Binary Black Hole Merger: Mass-Separation Relation and Intermediate Mass Black Holes

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Abstract

Intermediate Mass Black Holes (IMBHs) are an elusive category of black holes in the mass range of 100 to 100000 Solar Masses. Binary IMBHs might form due to mergers of Globular Clusters, Pair Instability Supernovae, and in Young Massive Star Clusters. In this Research Note, merger timescale, constraints on the separation based on the timescale, and other parameters of Binary IMBHs are calculated analytically and are discussed. The calculations were conducted using Newtonian and Einstienian dynamics. The timescale of a Binary IMBH system to reach maximum gravitational wave amplitude is also calculated ad discussed. We also present the relation between the combined Mass of a Binary Black Hole (BBH) System and the Separation between two BHs required for a BBH system to merge within a given timescale $\mathrm{t_c}$, solely due to Gravitational Radiation is a function of the total mass of the system. In this article, $\mathrm{t_c}$ is set equal to Hubble time $\mathrm{t_H}$. Now, the relation obtained is essentially the relation between separation of a BBH system (collide within $\mathrm{t_H}$) and its Mass . The calculations were conducted for all three categories of Black Holes: Stellar, Intermediate, and Supermassive. Time ahead, the relation might be used for determining whether a BBH merger would be observational. The relation is also solved for Intermediate Mass Black Holes (IMBHs), and t_c and separation for collision within t_H was calculated.

1 Introduction

Intermediate Mass Black Holes (IMBHs) are a class of black holes in the mass range of $10^2\mathrm{M}_\odot - 10^5\mathrm{M}_\odot$. Very few candidates of IMBHs are confirmed at present. Some observational evidences of IMBHs are low-luminosity Active Galactic Nuclei and Ultra Luminous X-Ray Sources.

In this Research Note, the parameters of Binary IMBHs are calculated and discussed. Binary IMBHs can form in Globular Clusters ([1]), Pair Instability Supernovae ([2], in Young Massive Star Clusters ([3], and by other astrophysical processes. Throughout the calculations, it is assumed that the black holes are orbiting each other in circular orbits; they have equal mass.

2 The Merger Parameters

Firstly, we calculate constraints on the separation of the Binary based on Hubble Time (t_H). The calculation were conducted by equating the following equation to t_H:

$$t_{\text{coals}} = \frac{5}{256}\frac{c^5}{G^3}\frac{r^4}{(M_1 M_2)(M_1+M_2)} \quad (1)$$

where $M_1 = M_2$ are the masses of the IMBHs, c is the velocity of light, and G is the Universal Gravitational Constant. The solutions obtained by equating t_{coals} to t_H **were in the range 1.48× 10^{-5} for 100M$_\odot$ IMBHs to 0.002 parsecs for 10^5M$_\odot$ IMBHs**.

These solutions can be applied to galaxy mergers and other astrophysical processes in order to predict whether an IMBH merger will be observed. For calculating the separation during maximum Gravitational Wave (GW) amplitude R_{max} is given by the equation ([4]) below:

$$R_{\text{max}} = \left(\frac{GM_{\text{T}}}{\omega^2{}_{\text{Kep}}}\right)^{\frac{1}{3}} \quad (2)$$

where M_T is the total mass of the system, ω_{Kep} is the orbital angular frequency. By solving (2) over the mass range of IMBHs, R_{max} is in the range 10^5 to 10^6 kilometers. The total energy (potential and kinetic combined) of a binary IMBH can be calculated using the equation below:

$$E = -\frac{GM_{\text{T}}\mu}{2r} \quad (3)$$

And the results obtained are approximately in the range of **2.99 × 10^{20} Yotta Joules to 1.7 × 10^{24} Yotta Joules.**

Now, we calculate a very important timescale (t_{dec}): The time taken for the separation of IMBHs in the range 1.48× 10^{-5} parsecs — 0.002 parsecs (the initial separations) to decrease down to R_{max}. The timescale was calculated based on equation (1), with r in (1) modified to ΔR: $\Delta R = R_{max} - R_{\text{H}}$ where R_H is the separation needed for the IMBHs to merge within t_H. The results obtained are interesting: **12.5 Gigayears for 100M$_\odot$ IMBHs and 4.12 Gigayears for 10^5M$_\odot$**. This timescale is very useful can be used to predict when we might be able to observe an IMBH merger.

3 Black Hole Mass – Separation Relation

The relation between M and r, was calculated using Python, and the Python code is uploaded to Github. The Python program plots the BBH System $M - r$ relation. It was assumed that both the component BHs of the system have equal mass. The relation was computed for Stellar Mass Black Holes (StMBHs), Intermediate Mass Black Holes(IMBHs), and Supermassive Black Holes (SMBHs).

Calculating the relation while setting the timescale limit equal to the age of the universe (Hubble Time t_H), we can obtain a relation which gives the separation two binary black holes (assuming their mass to be 1 need to be in order to merge within t_H, which happens to be **0.46 parsecs**. This result can explain the low number of observations of

The relation obtained can be applied to Galaxy Mergers. Binary SMBH Systems are predicted by the hierarchical model of galaxy formation. If the masses of the SMBHs of the merged galaxy is determined, the relation can be used to find out the separation. These relations can be used to determine whether a BBH System will merge within a given timescale. Based on the plot, one can determine whether a pair of BHs in a BBH System will collide or not.

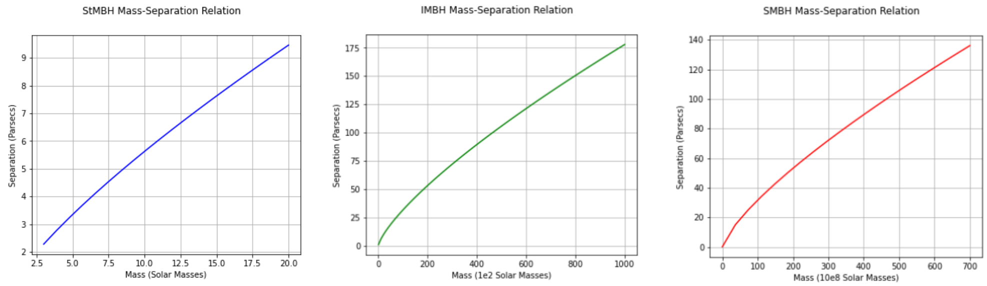

Figure 1. Binary Black Hole Mass-Separation Relation for StMBHs, IMBHs, and SMBHs.

4 Conclusions and Discussions

In this Research Note, merger parameters of Binary IMBH systems were calculated analytically. For a pair of IMBHs to merge within $t\ _H$, their separations need to be in the range **1.48$\times$ 10^{-5} for 100$M_\odot$ IMBHs to 0.002 parsecs for $10^5M_\odot$ IMBHs**. The separation at which GWs reach maximum amplitude for a pair of IMBHs is in the range 10^5 to 10^6 kilometers. The total energy of Binary IMBHs are approximately in the range of **2.99 $\times$ 10^{20} Yotta Joules to 1.7 $\times$ 10^{24} Yotta Joules**. Roughly 1% – 2% of this energy is converted into GWs. The time taken for a pair of IMBHs to reach the point of maximum GW amplitude is found to be **12.5 Gigayears for 100$M_\odot$ IMBHs and 4.12 Gigayears for $10^5M_\odot$**. These parameters can be used to predict timescales of Binary IMBH systems, energy released, and with various other demographics of these Binaries. We have also presented the relation between BBH system mass and separation and plotted it on using Python. The relation obtained can explain the low number of observations of some BBH mergers. Binary IMBH System calculations such as separation for collision within t_H and t_c were carried out and presented.

References

1. Alexander Rasskazov, Giacomo Fragione, and Bence Kocsis. Binary Intermediate-mass Black Hole Mergers in Globular Clusters. , 899(2):149, August 2020.

2. The LIGO Scientific Collaboration and the Virgo Collaboration. GW190521: A Binary Black Hole Merger with a Total Mass of 150 $M_\odot$. *arXiv e-prints*, page arXiv:2009.01075, September 2020.

3. Rizzuto et al. Intermediate Mass Black Hole Formation in compact Young Massive Star Clusters. *arXiv e-prints*, page arXiv:2008.09571, August 2020.

4. B.Abbott et al. The basic physics of the binary black hole merger GW150914. *Annalen der Physik*, 529(1-2):1600209, January 2017.