Supplementary Information

**Voltage-driven gigahertz frequency tuning of spin Hall nano-oscillators**

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**Note1. ST-FMR spectra for various frequencies**

Figure S1a shows the schematic illustration of the ST-FMR measurement [S1, S2], of which details are discussed in the Method section of the main text. Figures S1b-S1d show the ST-FMR spectra of the Co/Ni sample with different $V\_{g}$’s for various frequencies ranging from 10 to 21 GHz, measured while sweeping magnetic fields along the *z*-direction.

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**Figure S1. ST-FMR measurement of Co/Ni sample for various frequencies. a,** Schematic diagram of the device structure of the ST-FMR measurement set-up. **b-d,** ST-FMR spectra of the Co/Ni sample for various frequencies ranging from 10 to 21 GHz with sequentially applied gate voltages $V\_{g}$ = 0 V (initial) (b), $V\_{g}$ = +5 V (c), and $V\_{g}$ = -5 V (d). The dotted lines are the fitting curves based on Eq. (2) of the main text.

**Note 2. Voltage-driven frequency modulation in Co/Ni sample with a different thickness**

To show the reproducibility of the voltage-driven frequency modulation, we fabricated another Co/Ni device of Ta (3 nm)/Pt (5 nm)/[Co (0.4 nm)/Ni (0.6 nm)]7/Co (0.4 nm)/AlOx (2 nm), where a slightly thinner Co (0.4 nm) is used compared to the sample used in the main text has a [Ta (3 nm)/Pt (5 nm)/[Co (0.45 nm)/Ni (0.6 nm)]7/Co (0.45 nm)/AlOx (2 nm)]. We first check PMA of the sample using the ST-FMR measurement with the same procedure used in Fig. 2 of the main text. Figure S2 shows the resonance frequency ($f\_{res,z}$) of ST-FMR spectra as a function of the resonance field ($B\_{res,z}$) for two Co/Ni samples having different Co thicknesses. As the *y*-intercept indicates the PMA field ($B\_{k}$) according to the Kittel formula $f\_{res,z}=\frac{γ}{2π}(B\_{res, z}+B\_{k})$ [S3], demonstrating that the sample with a thinner Co has a stronger PMA.

We then fabricate an SHNO with a constriction width of 100 nm. The experimental procedure for the power spectral density (PSD) measurement is the same as used in Fig. 3 of the main text, except for a dc current ($I\_{dc}$) of 1.8 mA used. Figures S3a-S3c show the color plots of PSD as a function of a magnetic field ($B$), where gate voltages of +5 V and -3V were sequentially applied. The auto-oscillation peak is clearly observed, and its frequency is increased by the positive voltage and restored by the subsequent negative voltage. Figure S3d shows the auto-oscillation spectra for a magnetic field of $B=0.9 T$ for different gate voltages, extracted from Figs. S3c-S3g. The frequency modulation of the sample with a thinner Co is about a few GHz, confirming the reproducibility of the voltage-driven frequency modulation of the SHNO.



**Figure S2. ST-FMR measurement.** Resonance frequency ($f\_{res,z}$) of ST-FMR spectra as a function of the resonance field ($B\_{res,z}$) for the samples of Ta (3 nm)/Pt (5 nm)/[Co (0.45 nm)/Ni (0.6 nm)]7/Co (0.45 nm)/AlOx (2 nm) (black squares) and Ta (3 nm)/Pt (5 nm)/[Co (0.4 nm)/Ni (0.6 nm)]7/Co (0.4 nm)/AlOx (2 nm) structures (red circles).

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**Figure S3. Voltage-driven frequency modulation in SHNO of Ta (3 nm)/Pt (5 nm)/[Co (0.4 nm)/Ni (0.6 nm)]7/Co (0.4 nm)/AlOx (2 nm). a-c,** PSDs versus a magnetic field for sequentially applied gate voltages, $V\_{g}$ = 0 V (initial state) (**a**), $V\_{g}$ = +5 V (**b**), and $V\_{g}$ = -3 V (**c**). $I\_{dc}$= 1.8 mA. **d,** Auto-oscillation spectra for $B=0.9 T$ for different gate voltages, extracted from Figs. S3c-S3g. The yellow line is the Lorentz fit of the auto-oscillation spectra.

**Note 3. Gate voltage effect on the current-induced SOT**

We investigated the gate voltage effect on current-induced spin-orbit torque (SOT) using in-plane harmonic measurements. For the measurement, we fabricated a Hall bar device with a 10 μm × 10 μm cross using the Ta (3 nm)/Pt (5 nm)/[Co (0.45 nm)/Ni (0.6 nm)]7/Co (0.45 nm)/AlOx (2 nm) film. The first and second harmonic Hall resistance ($R\_{1ω}$ and $R\_{2ω}$) were simultaneously measured with an a.c. current of 15 mA and a frequency of 11 Hz while rotating the sample (azimuthal angle *φ*) under an in-plane magnetic field ($B\_{ext}$). The $B\_{ext}$ ranges from 0.3 T to 4.0 T, which is larger than the perpendicular magnetic anisotropy field $(B\_{k}$), so the magnetization is aligned in the magnetic field direction. The gate voltage was applied to the top electrode for 5 minutes at 150 ℃ before the measurement. Figure S4a shows the $R\_{2ω}$ as a function of $φ$ under a magnetic field of 0.6 T for various gate voltages. The $R\_{2ω}$ can be expressed as [S4, S5],

$R\_{2ω}\left(φ\right)=\left(R\_{AHE}\frac{B\_{DLT}}{B\_{eff}}+R\_{∇T}\right)cosφ+2R\_{PHE}\frac{B\_{FLT}}{B\_{ext}}\left(2cos^{3}φ-cosφ\right)$, (2)

where $R\_{AHE}$, $R\_{PHE}$ and $R\_{∇T}^{2ω}$ are the anomalous Hall resistance, planar Hall resistance and thermal effect contribution, respectively; $B\_{DLT}$ and $B\_{FLT}$ are the damping-like effective field and field-like effective field including Oersted field, respectively; $B\_{eff}$ is the effective magnetic field ($B\_{eff}=B\_{ext}-B\_{k})$. Figures S4b and S4c show the magnetic field-dependence of the $cosφ$ and ($2cos^{3}φ-cosφ$) components of $R\_{2ω}$, respectively. Figure S4d demonstrates that the extracted $B\_{DLT}$ and $B\_{FLT}$ values of the sample, which are not changed by the gate voltage.



**Figure S4. Gate voltage effect on current-induced SOT. a,** Second-harmonic Hall resistance $R\_{2ω}$ versus azimuthal angle $φ$ under an in-plane magnetic field of 0.6 T for sequentially applied gate voltages. **b,** $cosφ$ component of $R\_{2ω}$ as a function of $1/B\_{eff}$ for sequentially applied gate voltages. **c,** ($2cos^{3}φ-cosφ$) component of $R\_{2ω}$ as a function of $1/B\_{ext}$ for sequentially applied gate voltages. The gate voltages were applied in the sequence indicated by the black arrows in Figs. S4a-S4c. **d,** The variation of the SOT-induced $B\_{DLT}$ (black square) and $B\_{FLT}$ (black triangle) values with sequentially applied gate voltages.

**Note 4. Threshold current for current-induced magnetization auto-oscillation**

We determine the threshold current ($I\_{th}$) at which auto-oscillation begins to occur by a linear fit of the inverse of the PSD integral. Figures S5a-S5e show the (integral of PSD)-1 versus current for sequentially applied gate voltages, where *I*th is obtained by the *x*-intercept of the linear fit (solid red lines) [S6, S7]. Figure S4f displays the variation of $I\_{th}$ with gate voltages, which is the same as Fig. 4f in the main text.



**Figure S5. Threshold current for current-induced magnetization auto-oscillation. a-e,** (integral of PSD)-1 as a function of current for sequentially applied gate voltages, Vg = 0V (initial) (**a**), $V\_{g}$ = +4 V (**b**), $V\_{g}$ = +5 V (**c**), $V\_{g}$ = -2 V (**d**), and $V\_{g}$ = -3 V (**e**). $B=0.52 T$. **f,** $I\_{th}$ according to the sequentially applied gate voltages, extracted from Figs. S5a-S5e.

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