Modeling the electron orbital in transition subatomic structure

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Modeling the electron orbital in transition subatomic structure

The task "on infinity in extremum"

Mikhail K. Ivantsov

Abstract As part of the well-known task about a motion of charged particle in central forces field, a certain parallelism for electronic distribution between the atomic and subatomic "orbits" can be established. In this conjuncture the ground state of muonium atom as in transition electron-nuclear structure is highlighted. Moreover, there is specifically nuclear solution of fine-structure constant which with a hyper-fine structure, like of the Lamb shift of hydrogen atom, is unambiguously associated. Such a special approach, in the terms of electric interaction, may serve as an extension to the existing meson-boson classification. In particular, some idea about a versatility of the Higgs mechanism in nuclear reactions put forward for consideration here. But it would be just spatial abstraction, where subatomic matter expands as into infinity. And what would be beyond the edge of the universe?

Keywords wave function · stationary Schrödinger equation · Einstein equation

1 Introduction

In the context of quantum-mechanical task, an ambiguous situation of the current probabilistic event for electron may take place, when charged particle moves in empty spatial region.
Indeed, at limiting the electromagnetic interaction velocity, an instantaneous reinstallation of electric field in the whole space at infinity would be required. Essentially, an electron is manifested before itself in own field, like if in a future (a nonsense).
Whereby an apparent paradox of the mechanistic electron, which exists independently against a purely electric phenomenon, may arise.
Such an indistinguishably of electric charge is illustrated in Fig. 1 - where restoration of the orbital electron on a charged nucleus would be.

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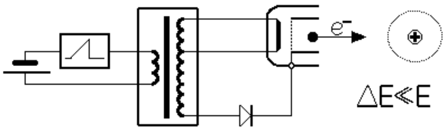

Fig. 1 This scheme shows the path of an electron which leaves the initial source in closed electric circuit, to go into open space through the electromagnetic field (of the transformer). The question is what is the electric current mean if the electron already falls out of the closed circuit? It turns out that a fantastic "device" of the electromagnetic transformation exists, as something source of mass-charge quantities, where a transportation for an electron occurs, ostensibly possessing mass $\mathrm{E} = \mathrm{mc}^2$

Therefore, special formulation of wave function is further reviewed, which implies the charge singularity in space at infinity.
It is one could suggest applied task from the stationary Schrödinger equation, where own electron value in accordance to Einstein's formula will be investigated $\mathrm{E} = \mathrm{mc}^2$
At a given reasoning, so called "electromagnetic interpretation" of wave function may be mentioned $\mathrm{e} \in |\psi|^2$ - the distribution of elementary charge in Coulomb field of nucleus (in nowadays unacknowledged Schrödinger work [1]).
In this presupposition the known stationary Schrödinger equation is considered

$$-\frac{\hbar^2}{2\mathrm{m}}\,\nabla^2\psi + V\,\psi = \mathcal{E}\,\psi$$

There is the postulated Coulomb interaction $V(r) \sim \mathsf{e}^2/r$ - from where the electronic levels there are $\mathcal{E} = \mathcal{E}(n)$ - in system of principal quantum numbers $n = 1, 2, \ldots$

Let given to the quantum-mechanical equation, provided the constant own value:

$$-\mathrm{E}\,\mathrm{r}^2\,\nabla^2\psi + V\,\psi = \mathrm{E}\,\psi \tag{1}$$

Here, respectively, electron value is redefined $\mathcal{E}(n) \in \mathrm{E}$ - under the symbolic Bohr radius $\mathrm{r} = r_0$ (Note below 1.2).

Then not postulated Coulomb's potential value can be derived (in $V(r) \in \mathrm{E}$)

$$V(r) \sim \mathrm{n}\,\frac{\mathsf{e}^2}{r} \tag{2}$$

- where corresponding charge coefficient from principal quantum numbers follows $\mathrm{n} = 1, 2, \ldots$

Formally, for the stationary Schrödinger equation, the orthogonality condition is not performed, since an applied dependency there is in form $\mathcal{E}(n) \in V$
The orthogonal solution should not be dependent on own numbers $\psi \neq \psi(n)$ - that only for constant value is defined $\mathrm{E} = const$ - in Eq. (1) (as per Note 1.1).

Hereinafter provides notes:

1.1 Note

There is orthogonally compatible solution of radial part in Eq. (1)

$$\psi(r) = Q_n^l(r) \cdot \exp(-\tfrac{1}{2}\, r)$$

- which recorded through the generalized orthogonal Laguerre polynomials

$$Q_n^l(r) = r^l\, L_{n+l}^{2l+1}(r)$$

There is a quadratically integrable expression

$$\int_0^\infty L_{n+l}^{2l+1}(t)\, L_{n'+l}^{2l+1}(t)\; t^{2l+1}\; \exp(-t)\, \mathrm{d}t = 0 \qquad (n' \neq n)$$

- where orbital numbers $l = 0, 1, \ldots, n-1$ - in system of principal numbers $n = 1, 2, \ldots$

1.2 Note

Symbolically, in the fundamental constants, a relativistic ratio at achievable limit velocity $\mathtt{c} = c_0$ - on main electronic orbit $\mathtt{r} = r_0$ - of Bohr's radius, is designated

$$\mathcal{C} = \frac{\grave{\mathtt{c}}}{\mathtt{c}} = \frac{\grave{\mathtt{r}}}{2\,\pi\,\mathtt{r}}$$

Here, according with the Compton wavelength $\grave{\mathtt{r}} = \frac{2\pi\hbar}{\mathtt{m\,c}}$ - there is corresponding offset for fine-structure constant, by the Sommerfeld work [2].
Is known a postulated law on "moment of momentum", in form the Planck constant

$$\mathtt{r\,m}\,\grave{\mathtt{c}} = \frac{1}{2\pi}\,\grave{\mathtt{r}}\,\mathtt{m\,c} = \hbar$$

- from where the rest energy for electron follows

$$\mathtt{E} = \mathtt{mc}^2 = \frac{2\pi\hbar\,\mathtt{c}}{\grave{\mathtt{r}}}$$

From the relative constant on electric interaction $C \sim \mathtt{e}^2$ - there is electron increment of the ground state of atom (ionization energy)

$$\mathit{\Delta}\mathrm{E} = \mathcal{E}(n=1) = \frac{1}{2}\, C^2\, \mathrm{E}$$

2 Assumption about scattering function (in central symmetry system)

Within the confines of an atom task, the known electrostatic theorem can be applied, where, for a flow of electric displacement through closed surface $F(r) \sim \frac{\mathrm{d}}{\mathrm{d}r} V(r)$ - there is charge proportionality by the Gauss-Ostrogradsky formula

$$\oint_S \mathbf{F} \cdot \mathrm{d}S \sim 4\pi\, q$$

So that, in the central forces field of Coulomb's attraction, a tension spherical surface of the repulsive forces, as if under electron shell that surrounds the charged kernel, can be revealed.
From this, some function by type the distribution of probability density is deduced (mean superficial value)

$$\chi \sim \oint_S [\psi]^2\, \mathbf{F} \cdot \mathrm{d}S \tag{3}$$

- for which, accordingly, radial dependence by the spherical functions is solved in Eq. (1)

$$\chi(r) = [\psi(r)]^2 = \int_0^{\pi} \mathrm{d}\theta \int_0^{2\pi} [\psi(r, \theta, \phi)]^2\, \mathrm{d}\phi$$

In fact, there it is required to install a special repulsion interaction, that implies indefinite solution for the wave function at infinity.
In that case, there can be a permissible mapping as on the complex plane, where for some quadratic-integrated function, the integral Cauchy representation has place (in work [3])

$$\tilde{f}(z') = \frac{1}{\pi \mathrm{i}} \int_{-\infty}^{+\infty} \frac{f(\zeta)}{\zeta - z'}\, \mathrm{d}\zeta$$

There is arithmetic mean of the boundary integral values of Cauchy type, as in strict condition of the Sokhotski-Plemelj formulas

$$\left.\begin{aligned} \varphi_i(z') - \varphi_e(z') &= \varphi(z') \\ \frac{1}{2\,\pi \mathrm{i}} \int_L \frac{\varphi(\zeta)}{\zeta - z'}\, \mathrm{d}\zeta &= \frac{\varphi_i(z') + \varphi_e(z')}{2} \end{aligned}\right\}$$

The value of boundary function is equal to difference of these values, inside $\varphi_i(z')$ - and outside $\varphi_e(z')$ - of integral contour on arbitrary point (in work [4]).
In particular, there is identical transformation for function of a real positive parameter on open contour in infinity

$$f(x) = \frac{1}{\pi \mathrm{i}} \int_0^{\infty} \frac{f(\zeta)}{\zeta - x}\, \mathrm{d}\zeta$$

Let the mapping for the reflected branch of wave function in the negative limit of integration exists

$$\chi(z) + \chi(-\bar{z}) = \frac{1}{\pi \mathrm{i}} \int_{-\infty}^{+\infty} \frac{\chi|t|}{t - z} \mathrm{d}t \tag{4}$$

- which from some base integral expression can be continued:

$$\mathrm{X} = \frac{1}{2\pi \mathrm{i}} \oint_{-\infty} \frac{\chi(t)}{t} \mathrm{d}t \tag{5}$$

There is the principal limited value of the Cauchy integral by the Sokhotski-Plemelj theorem (in work [5])

$$i \lim_{\epsilon \to 0^+} \int_{-\infty}^{+\infty} \frac{f(t)}{t - i\,\epsilon} \,\mathrm{d}t = -\pi f(0) + i\,\mathcal{P} \int_{-\infty}^{+\infty} \frac{f(t)}{t} \,\mathrm{d}t$$

By the integral Hankel representation, the bypass of integral loop is noted (in Fig. 2)

$$\frac{1}{\Gamma(1-\alpha)} = \frac{1}{2\pi \mathrm{i}} \oint_{-\infty} t^{\alpha} \frac{\exp(t)}{t} \mathrm{d}t$$

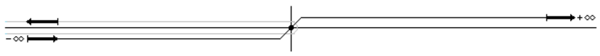

Fig. 2 Bypass at initial point on complex plane from infinity for Eq. (5) - as in discontinuity point for Eq. (4). It is some mathematical breach of steadiness, as in charge singularity.

3 Application in weighted characteristic at extremum

In such a narrowly assigned task, the initial condition of wave function can be performed in Eq. (3)

$$\chi(r) = [\psi(r)]^2 = [Q(r)]^2 \cdot \exp(-r)$$

- where corresponding system of the orthogonal solution is separated (as per Note 1.1).
So that, when passing through the initial point on complex-conjugate plane, the highlighted orthogonal system remains always on principal half-plane in form:

$$\left.\begin{matrix} \mathcal{I}(x) \\ \bar{\mathcal{I}}(x) \end{matrix}\right\} = \mathrm{Re} \left[Q^2(z) \cdot \left\{ \begin{matrix} \exp(-z) \\ \exp(\bar{z}) \end{matrix} \right. \right] \tag{6}$$

Here is the functional expression of real parts in the integral transformation (4)

$$\left.\begin{matrix} \mathcal{I}(x) \\ \bar{\mathcal{I}}(x) \end{matrix}\right\} = \mathrm{Re} \left\{ \begin{matrix} \chi(z(x)) \\ \chi(-\bar{z}(x)) \end{matrix} \right.$$

- which in the solution of parametric line on complex plane can be provided (for imaginary axis)
At the bypass in initial point on complex plane a replacement to opposite sign is required

$$\left.\begin{matrix} z(x) = x + \mathrm{i}\, y(x) \\ \bar{z}(x) = x - \mathrm{i}\, y(x) \end{matrix}\right\}$$

There is resolved explicitly the real parts in Eq. (6) (at the condition in initial point $\mathcal{I}(0)+\bar{\mathcal{I}}(0)=0$)

$$\mathcal{I}(x)+\bar{\mathcal{I}}(x)=\Big(\mathrm{Re}\left[Q^2(z)\right]\cdot\cos(y)+\mathrm{Im}\left[Q^2(z)\right]\cdot\sin(y)\Big)\cdot(\exp(-x)+\exp(x))=$$
$$=\frac{4}{\pi}xy\int_0^{\infty}\frac{Q^2(t)\,\exp(-t)\,t\,\mathrm{d}t}{t^4+2t^2\,(y^2-x^2)+(y^2+x^2)^2}$$

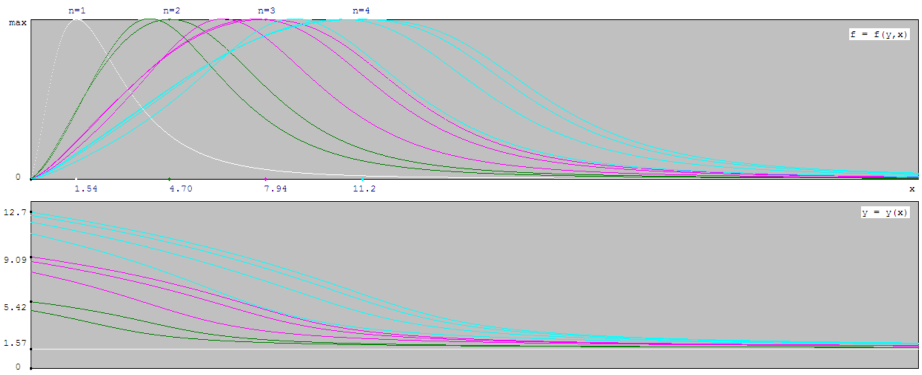

Fig. 3 The reduced curves of reflected branches in maximum $f(y,x)\sim\bar{\mathcal{I}}(x)$ - where approximate equality for points of maximum $x(n)\approx\frac{2n-1}{2}\pi$ (by principal numbers). In solution of complex plane $y=y(x)$ - for ground state, nearly straight line along the asymptote runs $y_{(n=1)}(\infty)\to y_{(n=1)}(0)=\frac{\pi}{2}$ (marked in white)

The specified expression of real parts (6) - may actually make meaning only for constant values (in point of maximum $x\to x_\imath$) - under offset order of principal numbers $\imath=\mathrm{n}-1=0,1,\dots$)

$$\mathcal{I}_\imath\in\bar{\mathcal{I}}_\imath\quad\left(\bar{\mathcal{I}}_\imath=\bar{\mathcal{I}}(x_\imath)\to\max\right)$$

In such an extreme position the reflected branch is depicted (in Fig. 3).

Let, in this extreme solution of real parts from Eq. (6) - some expression of spherical system, which normalized at the above basis (5) - is founded:

$$\left.\begin{matrix}\grave{\mathrm{r}}_\imath\\ \bar{\mathrm{r}}_\imath\end{matrix}\right\}\to\frac{4\pi\,\mathrm{r}_\imath}{\mathrm{X}_\imath}\left\{\begin{matrix}\mathcal{I}_\imath\\ \bar{\mathcal{I}}_\imath\end{matrix}\right.$$

Then, there is an obvious proportionality of radial displacement of relative moment (at extremum $\mathcal{M}\in\bar{\mathcal{I}}$):

$$\begin{cases}\mathcal{C}_\imath=\grave{\mathtt{c}}_\imath\,/\mathtt{c}_\imath\,=\grave{\mathtt{r}}_\imath\,/(2\pi\,\mathtt{r}_\imath)\\ \mathcal{M}_\imath=\grave{\mathtt{m}}_\imath\,/\mathtt{m}_\imath\,=\mathtt{r}_\imath\,/\bar{\mathtt{r}}_\imath\end{cases}\tag{7}$$

Here, provided the highlighted ground state, there is degenerate electronic value

$$m_\imath {c_\imath}^2=m_0c_0^2\equiv\mathtt{E}\quad(\imath=0,1,\dots)$$

- as, directly , in the law on moment of momentum conservation (by Note 1.2)

$$r_\imath \grave{m}_\imath \grave{c}_\imath = r_0 \grave{m}_0 \grave{c}_0$$

Thus, a ratio of electron moment is derived

$$\frac{m_\imath}{m_0} = \frac{{c_0}^2}{{c_\imath}^2} = \frac{{r_\imath}^2}{{r_0}^2} = \frac{\mathcal{M}_0\,\mathcal{C}_0}{\mathcal{M}_\imath\,\mathcal{C}_\imath} \tag{8}$$

- according with the relative weighted moment in extremum (6)

$$\frac{\mathcal{M}_0\,\mathcal{C}_0}{\mathcal{M}_\imath\,\mathcal{C}_\imath} = \frac{\mathcal{I}_0}{\bar{\mathcal{I}}_0}\,\frac{\bar{\mathcal{I}}_\imath}{\mathcal{I}_\imath} = \exp^2(x_\imath - x_0)$$

4 Energy representation

In the above system of moment (7) - the effective levels of kinetic energy type are deduced (by dissipation energy):

$$\varDelta E_\imath = \frac{1}{2}\,\mathcal{M}_\imath {\mathcal{C}_\imath}^2\,\mathtt{E} \tag{9}$$

Here is a highlighted solution of the ground state among the transition states in form (Table 1)

$$\mathcal{M}_\imath > 1 > \mathcal{M}_0 \quad (\mathcal{M} \in \max)$$

As it turned out, there is a relative expression on the center-of-mass system

$$\mathcal{M}_0 \to \frac{\mu}{\mu + \mathrm{m}} \quad (\mu \gg \mathrm{m})$$

- where reduced energy value by muonium atom type is obtained

$$\varDelta E_0 = \frac{1}{2}\,\mathcal{M}_0 {\mathcal{C}_0}^2\,\mathtt{E} \to \mathcal{M}_0 \varDelta\mathtt{E} \tag{10}$$

- in the approximation of fine-structure constant (Table 1).
Moreover, there is the transition state of energy displacement from Eq. (9):

$$\varDelta S(n=2) \to \frac{\varDelta E_1}{\varDelta E_0}\varDelta\mathtt{E} \tag{11}$$

- which may belong to leading line $S(n=2)$ - of fine structure of hydrogen-like atom, that with the Lamb shift is associated (in work [6]).
In the accordance with quantum state for first level, there is a moment of electron scattering at extremum (Table 1)

$$\frac{S(n=2)}{\varDelta S(n=2)} \to \mathcal{M}_1$$

In a proof of the proton state, the Compton Effect of scattering, from like fine-structure constant in Eq. (9) - can be supposed (the root mean square)

$$\overline{\mathcal{C}}_\imath{}^2 = \sqrt{\mathcal{M}_\imath {\mathcal{C}_\imath}^2 \cdot \mathcal{M}_0 {\mathcal{C}_0}^2}$$

Table 1 Solution of principal numbers in system $\mathcal{C}_\imath \in \mathcal{M}_\imath$ - in extremum $\mathcal{M}_\imath \in \max$ - according to Eq. (7)

$\imath = n-1$	0	1	2	3
$\mathcal{M}_\imath$	$\approx 1 - 1/207$	10.38	37.40	91.25
$\mathcal{C}_\imath$	7.297×10^{-3}	1.281×10^{-6}	5.361×10^{-10}	3.163×10^{-13}

As it turned out, there are the energy values

$$K_\imath = \frac{\mathtt{E}}{\pi\,\overline{\mathcal{C}}_\imath} \tag{12}$$

- where there is first transition level in accordance with proton (Table 2).

Presumably, from the above electron moment Eq. (8) - the special energy levels of electromagnetic energy can be characterized

$$T_\imath = m_\imath\,\mathtt{c}^2 \tag{13}$$

- where electron value $T_0 = \mathtt{E}$ - provided the achievable speed $c_0 = \mathtt{c}$
If the first level in accordance to charged π-meson there is, then the second level may be assigned to particle by boson type (Table 2).

For the testing of Eq. (13) - the extended expression has been composed

$$\frac{\overline{T}{}^l_\imath}{T_\imath} = \begin{cases} 1 & (l = 0) \\ \sqrt{\frac{T_\imath{}^2}{T_\imath{}^2 + T^{l\,2}_\imath}} & (l \geq 1) \end{cases} \tag{13$'$}$$

- as from projection of orbital numbers $l = 1, 2, \ldots, (n-1)$ - in shifted system of principal numbers $\imath = n-1$
There is possible a general order of the quantized charge $n - l - 1 = 0, 1, \ldots$ - both for the charged and neutral states.
Together with the π-mesons, the family of intermediate bosons can be identified, where a sequence to the spin momentum of particle occurs (Table 3).

Table 2 Comparative solution from the Eq. (12) - and Eq. (13) (in relative units of proton)

$\imath = n-1$	1	2	3
$K_\imath$	≈ 1.0	35.48	1169
$T_\imath$	0.14889	98.62	68521

Table 3 Regular diagonal series from the orbital projection in Eq. (13′) - at comparison with experimental data (in electron units)

$(n-1)\backslash l$	0	1	2
0	E	π^0	Z^0
1	$\pi^{\pm}$	$W^{\pm}$	
2	?		

$n-1$	l	$\frac{1}{2}\overline{T}^{l}_{n-1}$	experiment
1	0	273.38	273.13
	1	264.82	264.13
2	0	181081	–
	1	160886	158000
	2	180811	187000

5 Electron-nuclear correction

As far as can argued, in the description of electronic orbit, some amendment on self-perturbation should be taken into account (by a relativistic type).
Let the wave equation (1) - into the identical variants transformed, for both the electronic value and the potential value from Eq. (2) (from radial variable replacement)

$$-\mathtt{E}\,\mathtt{r}^2\,\nabla^2\psi + V\,\psi = \breve{\mathtt{E}}\,\psi \tag{1a}$$

$$-\mathtt{E}\,\mathtt{r}^2\,\nabla^2\psi + \breve{V}\,\psi = \mathtt{E}\,\psi \tag{1b}$$

These variants can be appropriately rewritten at relative λ-parameter

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2} + \frac{2}{t}\frac{\mathrm{d}\psi}{\mathrm{d}t} + \left[-\frac{1}{4}\frac{\breve{\mathtt{E}}}{\mathtt{E}} + \frac{\mathrm{n}}{t} - \frac{\lambda(\lambda+1)}{t^2}\right]\psi = 0$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}t^2} + \frac{2}{t}\frac{\mathrm{d}\psi}{\mathrm{d}t} + \left[-\frac{1}{4} + \frac{\mathrm{n}}{t}\sqrt{\frac{\mathtt{E}}{\breve{\mathtt{E}}}} - \frac{\lambda(\lambda+1)}{t^2}\right]\psi = 0$$

- where perturbed electronic value is redefined (in order of principal numbers $\mathrm{n} = \imath + 1 = 1, 2, \dots$)

$$\frac{\breve{\mathtt{E}}}{\mathtt{E}} = \left[\frac{\mathrm{n}}{\mathrm{n}+\lambda}\right]^2, \qquad \lambda = \mathrm{n}\left[\sqrt{\frac{\mathtt{E}}{\breve{\mathtt{E}}}} - 1\right]$$

Moreover, such an electronic perturbation is based under the above increment of energy (as per Note 1.2)

$$\breve{\mathtt{E}} = \mathtt{E} + \kappa^2\,\Delta\mathtt{E} \tag{14}$$

So that a small variation of electronic value is realized, for which corresponding relative charge coefficient given (some charge-factor).

As it turned out, there is a fractional charge-factor $\kappa = \pm\frac{1}{3}$ - where, for the highlighted ground state, an exceptional matching with the experimental muonium atom can be obtained (as per Table 4).

More accurate approximation, however, for potential variation is solved in Eq. (1*b*) - than in Eq. (1*a*).
Such a dissimilarity in the solutions may indicate for incomplete task (within the sixth sign of relative error).

Supposedly, the given charge-factor in Eq. (14) - toward the transition states can be continued (as per Table 5).
In this correspondence, with the Lamb shift of hydrogen atom, there is electron value as if from difference between the correlated and uncorrelated solutions for proton nucleus $K_1 - \breve{K}_1 \to \mathrm{E}$
There is a small variation of charge-factor for weak quantized dependence

$$\kappa_\imath = \kappa_{(n-1)}$$

- from where a correlation between the experimental data is simultaneously improved (as per Table 5).

In this case, perhaps, a slight correction with respect to electron in the transition nuclear structure can be introduced, as for upper limit of electronic neutrino (here, it would be within $\breve{\mathtt{E}}(\kappa_1) - \breve{\mathtt{E}}(\kappa_0) \approx 0.15\,\mathtt{eV}$).

Probably, such an imagination of the fractional electric charge, does not contradict the existing Quark Model of nucleus which is confirmed experimentally (in Fig. 4).

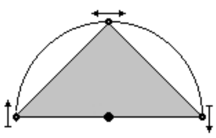

Fig. 4 Quasi-planar model for electron that is inscribed into nuclear "orbit", where elementary charge on three parts is disintegrated, as under a bound charge-pair. This can be shown from a property of the geometric plane which drawn only through three points.

Table 4 Solution of ground state from Eq. (10) - at comparison with experimental data of muonium atom, where relative expression of the center-of-mass system according to muon mass $\mu_0 = \frac{\mathcal{M}_0}{1-\mathcal{M}_0} \to \mathrm{m}_\mu$ - and fine-structure constant $\mathcal{C}_0 \to \alpha$ (relative error)

$\mathcal{M}_0 - \dfrac{\mathrm{m}_\mu}{\mathrm{m}_\mu + \mathrm{m_e}}$	$\dfrac{\mu_0 - \mathrm{m}_\mu}{\mathrm{m}_\mu}$	$\dfrac{\mathcal{C}_0 - \alpha}{\alpha}$	(variant)
-8.5×10^{-6}	1.8×10^{-3}	-4.9×10^{-5}	(1)
-1.0×10^{-8}	2.1×10^{-6}	5.1×10^{-6}	(1*a*)
-1.0×10^{-8}	2.1×10^{-6}	7.3×10^{-7}	(1*b*)

Table 5 Solution of transition state from the Eq. (13), Eq. (12) and Eq. (11) - in comparison with experimental data for charged π-meson, proton, also for the Lamb shift (relative error)

κ	0	$\frac{1}{3}$	$\frac{1.05}{3}$
$\dfrac{T_1 - 2\mathrm{m}_\pi \mathrm{c}^2}{2\mathrm{m}_\pi \mathrm{c}^2}$	9.0×10^{-4}	1.1×10^{-4}	3.0×10^{-5}
$\dfrac{K_1 - \mathrm{m_p c}^2}{\mathrm{m_p c}^2}$	5.2×10^{-4}	7.0×10^{-5}	2.6×10^{-5}
$1 - \dfrac{\Delta E_1}{\Delta E_0} \dfrac{\Delta \mathrm{E}}{\Delta S(n=2)}$	2×10^{-3}	2×10^{-4}	1×10^{-5}

6 Conclusion and discussion

It would be interesting to know what do means the hypothetical multiply-charged states in the nuclear structure.
Unlike the muon particle with charged kernel $\mathrm{n} = 1$ - the complex structure of stable proton must have twice-charged kernel $\mathrm{n} = 2$ - like surrounded by an electron "shell" in Eq. (2).

There, opposite the meson state of proton $K_1 > T_1$ - other boson states would have the anomalous nuclear nature in form $K_\imath < T_\imath$ (as per Table 2).
How does it correspond to reality, there are transient threshold state $K_2 + T_2 \sim 125.8\,\mathtt{GeV}$ - as for the experimental Higgs boson (in the account of proton units, as per Table 2).

For the purpose of confirmation, is possible to point to known experiment by the W-boson or Z-boson production, with a subsequent decay to quark pair (in work [7]).
After all, there can be a production of quark jets as with participation of intermediate "core", where one would observe to distinct splash (in Fig. 5).
As well, in this collision process of interacting nuclei, the resulting (double) value of electromagnetic energy can be demonstrated in form $2\,K_2 \to 15 + 52 \sim 67\,\mathtt{GeV}$ (in Fig. 6).

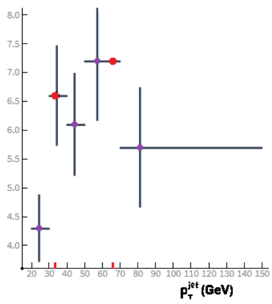

Fig. 5 The ratio of the W + c-jet to W + b-jet production cross sections for data, where the obtained values are marked $K_2 \sim 33\,\mathtt{GeV}$ - and $2\,K_2 \sim 67\,\mathtt{GeV}$ (in red marks). Taken from FIG. 8, by the FERMILAB-PUB-14-525-E

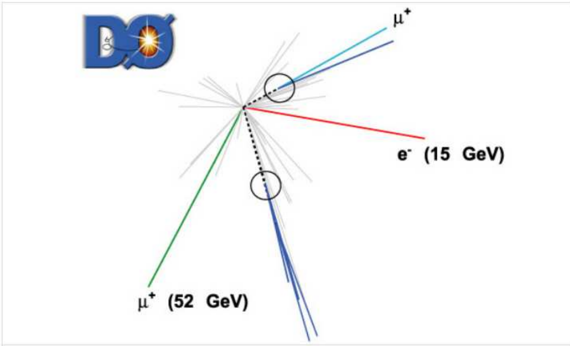

Fig. 6 Result of a restored event with two b-jets in experiment DZero at the Tevatron collider with the D0 detector. The picture is taken from site: www.fnal.gov

Perhaps, there is universal Higgs effect, which as resonant process inside a compound atomic nucleus can be detected. For example, a badly interpretable observation of the photoabsorption for actinides nuclei in region of Δ-resonance is noted, where total cross-section does not substantially match the so-called "universal curve" (in work [8]).
There may be charge exchange of some interconnected meson ensemble, a number of which is equal the nucleon number, as in accordance with the result of intermediate "core" $K_2 \approx 238 \cdot \pi^{\mp}$

It is necessary to note the known Rubbia hypothesis about solar neutrinos, where a slow reaction of nuclear fusion proceeds with participation of the heavy boson quanta (in work [9]).
Quite permissible that there are multiply-charged bosonic states with high energy levels that manifest themselves in stellar depths.
Then, the Higgs boson is a last stage for the heavy nuclei, among a number of the similar high-energy particles (as per Table 2).

The fact is evident that observed (thermonuclear) reaction of nuclear fusion, always occurs as avalanche due to excessive presence of actinides, that would be ineffective only with the light nuclei.
A similar effect can be both the most common in the space objects, where beyond some critical accumulation of heavy elements, the supernova explosion may happen.

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Conflict of interest

The authors declare that they have no conflict of interest.