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Xiaodong Ji (✉ jxd@ntu.edu.cn)

Nantong University Wenfeng Campus <https://orcid.org/0000-0002-1324-8051>

Tao Wang

Nantong University

Chong Qian

China Telecom

Chenhao Huang

China Telecom

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Power Minimization of UAV-Assisted Full-Duplex Mobile Relaying via Hybrid Trajectory Design

Xiaodong Ji^{*1}, Tao Wang¹, Chong Qian², Chenhao Huang³

1. School of Information Science and Technology, Nantong University, Nantong 226019, China

2. Wujiang Branch of China Telecom Group Co., Suzhou 215200, China

3. Gaoyou Branch of China Telecom Group Co., Yangzhou 225600, China

* Corresponding author: Xiaodong Ji, E-mail: jxd@ntu.edu.cn

Tao Wang, E-mail: 1290549851@qq.com

Chong Qian, E-mail: 865506980@qq.com

Chenhao Huang, E-mail: 121900180@qq.com

Abstract—This paper solved a propulsion power minimization problem subject to a total data-bits constraint of an unmanned aerial vehicle (UAV) enabled full-duplex mobile relaying system, where a full-duplex UAV relay is dispatched as a mobile relay to assist data transfer from a source to a destination by using three trajectory flying modes, namely, the UAV first flies in a circle above the source, next flies to the destination in a straight line, and finally flies in a circle above the destination until all the data bits has been transferred. Since the propulsion power minimization problem is a non-convex mixed integer programming problem and its closed-form solution is hard to obtain, it is transformed to three sub-problems so as to simplify its solution. After solving the three sub-problems, an iterative algorithm is proposed to achieve a sub-optimal solution to the propulsion power minimization problem, leading to a new hybrid circular/straight trajectory (HCST) design. Computer simulations are conducted and the results validated the proposed HCST design. It is shown that compared to the straight or circular flight trajectory design, the HCST performs well in terms of energy saving for the long distance and big data communication cases.

Index terms—UAV communication, trajectory design, power minimization

1. Introduction

The use of unmanned aerial vehicles (UAVs) for assisting wireless communications has attracted increasing interest recently [1]. Compared to the traditional terrestrial communications, UAV-assisted communications are more cost-effective by enabling on-demand operations, more flexible and swift for deployment due to the high mobility of UAVs. In addition, UAV-ground communication links are more likely to have line-of-sight (LoS) channels which offers higher capacity than that provided by terrestrial links. Thus, UAVs shall play an extremely important role in future wireless communications with applications such as temporary traffic offloading for cellular base stations [2]-[4], cooperative communications for mobile relaying [5]-[8], information dissemination

and data collection for Internet of Things [9], etc.

In this paper, we considered a full-duplex mobile relaying system consisting of a fixed-wing UAV, a source and a destination. The source would like to transfer data to the destination assisted by the UAV which acts as a decode-and-forward (DF) relay. The UAV relay has three trajectory flying modes, namely, the UAV first flies in a circle above the source, next flies to the destination in a straight line, and finally flies in a circle above the destination until all the data bits has been transferred. It is noted in [10] that an UAV's communication-related power is usually much smaller than its propulsion power. Therefore, the main goal of this paper is to minimize the total propulsion power of the UAV. We formulate the trajectory design as a non-convex mixed integer programming problem. To solve the problem, it is divided into a few sub-problems, by solving which a sub-optimal solution is acquired. Then, an iterative algorithm is proposed to obtain a hybrid circular/straight trajectory (HCST) design which can approach to the optimal trajectory minimizing the total propulsion power.

The rest of this paper is organized as follows: Section 2 describes the related works in literature. Section 3 introduces the system model under investigation and formulates a general optimization problem corresponding to the propulsion power minimization. By dividing the optimization problem into three sub-problems, the power minimization problem is solved in Section 4, leading to a new HCST design. Computer simulations of the proposed HCST design with performance comparisons are presented in Section 5. Section 6 concludes the paper.

2. Related works

Currently, there are many works in literature that concern the problem of maximizing information throughput or spectrum efficiency of UAV-assisted mobile relaying systems [11]-[16]. It is known that UAVs have limited on-board energy, and thus energy saving has been recognized as a more important factor for UAV communications. The authors of [17] investigated an energy-efficiency problem by optimizing the UAV's speed and load factors in circular trajectory. In [18], an energy-efficiency maximization problem was solved by jointly studying beamforming, power allocation, circular radius and flight speed of an UAV-assisted half-duplex mobile relaying system. In [19], a power consumption trade-off between ground terminal and UAVs was derived for circular and straight trajectories, respectively. The authors of [10] developed a load-carry-and-delivery scheme for a full-duplex UAV relaying network with the aim of enhancing the energy efficiency of the network. In [20], a rotary-wing UAV was used to communicate with multiple ground nodes, and an efficient algorithm was proposed to optimize the hovering locations and duration, as well as the flying trajectory so as to minimize the system energy consumption. The authors of [21] investigated an energy-efficiency problem with the aim of maximizing the data bits transferred per the UAV's propulsion energy via the trajectory design. It should be noted that most

of the existing works considered the circular or straight trajectory design separately. It is noted in [21] that there exists an upper bound of information throughput if a straight trajectory is adopted. It means that straight trajectory is not suitable for the case where a huge amount of data need to be transferred. In case that a circular trajectory is employed, the system information throughput will be poor if the source and destination nodes are located far away [17]. In view of the shortcomings of circular and straight trajectories, this paper investigates a HCST design for power minimization.

3. Method

Fig. 1 gives the system model, where an UAV denoted by R is dispatched as a mobile relay to assist data transfer from the source S to the destination D. Suppose that S and D are located far away, and thus no direct link exists between S and D due to high path loss caused by long distance and/or obstacles. Moreover, it is assumed that S would like to transfer a huge amount of data to D.

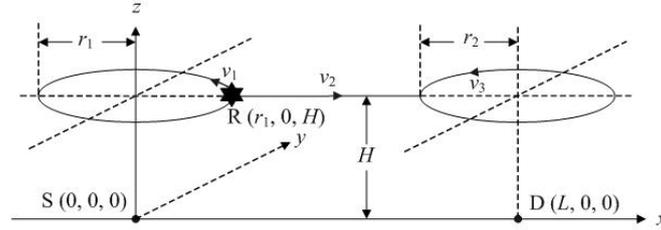


Fig. 1 UAV-enabled mobile relaying system with hybrid circular/straight trajectory

Without loss of generality, assume that the horizontal coordinates of S and D are $\mathbf{q}_S = [0, 0]^T$ and $\mathbf{q}_D = [L, 0]^T$, respectively, and the UAV flies at a fixed altitude H which corresponds to the minimum altitude required for terrain or building avoidance without frequent aircraft ascending or descending [10]. Denote by T the mission completion time. Throughout the whole mission time, the UAV needs to collect Q bits data from S and then forwards them to D. There exists three trajectory modes described as the following.

Mode 1: The UAV flies in a circle with center at $(0, 0, H)$ and radius r_1 . Denote by v_1 and T_1 the UAV's speed and duration of mode 1, respectively.

Mode 2: The UAV flies from $(r_1, 0, H)$ to $(L - r_2, 0, H)$ in a straight line. Denote by v_2 and T_2 the UAV's speed and duration of mode 2, respectively.

Mode 3: The UAV flies in a circle with center at $(L, 0, H)$ and radius r_2 . Denote by v_3 and T_3 the UAV's speed and duration of mode 3, respectively.

Suppose that the initial location of the UAV is $(r_1, 0, H)$. First, the UAV flies n laps according to mode 1. Then, the UAV flies along mode 2. Finally, the UAV flies according to mode 3 until Q bits data are completely transferred from S to D. Here, the duration of each mode must satisfy

$$T_1 = \frac{2\pi r_1}{v_1} n, (n = 1, 2, \dots), T_2 = \frac{L - r_1 - r_2}{v_2}, T_3 \geq 0. \quad (1)$$

During the whole mission time, a full-duplex relaying fashion is employed, namely, the UAV collects data from S and forwards them to D simultaneously. Note that for simplicity, the UAV's take-off and landing phases are

ignored. It is worth-mentioning that the UAV's trajectory should change between different modes smoothly, and the UAV's speed cannot be switched instantly between v_1 , v_2 and v_3 . However, since the total power consumption of an UAV is very large, the change of the UAV's kinetic energy between speeds v_1 , v_2 and v_3 and the difference between the practical trajectory and the trajectory in Fig. 1 can be omitted. Denote by $\mathbf{q}(t) \in \mathbf{R}^{2 \times 1}$ the UAV's trajectory on the horizontal coordinate, where $0 \leq t \leq T$. Then, the time-varying distance between S and the UAV R, and that between the UAV R and D can be, respectively, expressed as

$$d_{\text{SR}}(t) = \sqrt{H^2 + \|\mathbf{q}(t)\|^2}, \quad (2)$$

$$d_{\text{RD}}(t) = \sqrt{H^2 + \|(L, 0)^T - \mathbf{q}(t)\|^2}. \quad (3)$$

Here, it is assumed that the UAV R is equipped with a data-buffer of sufficiently large size and the communication links from R to S and D are dominated by the Line of Sight channels [11]. Moreover, the Doppler effect due to the UAV mobility is assumed to be perfectly compensated [11]. Then, the time-varying channel between S and R follows the free-space path loss model and can be written as

$$h_{\text{SR}}(t) = \beta_0 d_{\text{SR}}^{-2}(t) = \frac{\beta_0}{H^2 + \|\mathbf{q}(t)\|^2}. \quad (4)$$

Likewise, the time-varying channel between R and D can be expressed as

$$h_{\text{RD}}(t) = \beta_0 d_{\text{RD}}^{-2}(t) = \frac{\beta_0}{H^2 + \|(L, 0)^T - \mathbf{q}(t)\|^2}. \quad (5)$$

In (4) and (5), β_0 denotes the channel power at the reference distance $d_0 = 1\text{m}$. Assume that R collects and forwards data using different frequency bands, and thus there is no loop-interference at the UAV[11]. Then, the corresponding signals received by R and D can be, respectively, written as

$$y_{\text{R}}(t) = \sqrt{P_{\text{S}}} h_{\text{SR}}(t) x_{\text{S}}(t) + n_{\text{R}}(t), \quad (6)$$

$$y_{\text{D}}(t) = \sqrt{P_{\text{R}}} h_{\text{RD}}(t) x_{\text{R}}(t) + n_{\text{D}}(t). \quad (7)$$

In (6) and (7), $x_{\text{S}}(t)$ and $x_{\text{R}}(t)$ are the signals sent by S and R; P_{S} and P_{R} are the transmit power from S and R; $n_{\text{R}}(t)$ and $n_{\text{D}}(t)$ are the additive noises received by R and D, respectively.

The data of size $Q_{\text{R}}(t)$ and $Q_{\text{D}}(t)$ received by the UAV R and D at time t in bits/Hz can be, respectively, expressed as [10]

$$Q_{\text{R}}(t) = \int_0^t \log_2 \left(1 + \frac{P_{\text{S}} h_{\text{SR}}(\tau)}{\sigma^2} \right) d\tau = \int_0^t \log_2 \left(1 + \frac{P_{\text{S}} \gamma_0}{H^2 + \|\mathbf{q}(\tau)\|^2} \right) d\tau, \quad (8)$$

$$Q_{\text{D}}(t) = \int_0^t \log_2 \left(1 + \frac{P_{\text{R}} h_{\text{RD}}(\tau)}{\sigma^2} \right) d\tau = \int_0^t \log_2 \left(1 + \frac{P_{\text{R}} \gamma_0}{H^2 + \|(L, 0)^T - \mathbf{q}(\tau)\|^2} \right) d\tau, \quad (9)$$

where σ^2 denotes the noise power at the UAV R and D, and $\gamma_0 \triangleq \beta_0 / \sigma^2$. It is worth-mentioning that in the paper, data bits are normalized by bandwidth, and thus without specific instructions, bit means bit/Hz. In order to ensure Q bits data transfer during mission completion time T , (10) must be satisfied.

$$Q_R(T) \geq Q, Q_D(T) \geq Q \quad (10)$$

Moreover, the UAV can only forward data that has already been collected from S, and thus the following *information-causality constraint* must be satisfied [20]

$$Q_D(t) \leq Q_R(t), 0 \leq t \leq T. \quad (11)$$

It is well known that the total power consumption of an UAV consists of two parts, i.e., communication-related and propulsion power consumption. Here, only the UAV's propulsion power consumption is considered as the communication-related power consumption is usually much smaller than the propulsion power consumption [10]. According to [19], propulsion power consumption of a fixed-wing UAV with circular and straight trajectories can be modeled as $\left[c_1 + c_2 / (g^2 r^2) \right] v^3 + c_2 / v$ and $c_1 v^3 + c_2 / v$, respectively. Here, c_1 and c_2 are the parameters depending on the aircraft's weight, wing area, air density, etc., and g , r and v denote the acceleration of gravity, the radius of the circular and the velocity of the UAV, respectively [19]. Then, the optimization problem corresponding to the propulsion power minimization can be formulated as

$$\begin{aligned} (r_1^*, r_2^*, v_1^*, v_2^*, v_3^*, T_1^*, T_2^*, T_3^*, n^*) = \arg \min_{r_1, r_2, v_1, v_2, v_3, T_1, T_2, T_3, n} & T_1 \left[\left(c_1 + \frac{c_2}{g^2 r_1^2} \right) v_1^3 + \frac{c_2}{v_1} \right] + T_2 \left(c_1 v_2^3 + \frac{c_2}{v_2} \right) \\ & + T_3 \left[\left(c_1 + \frac{c_2}{g^2 r_2^2} \right) v_3^3 + \frac{c_2}{v_3} \right] \end{aligned} \quad (12a)$$

$$\text{s.t.} \quad r_1 + r_2 \leq L, r_1 > 0, r_2 > 0 \quad (12b)$$

$$V_{\min} \leq v_1, v_2, v_3 \leq V_{\max} \quad (12c)$$

$$T_1 = \frac{2\pi r_1}{v_1} n, (n = 1, 2, \dots), T_2 = \frac{L - r_1 - r_2}{v_2}, T_3 \geq 0 \quad (12d)$$

$$Q_R(T) \geq Q, Q_D(T) \geq Q \quad (12e)$$

$$Q_D(t) \leq Q_R(t), 0 \leq t \leq T \quad (12f)$$

$$T_1 + T_2 + T_3 = T \quad (12g)$$

where V_{\min} and V_{\max} are the UAV's the minimum and maximum flight velocity constraints, and (12b) and (12c) are the practical physical constraints.

4. Solution of Power Minimization Problem

In this section, the power minimization problem (12) is solved, leading to a HCST design. As a first step, the following lemma is presented so as to simplify the optimization problem.

Lemma 1: For problem (12), the constraints (12e) and (12f) can be rewritten as

$$Q_R(T) \geq Q_D(T) = Q \quad (13)$$

Proof: It is noticed that $Q_R(t)$ and $Q_D(t)$ are both monotonically increasing functions for $t \in [0, +\infty)$. Moreover, when S and D are located far away, $Q_R(t)$ can be seen as a convex function about t , while $Q_D(t)$ is a concave function. Since $Q_R(0) = Q_D(0) = 0$, (12f) can be satisfied if $Q_R(T) \geq Q_D(T)$ holds. Assume that $Q_D(T^*) > Q$ holds, where $T^* = T_1^* + T_2^* + T_3^*$ and T_1^* , T_2^* and T_3^* are the solutions of (12). Then, there must

exist a $\Delta > 0$ such that T_1^* , T_2^* and $T_3^* - \Delta$ can satisfy the constraints, and further reduces the power consumption, which contradicts the optimality of T_1^* , T_2^* and T_3^* . Therefore, the optimal solution of (12) must satisfy $Q_D(T^*) = Q$, and (12e) and (12f) can be rewritten as $Q_R(T) \geq Q_D(T) = Q$.

Denote by Q_R^{cir1} , Q_R^{str} and Q_R^{cir2} the sizes of data bits received by the UAV in the duration of modes 1, 2 and 3, respectively. Denote by Q_D^{cir1} , Q_D^{str} and Q_D^{cir2} the sizes of data bits received by D in the duration of modes 1, 2 and 3, respectively. Then, (13) can be rewritten as

$$Q_R^{\text{cir1}} + Q_R^{\text{str}} + Q_R^{\text{cir2}} \geq Q_D^{\text{cir1}} + Q_D^{\text{str}} + Q_D^{\text{cir2}} = Q, \quad (14)$$

and Q_R^{cir1} can be expressed as

$$Q_R^{\text{cir1}} = \frac{2n\pi r_1}{v_1} \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + r_1^2} \right). \quad (15)$$

According to [19], Q_R^{str} can be given by

$$Q_R^{\text{str}} = \frac{F_1(L - r_2) - F_1(r_1)}{v_2 \ln 2}, \quad (16)$$

where $F_1(x) = x \ln \left(1 + \frac{P_s \gamma_0}{H^2 + x^2} \right) - 2H \tan^{-1} \left(\frac{x}{H} \right) + 2\sqrt{H^2 + P_s \gamma_0} \tan^{-1} \left(\frac{x}{\sqrt{H^2 + P_s \gamma_0}} \right)$.

Since Q_R^{cir2} is mathematically intractable, a corresponding lower bound is considered. It should be noted that a lower bound of Q_R^{cir2} satisfying (14) can enable (12e) and (12f) to be hold. Using inequality (52) in [10], Q_R^{cir2} can be lower-bounded as

$$Q_R^{\text{cir2}} = \int_{-\pi}^{\frac{v_3 T_3 - \pi}{r_2}} \frac{r_2}{v_3} \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + L^2 + 2Lr_2 \cos x + r_2^2} \right) dx \geq T_3 B_1(r_2) + B_2(r_2, v_3), \quad (17)$$

where functions $B_1(x)$ and $B_2(x, y)$ are given as the following.

$$B_1(x) = \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + L^2 + x^2} \right),$$

$$B_2(x, y) = -\frac{2(\log_2 e) P_s \gamma_0 L x^2}{y(H^2 + L^2 + x^2)(H^2 + L^2 + x^2 + P_s \gamma_0)}.$$

Using the same approach, it is readily to obtain $Q_D^{\text{cir1}} \geq T_1 B_3(r_1) + B_4(r_1, v_1)$, $Q_D^{\text{str}} = \frac{F_2(L - r_1) - F_2(r_2)}{v_2 \ln 2}$ and

$Q_D^{\text{cir2}} = T_3 \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + r_2^2} \right)$, where functions $B_3(x)$, $B_4(x, y)$ and $F_2(x)$ are defined as the following.

$$B_3(x) = \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + L^2 + x^2} \right),$$

$$B_4(x, y) = -\frac{2(\log_2 e) P_R \gamma_0 L x^2}{y(H^2 + L^2 + x^2)(H^2 + L^2 + x^2 + P_R \gamma_0)}$$

$$F_2(x) = x \ln \left(1 + \frac{P_R \gamma_0}{H^2 + x^2} \right) - 2H \tan^{-1} \left(\frac{x}{H} \right) + 2\sqrt{H^2 + P_R \gamma_0} \tan^{-1} \left(\frac{x}{\sqrt{H^2 + P_R \gamma_0}} \right).$$

Since problem (12) is a non-convex mixed integer programming problem, it cannot be directly solved with standard convex optimization techniques. In the following, problem (12) is divided into three sub-problems by solving which a sub-optimal solution to the power minimization problem is acquired.

Recall that S and D are located far away and the UAV relay assists the data transfer between S and D by adopting three trajectory modes. In mode 1, the UAV is much closer to S than to D, and thus it has dominant traffic volume in collecting data from the source S. On the contrary, in mode 3 the traffic of the link from the UAV to D is dominant. Therefore, in order to simplify (12), two sub-problems corresponding to the power minimization in modes 1 and 3 are addressed first, where only the link having the dominant traffic volume is considered. Then, the two sub-problems can be formulated as (18) and (19).

$$(\tilde{r}_1, \tilde{v}_1) = \arg \min_{r_1, v_1} 2\pi n \left[\left(c_1 r_1 + \frac{c_2}{g^2 r_1^2} \right) v_1^2 + \frac{c_2 r_1}{v_1^2} \right] \quad (18a)$$

$$\text{s.t.} \quad Q_R^{\text{cir1}} = \frac{2\pi n r_1 \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + r_1^2} \right)}{v_1} \quad (18b)$$

$$v_{\min} \leq v_1 \leq v_{\max}, \quad 0 < r_1 < L \quad (18c)$$

$$(\tilde{r}_2, \tilde{v}_3) = \arg \min_{r_2, v_3} T_3 \left[\left(c_1 + \frac{c_2}{g^2 r_2^2} \right) v_3^3 + \frac{c_2}{v_3} \right] \quad (19a)$$

$$\text{s.t.} \quad Q_D^{\text{cir2}} = T_3 \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + r_2^2} \right) \quad (19b)$$

$$v_{\min} \leq v_3 \leq v_{\max}, \quad 0 < r_2 < L, \quad T_3 \geq 0 \quad (19c)$$

Of note is that in (18) and (19), parameters n , Q_R^{cir1} and Q_D^{cir2} are assumed to be fixed.

For problem (18), the optimal radius \tilde{r}_1 and velocity \tilde{v}_1 can be obtained via one-dimensional search by substituting (18b) into (18a). For problem (19), substituting (19b) into (19a) leads to

$$(\tilde{r}_2, \tilde{v}_3) = \arg \min_{r_2, v_3} \frac{\left(c_1 + \frac{c_2}{g^2 r_2^2} \right) v_3^3 + \frac{c_2}{v_3}}{\log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + r_2^2} \right)} \quad (20a)$$

$$\text{s.t.} \quad v_{\min} \leq v_3 \leq v_{\max}, \quad 0 < r_2 < L \quad (20b)$$

It can be easily found that $\left[c_1 + c_2 / (g^2 r_2^2) \right] v_3^3 + c_2 / v_3$ is a concave function for $v_3 \in (0, +\infty)$. Its extreme point is $v_3 = \left\{ c_2 / \left[3c_1 + 3c_2 / (g^2 r_2^2) \right] \right\}^{\frac{1}{4}}$. So, the minimum power on the numerator of (20a) can be achieved when (21) holds.

$$\tilde{v}_3 = \begin{cases} v_{\min}, & \text{if } v_{\min} > \left(\frac{c_2 g^2 r_2^2}{3c_1 g^2 r_2^2 + 3c_2} \right)^{\frac{1}{4}} \\ \left(\frac{c_2 g^2 r_2^2}{3c_1 g^2 r_2^2 + 3c_2} \right)^{\frac{1}{4}}, & \text{if } v_{\min} \leq \left(\frac{c_2 g^2 r_2^2}{3c_1 g^2 r_2^2 + 3c_2} \right)^{\frac{1}{4}} \leq v_{\max} \\ v_{\max}, & \text{if } v_{\max} < \left(\frac{c_2 g^2 r_2^2}{3c_1 g^2 r_2^2 + 3c_2} \right)^{\frac{1}{4}} \end{cases} \quad (21)$$

Substituting (21) into (20) and then the optimal solution of (20) can be obtained by using one-dimensional search over $0 < r_2 < L$.

Base on the above discussions, (12) can be transformed into

$$(\tilde{v}_2, \tilde{T}_1, \tilde{T}_3, \tilde{n}) = \arg \min_{v_2, \tilde{r}_1, \tilde{T}_3, n} T_1 \left[\left(c_1 + \frac{c_2}{g^2 \tilde{r}_1^2} \right) \tilde{v}_1^3 + \frac{c_2}{\tilde{v}_1} \right] + T_2 \left(c_1 v_2^3 + \frac{c_2}{v_2} \right) + T_3 \left[\left(c_1 + \frac{c_2}{g^2 \tilde{r}_2^2} \right) \tilde{v}_3^2 + \frac{c_2}{\tilde{v}_3^2} \right] \quad (22a)$$

$$\text{s.t.} \quad V_{\min} \leq v_2 \leq V_{\max} \quad (22b)$$

$$T_1 = \frac{2\pi \tilde{r}_1}{\tilde{v}_1} n, (n = 1, 2, \dots), T_2 = \frac{L - \tilde{r}_1 - \tilde{r}_2}{v_2}, T_3 \geq 0 \quad (22c)$$

$$Q_{\text{R}}^{\text{cir1}} + Q_{\text{R}}^{\text{str}} + B_1(\tilde{r}_2)T_3 + B_2(\tilde{r}_2, \tilde{v}_3) \geq Q \quad (22d)$$

$$B_3(\tilde{r}_1)T_1 + B_4(\tilde{r}_1, \tilde{v}_1) + Q_{\text{D}}^{\text{str}} + Q_{\text{D}}^{\text{cir2}} = Q \quad (22e)$$

In (22), \tilde{r}_1 and \tilde{v}_1 are the optimal solutions of (18), and \tilde{r}_2 and \tilde{v}_3 are the optimal solutions of (19).

However, the solution of problem (22) is still hard to achieve due to the integer constraint n . Then, the following two lemmas are proposed so as to simplify problem (22).

Lemma 2: For problem (22), the optimal solution of the number of the laps in mode 1 can be expressed as

$$\tilde{n} \in \left\{ \left\lfloor \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rfloor, \left\lceil \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rceil \right\}, \quad (24)$$

where \hat{T}_1 , \hat{v}_1 and \hat{r}_1 are given in Appendix A.

Proof: See APPENDIX A.

Lemma 3: Denote by $\tilde{Q}_{\text{R}}^{\text{cir1}}$ and $\tilde{Q}_{\text{R}}^{\text{str}}$ the optimal data bits received by the UAV in modes 1 and 2, respectively, which can be obtained by solving problem (22) and then substituting the obtained solutions into (15) and (16). Let $\hat{Q}_{\text{R}}^{\text{cir1}} = Q - \tilde{Q}_{\text{R}}^{\text{str}} - B_1(\tilde{r}_2)\tilde{T}_3 - B_2(\tilde{r}_2, \tilde{v}_3)$. Then, $\tilde{Q}_{\text{R}}^{\text{cir1}}$ can be expressed as

$$\tilde{Q}_{\text{R}}^{\text{cir1}} = \max \left\{ \hat{Q}_{\text{R}}^{\text{cir1}}, Q_{\text{ext}} \right\}, \quad (23)$$

where $Q_{\text{ext}} = 2n\pi\sqrt{\tilde{r}_1} \left(\frac{c_2 + c_1 g^2 \tilde{r}_1^2}{c_2 g^2} \right)^{\frac{1}{4}} \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + \tilde{r}_1^2} \right)$.

Proof: By fixing the value of the radius and the speed of mode 1, it can be easily found that the power consumption of the UAV is monotonically decreasing for $Q_{\text{R}}^{\text{cir1}} \in (0, Q_{\text{ext}})$ and monotonically increasing for $Q_{\text{R}}^{\text{cir1}} \in (Q_{\text{ext}}, +\infty)$. Since $B_1(\tilde{r}_2)\tilde{T}_3 + B_2(\tilde{r}_2, \tilde{v}_3)$ is a lower-bound of the data bits received by the UAV relay received in mode 3, $\tilde{Q}_{\text{R}}^{\text{cir1}} \geq \hat{Q}_{\text{R}}^{\text{cir1}}$ must be hold. For the case where $\hat{Q}_{\text{R}}^{\text{cir1}} \geq Q_{\text{ext}}$, if $\tilde{Q}_{\text{R}}^{\text{cir1}} - \hat{Q}_{\text{R}}^{\text{cir1}} > 0$ holds, we can always

find a smaller power consumption by reducing the power consumption of mode 1, while fixing the power consumption of modes 2 and 3. It contradicts the optimality assumption. So, $\tilde{Q}_R^{\text{cir1}} = \hat{Q}_R^{\text{cir1}}$ must hold in this situation. For that case where $\hat{Q}_R^{\text{cir1}} < Q_{\text{ext}}$, if $|\tilde{Q}_R^{\text{cir1}} - Q_{\text{ext}}| > 0$ holds, we can always find a smaller power consumption by reducing the power consumption of mode 1 while fixing the power consumption of modes 2 and 3. It also contradicts the optimality assumption. So, $\tilde{Q}_R^{\text{cir1}} = Q_{\text{ext}}$ must hold in order to achieve the minimum power consumption. In conclusion, $\tilde{Q}_R^{\text{cir1}} = \max\{\hat{Q}_R^{\text{cir1}}, Q_{\text{ext}}\}$ holds.

For the case that \tilde{n} is known, problem (22) can be rewritten as the other sub-problem, giving

$$(\tilde{v}_2, \tilde{T}_3) = \arg \min_{v_2, T_3} \frac{2\pi\tilde{r}_1}{\tilde{v}_1} \tilde{n} \left[\left(c_1 + \frac{c_2}{g^2\tilde{r}_1^2} \right) \tilde{v}_1^3 + \frac{c_2}{\tilde{v}_1} \right] + \frac{L - \tilde{r}_1 - \tilde{r}_2}{v_2} \left(c_1 v_2^3 + \frac{c_2}{v_2} \right) + T_3 \left[\left(c_1 + \frac{c_2}{g^2\tilde{r}_2^2} \right) \tilde{v}_3^2 + \frac{c_2}{\tilde{v}_3^2} \right] \quad (25a)$$

$$\text{s.t.} \quad V_{\min} \leq v_2 \leq V_{\max}, \quad T_3 \geq 0 \quad (25b)$$

$$\frac{2\tilde{n}\pi\tilde{r}_1}{\tilde{v}_1} \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + \tilde{r}_1^2} \right) + B_1(\tilde{r}_2)T_3 + B_2(\tilde{r}_2, \tilde{v}_3) + \frac{F_1(L - \tilde{r}_2) - F_1(\tilde{r}_1)}{v_2 \ln 2} \geq Q \quad (25c)$$

$$B_3(\tilde{r}_1)\tilde{T}_1 + B_4(\tilde{r}_1, \tilde{v}_1) + \frac{F_2(L - \tilde{r}_1) - F_2(\tilde{r}_2)}{v_2 \ln 2} + T_3 \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + \tilde{r}_2^2} \right) = Q \quad (25d)$$

In (25), \tilde{T}_1 can be achieved by substituting \tilde{r}_1 , \tilde{v}_1 and \tilde{n} into (1). It can be observed that the cost function and the constraint set in problem (25) are convex, and thus it is a convex optimization problem. The solution of (25) can be obtained by solving the Karush-Kuhn-Tucker (KKT) equations, and can be expressed as

$$\tilde{v}_2 = -0.5\sqrt{f_2} + 0.5\sqrt{-f_2 + \frac{a_9 a_{10}}{c_1 a_5 \sqrt{f_2} (L - \tilde{r}_1 - \tilde{r}_2)}} \quad (26)$$

$$\tilde{T}_3 = \frac{2f_2 k a_{10} - 4a_9 a_{10}^2 \tilde{v}_2 - 2\sqrt{f_2} \left[a_{10} (2a_9 a_{10} - k\tilde{v}_2) - 8c_2 a_5 (L - \tilde{r}_1 - \tilde{r}_2) (a_8 - b_3 \tilde{T}_1) \right]}{16c_2 a_5^2 \sqrt{f_2} (L - \tilde{r}_1 - \tilde{r}_2)} \quad (27)$$

$$+ \frac{4c_1 a_5 a_{10} \left[f_2 k - 24c_2 a_5 (L - \tilde{r}_1 - \tilde{r}_2) \right] (\sqrt{f_2} + \tilde{v}_2)}{16c_2 a_5^2 k}$$

where $k = \left\{ 3\sqrt{3} \sqrt{c_1^2 a_5^2 (L - \tilde{r}_1 - \tilde{r}_2)^2 c_1^2 \left[27a_9^4 a_{10}^4 + 4096c_1 c_2^3 a_5^4 (L - \tilde{r}_1 - \tilde{r}_2)^4 \right]} - 27c_1 a_5^2 a_9 a_{10}^2 (L - \tilde{r}_1 - \tilde{r}_2) \right\}^{\frac{1}{3}}$,

$$f_2 = \frac{8c_2 a_5 (L - \tilde{r}_1 - \tilde{r}_2)}{k} - \frac{k}{6c_1 a_5 (L - \tilde{r}_1 - \tilde{r}_2)}, \quad a_5 = \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + \tilde{r}_2^2} \right), \quad a_9 = \left(c_1 + \frac{c_2}{g^2 \tilde{r}_2^2} \right) \tilde{v}_3^2 + \frac{c_2}{\tilde{v}_3^2} \quad \text{and}$$

$$a_{10} = \frac{F_2(L - \tilde{r}_1) - F_2(\tilde{r}_2)}{\ln 2}.$$

According to *lemma 2* and *lemma 3*, an iterative algorithm is proposed to solve problem (22), leading to a sub-optimal solution to the propulsion power minimization problem (12). First, \tilde{r}_1 , \tilde{v}_1 , \tilde{r}_2 and \tilde{v}_3 are acquired by solving (18) and (20) with fixed Q_R^{cir1} and Q_D^{cir2} . Second, substituting \tilde{r}_1 , \tilde{v}_1 , \tilde{r}_2 and \tilde{v}_3 into (22) and then according to *lemma 3*, $\tilde{Q}_R^{\text{cir1}}$ can be achieved. Next, set $Q_R^{\text{cir1}} = \tilde{Q}_R^{\text{cir1}}$ and repeat the first two steps until a convergent solution is found. Then, the algorithm of the sub-optimal solution for the propulsion power

minimization problem can be given as follows.

<p>Algorithm: iteration algorithm for sub-optimal propulsion power minimization problem.</p> <p>Step 1: Initialize $Q_R^{\text{cir1}} = Q$, tolerance $\varepsilon > 0$.</p> <p>Step 2: Set the value of \tilde{n} according to <i>lemma 2</i>.</p> <p>Step 3: Solve problems (18) and (20) and then acquire \tilde{r}_1, \tilde{v}_1, \tilde{r}_2 and \tilde{v}_3.</p> <p>Step 4: Calculate $\tilde{Q}_R^{\text{cir1}}$ according (26), (27) and (23).</p> <p>Step 5: Calculate $\Delta Q = \tilde{Q}_R^{\text{cir1}} - Q_R^{\text{cir1}}$. If $\Delta Q > \varepsilon$, set $Q_R^{\text{cir1}} = \tilde{Q}_R^{\text{cir1}}$, and then go to Step 3; otherwise go to Step 6.</p> <p>Step 6: Output the sub-optimal radius \tilde{r}_1, velocity \tilde{v}_1 and duration \tilde{T}_1 in mode 1; output the sub-optimal velocity \tilde{v}_2 and duration \tilde{T}_2 in mode 2; output the sub-optimal radius \tilde{r}_2, velocity \tilde{v}_3 and duration \tilde{T}_3 in mode 3. Then calculate the total power consumption E according to (12a).</p>
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It is worth-mentioning that according to *lemma 2*, \tilde{n} has two possible solutions, i.e., $\left\lfloor \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rfloor$ and $\left\lceil \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rceil$.

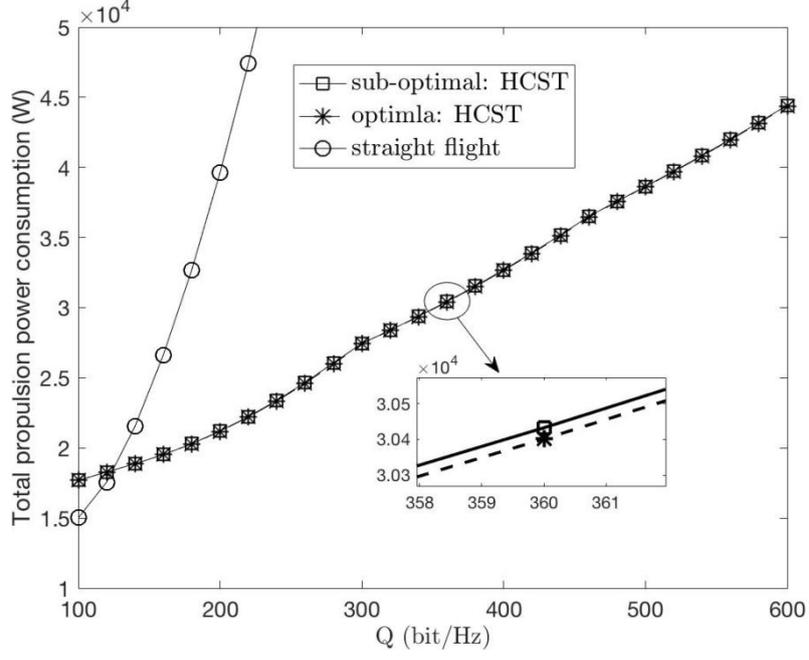
Therefore, two sets of solution could be achieved if substituting $\tilde{n} = \left\lfloor \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rfloor$ and $\tilde{n} = \left\lceil \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rceil$ into the iteration algorithm. Actually, the optimal solution of the algorithm is the one having the smaller power consumption.

5. Simulation Results and Discussions

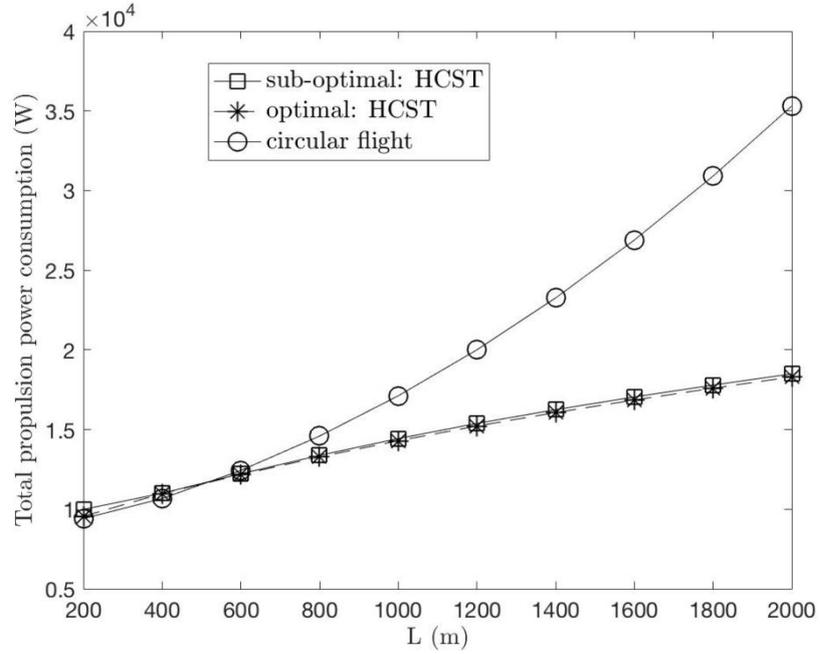
In this section, computer simulations are conducted to validate the proposed HCST design. In order to make comparisons, a straight flight and a circular flight trajectories are included. The UAV altitude is fixed at $H = 200$ m. The noise power is set to $\delta^2 = -110$ dBm and the reference channel power is set to $\beta_0 = -50$ dB. Furthermore, $c_1 = 9.26 \times 10^{-4}$, $c_2 = 2250$, $v_{\min} = 10$ m/s and $v_{\max} = 50$ m/s are assumed.

Fig. 2 makes the comparisons with the sub-optimal trajectory of HCS and optimal trajectory of HCS in terms of the circular radius and the speed of modes 1 and 3. The sub-optimal results are achieved by the iteration algorithm mentioned above. While a search algorithm is used to solve the problem (12) which leads to the optimal results. It is shown in Fig. 1 that the UAV's trajectory and total propulsion power consumption of sub-optimal and optimal are closed.

In Fig. 2, the distance between S and D is set to $L = 5000$ m, the transmit power of S and UAV relay is set to $P_S = P_R = 1$ W. As shown in Fig. 2 (a) and (b), the circular radius r_1 is monotonically increasing for $Q \in (200, 280)$ while the speed v_1 is monotonically decreasing for $Q \in (200, 280)$. That's because with the increasing of Q , the data bits Q_R^{cir1} transmitted from S node to UAV node must increase. When a larger Q_R^{cir1} data bits is need to be transmit from S node to the UAV, the circular radius r_1 and speed v_1 should be optimized jointly. Furthermore, a larger radius and lower speed lead to a smaller power consumption. Moreover, the value of circular radius r_1 and speed v_1 changed rapidly in $Q \in (280, 300)$. The reason is a larger number of lap in mode



(a)



(b)

Fig. 3 Total propulsion power consumption versus total data bits Q and the distance L

Figs. 3 make the comparisons with the HCS, straight flight and circular flight trajectory in terms of the total propulsion power consumption. It is shown in Fig. 3 that the HCS perform well in big data and long distance compared to straight and circular trajectory.

In Fig. 3 (a), the distance between S and D is set to $L = 5000\text{m}$, the transmit power of S and UAV relay is set to $P_s = P_r = 1\text{W}$. Fig. 3 (a) present the total propulsion power consumption of the HCS and straight flight trajectory, showing that the HCS can saving more energy compare to the straight flight trajectory when the total data bits Q is large. For instance, Fig. 3 (a) shows that the HCS can achieve about 18.4kW energy savings compared to the straight flight trajectory when $Q=200\text{bit}$. In Fig. 3 (b), the total data bits is set to $Q = 300\text{bit}$, the

transmit power of S and UAV relay is set to $P_s = P_r = 1W$. Fig. 3 (b) present the total propulsion power consumption of the HCS and circular flight trajectory, showing that the HCS can saving more energy compare to the circular flight trajectory when the distance L is large. For instance, Fig. 3 (b) shows that the HCS can achieve about 2.7kW energy savings compared to the circular flight trajectory when $L=1000m$.

6. Conclusions

This paper proposes a new HCS trajectory referring to the case of long distance and big data communication. And investigates the unmanned aerial vehicle's (UAV) propulsion power minimization problem subject to the total data bits constraint of a UAV-enabled full-duplex one-way mobile relaying system. Considering a sub-optimal problem, and the corresponding sub-optimal solutions are obtained by an iterative algorithm. The results show that the HCS performs better than straight or circular flight trajectory in terms of energy saving when long distance and big data communication is considered.

Abbreviations

DF: decode-and-forward; HCST: hybrid circular/straight trajectory; LoS: line-of-sight; KKT: Karush-Kuhn-Tucker; UAV: unmanned aerial vehicle;

Declarations

Availability of data and materials

Please contact the corresponding author for data requests.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

All the authors take part in the discussion of the work described in this paper. The author Xiaodong Ji wrote the first version of the paper. The author Tao Wang helped Xiaodong Ji revise the paper. The authors Chong Qian and Chenhao Huang did the experiments of the paper. All the four authors worked closely during the preparation and writing of the manuscript. All the authors read and approved the final manuscript.

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Appendix A

For problem (22), it is readily to find the total propulsion power consumption is monotonically decreasing at first, and then monotonically increasing for n . Firstly, we ignore the constraint that the UAV has to fly the whole laps in mode 1. Then, the sub-optimal trajectory of mode 1 can be derived by solving problem (28).

$$(\hat{r}_1, \hat{v}_1) = \arg \min_{r_1, v_1} \frac{\left(c_1 + \frac{c_2}{g^2 r_1^2} \right) v_1^3 + \frac{c_2}{v_1}}{\log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + r_1^2} \right)} \quad (28a)$$

$$\text{s.t.} \quad v_{\min} \leq v_1 \leq v_{\max}, \quad 0 < r_1 < L \quad (28b)$$

The minimum power on the numerator of (28a) can be achieved when (29) holds.

$$\hat{v}_1 = \begin{cases} v_{\min}, & \text{if } v_{\min} > \left(\frac{c_2}{3c_1 + \frac{3c_2}{g^2 r_1^2}} \right)^{\frac{1}{4}} \\ \left(\frac{c_2}{3c_1 + \frac{3c_2}{g^2 r_1^2}} \right)^{\frac{1}{4}}, & \text{if } v_{\min} \leq \left(\frac{c_2}{3c_1 + \frac{3c_2}{g^2 r_1^2}} \right)^{\frac{1}{4}} \leq v_{\max} \\ v_{\max}, & \text{if } v_{\max} < \left(\frac{c_2}{3c_1 + \frac{3c_2}{g^2 r_1^2}} \right)^{\frac{1}{4}} \end{cases} \quad (29)$$

Substituting (29) into (28) and then the optimal solution of (28) can be obtained by using one-dimensional search over $0 < r_1 < L$. Then, problem (22) can be written as a convex optimization problem as follows.

$$(\hat{v}_2, \hat{T}_1, \hat{T}_3) = \arg \min_{v_2, \tilde{r}_1, \tilde{T}_3} T_1 \left[\left(c_1 + \frac{c_2}{g^2 \hat{r}_1^2} \right) \hat{v}_1^2 + \frac{c_2}{\hat{v}_1^2} \right] + \frac{L - \hat{r}_1 - \tilde{r}_2}{v_2} \left(c_1 v_2^3 + \frac{c_2}{v_2} \right) + T_3 \left[\left(c_1 + \frac{c_2}{g^2 \tilde{r}_2^2} \right) \tilde{v}_3^2 + \frac{c_2}{\tilde{v}_3^2} \right] \quad (30a)$$

$$\text{s.t.} \quad V_{\min} \leq v_2 \leq V_{\max} \quad (30b)$$

$$T_1 \geq 0, T_3 \geq 0 \quad (30c)$$

$$Q_{\text{R}}^{\text{cir1}} + Q_{\text{R}}^{\text{str}} + B_1(\tilde{r}_2)T_3 + B_2(\tilde{r}_2, \tilde{v}_3) \geq Q \quad (30d)$$

$$B_3(\tilde{r}_1)T_1 + B_4(\tilde{r}_1, \tilde{v}_1) + Q_{\text{D}}^{\text{str}} + Q_{\text{D}}^{\text{cir2}} = Q \quad (30e)$$

The solution of problem (30) can be obtained by solving the KKT equations, and the optimal value of \tilde{n} can be given as

$$\tilde{n} = \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \quad (31)$$

$$\hat{T}_1 = \left\{ -192 \times 2^{1/3} a_2^2 (a_4 a_5 - a_6 b_1) (a_3 a_5 - b_1 b_3)^2 c_1 c_2 \sqrt{f_1} (\sqrt{f_1} + v_2) + 2(a_4 a_5 - a_6 b_1) k_1 (\sqrt{f_1} + v_2) \right. \\ \left. \times \left[-4(a_9 a_3 a_6 + a_1 a_4 a_5 - a_1 a_6 b_1 - a_9 a_4 b_3) + 2^{2/3} \sqrt{f_1} k_1 \right] + 8a_2 (a_3 a_5 - b_1 b_3) \sqrt{f_1} k_1 [4a_3 a_7 c_2 + a_4 a_5 c_1 f_1 (\sqrt{f_1} + v_2) \right. \right. \\ \left. \left. - 4b_1 a_8 c_2 - a_6 b_1 c_1 f_1 (\sqrt{f_1} + v_2) \right] \right\} / \left[32a_2 (a_3 a_5 - b_1 b_3)^2 c_2 \sqrt{f_1} k_1 \right]$$

,where

$$\hat{v}_2 = -1/2 \times \sqrt{f_1} + 1/2 \times \sqrt{(a_1 a_4 a_5 + a_9 a_3 a_6 - a_1 a_6 b_1 - a_9 a_4 b_3) / [a_2 (a_3 a_5 - b_1 b_3) c_1 \sqrt{f_1}] - f_1},$$

$$a_1 = \left(c_1 + \frac{c_2}{g^2 \hat{r}_1^2} \right) \hat{v}_1^2 + \frac{c_2}{\hat{v}_1^2},$$

$$a_2 = L - \hat{r}_1 - \tilde{r}_2,$$

$$a_3 = \log_2 \left(1 + \frac{P_s \gamma_0}{H^2 + \hat{r}_1^2} \right),$$

$$a_4 = \frac{F_1(L - \tilde{r}_2) - F_1(\hat{r}_1)}{\ln 2},$$

$$a_5 = \log_2 \left(1 + \frac{P_R \gamma_0}{H^2 + \tilde{r}_2^2} \right),$$

$$a_6 = \frac{F_2(L - \hat{r}_1) - F_2(\tilde{r}_2)}{\ln 2},$$

$$a_7 = Q - b_2,$$

$$a_8 = Q - b_4,$$

$$a_9 = \left(c_1 + \frac{c_2}{g^2 \tilde{r}_2^2} \right) \tilde{v}_3^2 + \frac{c_2}{\tilde{v}_3^2}.$$

Recall that the UAV has to fly the whole laps in mode 1, namely, \tilde{n} should be an integer, the optimal laps in

$$\text{mode} \text{a shall be } \left\lfloor \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rfloor \text{ or } \left\lceil \frac{\hat{T}_1 \hat{v}_1}{2\pi \hat{r}_1} \right\rceil.$$

Figure legends

Fig. 1 UAV-enabled mobile relaying system with hybrid circular/straight trajectory

Fig. 2 Trajectory versus total data bits Q

Fig. 3 Total propulsion power consumption versus total data bits Q and the distance L