

A New Input-Output Table Built from Neoclassical Analysis that Considers Economic Rent

Benjamin Leiva (✉ bnleiva@uvg.edu.gt)

Universidad del Valle de Guatemala

Research

Keywords: Input-output framework, neoclassical theory, economic rent, economic structure

Posted Date: October 11th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-948361/v1>

License:  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

A new input-output table built from neoclassical analysis that considers economic rent

September 29, 2021

Benjamin Leiva. bnleiva@uvg.edu.gt. Observatorio Económico Sostenible, Universidad del Valle de Guatemala, Guatemala. ORCID: 0000-0002-2312-2532.

Abstract

Despite the extended use of the input-output framework, its acceptance among economists is limited due to lack of microfoundations and imposed omission of economic rent. A new input-output table is built from basic neoclassical profit maximization, which explicitly considers economic rent. The new table holds eight differences with conventional open input-output tables which enhances the framework's theoretical basis, allows for economic rent, and yields new insights into economic structures.

Keywords: Input-output framework, neoclassical theory, economic rent, economic structure

JEL Codes: C67, D57

1 Introduction

Input-output (IO) analysis is extensive and multifaceted. After the original analysis of the US economy by Leontief (1936), the technique has been used in over 100 national economies (Rose and Miernyk, 1989), regional economies (Costa, 1984), and the entire world (Leontief et al., 1977; Almon, 1984). Moreover, IO's original focus has been extended to study development (Bulmer-Thomas, 1982), inequality (Reich, 2018), taxation (Golladay and Haveman, 1976), the environment (Leontief, 1970; Piñero et al., 2018), and among other topics technological change (Ayres and Shapanka, 1976). The technique's breadth has been documented in review articles by Rose and Miernyk (1989) and Xie et al. (2018).

Yet there are two features that limit the acceptance of IO analysis by many economists. One is that IO and neoclassical economics seem to be mostly incompatible (Kurz and Salvadori, 2000) and have little cross recognition of each perspective's merits (ten Raa, 1994; Raa, 2004). Given the hegemony of neoclassicism, this has been at least a contributing factor in the contemporary loss of IO's appeal (Dietzenbacher et al., 2013). The main sources of this disconnect are the assumption of fixed coefficients of production that eliminate the possibility of input substitution and optimization processes, and the assumption of constant returns to scale that rules out the diverse productive conditions found within and across industries (Christ, 1955). The resulting rigidity, alongside the perspective's lack of formal microfoundations, makes neoclassical economists perceive IO analysis as an imposed system of accounting identities (Rose and Miernyk, 1989) and "a mechanical manipulation of data" (ten Raa, 1994).

The second limiting feature is that IO analysis assumes economic rent away by imposing constant returns to scale. Given that economic rent is an integral part of the theory of distribution and the theory of cost (Wessel, 1967), this reduces IO's theoretical and practical appeal, specially regarding distributional issues. Although the framework has been used to study the relation between inequality and final demand, production, and employment, and how the factor distribution of income influences inequality (for example, in Steenge and Serrano (2012)), excluding rent handicaps the framework's capacity to interpret an increasingly unequal world.

Can IO tables be derived compatibly with neoclassical economics and explicitly consider economic rent? Hudson and Jorgenson (1974) and Lakshmanan and Lo (1972) mix neoclassical procedures and IO tables to relate industry behavior to economy-

wide balances, but they do so in an *ad-hoc* two-stage model. Similarly, the RCOT literature uses aggregate optimization procedures, but with a focus on technological heterogeneity and no interest in its relation to neoclassical theory. To the best of the author's knowledge, the only attempt to relate neoclassical procedures with IO tables has been done by Thijs ten Raa and Pierre Mohnen in a series of articles spanning a decade. In ten Raa (1994), Raa and Mohnen (2014), Raa (1995), Mohnen et al. (1997), Raa and Mohnen (2002) and Raa (2004) the authors use an aggregate neoclassical profit maximization process to derive technical coefficients and quantity and value equations. Throughout the papers they increasingly sophisticate the framework to include non-tradable commodities, a utility foundation, fluctuations of the economy about its frontier, total factor productivity and value shares of factor inputs. The approach proposed here shares the attempt to bridge neoclassical economics and IO, but little else. Starting from an individual firm's profit maximizing process, instead of an aggregate maximizing process as in ten Raa and Mohnen, makes the subsequent framework and results distinct. Moreover, the author ignores any attempt to meaningfully include economic rent in IO analysis.

This article derives IO tables from individual profit maximization. The approach provides microfoundations for such tables and the abandonment of the fix technical coefficients. The constant returns to scale assumptions is replaced with decreasing returns to scale, allowing for economic rent. Initially the new IO table is derived assuming no government, foreign trade, uncertainty, and market power to keep the exposition tractable, yet extensions are presented in which these assumptions are dropped. The resulting tables holds important differences with open and closed versions of conventional IO tables and with previous attempts to relate neoclassical economics and IO analysis. Furthermore, the inclusion of economic rent has important implications for the structure of economic systems.

The paper continues with the optimization basis of the new IO table in Section 2. Section 3 presents the basic version of the new IO table and section 4 compares this new table with a conventional open IO table used by the U.S. Bureau of Economic Analysis. Section 5 extends the new IO table to include government, foreign trade, uncertainty, and market power. Section 6 discusses the implications of the approach and section 7 concludes.

2 Methods

Consider a profit maximizing firm p producing a unique commodity k without uncertainty and market power. The firm's problem is

$$\max_{\mathbf{q}_{k,p}} \Pi_{k,p} = P_k f_{k,p}(\mathbf{q}_{k,p}) - \mathbf{P}' \mathbf{q}_{k,p}, \quad (1)$$

where $\Pi_{k,p}$ is the firm's profit, P_k is the commodity's price, $\mathbf{q}_{k,p} = [q_{1,k,p}, \dots, q_{I,k,p}]'$ is a vector of input quantities, $\mathbf{P} = [P_1, \dots, P_I]'$ is a vector of input price, and $f_{k,p}(\cdot)$ is a concave form above, continuous, and twice-differentiable production function. Consider that I includes *all* inputs such that the firm's profit is economic rent. The FOC to choose optimal quantity of input i yields the known value of production equation

$$P_k f_{i,k,p} = P_i, \quad \forall i \in I_p \quad (2)$$

which can be multiplied by $q_{i,k,p}$ and summed over all inputs to invoke Euler's homogeneous function theorem. If f is homogeneous of degree n , then (2) becomes

$$P_k n q_{k,p}^* = \mathbf{P}' \mathbf{q}_{k,p}^*, \quad (3)$$

where $q_{k,p}^*$ and $\mathbf{q}_{k,p}^*$ are optimal quantities as they satisfy the FOCs in (2). Next, define $q_{k,p}^b$ and $q_{k,p}^s$ as basal and surplus production respectively, such that

$$n q_{k,p} \equiv q_{k,p}^b \equiv q_{k,p} - q_{k,p}^s. \quad (4)$$

Basal production $q_{k,p}^b$ is the production level with which, at going prices, the firm pays for all inputs. Thus, if the firm's total production equals its basal production profits are zero. Using the first identity from (4) in (3) shows that

$$P_k q_{k,p}^{b*} = \mathbf{P}' \mathbf{q}_{k,p}^*. \quad (5)$$

Furthermore, surplus production is the portion of total production in excess of basal production. Thus, valued at going prices it provides the firm's rent. Using the second identity from (4) in (5) yields the optimal solution to (1) as

$$P_k q_{k,p}^* = \mathbf{P}' \mathbf{q}_{k,p}^* + \Pi_{k,p}^*, \quad (6)$$

where $P_k q_{k,p}^*$ is the firm's optimal income and $\Pi_{k,p}^* = P_k q_{k,p}^{s*}$ is the firm's maximum rent. Given a level of prices, rents are determined by the firm's returns to scale, as rearranging (4) shows that

$$q_{k,p}^s \equiv (1 - n) q_{k,p}. \quad (7)$$

Per (7), if the firm exhibits constant returns to scale (CRS), then $n = 1$, $q_{k,p}^s = 0$, and $\Pi_{k,p} = 0$. In words, factor payments deplete the value of the product and there is no economic rent. This case is the standard assumption in IO analysis, which forces economic rent away and incapacitates optimizing procedures because under CRS the rent-maximizing output $q_{k,p}^*$ is undefined. If the firm exhibits increasing returns to scale (IRS), then $n > 1$, $q_{k,p}^s < 0$, and $\Pi_{k,p} < 0$. Due to the sign inversion of the second order condition, $q_{k,p}^*$ is a rent-*minimizing* output. This case is also incompatible with optimizing procedures because under IRS the rent-*maximizing* output (and the economic rent) is infinite without additional structure such as capacity constraints and/or demand functions. The only scenario compatible with optimizing procedures given the current setting is decreasing returns to scale (DRS), where $n < 1$, $q_{k,p}^s > 0$, and $\Pi_{k,p} > 0$. In this context factor payments are less than the value of the product and the rent-maximizing output $q_{k,p}^*$ is well-defined.

Lastly, suppose that p 's industry is composed of N_k firms producing the same homogeneous commodity k . These producers can be widely heterogeneous in terms of their production functions, but all exhibit DRS and non face uncertainty nor exert market power. In such case, summing (6) over N_k yields the industry-wide rent maximizing solution

$$P_k q_k^* = \mathbf{P}' \mathbf{q}_k^* + \Pi_k^*, \quad (8)$$

where $q_k^* = \sum_{p=1}^{N_k} q_{k,p}^*$ is industry-wide optimal supply, $\Pi_k^* = \sum_{p=1}^{N_k} \Pi_{k,p}^*$ industry-wide maximum rent, and $\mathbf{q}_k^* = [q_{1,k}^*, \dots, q_{I,k}^*]'$ industry-wide optimal input demand (with $q_{i,k}^* = \sum_{p=1}^{N_k} q_{i,k,p}^*$). Equation (8) represents the rent maximizing solution for each of the K industries of an economy.

3 Results

An economy with K industries can be represented by displaying equation (8) in the columns of a new IO table (NT), with industries in the columns and inputs in the rows as shown in Table 1. The table distinguishes between input industries ($i \in I$) and final industries ($f \in F$), where only the former are required to reproduce the economy (more details on this distinction in section 3.1.7). As $I + F = K$, the table is non-square. In the first I rows and K columns, the entry in the i th row and k th column gives the value of the i th input used by the k th industry. The Rent and Income rows show industry-wide

rents and income respectively. A column of Direct Consumption lays after all industries representing inputs used directly by households as final consumption. Assuming no change of inventories, government nor foreign trade to simplify the exposition, the sum of the elements of each row represents the total expenditure on a given input, which is placed in the Expenses column. Given that “each revenue item of an enterprise or household must reappear as an outlay item in the account of some other enterprise or household” (Leontief, 1936), the elements in the Expenses column are by definition equivalent to the first I elements of the Income row.

Table 1. New IO table (NT)

		Input Industries			Final Industries			Direct Cons.	Expenses
		1	...	I	f	...	F		
Inputs	1	$P_1q_{1,1}^*$...	$P_1q_{1,I}^*$	$P_1q_{1,f}^*$...	$P_1q_{1,F}^*$	$P_1q_{1,d}^*$	$P_1q_1^*$
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots
	I	$P_Iq_{I,1}^*$...	$P_Iq_{I,I}^*$	$P_Iq_{I,f}^*$...	$P_Iq_{I,F}^*$	$P_Iq_{I,d}^*$	$P_Iq_I^*$
Rent		Π_1^*	...	Π_I^*	Π_f^*	...	Π_F^*		
Income		$P_1q_1^*$...	$P_Iq_I^*$	$P_fq_f^*$...	$P_Fq_F^*$		

A new table of coefficients (NTC) can be obtained in a similar way as the NT. Dividing (8) by $P_kq_k^*$ yields

$$1 = \sum_i^I a_{i,k} + \pi_k, \quad (9)$$

where $a_{i,k} = \frac{P_iq_{i,k}^*}{P_kq_k^*}$ are technical coefficients and $\pi_{i,k} = \frac{\Pi_k^*}{P_kq_k^*}$ are profit margins (i.e., rents per unit value of output). Placing this equation as done with equation (8) for the NT leads to the NTC in Table 2.

Table 2. New table of coefficients (NTC)

		Input Industries			Final Industries			Direct Cons.	Expenses
		1	...	I	f	...	F		
Inputs	1	$a_{1,1}$...	$a_{1,I}$	$a_{1,f}$...	$a_{1,F}$	-	-
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots
	I	$a_{I,1}$...	$a_{I,I}$	$a_{I,f}$...	$a_{I,F}$	-	-
Rent		π_1^*	...	π_I^*	π_f^*	...	π_F^*		
Income		1	...	1	1	...	1		

Considering any two industries $\{m, n\} \in K$, the NTC and equation (9) imply that the difference in profit margins between m and n is inversely related to the difference in their sum of technical coefficients, or in other words, that the higher the relative efficiency of an industry the higher the relative profit margin. In fact, using (9) for any two industries yields

$$\begin{aligned}\sum_i^I a_{i,m} + \pi_m &= \sum_i^I a_{i,n} + \pi_n \\ \pi_m - \pi_n &= \sum_i^I a_{i,n} - \sum_i^I a_{i,m}.\end{aligned}\tag{10}$$

Moreover, the NTC can be used alongside the NT to obtain a new leontief inverse with the identity

$$\mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{Y} + \mathbf{D} \equiv \mathbf{X},\tag{11}$$

where $\mathbf{A} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,I} \\ \vdots & \ddots & \vdots \\ a_{I,1} & \cdots & a_{I,I} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} a_{1,f} & \cdots & a_{1,F} \\ \vdots & \ddots & \vdots \\ a_{I,f} & \cdots & a_{I,F} \end{bmatrix}$ are Matrices of Coefficients of

input and final industries respectively, and $\mathbf{X} = [P_1q_1^*, \dots, P_Iq_I^*]'$ and $\mathbf{Y} = [P_fq_f^*, \dots, P_Fq_F^*]'$ are Income vectors of input and final industries at the bottom of the NT in Table 1. Recall that \mathbf{X} is identical to the Expenses column. Lastly, $\mathbf{D} = [P_1q_{1,d}^*, \dots, P_Iq_{I,d}^*]'$ is households' Direct Consumption vector. Solving for \mathbf{X} yields

$$\begin{aligned}\mathbf{X} - \mathbf{A}\mathbf{X} &\equiv \mathbf{B}\mathbf{Y} + \mathbf{D} \\ (\mathbf{I} - \mathbf{A})\mathbf{X} &\equiv \mathbf{B}\mathbf{Y} + \mathbf{D} \\ \mathbf{X} &\equiv (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{B}\mathbf{Y} + \mathbf{D}).\end{aligned}\tag{12}$$

This new leontief inverse has the same general structure and interpretation as the conventional one, yet the Final Demand vector has changed to include final industries' output ($\mathbf{B}\mathbf{Y}$). Final demand also includes household direct consumption of input goods (\mathbf{D}), which only contains inputs that are not needed to reproduce households (more details on this distinction in section 3.1.6). The inclusion of government and foreign trade would add these items to final demand as shown in sections 3.2.1 and 3.2.2.

Given that existing data does not suffice to build Table 1 and Table 2, a numerical illustration of both tables with three input goods and two final goods is presented as Supporting Information to illustrate the proposed framework and show that the equations work. This is not ideal, yet commonly done when new frameworks are incompatible with existing data as done in Sancho (2019), Steenge and Reyes (2020),

dons the fixed coefficients assumption. The CT is chiefly recognized and criticized for its fixed coefficients assumption. Moreover, the NT allows for heterogeneous production functions in an industry, while the CT imposes that each industry uses one representative technology (Steenge et al., 2018).

3.1.3 Returns to scale

The NT assumes DRS while the CT assumes CRS. CRS holds well in a stationary economy, yet DRS is more generally appropriate when studying a growing economy and when faced with upward sloping supply curves.

3.1.4 Labor and capital

The NT considers labor and capital as inputs of an economy produced by their own “industries” like the original closed IO table (Leontief, 1936). In the NT, this structure follows from the specification of the production function in (1), in which labor and capital are considered as no different from any other input. On the contrary, the CT is an open IO table that considers labor and capital as part of final demand.

Including labor and capital as inputs leads to the rows “compensation of employees” and “gross operating surplus” in the CT to be placed alongside payments to other inputs in the NT’s rows. Moreover, the columns “personal consumption expenditures” and “private fixed investment” in the CT are placed alongside other input industries in the NT’s columns.

This change addresses the natural drawback of considering expenditure on labor and capital (arguably the two most important inputs) as final use. Note however that the NT also addresses the drawback of closed IO tables that consider investment as an instantaneous activity devoid of future output considerations (?). In the NT the production of each capital good is motivated by its price, which is equal to the present value of future profits. The drawback that remains is that new capital goods (and specially employees) typically take much longer to produce than other inputs, and yet all are produced in the same period in the NT.

3.1.5 Leontief inverse

A leontief inverse can be derived from the CT and NT. In both cases the multiplier indicates how much inputs are associated with a given level of final demand. This

enables the use of the leontief inverse to evaluate policy changes, yet the precision of results depends on the stability of technical coefficients. As policy changes generally impact prices, the optimizing solution of individual firms, and therefore technical coefficients, the assumption of technical coefficients stability done by the IO framework is a standard critique to the CT.

The difference with the NT is that, as the leontief inverse is microeconomically founded, the stability of technical coefficients upon changing prices can be tested and not simply assumed. This provides higher confidence in the prescriptive capacity of the framework.

Another difference in the leontief inverse derived from the CT and NT is the Final Demand vector. In the CT the vector is exogenous while in the NT it is endogenous: The vector contains final industries' output value, which depend on rents generated in the production of inputs as shown in section 3.1.8 with equation (13). The vector also contains direct consumption of inputs by households, which appears endogenously in the table as an adjustment parameter that balances out expenditures and outlays. The exclusion of government and foreign trade in the Final Demand vector of the NT is not a structural difference with the CT, as they can be seemingly included as shown in sections 3.2.1 and 3.2.2.

3.1.6 Personal consumption expenditure

In the CT, personal consumption expenditure is a column in the Final Uses section, which is problematic because people are, apart from final consumers, inputs to the production process. In the NT, this expenditure is divided into two parts. One contains the level of consumption required to maintain people in regular productive conditions (e.g. normal diet, housing, clothing), which appears among the first I columns showing the inputs required to reproduce the workforce. This first portion exists in the closed IO table.

The other part contains the level of consumption over and above what is required to keep people productive (e.g., concerts, novels, videogames), which contains the output of final industries in the last F elements of the Income row, and the inputs consumed directly by individuals which are not necessary for reproduction (e.g. wood for a tree house) in the Direct Consumption column. This second part is entirely new to the IO literature.

3.1.7 Input and final industries

The NT distinguishes between input and final industries, whereas such distinction does not exist in the IO literature. Input industries are those required for an economy to physically reproduce everything being produced (e.g. steel, engines, employees), while final industries are those not required for such reproduction (e.g. tourism, concerts, videogames).

Focusing on the *reproduction* of an economy allows the distinction between input and final industries to remain positive. The observable nature of what was physically required to produce what has already been produced enables an objective assessment of which are input industries and which are not. The distinction between input and final industries only involves limited gray areas, such as functional luxury items. Even then, the portion of the value of a good that is required to reproduce an economy and the portion that goes beyond that can be disentangled by observing non-luxury versions of the same good or controlling for Veblen-effects (Bagwell and Bernheim, 1996).

This distinction leaves normative judgment within final industries, which is irrelevant to the construction and conceptualization of the NT. Final industries could be categorized as *useful* to expand the economy (e.g. scientific research), *significant* to enrich human experience (e.g. fine arts), or *superfluous* expressions of human desires (e.g. luxury). The only qualifying trait is that they are all unnecessary to reproduce what is currently being produced.

Note however that while final industries are physically unnecessary to reproduce the economy, they provide the demand needed to sell input industries' output. Thus, input industries *could* maintain output at the same level if final industries were stopped because their inputs would still be available, yet they *would not* given that part of their output could not be sold.

3.1.8 Final demand and rents

In the CT there are no rents given the assumption of CRS. Moreover, the table is usually used to show how one more unit of final demand “yields” additional gross output given the leontief inverse, with the relation going from final demand to gross output (Dietzenbacher et al., 2013). In the NT, final demand (direct consumption and final industries' output) depends on the magnitude of rents, with the relation going from

the latter to the former. This has a profound implication on how economic systems are structured.

Mathematically, this relation derives from the fact that in the NT (Table 1) the elements of the Expenses column (vector \mathbf{X}) are equal to the first I elements of the Income row, and that each of those elements are equal to the sum of their corresponding column and row. Equating and summing over all industries shows that the value of final demand (final industries and direct consumption) is determined by the value of the rents of all industries:

$$\sum_{f=1}^F P_f q_f^* + \sum_{i=1}^I P_i q_{i,d}^* = \sum_{k=1}^K P_k q_k^{s*}. \quad (13)$$

Conceptually, a society first uses inputs to produce inputs (thus securing its reproduction), and only the excess inputs are either used directly or employed to produce final goods. In simpler terms, this is the notion of surplus production. For example, if a producer of rice uses all the rice produced to maintain itself, there is no more to feed a writer. A writer can only exist because a rice producer secures a surplus that can feed the writer. In a world with more than one input, the balance between inputs and outputs is weighted by prices and the surplus takes the form of rents. This can be seen explicitly by subtracting final industries' rents from both sides of (13), which shows that the basal value of final industries (i.e. the value to cover their costs) plus the value of direct consumption of input goods equals the rents of input industries:

$$\sum_{f=1}^F P_f q_f^{b*} + \sum_{i=1}^I P_i q_{i,d}^* = \sum_{i=1}^I P_i q_i^{s*}. \quad (14)$$

Note that if all industries were to exhibit CRS such that all rents are zero, (aside the fact that profit maximization would not have a solution) then the value of all final industries and direct consumption would also be zero. Such NT would be undistinguishable from a closed IO table.

3.2 Extensions

3.2.1 Government

To include government in the NT consider a modified version of the firm's problem in (1)

$$\max_{\mathbf{q}_{k,p}} \Pi_{k,p} = (1 - \tau)[P_k f_{k,p}(\mathbf{q}_{k,p}) - \mathbf{P}' \mathbf{q}_{k,p} - F_{k,p}], \quad (15)$$

where $\tau \in (0,1)$ is a rent tax and $F_{k,p}$ is the sum of all lump sum taxes faced by the firm. A sales tax is omitted because it induces price distortions that make the introduction of government less tractable. The resolution of (15) yields the same FOC in (2), and Euler's theorem leads to the same relation for basal production in (5) given the neutral nature of these taxes. The difference appears in (6) as rent is now shared between the firm and government. Hence, (6) becomes

$$P_k q_{k,p}^* = \mathbf{P}' \mathbf{q}_{k,p}^* + T_{k,p} + \Pi_{k,p}^*, \quad (16)$$

where $\Pi_{k,p}^* = (1 - \tau)P_k q_{k,p}^{s*} - F_{k,p}$ is the maximum rent that the firm can secure given the rent and lump sum taxes (i.e., private rent), $T_{k,p} = \tau P_k q_{k,p}^{s*} + F_{k,p}$ is the firm's total tax payment (i.e., public rent), and $P_k q_{k,p}^{s*}$ is the total rent produced by the firm. As with equation (8), the summation of (16) over all firms in an industry yields the rent maximizing solution for each of the K industries of an economy, but now with government. Hence:

$$P_k q_k^* = \mathbf{P}' \mathbf{q}_k^* + T_k + \Pi_k^*, \quad (17)$$

As with equation (8), equation (17) can be used to represent an entire economy. Table 3 adds to Table 1 a row between inputs and rents and a column between final industries and direct consumption. The new row accounts for the cost for industries (including households and final industries) of maintaining a government, and the new column for the value of the inputs used by such government. The last element of the tax row and government column contains the sum of all taxes paid ($\sum T_k$). The summation of the tax and private rent rows yields the total rent produced by the economy.

Table 3. NT with government

		Input Industries			Final Industries			Direct Cons.	Gov.	Expenses
		1	...	I	f	...	F			
Inputs	1	$P_1 q_{1,1}^*$...	$P_1 q_{1,I}^*$	$P_1 q_{1,f}^*$...	$P_1 q_{1,F}^*$	$P_1 q_{1,c}^*$	$P_1 q_{1,g}^*$	$P_1 q_1^*$
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots
	I	$P_I q_{I,1}^*$...	$P_I q_{I,I}^*$	$P_I q_{I,f}^*$...	$P_I q_{I,F}^*$	$P_I q_{I,c}^*$	$P_I q_{I,g}^*$	$P_I q_I^*$
Tax		T_1	...	T_I	T_f	...	T_F			$\sum T_k$
Private Rent		Π_1^*	...	Π_I^*	Π_f^*	...	Π_F^*			
Income		$P_1 q_1^*$...	$P_I q_I^*$	$P_f q_f^*$...	$P_F q_F^*$		$\sum T_k$	

The new table of coefficients (NTC) with government can be obtained by dividing (17) by $P_k q_k^*$. Similar to equation (9), this yields

$$1 = \sum_i^I a_{i,k} + t_k + \pi_k, \quad (18)$$

where $t_k = \frac{T_k}{P_k q_k^*}$ is the tax burden. The resultant NTC with government is

Table 4. NTC with government

		Input Industries			Final Industries			Direct Cons.	Gov.	Expenses
		1	...	I	f	...	F			
Inputs	1	$a_{1,1}$...	$a_{1,I}$	$a_{1,f}$...	$a_{1,F}$	-	-	-
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots
	I	$a_{I,1}$...	$a_{I,I}$	$a_{I,f}$...	$a_{I,F}$	-	-	-
Tax		t_1	...	t_I	t_f	...	t_F			-
Private Rent		π_1^*	...	π_I^*	π_f^*	...	π_F^*			
Income		1	...	1	1	...	1		-	

The leontief inverse can be obtained from Table 3 and Table 4 in the same way that was done without government, starting from the identity

$$\mathbf{AX} + \mathbf{BY} + \mathbf{D} + \mathbf{G} \equiv \mathbf{X}, \quad (19)$$

where the only difference with (23) is $\mathbf{G} = [P_1 q_{1,g}^*, \dots, P_I q_{I,g}^*]'$. Accordingly, the leontief inverse with government is

$$\mathbf{X} \equiv (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{BY} + \mathbf{D} + \mathbf{G}). \quad (20)$$

Per the same logic that yields equation (13), the value of final demand (final industries, direct consumption and government expenditure) is determined by the value of total rents (private rents + taxes) of all industries

$$\sum_{f=1}^F P_f q_f^* + \sum_{i=1}^I P_i q_{i,d}^* + \sum_{i=1}^I P_i q_{i,g}^* = \sum_{k=1}^K P_k q_k^{s*}, \quad (21)$$

and per the same logic that yields equation (14), the basal value of final industries, plus the value of direct consumption of input goods, plus government expenditure equals the total rent of input industries

$$\sum_{f=1}^F P_f q_f^{b*} + \sum_{i=1}^I P_i q_{i,d}^* + \sum_{i=1}^I P_i q_{i,g}^* = \sum_{i=1}^I P_i q_i^{s*}. \quad (22)$$

Equations (21) and (22) imply that the existence and size of government depends on rents. If there are no rents, rent taxes are zero and lump sum taxes make firms insolvent if $F_{k,p} > (1 - \tau)P_k q_{k,p}^{s*}$ such that $q_{k,p}^{s*} = 0 \Rightarrow F_{k,p} = 0$. Thus, no rents implies

no taxes and no government. This extends the idea that a society first secures its reproduction, and only thereafter has the means to use the remaining inputs to fund final industries, direct consumption of inputs, and government.

Excluding government from the reproductive requirements of a society is contentious. To reproduce themselves, economies require at least some laws and institutions provided, such that the distinction between input industries and government is more convoluted than between input and final industries. This could be addressed arguing that the inputs that enable the functioning of firms and governments are only produced by input industries, and only their rents are available to maintain government activity, yet the natural counterargument is that inputs cannot be produced without minimum government services such as the rule of law.

A solution to this conundrum could distinguish between minimum government required for a functional society providing order, defence, judiciary, etc., and a discretionary government performing discretionary activities such as offensive war and space exploration. This approach enables the distinction between a tax bill for minimum government which would count as an input outlay, and a tax bill for discretionary government that would come from rent. The specific implementation of this idea, and a full discussion on the co-dependence between input industries and government in the context of Table 3 exceeds the scope of this paper. To illustrate the NT and NTC with government, Table 3 and Table 4 are also provided as exemplary calculations in the Supporting Information.

3.2.2 Foreign trade

Foreign trade does not change the equations representing profit maximization nor the industry-wide solutions without government in (8) and with government in (17). Allowing for exports and imports changes the origin of inputs and destiny of outputs, and thus must be accounted for directly in the NT. As shown in Table 5, the NT with foreign trade includes export industries ($e \in E$) such that $I + F + E = K$, and a negative imports column next to the government column ensures that the Expense vector remains equal to the first I elements of the Income vector. No tariffs are assumed without loss of generality.

Table 5. NT with government and foreign trade

		Input Industries			Final Industries			Export Industries			Direct Cons.	Gov.	Imp.	Expenses
		1	...	I	f	...	F	e	...	E				
Inputs	1	$P_1q_{1,1}^*$...	$P_1q_{1,I}^*$	$P_1q_{1,f}^*$...	$P_1q_{1,F}^*$	$P_1q_{1,e}^*$...	$P_1q_{1,E}^*$	$P_1q_{1,d}^*$	$P_1q_{1,g}^*$	$-P_1q_{1,m}^*$	$P_1q_1^*$
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots	\vdots
	I	$P_Iq_{I,1}^*$...	$P_Iq_{I,I}^*$	$P_Iq_{I,f}^*$...	$P_Iq_{I,F}^*$	$P_Iq_{I,e}^*$...	$P_Iq_{I,E}^*$	$P_Iq_{I,d}^*$	$P_Iq_{I,g}^*$	$-P_Iq_{I,m}^*$	$P_Iq_I^*$
Tax		T_1	...	T_I	T_f	...	T_F	T_e	...	T_E				ΣT_k
Private Rent		Π_1^*	...	Π_I^*	Π_f^*	...	Π_F^*	Π_e^*	...	Π_E^*				
Income		$P_1q_1^*$...	$P_Iq_I^*$	$P_fq_f^*$...	$P_Fq_F^*$	$P_eq_e^*$...	$P_Eq_E^*$		ΣT_k		

The table of coefficients with exports in Table 6 follows as before. The novelty are the technical coefficients of exports $a_{i,e}$.

Table 6. NTC with government and foreign trade

		Input Industries			Final Industries			Export Industries			Direct Cons.	Gov.	Imp.	Expenses
		1	...	I	f	...	F	e	...	E				
Inputs	1	$a_{1,1}^*$...	$a_{1,I}^*$	$a_{1,f}^*$...	$a_{1,F}^*$	$a_{1,e}^*$...	$a_{1,E}^*$	-	-	-	-
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots	\vdots	\vdots
	I	$a_{I,1}^*$...	$a_{I,I}^*$	$a_{I,f}^*$...	$a_{I,F}^*$	$a_{I,e}^*$...	$a_{I,E}^*$	-	-	-	-
Tax		t_1	...	t_I	t_f	...	t_F	t_e	...	t_E			-	-
Private Rent		π_1^*	...	π_I^*	π_f^*	...	π_F^*	π_e^*	...	π_E^*				
Income		1	...	1	1	...	1	1	...	1		-		

Combining this NTC (Table 6) with its corresponding NT (Table 5) yields the identity

$$\mathbf{AX} + \mathbf{BY} + \mathbf{CZ} + \mathbf{D} + \mathbf{G} - \mathbf{M} \equiv \mathbf{X}, \quad (23)$$

where $\mathbf{C} = \begin{bmatrix} a_{1,e} & \cdots & a_{1,E} \\ \vdots & \ddots & \vdots \\ a_{I,e} & \cdots & a_{I,E} \end{bmatrix}$ is the Matrix of Coefficients of export industries, $\mathbf{Z} = [P_eq_e^*, \dots, P_Eq_E^*]'$ is the Income vector of export industries, and $\mathbf{M} = [P_1q_{1,m}^*, \dots, P_Iq_{I,m}^*]'$ is a Imports vector. Solving for \mathbf{X} yields the leontief inverse

$$\mathbf{X} \equiv (\mathbf{I} - \mathbf{A})^{-1}(\mathbf{BY} + \mathbf{CZ} + \mathbf{D} + \mathbf{G} - \mathbf{M}). \quad (24)$$

where the Final Demand vector now contains the output of final industries, households' direct consumption, government expenditure, and net exports.

Per the same logic that yields equation (13) and (21), the value of final demand (final industries, direct consumption, government expenditure, exports minus imports) is determined by the value of total rents (private rents + taxes) of all industries

$$\sum_{f=1}^F P_f q_f^* + \sum_{i=1}^I P_i q_{i,d}^* + \sum_{i=1}^I P_i q_{i,g}^* + \sum_{e=1}^E P_e q_e^* - \sum_{i=1}^I P_i q_{i,m}^* = \sum_{k=1}^K P_k q_k^{s*}, \quad (25)$$

Given that imports are given exogenously and subtract the value of final demand, without further structure ever growing imports could sustain ever growing final demand. The simplest structure to avoid this is to impose trade balance such that the total value of exports equals the total value of imports, and that only input goods are imported. As this implies that $\sum_{e=1}^E P_e q_e^* = \sum_{i=1}^I P_i q_{i,m}^*$ equation (25) reduces to (21). This is unrealistically simple as in reality the balance of payments and not the trade balance must equal zero, yet for the purpose of this article the simplest structure suffices. Note that this assumption does not cancel out \mathbf{CZ} with \mathbf{M} in (23) and (24) because each element of both vectors are not the same *vis-à-vis*.

3.2.3 Uncertainty

To include uncertainty consider a modified version of the firm's problem in (15) of the form

$$\max_{\mathbf{q}_{k,p}} E(\Pi_{k,p}) = E\{(1 - \tau)[P_k f_{k,p}(\mathbf{q}_{k,p}) - \mathbf{P}' \mathbf{q}_{k,p} - F_{k,p}]\}, \quad (26)$$

where $E\{\cdot\}$ is the expectation operator. Assuming firms know the density function of prices and taxes, this statement leads to an IO table similar to Table 6 except for all prices, rents, taxes and income appearing in expected value. This is an *ex ante* table representing the aggregated result of firms' *ex ante* expectations. Additionally, under uncertainty there is an *ex post* table containing realized prices and taxes, and actual rents and income. If realized values turn out far from their expected means, firms can obtain unexpected gains or losses.

3.2.4 Market power

To include market power in the NT consider a modified version of the firm's problem in (1)

$$\max_{\mathbf{q}_{k,p}} \Pi_{k,p} = P_{k,p}(q_{k,p}^*) f_{k,p}(\mathbf{q}_{k,p}) - \mathbf{P}' \mathbf{q}_{k,p}, \quad (27)$$

where $P_{k,p}(q_{k,p}^*)$ is the commodity's price, which is no longer exogenous but a function of the firm's output $q_{k,p}^*$. To avoid unnecessary complexity, $\mathbf{q}_{k,p}$ contains all commodities

in I_p except the one produced by p (henceforth $I_{-k,p}$). The modification changes the FOC in (2) to

$$P_{k,p} f_{i,k,p} (1 + \epsilon_{k,p}) = P_i, \quad \forall i \in I_{-k,p} \quad (28)$$

where $\epsilon_{k,p} = \frac{\partial P_{k,p}}{\partial f_{k,p}(\mathbf{q}_{k,p})} \cdot \frac{f_{k,p}(\mathbf{q}_{k,p})}{P_{k,p}} \leq 0$ is the elasticity of the inverse demand function faced by the firm. Such elasticity represents the firm's degree of market power. If $\epsilon_{k,p} = 0$ the firm faces perfect competition, has no market power, and (28) reduces to (2). Otherwise, the higher $\epsilon_{k,p}$ in absolute value, the higher the degree of market power, and the stronger that the inequality $P_{k,p} f_{i,k,p} > P_i$ becomes.

As the elasticity does not change across inputs, the effect of market power on the equations in section 3 is straightforward. Moreover, to simplify interpretation, an auxiliary variable $\beta_{k,p} = -\epsilon_{k,p} P_{k,p}^* q_{k,p}^{b*}$ can be defined such that (5) becomes

$$P_{k,p}^* q_{k,p}^{b*} = \mathbf{P}' \mathbf{q}_{k,p}^* + \beta_{k,p}. \quad (29)$$

Note that now $P_{k,p}^*$ is the optimal price for the firm instead of an exogenous parameter. Given that $P_{k,p}^* q_{k,p}^{b*} > \mathbf{P}' \mathbf{q}_{k,p}^*$, this arrangement shows that even at basal production profits are positive. Similarly, equation (6) in the context of market power becomes

$$P_{k,p}^* q_{k,p}^* = \mathbf{P}' \mathbf{q}_{k,p}^* + \Pi_{k,p}^* + \beta_{k,p}, \quad (30)$$

where $\beta_{k,p}$ represents market power profits secured by the firm (i.e., sometimes called monopoly rents). Finding $q_{k,p}^*$ and with it $\mathbf{q}_{k,p}^*$, $\Pi_{k,p}^*$, $\beta_{k,p}$ and $P_{k,p}^*$ requires knowing the demand function and the use of numerical methods such as the Newton-Raphson method because under general conditions $q_{k,p}^*$ cannot be solved for directly. This provides a good approximation, but the small error implies that later, at the economy-wide level, the exact identity found for leontiev's inverse in prior sections is lost.

Adding firms within an industry is not as straightforward as in section 3 because each firm sets a distinct price given its unique elasticity of inverse demand and production function. Thus, equation (8) in the context of market power becomes

$$\sum_{p=1}^{N_k} P_{k,p}^* q_{k,p}^* = \mathbf{P}' \mathbf{q}_k^* + \Pi_k^* + \beta_k, \quad (31)$$

where $\sum_{p=1}^{N_k} P_{k,p} q_{k,p}$ is the industry's total income and $\beta_k = \sum_{p=1}^{N_k} \beta_{k,p}^*$ is industry-wide market power profits. Setting equation (31) as (8) in section 3 to obtain the NT with market power faces two problems. The first one is that (31) considers many output prices but only one input price, which works at the industry level but is insufficient at

the economy-wide level because the many output prices of one industry must become the unique input price for other industries. The simplest solution to this is considering the average of all output prices of an industry as the unique input price to all other industries. This approach, although simple, introduces errors to the table that further embroils the identity of leontiev's inverse. Yet, through this approach the NT with market power can be obtained as shown in Table 7 below.

Table 7. NT with market power

		Input Industries			Final Industries			Direct Cons.	Expenses
		1	...	I	f	...	F		
Inputs	1	$P_1q_{1,1}^*$...	$P_1q_{1,I}^*$	$P_1q_{1,f}^*$...	$P_1q_{1,F}^*$	$P_1q_{1,d}^*$	$\sum_{p=1}^{N_1} P_{1,p}q_{1,p}^*$
	\vdots	\vdots	\ddots	\vdots	\vdots	\ddots	\vdots	\vdots	\vdots
	I	$P_Iq_{I,1}^*$...	$P_Iq_{I,I}^*$	$P_Iq_{I,f}^*$...	$P_Iq_{I,F}^*$	$P_Iq_{I,d}^*$	$\sum_{p=1}^{N_I} P_{I,p}q_{I,p}^*$
Rent		$\Pi_1^* + \beta_1$...	$\Pi_I^* + \beta_I$	$\Pi_f^* + \beta_f$...	$\Pi_F^* + \beta_F$		
Income		$\sum_{p=1}^{N_1} P_{1,p}q_{1,p}^*$...	$\sum_{p=1}^{N_I} P_{I,p}q_{I,p}^*$	$\sum_{p=1}^{N_f} P_{f,p}q_{f,p}^*$...	$\sum_{p=1}^{N_F} P_{F,p}q_{F,p}^*$		

The second problem with this NT with market power is that, although input prices are exogenous at the industry level, they are endogenous at the economy-wide level. Thus, the resolution of the optimal quantity and average price for one industry changes the optimal quantity and average price of all other industries, which feed back to the original industry. This situation requires iterative procedures that optimize each industry against the optimization of all other industries until an equilibrium is found.

4 Discussion

The new IO table (NT) can be built from the most basic profit maximization process. The NT 1) has microfoundations, 2) does not require the assumption of fixed coefficients, 3) is based on decreasing instead of constant returns to scale 4) considers labor and capital as inputs, 5) restates the leontief inverse, 6) modifies the definition of personal consumption expenditure, 7) distinguishes between input and final industries, and 8) links the existence of final demand and government to rents.

Features 1-5 harmonize IO analysis with neoclassical theory, while those from 6-8 show the role of rent in an economy. The basis of features 1-5 is the existence of well-behaved production functions, Euler's homogeneous function theorem, and the

recognition that everything bought and sold in an economy is produced and purchased by someone else. The only key assumption is decreasing returns to scale.

The NT does not require fixed coefficient to do analysis, yet demands knowing every firm's production function and input cost (including capital cost). If this is known, the impact of changes in the output price of industry k in the output, income, and rent of all industries can be predicted. Obtaining the data to do this can be viewed as future research and/or a drawback of the NT, specially in the context of permanent technological revolutions. Alternatively, the NT can be thought of as a theoretical basis for the conventional IO framework. Such approach would be imprecise given that the economic structure underlying both IO tables are essentially different, but has the appeal that once a table is built, IO analysis can be done by working directly with the NT and leontiev inverse. This requires invoking fixed coefficients which should be tested rather than assumed, yet provides a way to analyze policy without building the table from the ground up. As with conventional IO tables, this approach would be most meaningful for short-term analyses.

The consideration of labor and capital as inputs produced by industries as implied by equation (1) was discussed by Leontief (1937) in his original article, and implemented in his closed model (Leontief, 1941, 1936). This approach has been used sparsely compared to the open model depicted in Figure 1. Considering capital as produced by one or many industries is hardly controversial, yet doing so with labor implies reducing parenting to an unrealistic cold calculation of gains. The approach might be acceptable by highlighting that profit maximization need not be an explicit objective of families but a way to model how parents behave, similar to how Malthusianism understood population dynamics through forces beyond people's conscious objectives.

At the core of features 6-8 is a novel understating of the structure of economic systems based on rents. They sustain personal expenditures that are not required for household reproduction, final industries that are not required for economic reproduction, and government activity above and beyond what is required to maintain the institutions that enable economic reproduction. Thus, rents must be ubiquitous in complex economies. This role of rents contrasts with the anecdotal treatment that they generally receive in IO analysis and neoclassical theory.

The way rents are defined by equation (7) implies that they are chiefly determined by the degree of decreasing returns to scale: the lower the returns, the higher the rents. This counter-intuitive result is due to lower returns to scale implying higher-sloping

supply curves and therefore higher prices *ceteris paribus*. As the relation goes from technology to prices, an individual firm cannot obtain higher rents by lowering its returns to scale given a vector of prices. Such a foolish firm would reduce its rents by falling into a reverse case of the fallacy of composition.

Rents as a consequence of diminishing returns implies that they grow as industries expand production and reach technological maturity. Without further technological progress, competition, or political intervention, the rise in rents enables more aggressive rent-seeking and massive transfers of income and wealth to those that succeed. Such tendency can be countered by a new wave in the Kondratiev cycle, higher competition through competition policy and/or trade policy, and taxes, unions and/or rent-hindering institutions. The resultant dynamic dialogues well with the idea of Kuznets waves put forth by Milanovic (2016).

Lastly, rents in the NT are the basis of final industries as shown by equation (13) and (14). The distinction between input and final industries is generally relevant to understand the structure of an economy, yet has become more relevant throughout the COVID-19 pandemic given the widespread use of the term “essential industries”. While neoclassical theory and conventional IO analysis provides no guidance to identify such industries, the NT provides a clear distinction between what is essential (i.e., input industries) and non-essential (i.e., final industries).

Drawbacks of the NT relate to the exogeneity of prices and data collection. The exogeneity of prices implies no market power, omits demand, imposes zero pass-through of costs and therefore does not mathematically guarantee that direct consumption is positive. Dropping the no market power assumption by making prices endogenous deals with these issues, but makes the analysis considerably more complex and the NT less precise. Moreover, endogeneity of prices requires knowing demand functions. Linking demand functions to income given firm’s profits and factor payments is an interesting avenue to study general equilibrium, yet exceeds the scope of this article and remains as future research.

Regarding data collection to build the NT, new protocols are required to distinguish between input and final industries, and both portions of personal consumption expenditure. While such effort is challenging because ambiguous distinctions are likely to arise, building a minimum consensus on how economies are structured in terms of industries that are required to physically reproduce an economy and those that are not would be a valuable task, specially in times of COVID-19. Implementing these new protocols

to build the NT would be costly, although conversion methods that allow changes in classification as in Rueda-Cantuche et al. (2020) might help. In any case, these new data collection protocols could provide a new and enriched picture of the structure of an economy and a valuable tool to assess policy impacts.

5 Conclusions

A new input-output table provides microfoundations to the input-output framework and recognizes the origin and role of economic rent in the structure of economic systems. This table is built from a basic profit maximization process, Euler's theorem, and the logical connection between revenues and outlays. The only required assumption is decreasing returns to scale. The approach addresses common critiques to input-output analysis related to lack of microfoundations, fixed coefficients and homogeneous production functions. The changes to conventional input-output tables entail challenges to data collection, yet might be useful to enhance our understanding of economic structures and expand the acceptability and use of the input-output framework.

6 Declarations

6.1 Availability of data and materials

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study. The numerical illustrations are presented as supplementary information files.

6.2 Competing interests

The author declares having no competing interests

6.3 Funding

The author received funding while doing this work from CONICYT/ANID under grant PFCHA/DOCTORADO BECAS CHILE/2015 – 72160256.

The funding was not linked to this specific research, and the source was not involved in the conduct of the research, preparation of the article, study design, writing of the

report or in the decision to submit the article for publication. The views expressed here, as well as any errors and/or omissions, are entirely the responsibility of the author.

6.4 Author's contributions

BL designed, drafted and revised this work. BL approves the submitted version and agrees both to be personally accountable for the author's own contributions and to ensure that questions related to the accuracy or integrity of any part of the work, even ones in which the author was not personally involved, are appropriately investigated, resolved, and the resolution documented in the literature.

6.5 Acknowledgements

Not applicable

References

- Almon, C. J. (1984). The INFORUM-IIASA International Systems of Input-Output Models. In *Proceedings of the Seventh International Conference on Input-Output Techniques*, New York. UN.
- Ayres, R. and Shapanka, A. (1976). Explicit technological substitution forecasts in long-range input-output models. *Technological Forecasting and Social Change*, 9:113–38.
- Bagwell, L. S. and Bernheim, D. (1996). Veblen Effects in a Theory of Conspicuous Consumption. *The American Economic Review*, 86(3):349–373.
- Bulmer-Thomas, V. (1982). *Input-Output Analysis in Developing Countries*. Wiley, New York.
- Christ, C. F. (1955). Chapter Title: A Review of Input-Output Analysis. In *Input-Output Analysis: An Appraisal*, pages 137 – 182. Princeton University Press.
- Costa, A. (1984). United Nations Global Modeling: Experimental Projection on the Basis of Alternative Procedures. In *Proceedings of the Seventh International Conference on Input-Output Techniques*, New York. UN.
- Dietzenbacher, E., Lenzen, M., Los, B., Guan, D., Lahr, M. L., Sancho, F., Suh, S., and Yang, C. (2013). Input-output analysis: the next 25 years. *Economic Systems Research*, 25(4):369–389.

- Golladay, F. and Haveman, R. (1976). Regional and distributional effects of a negative income tax. *American Economic Review*, 66:629–41.
- Guerra, A.-I. and Sancho, F. (2011). Revisiting the original ghosh model: can it be made more plausible? *Economic Systems Research*, 23(3):319–328.
- Hudson, E. A. and Jorgenson, D. W. (1974). U. S. Energy Policy and Economic Growth, 1975-2000. *The Bell Journal of Economics and Management Science*, 5(2):461–514.
- Kurz, H. D. and Salvadori, N. (2000). 'Classical' roots of input-output analysis: A short account of its long prehistory. *Economic Systems Research*, 12(2):153–179.
- Lakshmanan, T. R. and Lo, F.-C. (1972). A regional economic model for the assessment of effects of air pollution abatement. Technical report.
- Leontief, W. (1941). *The Structure of American Economy, 1919-1939: An Empirical Application of Equilibrium Analysis*. International Arts and Sciences Press, White Plains, N. Y., 2nd enlarg edition.
- Leontief, W. (1970). Environmental repercussions and the economic system. *Review of Economics and Statistics*, 52:262–72.
- Leontief, W., Carter, A., and Petri, P. (1977). *The Future of the World Economy*. Oxford University Press, New York.
- Leontief, W. W. (1936). Quantitative Input and Output Relations in the Economic Systems of the United States. *The Review of Economics and Statistics*, 18(3):105–125.
- Leontief, W. W. (1937). Interrelation of Prices, Output, Savings, and Investment. Technical Report 3.
- Manresa, A. and Sancho, F. (2020). A follow-up note on the plausibility of the leontief and ghosh closed models. *Economic Systems Research*, 32(1):166–172.
- Milanovic, B. (2016). *Global inequality: A new approach for the age of globalization*. Harvard University Press.
- Mohnen, P., Raa, T. T., and Bourque, G. (1997). Mesures de la croissance de la productivité dans un cadre d'équilibre général: L'économie du québec entre 1978 et 1984. *Canadian Journal of Economics*, 30:295–307.
- Piñero, P., Bruckner, M., Wieland, H., Pongrácz, E., and Giljum, S. (2018). The raw material basis of global value chains: allocating environmental responsibility based on value generation. *Economic Systems Research*.
- Raa, T. T. (1995). *Linear Analysis of Competitive Economies*. New York, Harvester Wheatsheaf.

- Raa, T. T. (2004). A Neoclassical Analysis of TFP Using Input-Output Prices. In *Wassily Leontief and Input-Output Economics*, pages 151 – 165. Cambridge University Press.
- Raa, T. T. and Mohnen, P. (2002). Neoclassical growth accounting and frontier analysis: a synthesis. *Journal of Productivity Analysis*, 18:111–128.
- Raa, T. T. and Mohnen, P. (2014). Neoclassical input-output analysis. *Regional Science and Urban Economics*, 24:135–158.
- Reich, U.-P. (2018). Who pays for whom? Elements of a macroeconomic approach to income inequality. *Economic Systems Research*, 30(2):201–218.
- Rose, A. and Miernyk, W. (1989). Input-Output Analysis: The First Fifty Years. *Economic Systems Research*, 1(2):229–272.
- Rueda-Cantuche, J. M., Amores, A. F., and Remond-Tiedrez, I. (2020). Can supply, use and input–output tables be converted to a different classification with aggregate information? *Economic Systems Research*, 32(1):145–165.
- Sancho, F. (2019). An armington–leontief model. *Economic Structures*, 8(25):111–128.
- Steenge, A. and Reyes, R. (2020). Return of the capital coefficients matrix. *Economic Systems Research*, 32(4):439–450.
- Steenge, A. E., Bouwmeester, M. C., and Incera, A. C. (2018). Rents, resources, and multiple technologies; Ricardian mechanisms in input-output modelling. *Economic Systems Research*.
- Steenge, A. E. and Serrano, M. (2012). Income distributions in input-output models. *Economic Systems Research*, 24(4):391–412.
- ten Raa, T. (1994). On the methodology of input-output analysis. *Regional Science and Urban Economics*, 24:3–25.
- Wessel, R. (1967). A Note on Economic Rent. *The American Economic Review*, 57(5):1221–1226.
- Xie, Y., Ji, L., Zhang, B., and Huang, G. (2018). Evolution of the Scientific Literature on Input – Output Analysis : A Bibliometric Analysis of 1990 – 2017. *Sustainability*, 10(9).

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Numericalillustrations.xlsx](#)
- [SI.pdf](#)