

# A Mathematical Modeling Framework To Detect The Optimal Financial Turning Points

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## Research

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# Abstract

The financial markets have always witnessed the competition of their participants for gaining high and stable profits. The realization extent of this goal depends on the profitability of the trading points or turning points (TPs) ahead. TPs prediction problem is one of the most challenging yet important problems in the financial discipline. The first step towards predicting financial TPs is to detect TPs from the history of the corresponding financial time series. Literature indicates that the profitability of the predicted financial TPs depends on the profitability of the detected TPs. Given this, numerous efforts have been devoted to enhancing the profitability of the detected financial TPs. Nevertheless, to the best of our knowledge, none of the existing detection methods can detect the most profitable or the optimal TPs from the history of financial time series. The present study concerns this research gap and ensures detecting the optimal financial TPs by proposing a mathematical modeling framework. The proposed optimal TPs detection model in this paper will be structured concerning the three following assumptions. First, short-selling the financial asset is possible. Second, the time value for the investment money is not considered. Third, detecting consecutive buying TPs and consecutive selling TPs is not allowed. Empirical results with twenty real data sets indicate that the proposed model, in contrast to the existing TPs detection methods, detects the optimal TPs from the history of the financial time series.

## Introduction

The financial market is a complex system driven by fundamental, financial, economic, political, and human agents. Given this, it has always been challenging for the financial market investors and traders to identify the right times to trade, i.e., profitable TPs. Predicting financial asset's TPs, which is known as a means to realize the above-mentioned goal, is one of the most challenging problems in financial discipline. Because the financial time series are characterized by features such as noise, non-stationarity, unstructured nature, high degree of uncertainty, and hidden relationships [1]. One of the other factors that make it quite difficult to predict financial TPs is that the final goal of this prediction isn't to minimize the prediction error but to maximize the profitability [2, 3]. Despite the difficulty of the TPs prediction problem, this issue is of undeniable importance to academic researchers and industrial practitioners. Shih, Shih *et al.* [4] declare that most recent research on predicting asset price has only been concerned with the prediction of price; yet the problem of predicting TPs is more practical than price prediction. Besides, the corresponding literature indicates that investors are more concerned with the prediction of TPs than that of the prices ahead [5].

TPs prediction subject is not only well known within the financial discipline [6] but also has applications in other fields. The TPs-based policies result in stabilizing effects for business cycles. While ill-timed fiscal and monetary policies may result in some unintended destabilizing ones [7]. TPs in tourism demand will occur when growth rates move from an expansion period to a recession or vice versa. During the positive growth period, resources are in high demand. While in the negative growth period, resources are in low demand. Such a change in demand for resources requires developing an appropriate risk management strategy in tourism destinations. Also, the government and various tourism industry sectors

need prior knowledge of TPs in tourism demand for investment and planning [8]. Concerning the housing market, prices have rebounded rapidly in the low-interest-rate environment in some countries. The issue of whether prices are now close to TP is of considerable policy interest [9]. Inflation targeting is an important part of economic policy. Regarding the monetary targeting experiments, the UK government reformulated the monetary policy's goal in terms of a direct target for TPs in the inflation cycle [10].

In the financial discipline, the first step towards predicting TPs is to detect TPs from the history of time series. Thereafter, the detected financial TPs can be used to train the classifier of the TPs prediction model and predict financial TPs. Accordingly, and as shown in Fig. 1, it can be concluded that the two problems of financial TPs detection and prediction are not the same. The problem addressed by the current paper is the financial TPs detection problem, not the TPs prediction problem. The remainder of the article is organized as follows. Section 2 provides a review of the literature relevant to this study. Our proposed thematical model for optimizing the financial TPs detection process is given in Sect. 3. After that, Sect. 4 introduces the data sets and the performance indicator for evaluating and confirming the proposed method's performance in the problem of detecting financial TPs. Finally, the conclusions will be presented in Sect. 5.

## Literature Review

On the undeniable importance of the financial TPs detection problem in the process of predicting TPs, it can be said that a great part of the profitability of the predicted TPs will be provided through the profitability of the detected TPs [11]. Besides, Tang *et al.* [3] indicate that under a constant TPs prediction model, enhancing the profitability of the detected TPs leads to an increment to the profitability of the predicted TPs. Notably, the two terms of the profitability of the detected TPs and the profitability of the TPs detection method are equal. In the literature of financial TPs, the performance indicator for evaluating a TPs detection method is the profitability of TPs detected by that method. Owing to the direct relationship between the profitability of the detected TPs and that of the predicted TPs, never efforts into ways of improving the performance of financial TPs detection methods have been given up. Generally speaking, these efforts can be categorized as time series smoothing methods and time series segmentation-based methods.

The first category of financial TPs detection methods comprises the time series smoothing methods. Smoothing methods are of high popularity within the financial discipline; since they discover financial time series' trends by smoothing the prices, and investors can earn profit through these discovered trends. These methods believe that shifts in the existing trends of the financial time series can be used as an indicator to detect TPs. Among all the smoothing methods, moving average (MA) is considered the simplest and most useful one [12]. In this regard, Brock *et al.* [13] applied the MA-based oscillator to detect the TPs from the history of the Dow Jones Industrial Average (DJIA) time series. According to the MA rule, buy and sell TPs will be detected by two long-period and short-period MAs. This strategy, in its simplest form, is expressed as buying (or selling) when the short-period MA rises above (or falls below)

the long-period MA. The rationale behind this idea is that when the short-period MA penetrates the long-period one, a trend is considered to be initiated.

Despite the simplicity and usefulness of MA-based TPs detection methods, exponential moving average (EMA) is considered more adaptive because it puts more weight on recent prices while putting fewer weights on earlier days [12, 14]. In this regard, Grillenzoni [15] compared three methods of double exponential smoothing, time-varying parameters, and prediction errors statistics, for detecting TPs in stock values and deciding the time to trade and reported satisfactory performances of these methods in the process of detecting financial TPs. Thereafter, Grillenzoni [6] assessed the ability of various exponential smoothers to detect TPs of financial time series. His novel idea was to select smoothing and alarm coefficients by maximizing the profit computed on the historical data. He believed that due to the occurrence of maximum gain in correspondence with actual TPs, it follows unbiased detection of this method. Empirical findings with S&P 500, indicate the nearly equivalent performance of different smoothing coefficients.

The first category of financial TPs detection methods has the following deficiencies. Smoothing methods tend to identify TPs based on trend-cycle components estimated on the entire data set [15]. This feature involves the problems of detecting profitable financial TPs [16]. Although Dash and Dash [17] believe that since these methods are exclusively produced based on historical stock data, detecting financial TPs through these types of methods, may not always be profitable.

The second category of financial TPs detection methods includes the time series segmentation-based methods. Piecewise linear representation (PLR)-based TPs detection methods, as the representatives of the second category, are one of the most widely used and well-studied TPs detection methods. Literature indicates that PLR is one of the most successful TPs detection methods existing in the corresponding literature [18]. The PLR can be used as a sliding window to detect the financial time series TPs by representing trends. This method determines the sliding window's size and labels the local maximum or minimum data points within the time window as TPs. A larger PLR's sliding window will create long trend patterns; while the patterns will be sensitive when the sliding window is very small. Notably, the profitability of the detected TPs by PLR depends on the sliding window, and PLR itself doesn't have the feature of selecting the sliding window automatically [19]. Hence, the corresponding literature has witnessed efforts towards enhancing the profitability of the detected TPs, by setting the appropriate sliding window.

The research on selecting the sliding window began by Chang *et al.* [20] in which they used a random sliding window for PLR. Thereafter and in line with improving the performance of PLR-based TPs detection methods, Chang *et al.* [21] and Lin *et al.* [22] adapted a genetic algorithm to fine-tune the value of window size for PLR. Numerical findings reported in these studies indicate the outperformance of GA-PLR in comparison to PLR with random sliding window and Buy-Hold Strategy. Luo *et al.* [23] studied the relationship between the profitability of the detected TPs and the sliding window and thereafter used the percentage of TPs in the historical data as the PLR's sliding window. The idea of this study was inspired

by the fact that it would be more reasonable to specify different sliding windows for different financial assets or different price fluctuations. Although this method outperforms the previous TPs detection methods, determining the optimal percentage of the occurred TPs is still an open issue. Tang *et al.* [19] proposed a new dynamic sliding window setting algorithm to improve the performance of PLR. This algorithm dynamically sets the sliding window for different financial time series, different periods, and different price fluctuations. Tang *et al.* [3] presented a fitness function to overcome the problem of setting the same threshold for different price fluctuations. The proposed fitness function can automatically select the PLR's window size, by maximizing the profit of trading in TPs generated by the PLR.

There are some drawbacks regarding the best detection approach existing in the literature (i.e., PLR), which make this method unable to detect the most profitable TPs or the optimal TPs from the history of financial time series. The profitability of the detected TPs by PLR depends on the sliding window, and PLR itself doesn't have the feature of selecting the sliding window automatically [19]. Besides, Chang *et al.* [21] believe that if the window size is not properly chosen, the sub-segments generated by the PLR may lead to the wrong trading decisions [21].

Despite all the efforts devoted to enhancing the performance of the TPs detection methods, to the best of our knowledge, the corresponding literature still lacks a method for detecting the optimal TPs. This paper aims to address the above-mentioned research gap by implementing the financial TPs detection problem in the context of a mathematical modeling framework. In contrast to the literature's TPs detection methods, the proposed financial TPs detection method in this paper, due to its essence, can detect the optimal TPs from the history of the financial time series.

## Research Method

The financial markets have always witnessed the competition of their participants for more profit. In such environments, gaining high and stable profits requires adopting profitable TPs ahead, as the trading strategy. To take the first step towards predicting financial TPs, the TPs existing in the history of financial time series should be detected. Given the direct relationship between the profitability of the predicted TPs and the profitability of the detected TPs, never research into ways of improving the TPs detection methods has been stopped. Nonetheless, to the best of our knowledge, the ever-existing detection methods can't detect the optimal TPs. To address this research gap, we propose a mathematical modeling framework characterized by detecting the optimal TPs from the history of the financial time series. The assumptions considered for modeling and solving the corresponding detection problems are as follows. First; short-selling the financial asset is possible. Second; time value for the investment money isn't considered. Third; it is impossible to detect consecutive buying TPs and consecutive selling TPs. To present the proposed mathematical modeling framework, let's first define the following.

**Definition 1.** As represented in Fig. 2, the breakpoints (BPs) existing in the history of financial time series are featured by disturbing the existing trends in one of the four following ways: up-trend to down-trend, up-trend to steady-trend, down-trend to up-trend, and down-trend to steady-trend. Excluding the BPs, the

remained data points will be named ordinary points (OPs). Accordingly, it can be concluded that in contrast to the BPs, the OPs lack trading value. Hence to achieve the optimal TPs, the BPs set will be used as the input of the proposed detection model.

**Definition 2.** The trading strategy, i.e., the trading system will be constructed from the sequence of the BPs. The distinct sequences of the existing BPs result in different trading strategies.

**Definition 3.** Let's define  $F = \{p_{f1}, p_{f2}, \dots, p_{fm}\}$  as the BPs set, i.e., the feasible TPs set; where  $p_{fi}$ ;  $i = 1, 2, \dots, m$  indicates the financial asset's price in the  $i^{\text{th}}$  BP and  $m$  represents the number of BPs existing in the history of the corresponding financial time series.

**Definition 4.** Let's define  $R_{f_i f_j}^q$  as the maximum profit obtained from trading in the  $(f_i, f_j)$  pair of BPs, using exactly intermediate BP(s) ( $i = 1, 2, \dots, m - 1, j = i + 1, \dots, m, q = 0, 1, \dots, m - 2$ ). To better understanding, Fig. 3, illustrates the  $(f_i, f_j)$  pairs of BPs using zero, one, and two intermediate BPs.

**Definition 5.** Let's define  $R$  as the maximum profit obtained from adopting the entire trading strategies existing in the history of the corresponding financial time series.

Considering the above assumptions and definitions, the proposed framework will find the optimal TPs set through the following three stages. First, the proposed model compares the profitabilities of all trading strategies. Second, the strategy with the maximum profit will be reported as the optimal trading system. Third, the entire TPs existing in the optimal trading strategy will be traced and thereafter reported as the optimal TPs existing in the history of the corresponding financial time series.

To solve the optimal TPs detection problem, Eq. (1) compares the entire trading strategies existing in the history of financial time series.

$$R = \max \left\{ R_{f_i f_j}^0, \max_{\substack{1 \leq q \leq j-i-1 \\ q+2 \leq k \leq j-1}} \left\{ R_{f_i f_k}^{q-1} + R_{f_k f_j}^0 \right\} \right\}; \quad i = 1, 2, \dots, m-1, \quad j = i+1, \dots, m \quad (1)$$

This comparison will be realized considering equations (2), (3), and (4). Through this context, the profits of the entire existing trading strategies will be compared and thereafter the optimal trading strategy will be identified. Then, the optimal trading strategy should be tracked to find its constructive BPs; which are in other terms, the optimal TPs. Figure 4, by illustrating the entire trading strategies existing in the history of time series, helps to better understand the process of the proposed mathematical modeling framework. Notably, Eq. (2) maximizes the profits of buying and short-selling the financial asset, considering the entire pairs of BPs. Since the time value of the investment money is not considered here, the collateral of the short-selling position won't be included in the calculations of Eq. (2). The parameters of  $TC_b$  and  $TC_s$ , used in Eq. (2), are respectively the financial asset's buying and selling transaction costs (TCs).

$$R_{f_i f_j}^0 = \max \left\{ \begin{array}{l} (1-TC_s)p_{f_j} - (1+TC_b)p_{f_i} \\ (1-TC_s)p_{f_i} - (1+TC_b)p_{f_j} \end{array} \right\}; \quad i=1,2,\dots,m-1, \quad j=i+1,\dots,m \quad (2)$$

$$R_{f_i f_j}^q = \left\{ R_{f_i f_k}^{q'} + R_{f_k f_j}^0 \right\}, \quad i < k < j \quad (3)$$

$$q' = \begin{cases} q-1 & \text{if } k-i \geq q \\ \text{Not counting} & \text{o.w.} \end{cases} \quad (4)$$

## Numerical Results

In this section, the proposed TPs detection model is applied to twenty real financial time series, to demonstrate its efficiency and performance in the problem of TPs detection. Thereafter, the proposed detection model's performance will be compared with that of other financial TPs detection methods. Finally, the sensitivity of the applied TPs detection methods to TCs will be analyzed.

### 4.1. Application of the proposed model to optimal turning points (TPs) detection

The twenty data sets chosen for demonstrating the efficiency and performance of the proposed model, in the face of financial TPs detection problem, are recognized by the following codes: 600736, 600197, 600211, 600694, 600351, 600488, 600054, 600019, 600058, 600682, 600597, 600066, 600881, 600228, 600697, 600107, 600053, 600051, 600163, and 600167. These financial assets have been selected from the Shanghai Stock Exchange in China and are the same data sets as those addressed by Tang *et al.* [3]. The time horizon considered for solving the TPs detection problem is from September 1, 2019, to April 1, 2021. During this horizon, the introduced financial assets experience 382 trading days. As mentioned earlier, the proposed TPs detection model can detect the optimal TPs set existing in the history of financial time series. The results obtained from applying the proposed TPs detection model to the corresponding data sets have been plotted in Fig. 5. Notably, the obtained plot for 600197 is brought here and the plots of the rest of the data sets have been reported in the appendix. In this figure, the red and blue marks, respectively indicate the optimal buying TPs and the optimal selling TPs, existing in the history of the corresponding financial time series.

### 4.2. Comparative assessment

In this section, the performance of the proposed TPs detection model will be compared with that of other TPs detection methods existing in the corresponding literature. As mentioned before, the term of performance, regarding financial TPs detection methods, implies the profitability of the detected TPs by the applied detection method. This section considers the annual rate of return (AROR) as the performance indicator. This indicator evaluates the profitability of the detected TPs from the history of financial time

series. The TPs detection methods used for comparing with the proposed method will be selected considering the TPs detection methods categorization, conducted by the current paper. Accordingly, the TPs detection methods presented by Tang *et al.* [3], and Luo *et al.* [23] belong to the category of time series segmentation-based methods, and the method offered by Zhu and Wang [12] is one of the time series smoothing-based TPs detection methods. Notably, the methods of Tang *et al.* [3], and Luo *et al.* [23] are structured based on PLR; which is known as the best detection approach in the literature. All these methods have been implemented under the same assumptions and conditions as the proposed method. It's worth noting that these studies address both problems of the TPs detection and prediction; yet since the current paper merely addresses the TPs detection problem, only the TPs detection methods of these studies will be considered to compare with the proposed model.

Table 1 reports the AROR amounts obtained from trading in TPs detected by the applied detection methods. According to the obtained results (Table 1), it can be realized that our proposed TPs detection model has the best performance in comparison with other detection methods. Besides, given the optimality of the proposed model's structure, it has the best performance in comparison with the entire TPs detection methods existing in the literature. As mentioned earlier, the claim expressing that none of the existing detection methods can detect the optimal TPs arises from a complete review of the literature by the researchers. Numerical results reported in Table 1, indicate that the performances of the literature's TPs detection methods are not optimal and these methods, due to their essence can't detect the optimal trading system existing in the history of time series. As mentioned before, according to Dash and Dash [17], the application of time series smoothing detection methods, may not always be profitable. The TPs detection method proposed by Zhu and Wang [12] which belongs to this category of TPs detection method, has the mentioned disadvantage; this disadvantage is also noticeable from this method's poor performance reported in Table 1. According to Table 1, the average amounts of AROR, obtained by Zhu and Wang [12], Tang *et al.* [3], Luo *et al.* [23], and our proposed model are respectively equal to -11.2231, 353.7912, 398.1214, and 470.3006.

**Table (1):** Comparison results of the performance of the TPs detection methods.

The financial asset	Our proposed model	Tang <i>et al.</i> [3]	Luo <i>et al.</i> [23]	Zhu and Wang [12]
600736	342.0411	224.2038	284.0687	-50.5108
600197	646.3684	512.6795	570.0529	-2.7654
600211	904.2170	702.7002	729.7811	205.2740
600694	278.6416	227.3493	206.8293	8.7571
600351	403.6573	297.9638	324.1591	4.0382
600488	370.7245	253.9410	303.6071	-70.4748
600054	303.2182	224.3793	254.3489	-48.8989
600019	391.9104	248.6832	321.0309	-23.5792
600058	350.6969	262.9485	282.6061	-79.1884
600682	580.4696	451.2373	512.8873	-13.1626
600597	535.7783	409.8536	471.0560	37.2391
600066	523.9291	417.3439	454.5738	-50.0897
600881	269.1414	158.6351	222.7265	-49.0375
600228	612.4444	501.3452	533.1490	18.2010
600697	259.1431	201.9443	196.6563	-16.1070
600107	697.9683	536.6281	607.4771	-26.3360
600053	623.1925	506.4608	557.3941	-37.6711
600051	484.2724	337.9676	410.4888	52.0791
600163	450.9516	309.7698	393.4788	-40.6744
600167	377.2453	289.7904	326.0567	-41.5541
<b>Average</b>	<b>470.3006</b>	<b>353.7912</b>	<b>398.1214</b>	<b>-11.2231</b>

Table 2 reports the numerical results of pairwise comparison of the TPs detection methods, obtained for the entire data sets. The number of improvement(s) indicates the number of the financial assets for which one TPs detection method outperforms the other detection method. Our proposed model is characterized by the entire number of improvements in its row, equal to the number of all the data sets. It implies that the proposed detection model has the best performance in comparison with other detection methods. Quite the opposite of the proposed model, Zhu, and Wang [16], which is featured with all the number of improvements in its row, equal to zero, has the worst performance compared to the other TPs detection methods.

Table (2): Pairwise comparison results for TPs detection methods.

Detection method		Our proposed model	Tang <i>et al.</i> [3]	Luo <i>et al.</i> [23]	Zhu and Wang [12]
Detection method					
Our proposed model	Number of improvement(s)	-	20	20	20
	Average improvement	-	116.5093%	72.1791%	481.5236
Tang <i>et al.</i> [3]	Number of improvement(s)	0	-	2	20
	Average improvement	-	-	54.2455%	365.0143%
Luo <i>et al.</i> [23]	Number of improvement(s)	0	18	-	20
	Average improvement	-	50.6895%	-	409.3445%
Zhu and Wang [12]	Number of improvement(s)	0	0	0	-
	Average improvement	-	-	-	-

### 4.3. Sensitivity analysis to transaction costs (TCs)

The entire numerical results reported up to this section, have been obtained under constant buying and selling TCs. This section analyzes the performance of the detection methods in the face of changing TCs. In this regard, Table 3 reports the performance indicator of TPs detection methods, obtained under different buying and selling TCs (for the case of 600107). According to the results reported in Table 3, it can be concluded that although the method proposed by Luo *et al.* [23] performs better than Tang *et al.* [3] at low TCs, it loses its efficiency as buying and selling TCs increase. This happens in such a way that for high TCs, the performance of Tang *et al.* [3] surpass Luo *et al.* [23], yet can't achieve the optimal performance (which belongs to our proposed model). Additionally, it can be concluded that Luo *et al.* [23] have the highest level of sensitivity to TCs changes. This finding is observable from this method's variation range in the face of different TCs, which is the widest among the applied methods. In contrast to Luo *et al.* [23], the method of Tang *et al.* [3] is featured with the least variation range in the face of changing TCs. Our proposed model not only has the best possible performance in the problem of

detecting financial TPs but also maintains this advantage under any circumstances, including changing TCs.

Table (3): Sensitivity of the detection method to TCs, analyzed for 600107.

The buying and selling TCs	Our proposed model	Tang <i>et al.</i> [3]	Luo <i>et al.</i> [23]	Zhu and Wang [12]
$TC'_b = 0; TC'_s = 0$	815.3245	616.1274	749.5978	-13.3023
$TC'_b = 0.25 \times TC_b; TC'_s = 0.25 \times TC_s$	786.5651	596.5097	714.1792	-16.5116
$TC'_b = 4 \times TC_b; TC'_s = 0.25 \times TC_s$	757.7444	580.3093	683.7899	-19.5331
$TC'_b = TC_b; TC'_s = TC_s$	697.9683	536.6281	607.4771	-26.3360
$TC'_b = 0.25 \times TC_b; TC'_s = 4 \times TC_s$	496.9252	381.2706	203.0099	-63.1131
$TC'_b = 4 \times TC_b; TC'_s = 4 \times TC_s$	487.0184	371.8860	171.1360	-66.3127

## Conclusions

Analyzing financial markets has always been a challenging field due to human decision-making. Investment decisions taken in this environment may increase or decrease the return on investment. The trading points, i.e., TPs of the financial asset, play a critical role in changing the amount of investment. Prediction of financial TPs is known as means for achieving the occurrence time of TPs ahead. The first step towards predicting financial TPs is to detect TPs from the history of the corresponding financial time series. According to the literature, the profitability of the predicted TPs depends on the profitability of the detected TPs. Given this relationship, never research into ways of enhancing the profitability of TPs detection methods has been given up. Nonetheless, to the best of our knowledge, none of the existing methods can detect the optimal TPs. To address this research gap, this paper proposes a mathematical modeling framework characterized by detecting the optimal TPs from the history of the financial time series. Notably, our mathematical modeling framework should consider three assumptions in its process. These assumptions include the possibility of short-selling the financial asset, considering no time value for the investment money, and the prohibition of detecting consecutive buying TPs and consecutive selling TPs. The numerical results obtained from applying our proposed detection model to real data sets and comparing its performance with that of other TPs detection methods, verify the efficiency of the proposed model in the problem of detecting financial TPs.

## Declarations

### Availability of data and materials

The data that support the findings of this study are available upon request from the corresponding author.

### Conflict of interests

The authors declare that they have no competing interests.

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## Authors' contributions

All authors have equally contributed to this work and approve of this submits.

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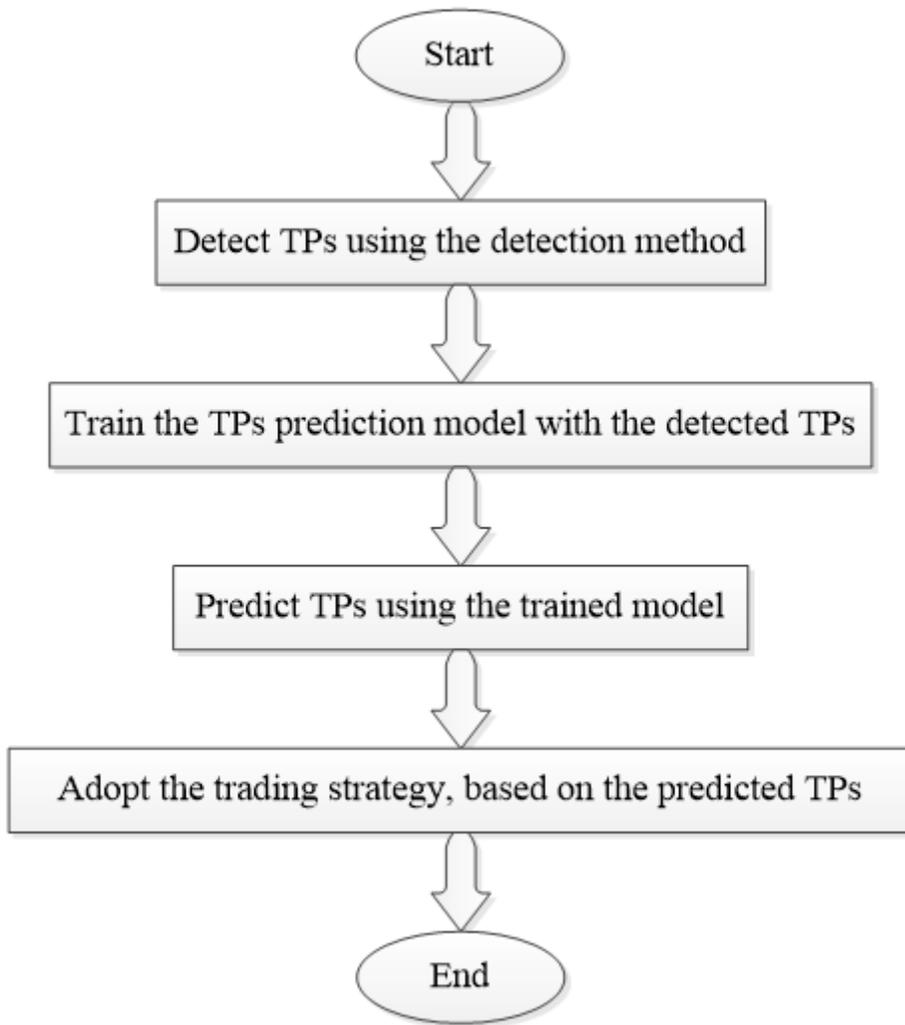
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## Figures



**Figure 1**

Relation of the TPs detection problem and the TPs prediction problem.

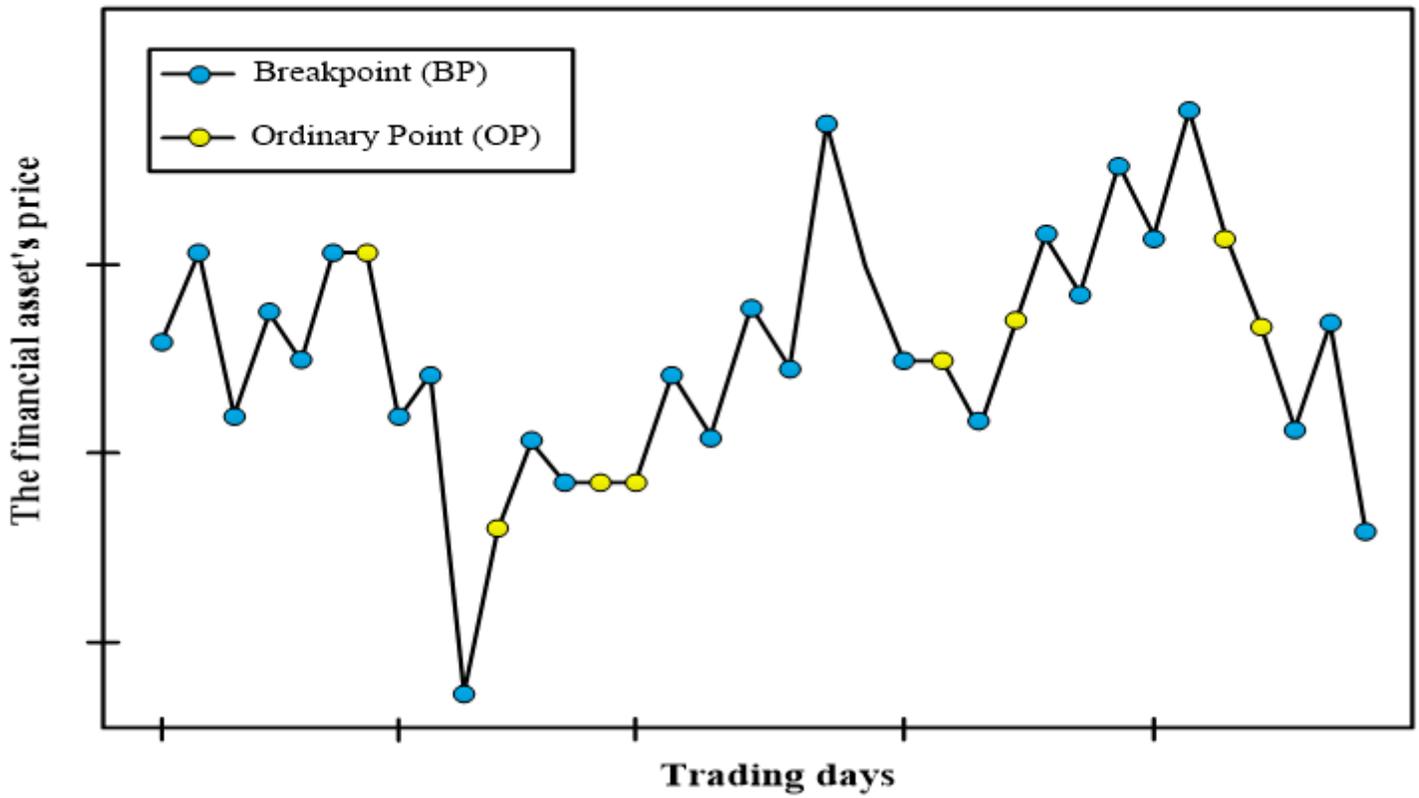


Figure 2

Representation of the BPs and the OPs, existing in the history of financial time series.

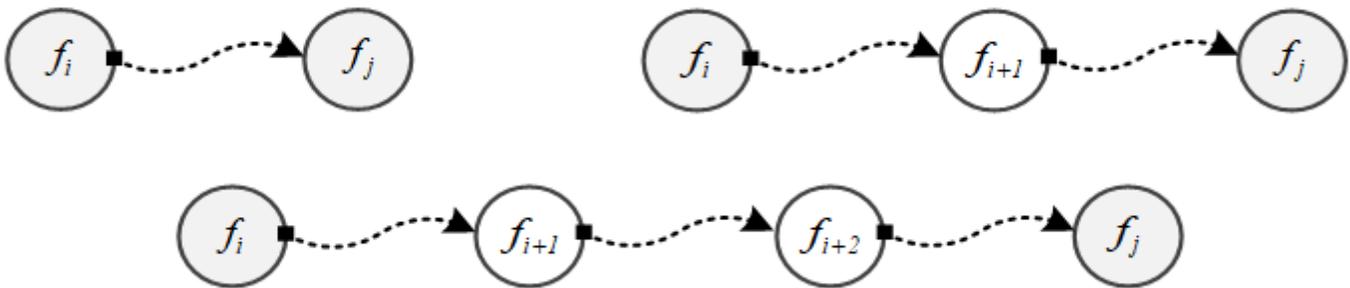


Figure 3

Examples of pairs of BPs, using the different number of intermediate BPs.

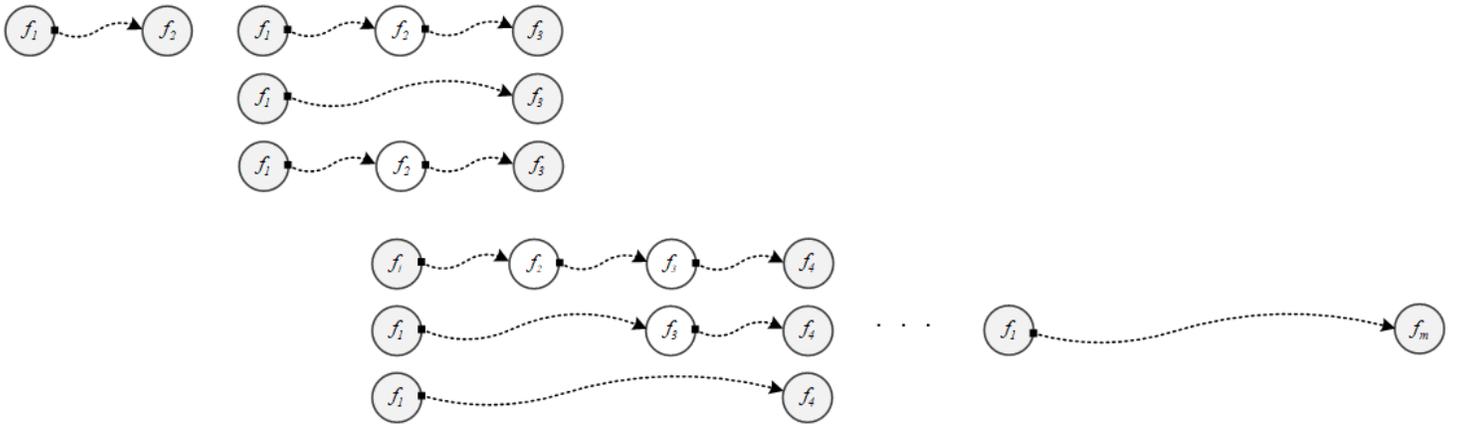


Figure 4

Representation of the trading strategies, existing in the history of financial time series.

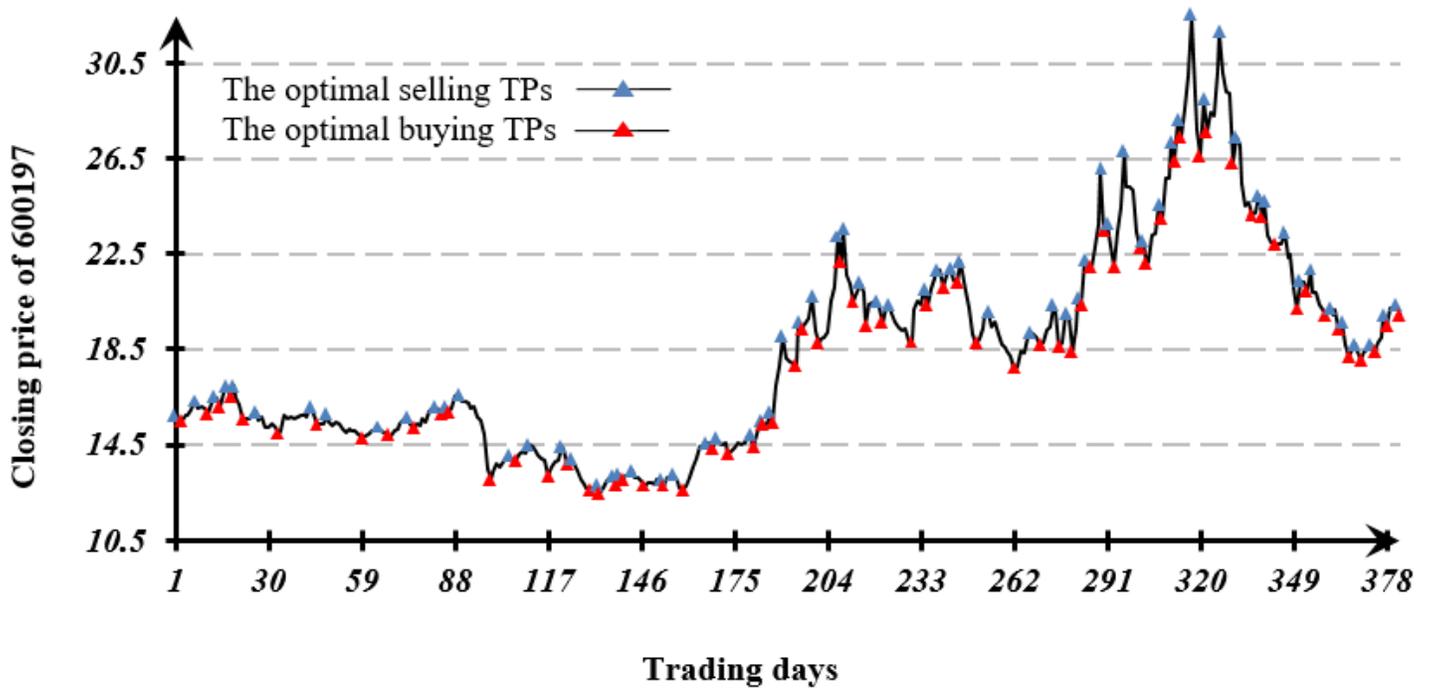


Figure 5

The optimal TPs detected by the proposed model.

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Appendix.docx](#)