natural of the assets prices and the cash flows. The report may help the reader know the value of the nation’s wealth; but it may not help the decision maker in this country to know the optimal time to a special decision. The World Bank report does not enable us to consider any other factor affects the assets. For example, the education level could not be considered in this model when we price the state assets. The traditional way for evaluating the assets of the country does not allow to considering the market effect.

In this paper we solve these problems by using the real option model, which enables the user of the model to achieve the optimality. We used the Ornstein-Uhlenbeck process which
enables us to price other factors in the model. We presented a new contribution by considering the strength of the relationship between the asset price and the priced factors. The strength here refers to the regression coefficient between the asset price and the priced factors. In this paper we call the strength coefficient as a contribution because Ornstein-Uhlenbeck process is not considering this coefficient. The model of the study satisfies the risk-neutral approach, which considers the market effect in the pricing model to make it more accurate.

3. The model:
According to the World Bank report (2018), the state value includes Natural capital, Produced capital, human capital, and net foreign assets. We will evaluate each component separately using our model considering the real-option approach:

3.1. The Natural capital:
The Natural capital includes many components such as the agriculture assets, the energy and mineral resources, and the protected areas and forests. We will price each asset in the following sections.

3.1.1. the agriculture assets:
Assume that the agriculture assets price follow the next geometrical Brownian motion

\[ dS(t) = \mu S(t)dt + \eta S(t)dW(t) \] ...

(1)

\[ d\ k\beta_1(Ar - ar)ds + \sigma dZ(s) \] ...

(2)

Where \( \mu \) is the drift of the asset price model, \( \eta \) represents the volatility of assets, and \( dW(t) \) is the increment to the Brownian motion of the asset price.

The price of asset is affected by the shocks in agricultural expands. The agricultural expands satisfy the Ornstein-Uhlenbeck process as the equation (2) reflects.

In equation (2), \( k \) is the speed of mean reversion of the agricultural expands return \( ar \) to its long-run mean \( Ar \). \( ar \) and \( Ar \) represent the instantaneous marginal yield of agricultural expands, and long-run mean, respectively; so \( ar \) revert to \( Ar \). \( dZ(s) \) reflects the increment to the Brownian motion of the agricultural expands.

The contribution of this paper is \( \beta_1 \) which represents the sensitivity coefficient of the agricultural expands toward the external factors 1,2,3,...,i (sub-determinants), such as, the inflation rate, exchange rate ..., etc. we can estimate the \( \beta_1 \) coefficient via the following equation:

\[ \beta_1 = \beta_{11} \times \beta_{12} \times \beta_{13} \times ... \times \beta_{1i} \]

Where \( i \) represents any important external factor affects the agricultural expands such as, the inflation rate, exchange rate... etc. the external factor may include the reputation which could be represented by the indicators such as Reputation Institute Index.

Including \( \beta_1 \) in the model change the state of the \( ar \) variable from an exogenous variable to be an endogenous variable, so its value is estimated outside the model. We think the pricing model will be more realistic after adding \( \beta_1 \).

We can integral equation (2) as:

\[ \int_t^T d\ k\beta_1(Ar - ar)ds + \int_t^T \sigma dZ(s) \] ...

(3)
The previous equation may be rewritten as following:

\[
\int_t^T d ar(s) = k\beta_1 Ar(T - t) - k\beta_1 \int_t^T ar(s) \, ds + \sigma \int_t^T d Z(s)
\]

So the cumulative yield of agricultural expands from the date \( t \) until \( T \) is:

\[
ar(T) - ar(t) = \int_t^T ar(s) \ldots (4)
\]

As in Bjerksund (1991), we assumed that cumulative agricultural expands yield could be expressed as following:

\[
X(t) \equiv \int_0^T ar(s) \, ds
\]

Where \( X(t) \) is the agricultural expands cumulative return for the period from 0 to \( t \).

The agricultural expands cumulative return, for the period from 0 to \( T \), \( X(T) \) may be calculated by inserting \( X(t) \) into equation (4) results in

\[
X(T) - X(t) \equiv \int_t^T ar(s) \, ds \ldots (5)
\]

Substituting for equations (4), (5) in equation (3) we get

\[
ar(T) - ar(t) = k\beta_1 Ar(T - t) - k\beta_1(X(T) - X(t)) + \sigma \int_t^T d Z(s) \ldots (6)
\]

The equation (6) gives us the agricultural expands cumulative return for the period from \( t \) to \( T \). Now it is easy to drive the asset value as we will see.

3.1.1.1. Evaluating the asset value and the variance considering the agricultural expands:

We may write the initial formula for evaluating the asset \( S \) on the date \( T \):

\[
V[S(T)] = e^{-r(T-t)}E[S(T)]
\]

According to Bjerksund (1991), the discounted future value of the asset can be expressed as follows:

\[
e^{-r(T-t)}S(T) = S(t) \exp \left\{ \left( \mu - r + \frac{1}{2} \eta^2 \right) (T - t) \right\} + \eta \int_t^T d W(s) \ldots (7)
\]

In Appendix (1), we derived the integral of the Brownian motion \( \int_t^T d w(s) \) of the assets price as:

\[
\int_t^T d w(s) = \int_t^T d w^* (s) - \frac{\mu - r}{\eta} (T - t) - \frac{1}{\eta} \left( X(T) - X(t) \right) \ldots (8)
\]

Multiplying the equation (8) by \( \eta \) and using the equation (9) (from Appendix (3)) to Substitute for \( X(T) - X(t) \)

This results in:

\[
\eta \int_t^T d W(s) = \eta \left[ \int_t^T d W^* (s) \right] - \frac{\mu - r}{\eta} (T - t) - \frac{1}{\eta} \left( X(T) - X(t) \right)
\]

\[
\int_t^T d W(s) = \eta \int_t^T d W^* (s) - (\mu - r)(T - t) - (X(T) - X(t))
\]

\[\text{In equation (4) we use} \int_t^T ar(s), \text{while in equation (5) we use} \int_t^T ar(s)ds.\]
Where we get the value \((X(T) - X(t))\) of the equation (9) and we substitute it in the previous equation and thus we get:

\[
\begin{align*}
\int_t^T d W(s) &= \eta \int_t^T d W^*(s) - (\mu - r)(T - t) - A r(T - t) - \frac{1}{k \beta_1} \sigma \int_t^T d Z(s) \\
&+ \frac{1}{k \beta_1} e^{-k \beta_1 (T - t)} a r(t) - \frac{1}{k \beta_1} A r(1 - e^{-k \beta_1 (T - t)}) \\
&+ \frac{\sigma}{k \beta_1} e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d Z(s) - \frac{a r(t)}{k \beta_1} \quad (10)
\end{align*}
\]

Inserting the equation (10) in the equation (7) we get the future discounted pay-off.

\[
\begin{align*}
e^{-r(T-t)} S(T) &= S(t) \exp \left( \left( \mu - r + a r - \frac{1}{2} \eta^2 \right) (T - t) + \eta \int_t^T d W^* (s) - (\mu - r) (T - t) \\
&- A r(T - t) - \frac{1}{k \beta_1} \sigma \int_t^T d z(s) + \frac{1}{k \beta_1} e^{-k \beta_1 (T - t)} a r(t) \\
&- \frac{1}{k \beta_1} A r(1 - e^{-k \beta_1 (T - t)}) + \frac{\sigma}{k \beta_1} e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d z(s) - \frac{a r(t)}{k \beta_1} \quad (11) \right)
\end{align*}
\]

Rearranging the previous equation, we get:

\[
\begin{align*}
e^{-r(T-t)} S(T) &= S(t) \exp \left( (T - t) \left( \mu - r + a r - \frac{1}{2} \eta^2 - \mu + r - A r \right) + \eta \int_t^T d W^* (s) \\
&- \frac{1}{k \beta_1} \sigma \int_t^T d Z(s) + \frac{1}{k \beta_1} \left( e^{-k \beta_1 (T - t)} a r(t) - A r(1 - e^{-k \beta_1 (T - t)}) - a r(t) \right) \\
&+ \frac{\sigma}{k \beta_1} e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d Z(s) \right)
\end{align*}
\]

\[
= S(t) \exp \left( (T - t) \left( a r - \frac{1}{2} \eta^2 - A r \right) + \frac{1}{k \beta_1} \left( e^{-k \beta_1 (T - t)} a r(t) - A r(1 - e^{-k \beta_1 (T - t)}) \right) - a r(t) \right) + \eta \int_t^T d W^* (s) - \frac{1}{k \beta_1} \left( \sigma \int_t^T d Z(s) - \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d Z(s) \right)
\]

From equation (8) we can easily get the value of \(\sigma \int_t^T d z(s)\). Thus, the future discounted pay-off may be written as:

\[
\begin{align*}
e^{-r(T-t)} S(T) &= S(t) \exp \left( (T - t) \left( a r - \frac{1}{2} \eta^2 - A r \right) \\
&+ \frac{1}{k \beta_1} \left( e^{-k \beta_1 (T - t)} a r(t) - A r(1 - e^{-k \beta_1 (T - t)}) - a r(t) \right) + \eta \int_t^T d W^* (s) \\
&- \frac{1}{k \beta_1} \left( \sigma \int_t^T d Z^* (s) - \sigma \int_t^T \lambda_1 d(s) - \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d Z^*(s) \right) \\
&+ \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} \lambda_1 d(s) \right) \quad (11)
\end{align*}
\]

Where \(\lambda_1\) is the market price per unit risk of agricultural return.

---

\(2\) Assuming that \(W^* = W - \bar{W}\), \(Z^* = Z - \bar{Z}\), the correlation between the change of the two Brownian motions \(W^*\) and \(Z^*\) is 

\[W^* \times Z^* = \rho\]
Thus, the expected value of this asset (agriculture, agricultural sector) can be calculated as follows:

\[
E = S(t)\exp\left\{ (T - t) \left( ar - \frac{1}{2} \eta^2 - Ar \right) + \frac{1}{k \beta_1} \left( e^{-k \beta_1 (T - t)} ar(t) - Ar(1 - e^{-k \beta_1 (T - t)}) - ar(t) \right) \right\}
\]

On the other hand, the variance is,

\[
\text{var} = \bar{\sigma}^2 \equiv E_t^\prime [(Z^\prime)^2] - (E_t^\prime [Z^\prime])^2
\]

Where

\[
(E_t^\prime [Z^\prime])^2 = \mu^2
\]

The previous magnitude represents the average of the X square.

\(E_t^\prime [(Z^\prime)^2]\) represents the mean of the square of \(Z^\prime\).

Where \(\mu\) is obtained from the equation (11) when we remove from it the variance.

\[
E_t^\prime [(Z^\prime)^2] = E_t\left[ \left( \eta \int_t^T d W^*(s) - \frac{1}{k \beta_1} \sigma \int_t^T d Z^*(s) - \frac{1}{k \beta_1} \sigma e^{-k \beta_1 r} \int_t^T e^{k \beta_1 s} d Z^*(s) \right)^2 \right]
\]

From Bjerksund (1991) we extract the following values:

- \(E_t^\prime = \left[ \eta \int_t^T d W^*(s) \right]^2 = \eta^2 (T - t)\)
- \(E_t^\prime = \left[ \frac{1}{k \beta_1} \sigma \int_t^T d Z^*(s) \right]^2 = \left( \frac{1}{k \beta_1} \right)^2 \sigma^2 (T - t)\)
- \(E_t^\prime = \left[ \frac{1}{k \beta_1} \sigma e^{-k \beta_1 r} \int_t^T e^{k \beta_1 s} d Z^*(s) \right]^2 = \left( \frac{1}{k \beta_1} \right)^2 \frac{\sigma^2}{2k \beta_1} (1 - (\theta)^2) = \left( \frac{1}{k \beta_1} \right)^2 \frac{\sigma^2}{2k \beta_1} (1 - (e^{-k \beta_1 (T - t)}))\)
- \(E_t^\prime = \left[ \left( \eta \int_t^T d W^*(s) \right) \left( \frac{1}{k \beta_1} \sigma \int_t^T d Z^*(s) \right) \right] = \frac{1}{k \beta_1} \eta \sigma \rho (T - t)\)
- \(E_t^\prime = \left[ \left( \eta \int_t^T d W^*(s) \right) \left( \frac{1}{k \beta_1} \sigma e^{-k \beta_1 r} \int_t^T e^{k \beta_1 s} d Z^*(s) \right) \right] = \left( \frac{1}{k \beta_1} \right)^2 \eta \sigma \rho (1 - \theta) = \left( \frac{1}{k \beta_1} \right)^2 \eta \sigma \rho (1 - e^{-k \beta_1 (T - t)})\)

\[
E_t^\prime = \left[ \frac{1}{k \beta_1} \sigma \int_t^T d Z^*(s) \left( \frac{1}{k \beta_1} \sigma e^{-k \beta_1 r} \int_t^T e^{k \beta_1 s} d Z^*(s) \right) \right] = \left( \frac{1}{k \beta_1} \right)^3 \sigma^2 (1 - e^{-k \beta_1 (T - t)})
\]

In order to calculate the variance, the following values must be calculated:

\[
\text{var} = \bar{\sigma}^2 \equiv E_t^\prime [(Z^\prime)^2] - (E_t^\prime [Z^\prime])^2
\]

Where

\[
E_t^\prime [(Z^\prime)^2] = \left[ \eta^2 (T - t) - \frac{1}{k \beta_1} \sigma \eta \rho (T - t) - \left( \frac{1}{k \beta_1} \right)^2 \eta \sigma \rho (1 - e^{-k \beta_1 (T - t)}) \right] - \frac{1}{k \beta_1} \eta \sigma \rho (T - t) + \left( \frac{1}{k \beta_1} \right)^2 \sigma^2 (T - t) + \left( \frac{1}{k \beta_1} \right)^3 \sigma^2 (1 - e^{-k \beta_1 (T - t)}) - \left( \frac{1}{k \beta_1} \right)^2 \sigma \rho (1 - e^{-k \beta_1 (T - t)}) + \left( \frac{1}{k \beta_1} \right)^3 \sigma^2 (1 - e^{-k \beta_1 (T - t)}) + \frac{1}{2k \beta_1} \sigma^2 (1 - e^{-2k \beta_1 (T - t)})
\]
Thus variance can be calculated \( \text{var} \) or \( \sigma^2 \) as follows:

\[
\text{var} = \sigma^2 = E_i[(Z')^2] - (E_i[Z'])^2
\]

The value of the asset is:

\[
V^*[S(T)] = E_i\left[e^{-k\beta_1(T-t)}S(T)\right] = E_i[S(t)\exp(Z')] = S(t)\exp\left\{ \hat{\mu} + \frac{1}{2} \sigma^2 \right\} \quad \ldots(13)
\]

### 3.1.1.2. Evaluating the state assets considering three factors:

In the previous discussion, we examined the state value considering only the effect of agricultural expands yields. We will now study the state value bearing in mind the technology effect.

The successive global developments have led to increased interest in innovation, invention, creativity, and technological developments. Despite the similarity of its daily use, in academic terms, there is a difference between them. The invention is the first occurrence of an idea for a new product or process, while innovation is the first attempt to carry it out into practice (Jan Fagerberg, 2004). Creativity is thinking about new things, while innovation is making new things. Creativity is about developing or thinking of new things and ways of facing problems and possibilities, while innovation is about making new things and the ability to perform creative solutions to enhance people’s life. Joseph Schumpeter (2002) defines innovation as an activity which leads to new producing. He divides it as follows: Introducing a new product, introducing a new method of production, opening new markets, finding of appropriate sources of raw materials, establishing a new organization in the industry.

Innovation, invention, creativity, and technology are considered as a key factor for sustainable growth and economic development. Where they help to provide various goods and services with the highest quality, raise the levels of citizen’s lives, increase employment opportunities and raises wages, help businesses to grow and continue to achieve profits, increase their competitiveness at the local and international levels, lead to productivity improvements, which in turn may improve economic performance (Martin et al., 1993; Graham, 2000), and help the prosperity of the national economy as a whole. It is noted that the beneficiary and the biggest buyer of new technology and products are financial, communications and services sectors, wholesale and retail trade, and transport, and create new jobs (Brett et al., 1991; Feldman, 1994; Dietz, 1998; De La Mothe and Paquet, 1998; Varga, 1998; Bozeman, 2000; Tornatzky
et al., 2002; Shane, 2005). Contrariwise, Papaconstantinou (1995) found that technology may lead to job losses, bring poor nations out of poverty (Malecki, 1997).

Bernstein (2004) sees that economic growth requires four factors: property rights, scientific rationalism, capital markets, and fast and efficient communication. Therefore, technology is a necessary condition for development, but not enough. It is one of the main explanatory variables for economic location success (Wever and Stam, 1999). Technology is a central for regional change, which affects the cumulative wealth of nations.

In this section we investigate the value of the state assets considering agricultural expands yields, Technological effect. The technological effect may be expressed through education level.

The agriculture assets price satisfies the next geometrical Brownian motion:

\[
dS(t) = \mu S(t) dt + \eta S(t) dW(t) \tag{1}
\]
\[
dar(s) = k \beta_s (Ar - ar) ds + \sigma dZ(s) \tag{2}
\]
\[
edlev = a \beta_2 (EdLev - edlev) ds + \gamma dh(s) \tag{14}
\]

Where

- \(edlev\) represents the instantaneous marginal yield of technological effect.
- \(EdLev\) reflects long-run mean of the technological effect yield and which the \(EdLev\) revert to it
- \(\alpha\) is the speed of mean reversion of the technological effect return \(edlev\) to its long-run mean \(EdLev\),
- \(\gamma dh(s)\) reflects the increment to the Brownian motion of the technological effect.
- \(\beta_2\) represents the sensitivity coefficient of the technological effect toward the external factors 1,2,3,...,i (sub-determinants). As we showed in the two factor pricing model, the \(\beta_2\) coefficient may be calculated by:

\[
\beta_2 = \beta_{21} \times \beta_{22} \times \beta_{23} \times \ldots \times \beta_{2i}
\]

The price of asset is affected by the shocks in agricultural expands and the technological effect. The technological effect return satisfies the Ornstein-Uhlenbeck process as the equation (14) showed.

The integral of the instantaneous marginal yield of technological effect equation may be obtained by get the integral of equation (14) represented as following:

\[
\int_t^T d edlev(s) = \int_t^T a \beta_2 (EdLev - edlev) ds + \gamma \int_t^T dh(s) \tag{15}
\]

From the previous equation, we could easily extract the following equations:

\[
Edlev(T) - edlev(t) = \int_t^T d edlev(s) \tag{16}
\]
\[
Y(T) - Y(t) = \int_t^T edlev(s) ds \tag{17}
\]
\[
edlev(T) - edlev(t) = a \beta_2 EdLev (T-t) - a \beta_2 (Y(T) - Y(t)) + \gamma \int_t^T dh(s) \tag{18}
\]
It is easy now to calculate the asset value considering the considering agricultural expands yields and technological effect.

\[ V[S(T)] = e^{-r(T-t)}E^*[S(T)] \quad ... (19) \]

The previous equation represents the initial formula for evaluating the asset \( S \) on the date \( T \).

From Bjerksund (1991), \footnote{4} the future value is:

\[ e^{-r(T-t)}S(T) = S(t)\exp \left\{ \left( \mu - r \alpha r + edlev - \frac{1}{\eta} \right)(T - t) + \eta \int_t^T dW(s) \right\} \quad ... (20) \]

Multiplying the \( \int_t^T dW(s) \) value\(^3\) by the \( \eta \) and using the Appendix (4) to Substitute for \( X(T) - X(t) \), we get the value of \( \eta \int_t^T dW(s) \) in equation (21) as following:

\[ \eta \int_t^T dW(s) = \eta \left[ \int_t^T dW^* - \left( \frac{\mu - r}{\eta} \right)(T - t) - \frac{1}{\eta} \left( X(T) - X(t) \right) - \frac{1}{\eta} \left( Y(T) - Y(t) \right) \right] \]

\[ \int_t^T dW(s) = \int_t^T dW^* - (\mu - r)(T - t) - (X(T) - X(t)) - (Y(T) - Y(t)) \]

Where \( (X(T) - X(t)) \) could be get from equation(9), \( (Y(T) - Y(t)) \) could be obtained from equation (6) Appendix (4). Substituting with the value of both \( (X(T) - X(t)) \) and \( (Y(T) - Y(t)) \) it in the previous equation, we get:

\[ \eta \int_t^T dW(s) = \int_t^T dW^* - (\mu - r)(T - t) - Ar(T - t) - \frac{1}{k\beta_1} \sigma \int_t^T dZ(s) \]

\[ + \frac{1}{k\beta_1} e^{-k\beta_1(T-t)} ar(t) - \frac{1}{k\beta_1} Ar(1 - e^{-k\beta_1(T-t)}) \]

\[ + \frac{\sigma}{k\beta_1} e^{-k\beta_1 T} \int_t^T e^{k\beta_1 s} dZ(s) \]

\[ \int_t^T dW^* - \frac{ar(t)}{k\beta_2} = EdLev(T - t) - \frac{1}{\alpha\beta_2} \gamma \int_t^T dZ(s) \]

\[ + \frac{1}{\alpha\beta_2} e^{-a\beta_2(T-t)} edlev(t) - \frac{1}{\alpha\beta_2} EdLev(1 - e^{-a\beta_2(T-t)}) \]

\[ + \frac{\gamma}{\alpha\beta_2} e^{-a\beta_2 T} \int_t^T e^{a\beta_2 s} dh(s) - \frac{edlev(t)}{\alpha\beta_2} \quad ... (21) \]

Inserting the equation (21) in equation (20) we get the future discounted pay-off as following:

\[ \eta \int_t^T dW(s) = \int_t^T dW^* - \left( \frac{\mu - r}{\eta} \right)(T - t) - \frac{1}{\eta} (X(T) - X(t)) - \frac{1}{\eta} (Y(T) - Y(t)) \quad ... (8) \]

\(^3\) Using the same logic of deriving the equation (8) in the Appendix (1), the \( \int_t^T dW(s) \) may be written as following:

\[ \eta \int_t^T dW(s) = \int_t^T dW^* - \left( \frac{\mu - r}{\eta} \right)(T - t) - \frac{1}{\eta} (X(T) - X(t)) - \frac{1}{\eta} (Y(T) - Y(t)) \quad ... (8) \]
Rearranging the previous equation,

\[ e^{-r(T-t)}s(T) = s(t) \exp \left( \mu - r + ar + edlev - \frac{1}{2} \eta^2 \right) (T-t) + \eta \int_t^T dW^*(s) \]

\[ - (\mu - r)(T-t) - Ar (T-t) - \frac{1}{k\beta_1} \sigma \int_t^T dZ(s) + \frac{1}{k\beta_1} e^{-k\beta_1 (T-t)} ar(t) \]

\[ - \frac{1}{k\beta_1} Ar \left( 1 - e^{-k\beta_1 (T-t)} \right) + \frac{\sigma}{k\beta_1} e^{-k\beta_1 T} \int_t^T e^{k\beta_1 s} dZ(s) - \frac{ar(t)}{k\beta_1} \]

\[ - EdLev (T-t) - \frac{1}{\alpha\beta_2} \eta \int_t^T dh(s) + \frac{1}{\alpha\beta_2} e^{-\alpha\beta_2 (T-t)} edlev(t) \]

\[ - \frac{1}{\alpha\beta_2} EdLev \left( 1 - e^{-\alpha\beta_2 (T-t)} \right) + \frac{\gamma}{\alpha\beta_2} e^{-\alpha\beta_2 T} \int_t^T e^{\alpha\beta_2 S} dh(s) \]

\[ - \frac{edlev (t)}{\alpha\beta_2} \]

In other words, we can calculate the expected value of the asset by the next formula:
\[
e^{-r(T-t)}S(T) = S(t) \exp \left\{ (T - t) \left( ar + edlev - \frac{1}{2} \eta^2 - Ar - Edlev \right) \\
+ \frac{1}{k \beta_1} \left( e^{-k \beta_1 (T-t)} ar(t) - Ar \left( 1 - e^{-k \beta_1 (T-t)} \right) - ar(t) \right) \\
+ \frac{1}{\alpha \beta_2} \left( e^{-\alpha \beta_2 (T-t)} edlev(t) - Edlev \left( 1 - e^{-\alpha \beta_2 (T-t)} \right) - edlev(t) \right) \\
+ \eta \int_t^T dW^* (s) \\
- \frac{1}{k \beta_1} \left( \sigma \int_t^T dZ^*(s) - \sigma \int_t^T \lambda_1 d(s) \right) \\
- \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} dZ^*(s) + \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} d(s) \right) \\
- \frac{1}{\alpha \beta_2} \left( \gamma \int_t^T dh^*(s) \right) \\
- \gamma \int_t^T \lambda_2 d(s) - \gamma e^{-\alpha \beta_2 T} \int_t^T e^{\alpha \beta_2 s} dh^*(s) \\
+ \gamma e^{-\alpha \beta_2 T} \int_t^T e^{\alpha \beta_2 s} \lambda_2 d(s) \right) \right) \tag{22} \]

Where, \( \lambda_1, \lambda_2 \) are the market price per unit risk of agricultural expands yields and technological effect.

In order to calculate variance, the following values must be calculated:

\[
\text{var} = \hat{\sigma}^2 \equiv \mathbb{E}_t [(Z^*)^2] - (\mathbb{E}_t [Z^*])^2
\]

As we previously showed, \( \mu \) is obtained from the modified equation (22) when we remove the variance from it.

\[
\mathbb{E}_t [(Z^*)^2] = \mathbb{E}^* \left[ \left( \eta \int_t^T dW^* (s) \\
- \frac{1}{k \beta_1} \sigma \int_t^T dZ^*(s) + \frac{1}{k \beta_1} \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} dZ^*(s) - \frac{1}{\alpha \beta_2} \gamma \int_t^T dh^*(s) \\
+ \frac{1}{\alpha \beta_2} \gamma e^{-\alpha \beta_2 T} \int_t^T e^{\alpha \beta_2 s} dh^*(s) \right)^2 \right] \]

Using Bjerksund's (1991) we extract the following values:

- \( \mathbb{E}_t \left[ (\int_t^T dW^* (s))^2 \right] = \eta^2 (T - t) \)
- \( \mathbb{E}_t \left[ (\frac{1}{k \beta_1} \sigma \int_t^T dZ^*(s))^2 \right] = \left( \frac{1}{k \beta_1} \right)^2 \sigma^2 (T - t) \)
- \( \mathbb{E}_t \left[ \left( \frac{1}{\alpha \beta_2} \gamma \int_t^T dh^*(s) \right)^2 \right] = \left( \frac{1}{\alpha \beta_2} \right)^2 \gamma^2 (T - t) \)
- \( \mathbb{E}_t \left[ \left( \frac{1}{k \beta_1} \sigma e^{-k \beta_1 T} \int_t^T e^{k \beta_1 s} dZ^*(s) \right)^2 \right] = \left( \frac{1}{k \beta_1} \right)^2 \left( \frac{\sigma^2}{2k} \right) (1 - \theta^2) \)
\[ -\theta_2^2 = (e^{-\mu_1(T-t)})^2 = (e^{-2\mu_1(T-t)}) \]

\[ E_t\left[ \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right)^2 \right] = \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma^2 \left( 1 - \theta_2^2 \right) - \theta_2^2 = (e^{-a_{\beta_2}(T-t)})^2 = (e^{-2a_{\beta_2}(T-t)}) \]

\[ E_t\left[ \left( \eta \int_t^T dW^*(s) \left( \frac{1}{k_{\beta_1}} \int_t^T dZ^*(s) \right) \right) = \frac{1}{k_{\beta_1}} \sigma \eta \rho_{12}(T-t) \right] \]

\[ E_t\left[ \left( \eta \int_t^T dW^*(s) \left( \frac{1}{k_{\beta_1}} \sigma e^{-k_{\beta_1} T} \int_t^T e^{k_{\beta_1} s} dZ^*(s) \right) \right) = \left( \frac{1}{k_{\beta_1}} \right)^2 \sigma \eta \rho_{12}(1 - e^{-k_{\beta_1}(T-t)}) \right] \]

\[ E_t\left[ \left( \eta \int_t^T dW^*(s) \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right) \right) = \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13}(1 - e^{-a_{\beta_2}(T-t)}) \right] \]

\[ E_t\left[ \left( \frac{1}{k_{\beta_1}} \sigma e^{-k_{\beta_1} T} \int_t^T e^{k_{\beta_1} s} dZ^*(s) \right) \gamma \int_t^T dh^*(s) \right] = \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13}(1 - e^{-a_{\beta_2}(T-t)}) \right] \]

\[ E_t\left[ \left( \frac{1}{k_{\beta_1}} \sigma e^{-k_{\beta_1} T} \int_t^T e^{k_{\beta_1} s} dZ^*(s) \right) \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right) \right] = \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13}(1 - e^{-a_{\beta_2}(T-t)}) \right] \]

To see the illustration of the expected values:

\[ E_t\left[ \left( \frac{1}{k_{\beta_1}} \sigma e^{-k_{\beta_1} T} \int_t^T e^{k_{\beta_1} s} dZ^*(s) \right) \gamma \int_t^T dh^*(s) \right] \]

\[ E_t\left[ \left( \frac{1}{k_{\beta_1}} \sigma e^{-k_{\beta_1} T} \int_t^T e^{k_{\beta_1} s} dZ^*(s) \right) \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right) \right] \]

\[ E_t\left[ \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right) \left( \frac{1}{a_{\beta_2}} \gamma e^{-a_{\beta_2} T} \int_t^T e^{a_{\beta_2} s} dh^*(s) \right) \right] \]

In order to calculate the variance, the following values must be calculated:

\[ E_t'[(\varepsilon^*)^2] = \eta^2(T-t) - \frac{1}{k_{\beta_1}} \sigma \eta \rho_{12} (T-t) + \left( \frac{1}{k_{\beta_1}} \right)^2 \sigma \eta \rho_{12} (1 - e^{-k_{\beta_1}(T-t)}) - \]

\[ \frac{1}{a_{\beta_2}} \gamma \eta \rho_{13} (T-t) + \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13} (1 - e^{-a_{\beta_2}(T-t)}) - \frac{1}{k_{\beta_1}} \sigma \gamma \rho_{23} (T-t) - \left( \frac{1}{k_{\beta_1}} \right)^2 \sigma \gamma \rho_{23} (1 - e^{-k_{\beta_1}(T-t)}) - \]

\[ \frac{1}{k_{\beta_1}} \sigma \gamma \rho_{23} (1 - e^{-k_{\beta_1}(T-t)}) + \frac{1}{k_{\beta_1}} \sigma \gamma \rho_{23} (1 - e^{-k_{\beta_1}(T-t)}) - (1 - e^{-k_{\beta_1}(T-t)}) - \]

\[ (1 - e^{-k_{\beta_1}(T-t)}) + \left( \frac{1}{k_{\beta_1}} \right)^2 \sigma \gamma \rho_{23} (1 - e^{-k_{\beta_1}(T-t)}) - \frac{1}{a_{\beta_2}} \gamma \eta \rho_{13} (T-t) + \left( \frac{1}{k_{\beta_1}} \right) \sigma \gamma \rho_{23} (T-t) + \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13} (1 - e^{-a_{\beta_2}(T-t)}) - \]

\[ \left( \frac{1}{k_{\beta_1}} \right)^2 \sigma \gamma \rho_{23} (1 - e^{-k_{\beta_1}(T-t)}) - \frac{1}{a_{\beta_2}} \gamma \eta \rho_{13} (T-t) + \left( \frac{1}{k_{\beta_1}} \right) \sigma \gamma \rho_{23} (T-t) + \left( \frac{1}{a_{\beta_2}} \right)^2 \gamma \eta \rho_{13} (1 - e^{-a_{\beta_2}(T-t)}) - \]
Thus, the variance value or \( \hat{\sigma} \) can be calculated as follows:

\[
\text{var} = \hat{\sigma}^2 \equiv E^*[Z^*]^2 - (E^*[Z]^2)
\]

The value of the asset \( V \) is:

\[
V^*[s(T)] = E^*[e^{-r(T-t)}s(T)] = E^*[s(t)]e(\hat{\sigma}) = s(t)\exp\left\{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2\right\} = E^*[s(T)e(\hat{\sigma})]
\]

\[S(t)\exp\left\{\hat{\mu} + \frac{1}{2}\hat{\sigma}^2\right\} (24)\]

### 3.1.2. Energy and Mineral assets

Evaluating the energy and mineral resources projects requires similar steps such as in the agriculture assets evaluation. Here, the method such as in equations (1), (2), and (14) will be used. The symbols in the previous equations would be replaced to fit the evaluation of the energy and mineral assets.

\( ar \) and \( Ar \) will represent the instantaneous marginal convenience yield of the energy and mineral resources projects, and long-run mean, respectively. Thus, the \( ar \) revert to \( Ar \). The symbol \( k \) will represents the speed of mean reversion of the convenience yield return \( ar \) to its long-run mean \( Ar \). The symbol \( edlev \) represents the instantaneous marginal yield of technological effect and \( EdLev \) reflects long-run mean of the technological effect yield which the \( edlev \) reverts to it. The symbol \( \alpha \) reflects the speed of mean reversion of the technological effect return return \( edlev \) to its long-run mean \( EdLev \).

\( \beta_1 \) represents the sensitivity coefficient of the convenience yield toward the external factors \( 1,2,3,...,i \) such as, the economic geography, governance, reputation, inflation.

The coefficient \( \beta_2 \) show the sensitivity of the technological effect yield toward the external factors \( 1,2,3,...,i \). We may include the governance, reputation, the public expensive on the education, …, etc. \( dZ(s) \) represents the increment to the Brownian motion of the convenience yield.

Note: the education reputation in this case may be measured via a weighted average of the score of the educational institutions regarding to any respected ranking such as QS ranking.
3.1.3. **Protected areas and forests**

The value of the forest resources may be calculated by the capitalized value of rents from timber and non-timber services. We can easily estimate the value of the protected areas by capitalizing the rents of these assets.

Since the rents of the forest resources and the protected areas satisfy a geometric Brownian motion, the value of these assets could be estimated using the equations (1), (2), and (14).

The value of the natural capital could be calculated by the summation of agricultural land, the energy and mineral assets value, the forest resources and the protected areas. The total value of the natural capital may be represented by the symbol $NC$.

3.2. **Produced Capital**:

The second component of the state value is the produced capital. The World Bank defined the produced capital as the manufactured or built assets such as machinery, equipment, and physical structures.

In general, there are several methods for evaluating the produced capital. For example, the perpetual inventory method is the certified method by the Organization for Economic Co-operation and Development (OECD) countries. One of the popular methods to evaluating the produced capital is to obtain it directly from the data of the national accounts data. Here, the popular formula of the GDP will be used:

$$GDP = private\ consumption + gross\ investment + government\ investment$$

$$+ government\ spending + (exports - imports).$$

Whatever the method used to calculate GDP, the GDP series will satisfy the Brownian motion equation and we can simply use the equations (13), and (24) to calculate the produced capital. Here, GDP represents the cash flows of the assets. In our model, we can represent the produced capital by the character $PC$.

3.3. **The Net Foreign Assets**:

According to the World Bank methodology for calculating the foreign assets value, it may be estimated by the Net foreign assets value. The Net foreign assets value is calculated by subtracting total foreign liabilities from total foreign assets. The foreign assets include equity, FDI, debt, derivative, and Forex. In this paper, we can evaluate equity, FDI, debt, and derivative using the equations (13), and (24). Regarding to Forex, we only need to translate it to the national currency. We can reflect the foreign assets value with $FA$.

3.4. **Human capital**:

Human resources must be developed continuously, and managed wisely to achieve sustainable development. For example, the citizens stack in low-skill intensive natural-resource-based industries, fail to develop their own or their children’s education. This may lead to negligent the accumulation of human capital.

The countries can accumulate their human capital by developing the educational system and increase the quality of the health system. The wasteful negligence which allows genius that born of lowly parentage to hang in lowly work, obstacles the national growth (Marshall, 1920). Hence, there is a relation between the nation’s capability to productively use physical

25
capital and the level of human capital, as weak human capital could hinder economic development. Unless the state invests in human capital, the investor will not invest in it, because foreign investors tend to invest in physical capital, not in human capital (Schultz, 1961). Centuries ago, Adam Smith [1776] stated the human capital, as one of the four sorts of fixed capital that contribute to production in a national economy. More and better Education is, therefore, an engine of economic growth around the world, as Adam Smith, John Stuart Mill, and Alfred Marshall knew. In contrast, Thorvaldur Gylfason (2001) found that rich resources countries can live well for longtime, even under the poor economic policies and a weak commitment to education. The poor resources nations may have a smaller margin for error, and thus less likely to make this mistake.

This highlights on the importance of the accumulation of human capital as the engine for the economic growth alongside the other wealth factors. The improving of education system speeds economic growth and improves citizen’s standard of living through many channels: it increases the efficiency and so the productive and so salaries (Psacharopoulos and Patrinos, 2004), enhancing democracy (Barro, 1997), creating a good environment for good governance, increasing equality, and bettering health system (Aghion et al., 1999). The impact of improving the health sector via on three channels, the first is the relation between the increase in the worker's productivity with a high level of health. The labor’s health effects on productivity, and so economic growth, which led to increase in per capita income (Knowles and Owen, 1997; Webber, 2002; Bloom et al., 2004; Acemoglu and Johnson, 2006). The second is linked to education by increasing the desire of individuals to invest in education with an increase in their expected life because of improving their health (Ehrlich and Lui, 1991; Kalemli-Ozcan et al., 2000; Blackburn and Cipriani, 2002; Chakraborty, 2004; Finlay, 2005). Third, it is directing the amounts that the state was supposed to spend on treatment to other spending aspects that help economic growth.

The empirical literature (Knowles and Owen, 1997; Webber, 2002; Bloom et al., 2004; Acemoglu and Johnson, 2006) focus on the labor productivity effects of health on economic growth where improvements in health increase per capita income directly because of the increasing of the production. The theoretical models, however, explore the relationship between health and economic growth via an indirect incentive effect on education investment (Blackburn and Cipriani, 2002; Chakraborty, 2004; Ehrlich and Lui, 1991; Finlay, 2005; Kalemli-Ozcan et al., 2000). Lorentzen, McMillan, and Wacziarg (2008) concluded positive effects of longevity on growth. There is a positive relation between market access and the accumulation of human capital (Redding and Schott, 2003; Breinlich, 2006). Consequently, Investing in the human element through education and health is one element of the wealth of nations and should be used to promote other wealth factors.

In the World’s Bank report (2018), and Jorgenson and Fraumeni (1992), these studies priced the human capital considering the real probabilities of employing and the discount rate. In this paper we used the risk-neutral approach which prices the human capital assets as the discounted expected value of the future payoff. The pricing model in the equations(13), and
used risk-neutral measure, so that we only need to extract the expected earnings of each employee category $H_{ae}$. Each category of employee has an expected earning $H$, years of schooling by $e$, and age $a$. Each expected earnings will satisfy the Brownian motion and the equations (13), and (24). We can use the marginal rate of experience years and the mean of the technological effect yield as pricing factors.

The coefficient $\beta_1$ represents the sensitivity coefficient of the marginal rate of experience years to the external factors $1,2,3,\ldots,i$ such as unemployment, inflation rate, exchange rate…etc. the $\beta_2$ represents the sensitivity coefficient of the technological effect to the factors $1,2,3,\ldots,i$ such as, public expenses on the educations, research and developments…etc.

The value of the human capital could be reflected by $HC$.

4. The state value:

In the previous sections, we displayed the calculation method for the fair value of each part of the state value. We used the risk-neutral approach to estimate the economic value of state. The risk-neutral approach enables us from considering the discounted value of the expected value of country assets instead of the traditional evaluation methods which consider the real probabilities and the discounted value without considering the market premiums of the factors affects the value of these assets. We can get the total state value using the following equation:

$$SV = NV + PC + FA + HC \ldots (25)$$

The previous equation enables us from evaluating the performance of the government and its ability to maximizing the wealth of the nation. The decision making will be a simple real-option model and we can use the smooth pasting method to get the optimal decision point. We can find the optimization point by find the first derivative of the state value function regarding any component of the state value (such as the agricultural land if the decision relate to the agricultural land) and equate it with zero.

5. The future research:

In this paper we investigate the state value using the same way of the corporate finance. We used the real-option approach of pricing the firm assets to evaluate the value of the assets of the country. We included several continuous factors in the pricing process such as the innovation, inflation, and the asset expenditure. However, for simplification we did not consider the discrete factors in the pricing model, such as the geographical variables, and democracy. We expect to extend our future research to include the discrete variables in the future research.

6. Conclusion:

In this paper, we considered the state value as the value of any economic institution, which we can easily price its assets. We divided the determinants of the state value into two classes to evaluate them. The first is the determinants of state value which mentioned within the model. It included (natural capital, product capital, the net foreign assets, human capital). The second is the sub-determinants of the state value, which may have an impact on the first determinants. It included (country’s reputation, economic geography, economic and social infrastructure, governance, market efficiency, institutions, democracy, and exchange rate). The contribution
of our paper is that we used the real-option approach to calculate the state value. Thus, we introduce the risk-neutral approach in evaluation the wealth of the country.

Evaluating the state assets enables us from making decisions by calculating the value before and after making any decision. Determining the value enables the decision maker from detecting the optimal time and the optimal value of the assets for any decision. We examined several cases of the state value. First, we modeled the pricing of the assets considering two factors, and then we include a third factor in the pricing model. We adjusted the model of Bjerkusund (1991) to be suitable for pricing the state assets. The contribution of this paper includes also the possibility of include unlimited factors influencing the value of the assets.

Reference


Adam Smith (March 9, 1776 ), wealth of nations: complete, Scotland, Kingdom of Great Britain.


National Aboriginal Capital Corporation Association (NACCA), May 2005, The Role of Housing in an Economy.


Uwe Deichmann, Somik V. Lall, Stephen J. Redding, and Anthony J. Venables, (2008). Industrial Location in Developing Countries, Published by Oxford University Press.


Appendix (1)

The derivation of the integral of the assets price's Brownian motion $\int_t^T dw(s)$

According to Gibson and Schwartz (1990) and Bjerkund (1991), we can simply get $dW(s)$ by subtracting the risk premium for the incremental return on risk free return $\frac{\mu - r}{\eta} (T-t)$ from the expected value of the Martingale risk measure $\int_t^T dW^*(s)$. We need to consider the agricultural expansion factor in pricing the asset. Thus, we also will subtract risk premium for agricultural expansion $\frac{1}{\eta} (X(T) - X(t))$ from $\int_t^T dW^*(s)$. We define the integral of the assets price's Brownian motion (The change in the price risk of the asset $\int_t^T dw(s)$) as following:

$$\int_t^T dw(s) = \int_t^T dW(s) - \frac{\mu - r}{\eta} (T-t) - \frac{1}{\eta} (X(T) - X(t)) \ldots (8)$$

We can get $(X(T) - X(t))$ using equation (3).

The value of the Martingale risk measure for the asset $S$ is $\int_t^T dW^*(s)$.

We subtracted the risk premium for additional asset over risk free return $\frac{\mu - r}{\eta} (T-t)$ from the Martingale risk measure $\int_t^T dW^*(s)$ then we subtracted the Agricultural expansion dividend risk premium $\frac{1}{\eta} (X(T) - X(t))$.

Appendix (2)

Derivation of the agricultural expands cumulative return, for the period from 0 to $T$ ($X(T)$) (equation (9)):

According to the equation (18), the agricultural expands cumulative return for the period from $t$ to $T$ may be written as:

$$ar(T) - ar(t) = k\beta_1 Ar(T-t) - k\beta_2 (X(T) - X(t)) + \sigma \int_t^T dZ(s)$$
By inserting the modified $ar(T)$ equation (5) in the left side of the equation (18) we can get the equation (6) as follows:

$$e^{-k\beta_1(T-t)}ar(t) - (1 - e^{-k\beta_1(T-t)})A_1 + \sigma \int_t^T e^{k\beta_1 s} dZ(s) - ar(t)$$

$$= k\beta_1 Ar(T - t) - k\beta_1 \left(X(T) - X(t)\right) + \sigma \int_t^T dZ(s)$$

Rearranging the previous equation, we get the value of $k\beta_1 \left(X(T) - X(t)\right)$:

$$k\beta_1 \left(X(T) - X(t)\right) = k\beta_1 Ar(T - t) + \sigma \int_t^T dZ(s) - e^{-k\beta_1(T-t)}ar(t) + (1 - e^{-k\beta_1(T-t)})Ar - \sigma e^{-k\beta_1 T} \int_t^T e^{k\beta_1 s} dZ(s) + ar(t)$$

Dividing both sides by $k\beta_1$ and rearranging, we can calculate $X(T)$:

$$X(T) = X(t) Ar(T - t) + \frac{1}{k\beta_1^2} \sigma \int_t^T dZ(s) - \frac{1}{k\beta_1} e^{-k\beta_1(T-t)}ar(t) + \frac{1}{k\beta_1} Ar(1 - e^{-k\beta_1(T-t)}) - \frac{\sigma}{k\beta_1} e^{-k\beta_1 T} \int_t^T e^{k\beta_1 s} dZ(s) + \frac{ar(t)}{k\beta_1} \cdots (9)$$

**Appendix (4)**

**Deriving the agricultural expands cumulative return and the technological effect yield, for the period from 0 to T ($X(t)$) (equation (6)):**

Remember the equation (18)

$$edlev(T) - edlev(t) = \alpha_2 Edlev (T - t) - \alpha_2(Y(T) - Y(t)) + \gamma \int_t^T dh(s) \cdots (18)$$

Where $edlev(T)$ may be extracted from Merton(1971) and Bjersund (1991) as following:

$$edlev(T) = e^{-k\beta_2(T-t)}edlev(t) - (1 - e^{-\alpha_2(T-t)})Edlev$$

$$+ \gamma e^{-\alpha_2 T} \int_t^T e^{\alpha_2 s} dh(s) \cdots (5)$$

Substituting with equation (5) in the left side of the equation (18), it will be easy to get equation (6):

$$e^{-k\beta_2(T-t)}edlev(t) - (1 - e^{-\alpha_2(T-t)})Edlev + \gamma e^{-\alpha_2 T} \int_t^T e^{\alpha_2 s} dh(s) - edlev(t)$$

$$= \alpha_2 Edlev (T - t) - \alpha_2(Y(T) - Y(t)) + \gamma \int_t^T dh(s)$$

By rearranging, we can get $\alpha_2(Y(T) - Y(t))$ as following:

\[\text{The value of } ar(T) \text{ may be extracted from Merton(1971) and Bjersund (1991)}\]

$$ar(T) = e^{-k\beta_1(T-t)}ar(t) - (1 - e^{-k\beta_1(T-t)})Ar$$

$$+ \sigma e^{-k\beta_1 T} \int_t^T e^{k\beta_1 s} dZ(s) \cdots (5)$$
$$\alpha \beta_2 (Y(T) - Y(t)) = \alpha \beta_2 EdLev(T - t) + \gamma \int_t^T dh(s) - e^{-k\beta_2 (T-t)} Edlev(t)$$

$$+ (1 - e^{-a \beta_2 (T-t)}) EdLev - \gamma e^{-a \beta_2 T} \int_t^T e^{a \beta_2 s} dh(s) + edlev(t)$$

By dividing the sides by $\alpha \beta_2$ and rearranging we get $Y(T)$ as follows:

$$Y(T) = Y(t) + EdLev(T - t) + \frac{1}{\alpha \beta_2} \gamma \int_t^T dh(s) - \frac{1}{\alpha \beta_2} e^{-k \beta_2 (T-t)} Edlev(t)$$

$$+ \frac{1}{\alpha \beta_2} EdLev(1 - e^{-a \beta_2 (T-t)}) - \frac{\gamma}{\alpha \beta_2} e^{-a \beta_2 T} \int_t^T e^{a \beta_2 s} dh(s)$$

$$+ \frac{edlev(t)}{\alpha \beta_2} \ldots (6)$$