**Table 1.** Mass balance equations for four models

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| **Model** | **Governing Equations** | **Remarks** |
| PFM | $$\frac{dX}{dZ}+ΩNTU\_{od} \left(X-Y\right)=0$$ | Continuous phase mass balance |
|  | $$\frac{dY}{dZ}+NTU\_{od}\left(X-Y\right)=0 $$ | Dispersed phase mass balance |
|  | $$Z=0\rightarrow X^{0}=X^{in}=1$$ | Boundary Conditions at top of column (z=0) |
|  | $$Z=1 \rightarrow Y^{1}=Y^{in}=0$$ | Boundary Conditions at bottem of column (z=H) |
| BFM | $$\left(1+α\right) X\_{n-1}-\left(1+2α\right) X\_{n}+α X\_{n+1}-\frac{ ΩNTU\_{od}}{N}\left(X\_{n}-Y\_{n}\right)=0$$ | Continuous phase mass balance |
| $$\left(1+β\right) Y\_{n+1}-\left(1+2β\right) Y\_{n}+β Y\_{n-1}+\frac{NTU\_{od}}{N} (X\_{n}-Y\_{n})=0$$ | Dispersed phase mass balance |
| $$Z=0 \rightarrow \left\{\begin{matrix}X\_{0}+α(X\_{0}-X\_{1})=1\\Y^{0}=Y\_{0}=Y\_{1} \end{matrix}\right.$$ | Boundary Conditions at top of column (z=0) |
| $$Z=1 \rightarrow \left\{\begin{matrix}X^{N+1}=X\_{N+1}=X\_{N} \\Y\_{N+1}-β\left(Y\_{N}-Y\_{N+1}\right)=0\end{matrix}\right.$$ | Boundary Conditions at bottem of column (z=H) |
| ADM | $$\frac{dX}{dZ}-\frac{1}{Pe\_{c}}\frac{d^{2}X}{dZ^{2}}+ΩNTU\_{od}\left(X-Y\right)=0$$ | Continuous phase mass balance |
| $$\frac{dY}{dZ}+\frac{1}{Pe\_{d}} \frac{d^{2}Y}{dZ^{2}}+NTU\_{od}\left(X-Y\right)=0$$ | Dispersed phase mass balance |
| $$Z=0 \rightarrow \left\{\begin{matrix}\left(\frac{U\_{c}}{E\_{c}}\right) \left(1-X^{0}\right)=-\left.\frac{dX}{dZ}\right|\_{0} \\\left.\frac{dY}{dZ}\right|\_{0}=0 \rightarrow Y^{0}=Y^{out}\end{matrix}\right.$$ | Boundary Conditions at top of column (z=0) |
| $$Z=1 \rightarrow \left\{\begin{matrix}\left.\frac{dX}{dZ}\right|\_{1}=0\rightarrow X^{1}=X^{out}\\\left(\frac{U\_{d}}{E\_{d}}\right) \left(Y^{1}\right)=-\left.\frac{dX}{dZ}\right|\_{1} \end{matrix}\right.$$ | Boundary Conditions at bottem of column (z=H) |
| FMM | $$\frac{dX}{dZ}-\frac{1}{Pe\_{c}} \frac{d^{2}X}{dZ^{2}}+Ω\sum\_{i=1}^{N} NTU\_{od,i} \left(X-Y\_{i}\right)=0$$ | Continuous phase mass balance |
| $$\frac{dY\_{i}}{dZ}+\frac{NTU\_{od,i}}{g\_{i}} \left(X-Y\_{i}\right)=0 (i=1, 2,…N)$$ | Dispersed phase mass balance |
| $$Z=0 \rightarrow \left\{ \begin{array}{c}\left(\frac{U\_{c}}{E\_{c}}\right) (1-X^{0})=-\left.\frac{dX}{dZ}\right|\_{0} \\ \\ Y\_{i}^{0}=Y\_{i}^{out} \left(i=1, 2,…N\right) \end{array}\right.$$ | Boundary Conditions at top of column (z=0) |
| $$Z=1 \rightarrow \left\{\begin{array}{c}\left.\frac{dX}{dZ}\right|\_{1}=0 \rightarrow X^{1}=X^{out} \\\genfrac{}{}{0pt}{}{ }{Y\_{i}^{1}=Y^{in}=0 \left(i=1, 2,…N\right)}\end{array}\right.$$ | Boundary Conditions at bottem of column (z=H) |
| $$g\_{i}=\frac{f\_{i} u\_{i}}{\sum\_{j=1}^{N}f\_{j} u\_{j}}$$ | Dynamic drop size distribution |
| $$u\_{i}=\frac{d\_{i}}{d\_{43}} U\_{slip}-\frac{U\_{c}}{1-ϕ}$$ | Drop velocity |
| $$d\_{43}=\sum\_{i=1}^{N}\frac{n\_{i} d\_{i}^{4}}{\sum\_{j=1}^{N}n\_{j} d\_{j}^{3}}$$ |  |
| $$f\_{i}=\frac{P\_{i}^{3}}{\sum\_{i=1}^{N}P\_{i}^{3}}$$ | Volumetric drop size distribution |
| $$a\_{i}= \frac{6 ϕ\_{i}}{d\_{i}}$$ | Drop specific surface area |
| $$ϕ\_{i}=ϕ f\_{i}$$ | Drop holdup |