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Moritz Heimpel (mheimpel@ualberta.ca)  
University of Alberta

Rakesh Yadav  
Harvard University  https://orcid.org/0000-0002-9569-2438

Nick Featherstone  
Southwest Research Institute

Jonathan Aurnou  
UCLA  https://orcid.org/0000-0002-8642-2962

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Polar and mid-latitude vortices and zonal flows on Jupiter and Saturn

Moritz Heimpel\textsuperscript{1,*} Rakesh Yadav\textsuperscript{2} Nick Featherstone\textsuperscript{3} and Jon Aurnou\textsuperscript{4}
\textsuperscript{1}University of Alberta, Department of Physics, Edmonton, Alberta, T6G 2E1, Canada
\textsuperscript{2}Department of Earth and Planetary Sciences, Harvard University, Cambridge, MA 02138, USA
\textsuperscript{3}Southwest Research Institute, 1050 Walnut Street, Suite 426, Boulder, CO 80302, USA
\textsuperscript{4}Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles, CA 90095, USA
\textsuperscript{*}e-mail: mheimpel@ualberta.ca

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Abstract

Zonal flow on Jupiter and Saturn consists of equatorial super–rotation and alternating East-West jet streams at higher latitudes. Interacting with these zonal flows, numerous vortices occur with various sizes and lifetimes. The Juno mission and Cassini’s grand finale have shown that the zonal jets of Jupiter and Saturn extend deeply into their molecular envelopes. On Jupiter, the vast majority of low and mid-latitude jovian vortices are anticyclonic, whereas cyclones appear at polar latitudes. Cassini mission observations revealed a similar pattern on Saturn; its North and South polar vortices are cyclonic, whereas anticyclones occur at mid-latitudes. We use the recently developed code Rayleigh to run high-resolution simulations of rotating convection in 3D spherical shells. Four model runs are presented that result in dynamical flows that are comparable to those on the giant planets. We confirm previous results, finding that deep convective turbulence can explain the structure of jets. However, the strength and depth of stable stratification, and the latitude, can modify jet morphologies and affect the formation and dynamics of vortices. Lower latitudes favour shallow anticyclonic vortices that form due to upward and divergent flow near the outer boundary. These anticyclones are typically shielded by cyclonic filaments associated with downwelling return flow. In contrast, a single polar cyclone, or clusters of cyclones form near the poles. All of our simulations have this global pattern; a strong preference for shallow anticyclones in the first anticyclonic shear zone away from the equatorial jet (corresponding to the region of the great red spot on Jupiter and
Storm Alley on Saturn), cyclonic and anticyclonic vortices at higher mid-latitudes, and a deeply seated cyclone or cyclone clusters at each pole. Our results show that Juno and Cassini observations of cloud-level flow can be explained in terms of deep convective dynamics in the molecular envelopes of Jupiter and Saturn.

1 Introduction

The gas giants Jupiter and Saturn, are composed mostly of fluid Hydrogen with minor amounts of Helium and traces of other elements. Electrical conductivity is dynamically negligible in much of the molecular envelope but increases by several orders of magnitude as molecular Hydrogen dissociates progressively to a depth where a metallic state is reached. The depth of transition to a metallic fluid has been estimated for Jupiter (roughly 80% - 90% of Jupiter’s radius) and Saturn (60% - 75% of Saturn’s radius) based on laboratory experiments [1], ab-initio computational models [2], and from inversions of magnetic field data from the Cassini [3] and Juno [4] missions. Precipitation of Helium near or within the metallic transition region of both planets is expected to affect heat flow and buoyant stability [5]–[7].

The global magnetic fields of Jupiter and Saturn are sustained by the dynamo process, which is driven by convection. At great depths, and in the transition region of the lower molecular envelope, magnetic Lorentz forces limit flow velocities. Maximum flow velocities in the dynamo regions of Jupiter and Saturn have been estimated to be of order of 1 cm/s [8], which is much lower than the order 100 m/s velocities of cloud level zonal flows. For Jupiter and Saturn, the maximum depths of fast zonal flow (of order 10 m/s) have been estimated at $0.96 R_J$ and $0.86 R_S$, respectively [9], [10].

Recent results from the Juno mission to Jupiter and the Cassini grand finale Saturn observations indicate that zonal jets extend deeply within the molecular envelopes of both planets [11]–[13]. Although it is not fully understood how high latitude atmospheric jets couple to the metallic interior fluid, computational models have shown that flow in the dynamo region is slow, whereas convection extending outward into the semiconducting molecular envelope can drive fast equatorial zonal flows[10], [14], [15].

Comparing the atmospheric dynamical features of Jupiter and Saturn, particularly the zonal jets and vortices, the similarities and differences are equally compelling. Each planet has a dominant prograde equatorial jet and several high latitude jets. Saturn’s jets are broader and faster than those of Jupiter. However, the Rossby number based on a length scale equal to the depth of the molecular envelope is roughly similar for Jupiter and Saturn. Saturn’s jets are relatively symmetrical
about the equator, whereas significant North-South asymmetry is observed for Jupiter. There have been numerous observations consistent with a preference for anticyclones over cyclones for the mid-latitudes of the gas giants, including Jupiter’s great red spot (GRS) and white ovals, and Saturn’s great storm of 2010 – 2011[16], [17]. Although the mid-latitude anticyclone preference is more clear for Jupiter than for Saturn, each planet has a highly active region that hosts anticyclones at latitudes corresponding to the first anticyclonic shear zone away from the equator. For Saturn this zone has been referred to as ‘storm alley’ [18]. However, both planets have cyclonic vortices at the poles. Saturn has a single cyclone at each pole, while Jupiter has clusters of several cyclones in the North and South polar regions [19]. Figure 1 shows images of anticyclones in Saturn’s storm alley, the southern polar cyclone, and several cyclones in Jupiter’s south polar region.

As in previous non-magnetic rotating convection models we restrict our attention to the molecular envelope[20]–[25], and choose inner boundary radii representative of the transition from fast to slow zonal flow, in the region of the molecular layer of Jupiter, outside the region of strongly affected by Lorentz forces. We present one numerical run with \( r_i = 0.9 r_o \), and three runs with \( r_i = 0.95 r_o \), where \( r_i \) and \( r_o \) are the inner and outer boundary radii, respectively. In addition we include the transition between Jupiter’s convective interior and its stably stratified atmosphere. This is done by applying uniform superadiabatic (convective) entropy flux at the inner boundary of the spherical shell, and uniform subadiabatic (stabilizing) flux at the outer boundary [21]. A uniform entropy sink balances the boundary fluxes to conserve total heat flux. For the four runs presented, we use different types of heating conditions which are coupled to the polytropic equation of state. These forcing conditions are set to yield zonal flow with velocity (scaled to the rotation rate) comparable to the zonal flow velocity of Jupiter.

2 Model description

2.1 Computational Code

The numerical dynamo code Rayleigh solves the governing equations of magnetohydrodynamics in spherical geometry under the anelastic approximation [26], [27]. MPI parallelization of Rayleigh is done in two dimensions, allowing models to run on the massively parallel IBM Blue Gene supercomputer Mira, at Argonne National Laboratory. We run Rayleigh in non-magnetic, rotating convection mode.
2.2 Governing equations

Consider a compressible fluid in a spherical shell rotating at a constant rotation rate $\Omega$ about the $z$-axis. The anelastic approximation allows for the incorporation of density stratification while filtering out fast acoustic waves. In our anelastic formulation entropy serves as the single fluctuating thermodynamic variable [28], [29]. We adopt a dimensionless formulation using $\Omega^{-1}$ as the time unit, and the shell thickness $d = r_o - r_i$ as the reference length scale. Density, temperature and gravity are non–dimensionalised using their values at the outer boundary $r_o$. The hydrostatic and adiabatic reference state is given by $\frac{dT(r)}{dr} = -Di g$, where $Di$ is the dissipation number

$$Di = \frac{g_o d}{c_p T_o}. \quad (1)$$

Assuming an ideal gas leads to a polytropic equation of state given by $\tilde{\rho}(r) = \tilde{T}(r)^n$, we use polytropic indices $n = 1$ and $n = 2$ here. A polytropic index $n = 1$ gives a linear increase in density, which has been used as a good approximation for Jupiter [30], whereas $n = 2$ was used in more recent numerical models [23], [29], [31]. We assume that the mass is concentrated in the inner part, such that $g \propto 1/r^2$ provides a good first-order approximation of the gravity profile in the molecular envelope of a giant planet[23], [29]. This leads to the following background temperature $\tilde{T}(r)$ and density $\tilde{\rho}(r)$ profiles

$$\tilde{T}(r) = \frac{Di}{(1-\eta)^2 r} + 1 - \frac{Di}{1-\eta}, \quad \tilde{\rho}(r) = \tilde{T}(r)^n \quad \text{and} \quad Di = \eta \left( \frac{N_\rho}{n} - 1 \right) \quad (2)$$

where $g_o$ and $T_o$ are the reference gravity and temperature at the outer boundary, respectively, $N_\rho = \ln \tilde{\rho}(r_i)/\tilde{\rho}(r_o)$ is the number of density scale heights of the background density profile $\tilde{\rho}(r)$, and $\eta = r_i/r_o$ is the radius ratio of the spherical shell.

The equations that govern compressible convection under the anelastic approximation are given by

$$\nabla \cdot \left[ \tilde{\rho}(r) \mathbf{v} \right] = 0, \quad (3)$$

$$\begin{align*}
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + 2 \mathbf{e}_z \times \mathbf{v} &= -\nabla \frac{p}{\tilde{\rho}(r)} + Ra^* \frac{r_o^2 s}{r^2} \mathbf{e}_r + \frac{Ek}{\tilde{\rho}(r)} \nabla \cdot D, \quad (4)
\end{align*}$$

$$\begin{align*}
\tilde{\rho}(r)\tilde{T}(r) \left( \frac{\partial s}{\partial t} + \mathbf{v} \cdot \nabla s \right) &= E_k \frac{p}{Pr} \nabla \cdot \left[ \tilde{\rho}(r)\tilde{T}(r) \nabla s \right] + Q(r) + \frac{Ek Di}{Ra^*} \Phi, \quad (5)
\end{align*}$$
where \( \mathbf{v}, p \) and \( s \) are velocity, pressure, and entropy, respectively, and \( Q(r) \) is an entropy source or sink. The components of the traceless rate-of-strain tensor \( \mathbf{D} \) are given by

\[
\mathbf{D}_{ij} = 2\tilde{\rho}(r)\left( \mathbf{e}_{ij} - \frac{1}{3}\delta_{ij} \nabla \cdot \mathbf{v} \right) \quad \text{with} \quad \mathbf{e}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right),
\]

where \( \delta_{ij} \) are the components of the identity matrix, and \( \Phi(r, \theta, \phi) \) is the viscous heating contribution expressed by

\[
\Phi = 2\tilde{\rho}(r)\left[ \mathbf{e}_{ij} \mathbf{e}_{ji} - \frac{1}{3}(\nabla \cdot \mathbf{v})^2 \right].
\]

The conductive entropy profile \( s_c(r) \) is obtained from the entropy equation by solving the thermal equilibrium equation:

\[
\nabla \cdot \left[ \tilde{\rho}(r)\tilde{T}(r)\nabla s \right] = -\frac{Pr}{Ek}Q(r)
\]

with the chosen boundary conditions. The value of the volumetric entropy sink \( Q(r) \), is set to balance the entropy flux boundary conditions such that thermal energy conservation is ensured.

The system of equations (3-5) is controlled by three non-dimensional numbers: the Ekman number \( Ek = \nu/\Omega d^2 \); the Prandtl number \( Pr = \nu/\kappa \) and the Rayleigh number \( Ra^* = -g_0\beta_i/c_p\Omega^2 \), where \( \beta_i \) is the entropy gradient \( ds_c/dr \) at \( r_i \), the inner boundary radius. We use constant entropy flux and free-slip velocity boundary conditions at the top spherical shell boundary \( r_o \). Free-slip or no-slip boundary conditions are used at the bottom boundary \( r_i \). The entropy flux boundary conditions perturb the adiabatic background state, yielding positive (superadiabatic) buoyancy and convection at the inner boundary of the spherical shell, and stable (subadiabatic) stratification at the outer boundary. Through the volume of the spherical shell, convection and stability are forced by the conductive entropy gradient \( ds_c/dr \). Where \( ds_c/dr \) is negative, the forcing is convective. Stably stratified forcing occurs where \( ds_c/dr \) is positive.

We use high resolution grids (See Table 1). Nevertheless, given our small Ekman numbers and large Rayleigh numbers, the convergence of these numerical models still requires the use of hyperdiffusivity. Thus, the diffusive terms entering in Eqs. (4-5) are multiplied by an operator of the functional form

\[
\epsilon(\ell) = \left( 1 + \alpha \left[ \frac{\ell - 1}{\ell_{\text{max}} - 1} \right]^\beta \right).
\]
Here, $\epsilon(\ell)$ is the hyperdiffusivity function that depends on the spherical harmonic degree $\ell$, $\alpha$ is the hyperdiffusion amplitude and $\beta = 3$ is the hyperdiffusion exponent[32]. Hyperdiffusion can potentially introduce anisotropy between the horizontal and the radial directions as $\epsilon(\ell)$ depends on the horizontal scale only, and it yields artificial viscous heating that can affect the heat transport balance.

3 Model simulations

3.1 Model setup

We present four model simulations. Table 1 shows parameters and conditions for the four runs. Run 1 has the same parameters as the model of Heimpel et al. (2015). However, in that previous work a four-fold longitudinal symmetry condition was used due to the constraints on computational resources [21]. Here, all four runs are computed in a full sphere with no longitudinal symmetry conditions. All four runs have convective bottom heat flux ($\frac{ds_c}{dr} < 0$) and stable top heat flux ($\frac{ds_c}{dr} > 0$) boundary conditions. Runs 1, 2 and 3 have top and bottom free-slip (FS) velocity boundary conditions. Run 4 has mixed velocity boundary conditions, with FS at the top ($r = r_o$) and no-slip (NS) and the bottom ($r = r_i$). Two different heating types are used. For Run 1 we implement a constant heat sink (internal cooling)

$$Q = \gamma.$$  

This heating type was used in [21] and provides forcing that is variable in radius and inversely proportional to $\tilde{\rho}(r)\tilde{T}(r)$. For subadiabatic heat flux boundary conditions at the outer boundary this heating type can result in a peak in the convective forcing at shallow depth, with decreasing convective forcing at greater depths (See Table 1 and Figure 2). For Runs 2, 3 and 4 the heating profile is

$$Q(r) = \gamma\tilde{\rho}(r)\tilde{T}(r).$$  

This profile has been used for emulating radiative cooling in a stellar convection zone, and in models of Jupiters dynamo. It is based on the assumption that specific entropy decreases uniformly in space over time [14]. For subadiabatic heat flux boundary conditions this heating condition provides increasing convective forcing with depth (See Table 1 and Figure 1).

Figure 2 shows the four heating profiles (i.e. conductive entropy gradient profiles $\frac{ds_c}{dr}$) for the
four model runs. The difference between the constant internal cooling (Run 1) and radiative cooling (Runs 2, 3 and 4) is apparent. For Run 1, convective forcing is relatively shallow – neutral buoyancy \( (\frac{ds_c}{dr} = 0) \) occurs at \( r \approx 0.995 \) and convective forcing peaks at \( r \approx 0.985 \). Also apparent is the difference that different choices of the polytropic index have on thermal forcing (See Table 1). Run 2 and Run 3 have polytropic indices \( n = 1 \) and \( n = 2 \), respectively, with the same entropy gradient boundary condition at the outer boundary \( (\frac{ds}{dr} = 250) \). This difference in polytropic index results in a deeper stably stratified layer for Run 2.

Figure 3 shows the final hyperdiffusivity functions in terms of the harmonic degree dependent Ekman number \( E(l) = \epsilon(l)Ek \) used for the four runs (see also Equation 9 and Table 1). Initial amplitudes of hyperdiffusion have been reduced stepwise as the simulations approached a statistically steady-state.

### 3.2 Model results

Figure 4 shows zonal flow profile, at the outer boundary and at mid-depth of the spherical shell, for each of the four runs. To compare with planetary flows the velocities are scaled in units of Rossby number \( Ro = \frac{V_\phi}{(\Omega r_o)} \), which is the flow velocity in the rotating reference frame divided by the outer boundary rotation velocity. The peak zonal flow velocities on Jupiter \( (Ro \approx 0.01) \) and Saturn \( (Ro \approx 0.04) \) occur near the equator. Noting that Run 1 has radius ratio \( r_i/r_o = 0.9 \) and Runs 2, 3, and 4 have \( r_i/r_o = 0.95 \), and Jupiter and Saturn have estimated maximum depths of fast zonal flow at roughly \( r/R_J \approx 0.96 \) and \( r/R_S \approx 0.86 \), respectively [9], [10], we see that the flow velocities are in a range consistent with that of the gas giants. For all four runs, the similarity in strength of zonal flows at the outer boundary and at mid-shell depth shows that the jets are deeply seated. For Run 3 retrograde equatorial zonal flow at the outer boundary contrasts with strongly prograde flow at mid-shell depth. As mentioned above, the parameters for Run 1 are the same as for a previously published model [21], except that here, Run 1 was computed in a full sphere. Run 1 differs from the other runs mainly in the radius ratio and the heating function, which concentrates buoyancy production at shallow levels below a thin stably stratified layer near the outer boundary. In contrast, Runs 2, 3 and 4 have heating functions that do not peak at shallow levels. These runs have relatively deep stably stratified layers, as indicated by the neutral stability radius, NSR (Table 1). In the deeper convective region, forcing increases with further depth toward the bottom boundary (Figure 2). Thus Runs 2 and 3, with deep stability in a relatively thin shell and a free-slip bottom boundary, yield zonal flows with a less dominant equatorial jet and relatively strong high latitude jets. For Run 3 the retrograde zonal flow at the outer boundary contrasts with the strongly
prograde flow at mid-shell depth. For the three cases with free-slip bottom boundary conditions, the prominence of an equatorial dimple (relative retrograde flow) increases with the depth of the NSR. Run 4 differs from the others in that it has a no-slip bottom boundary condition. This bottom condition efficiently kills zonal flows at high latitudes. This is consistent with previous studies that compare free-slip and mixed boundary conditions [23], [33].

Each of the Figures 5 - 8 show images of the thermal and flow structure of the four runs. The runs represent a range of depths, velocity boundary conditions, heating types, and Rayleigh & Ekman numbers (Table 1).

The width of the equatorial jet depends primarily on the shell depth (Run 1 has $r_i/r_o = 0.9$ while Runs 2, 3 and 4 have $r_i/r_o = 0.95$). That width is a direct result of the 2-dimensionalisation of quasigeostrophic flow in a rapidly rotating fluid spherical shell. Prograde flow tends to fill the volume outside the tangent cylinder (TC), which is the imaginary axial cylinder tangent to the inner spherical surface. This can be seen clearly in the $\phi$ - slice images of axisymmetric zonal velocity in Figures (5d - 8d). Comparing Figures 5 – 8 to Figure 4 shows that the latitude that bounds the equatorial jet at the outer boundary is roughly given by the latitude of intersection of the TC with the outer boundary, $\lambda_{TC} = \cos^{-1}(r_i/r_o)$, i.e. $\lambda_{TC} \approx 26^\circ$ for $r_i/r_o = 0.9$ and $\lambda_{TC} \approx 18^\circ$ for $r_i/r_o = 0.95$. Comparing the zonal flow profiles in Figure 4 seems to show an anomalous equatorial jet for Run 3, with a narrow retrograde equatorial jet. However, comparing with Figure 7d, which shows a $\phi$ - slice of the zonal flow for Run 3, it is clear that the retrograde equatorial flow can be interpreted as a strong dimple on the prograde equatorial flow bounded by the TC. Weaker equatorial dimples are also evident for Runs 1 and 2, while Run4 shows a different prograde equatorial flow structure.

Zonal flow is strongly cylindrical in all four runs, as seen in Figures 5d – 8d. Furthermore, the strength of the vorticity is relatively constant for Runs 2 and 3, which both have relatively deep stable stratification and free-slip bottom boundary conditions. This is seen by comparing the radial vorticity near the outer boundary and at mid-shell (Figures 5c & 5f and 6c & 6f). Run 1 has stronger vorticity near the outer boundary than at mid-depth. This is due to the internal heating, which in contrast to the other three runs, features a forcing function (the outward conductive entropy gradient $-ds_c/dr$) that peaks near the outer boundary (see Figure 2). Runs 1, 2, and 3, which have free-slip top and bottom boundaries, have coherent zonal jets at all latitudes. This is evident in Figure 4 and clearly shown in Figures 5-7. As mentioned above, only the equatorial jet is coherent for Run 4. Figure 8e shows that strong azimuthal flow velocity occurs in patches for Run 4, but this azimuthal flow is not axisymmetric, and does not form coherent zonal flows.
A central finding of this work concerns the variation of occurrence of cyclonic and anticyclonic vortices with latitude and depth in our models. Images of radial vorticity $\omega_r$ are shown near the outer boundary and at mid-depth for all four runs (Figs. 5 – 8, c & f). These images show the prevalence of anticyclonic vorticity at shallow depths and low latitudes, and the prevalence of cyclonic vorticity near the poles. Comparing Fig. 4 (zonal flow profiles) to Figs. 5c – 5f ($\omega_r$ near the outer boundary), cyclonic vortices are seen to occur at latitudes greater than the equatorial jet. Anticyclonic vorticity is favoured within the first anticyclonic region of shear outside the equatorial jet. A strong decrease in radial anticyclonic vorticity is particularly prevalent at low latitudes for Runs 1-3. All four runs also show the tendency for anticyclones to be shielded by filaments of cyclonic vorticity, especially at mid-latitudes. These shielded vortices were previously described in numerical models [21] and observational models of Jupiter’s atmosphere [34].

**Discussion and Conclusions**

Our runs represent a range of atmospheric stability conditions, from relatively shallow to deeper stable stratification, and two different bottom velocity boundary conditions (free-slip and no-slip). For all cases we find that surface jets project into the interior cylindrically, which has been shown in previous Boussinesq models [25] and for anelastic models with strong density stratification (e.g., [21], [22]). As for those previous results, our current runs show deep jets with mostly strong symmetry about the equator. However, as Run 3 shows, differences in the zonal flow velocities of mid-latitude jets, which is consistent with significant asymmetry in the kinetic energy spectrum. This run also has the deepest stability conditions, and shallow retrograde flow at the equator, perhaps comparable with the features of Jupiter's atmosphere: strong stability conditions, jet asymmetry, and a pronounced dimple of slower prograde zonal velocity centred on the equatorial jet.

Our simulations confirm that anticyclonic vortices form preferentially at low latitudes, especially within the first anticyclonic shear away from the equatorial jet [21]. Previous models have shown that cyclonic vortices form preferentially at the poles [20], [35], [36]. A new result here is that we demonstrate, for all four cases, that deep convection leads to a variable pattern of vorticity, with shallow anticyclonic vortices favoured at mid-latitudes and deeply seated cyclones near the poles. Consistent with [21] we find that the mid-latitude anticyclones are much stronger at shallow depths than at mid-shell levels. This is explained as a boundary effect; the anticyclones are formed by upwellings that originate in the convective zone and diverge near and at the top boundary. In contrast, we find that the polar and high latitude cyclones are deeply seated, with little or reduction...
of vorticity with depth (or increasing vorticity with depth for Run 4). The full depth extent of a polar vortex may be explained by considering it as a prograde (cyclonic) jet collapsed onto the polar axis. This is clearly seen by examining the zonal velocity in Figure 4. Runs 1 – 3 each have a polar jet with prograde velocity declining to zero at the pole. This relationship between polar jet and deep seated cyclonic vortex may also be seen in Figures 5 – 8, by examining the velocity to vorticity fields, especially for Runs 2 and 3. (Compare Figures 6c to 6e, 6f and 7e to 7c, 7f.) Runs 1 – 3 all show a single polar cyclone at each pole, comparable to Saturn. However, Run 4 which has the strongest convective forcing has several cyclones at each pole, comparable to Jupiter. Obviously these cyclones are not centred on the polar axis and cannot be interpreted as collapsed polar jets. However, the greater depth extent of polar cyclones, and high latitude anticyclones is due to the alignment of thermal plumes with the rotation axis in the polar regions [37]. (For an example of a deeply seated anticyclone in the polar region see Figure 6c and 6f.)

We find that Run 4, the model with a no slip bottom boundary condition, results in relatively weak and non-axisymmetric zonal flows at high latitudes. The inhibition of zonal flow by no-slip bottom boundaries was shown previously [33]. Dynamo models of the gas giants have shown a similar effect: the deep dynamo disallows fast zonal flows to develop in the deep interior, and at high latitudes [10], [15]. However, laboratory experiments have shown that strong jets may form in the presence of a zero bottom boundary velocity (no-slip) condition [38], [39]. Rapid progress in understanding the deep dynamic of Jupiter and Saturn has come from Juno and Cassini missions. However, much uncertainty exists about the radial profiles of Hydrogen dissociation, stability stratification, and the role of Helium. Future modelling that explores the truncation of cylindrical zonal flows in the transition from the molecular envelope to the deeper dynamo will help us gain further understanding of the structure and dynamics of the deep interiors and atmospheres of the giant planets.
<table>
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Table 1: List of numerical simulations. The number of latitudinal grid points $n_\theta = n_\phi/2 = 3(l_{max} + 1)/2$, where $n_\phi$ is the number of longitudinal grid points and $l_{max}$ is the maximum number spherical harmonic degrees. Neutral Stability Radius (NSR) is the radius at which the conductive entropy gradient $ds_c/dr = 0$ (see Figure 2). Internal heating types for Rayleigh. The number 4 corresponds to constant heat sink (10), 1 corresponds to radiative cooling (11). For all runs the top mechanical boundary conditions are free-slip. The bottom boundary conditions listed are either free-slip (FS) or no-slip (NS).
References


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Author Contributions

M.H. ran the simulation, wrote the manuscript and created the figures. N.F. developed the computational dynamo code Rayleigh. R.Y., N.F., and J.A. helped write and edit the manuscript.

Competing financial interests

The authors declare no competing financial interests.
Figures

Figure 1: Top left: NASA Cassini 2008 image PIA10411 of Saturn's southern mid-latitudes shows anticyclones in the anticyclonic shear band near the equatorial jet. Bottom left: NASA Cassini 2005 PIA07585 image shows the south polar cyclone. Right: NASA Juno 2017 PIA21382 image shows several cyclonic vortices near Jupiter’s south pole.
Run 1, $ds_c/dr$ at $r = 1.0 = 86$
Run 2, $ds_c/dr$ at $r = 1.0 = 250$
Run 3, $ds_c/dr$ at $r = 1.0 = 250$
Run 4, $ds_c/dr$ at $r = 1.0 = 100$

Figure 2: Conductive entropy gradient $ds_c/dr$ for the four simulations (Note the inverted vertical axis). The shape of each forcing function is controlled by the equation of state and boundary conditions (constant $ds_c/dr$ at the inner and outer boundaries). The gray dashed lines indicate $ds_c/dr = -1$ (convective inner boundary condition) and $ds_c/dr = 0$ (neutral stratification). For Run 1, the inner boundary is at $r = r_i = 0.90$. For Runs 2, 3 and 4 the inner boundary is at $r = 0.95$. For all runs the entropy gradient is stably stratified at the outer boundary ($ds_c/dr > 0$ at $r = r_o = 1$).
Figure 3: Hyperdiffusion models shown as Ekman number $E(l) = \epsilon(l)Ek$ as a function of spherical harmonic degree $l$ for the four simulations.
Figure 4: Zonal flow velocity for the four runs. Velocities are scaled as the planetary Rossby number, $Ro = V_\phi/(\Omega r_o)$, where $V_\phi$ is the axisymmetric azimuthal velocity (zonal velocity).
Figure 5: Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 1.

a. Azimuthally averaged radial entropy gradient. Blue indicates stable stratification, red deeper convection. b, Perspective south polar view (camera at −30° latitude) of radial vorticity near the outer boundary. Cyclonic (anticyclonic) vorticity is red (blue) in the northern hemisphere, and blue (red) in the southern hemisphere. c, North polar view (camera at 90° latitude) of radial vorticity near the outer boundary. d, Azimuthally averaged azimuthal (zonal) velocity in Rossby number units based on the shell depth. e, Perspective north polar view (camera at 30° latitude) of azimuthal (zonal) velocity near outer boundary. (Color bar is shared between d and e). f, North polar view of radial vorticity near at mid-shell depth. (Color bar is shared between c and f).
Figure 6: Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 2. See Figure 5 for image descriptions.
Figure 7: Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 3. See Figure 5 for image descriptions.
Figure 8: Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 4. See Figure 5 for image descriptions. In this figure the color bars differ between e and d.
Top left: NASA Cassini 2008 image PIA10411 of Saturn’s southern mid-latitudes shows anticyclones in the anticyclonic shear band near the equatorial jet. Bottom left NASA Cassini 2005 PIA07585 image shows the south polar cyclone. Right: NASA Juno 2017 PIA21382 image shows several cyclonic vortices near Jupiter’s south pole.
Conductive entropy gradient $dsc/dr$ for the four simulations (Note the inverted vertical axis). The shape of each forcing function is controlled by the equation of state and boundary conditions (constant $dsc/dr$ at the inner and outer boundaries). The gray dashed lines indicate $dsc/dr = -1$ (convective inner boundary condition) and $dsc/dr = 0$ (neutral stratification). For Run 1, the inner boundary is at $r = ri = 0.90$. For Runs 2, 3 and 4 the inner boundary is at $r = 0.95$. For all runs the entropy gradient is stably stratified at the outer boundary ($dsc/dr > 0$ at $r = ro = 1$).
Figure 3

Hyperdiffusion models shown as Ekman number $E(l) = e(l)E_k$ as a function of spherical harmonic degree $l$ for the four simulations.
Figure 4

Zonal flow velocity for the four runs. Velocities are scaled as the planetary Rossby number, \( Ro = V\Phi/(ro) \), where \( V\Phi \) is the axisymmetric azimuthal velocity (zonal velocity).
Figure 5

Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 1. a, Azimuthally averaged radial entropy gradient. Blue indicates stable stratification, red deeper convection. b, Perspective south polar view (camera at −30° latitude) of radial vorticity near the outer boundary. Cyclonic (anticyclonic) vorticity is red (blue) in the northern hemisphere, and blue (red) in the southern hemisphere. c, North polar view (camera at 90° latitude) of radial vorticity near the outer boundary. d, Azimuthally averaged azimuthal (zonal) velocity in Rossby number units based on the shell depth. e, Perspective north polar view (camera at 30° latitude) of azimuthal (zonal) velocity near outer boundary. (Color bar is shared between d and e). f, North polar view of radial vorticity near at mid-shell depth. (Color bar is shared between c and f).
Figure 6

Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 2. See Figure 5 for image descriptions.
Figure 7

Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 3. See Figure 5 for image descriptions.
Figure 8

Images of fluid flow and entropy gradient at a snapshot in time, near the end of Run 4. See Figure 5 for image descriptions. In this figure the color bars differ between e and d.