Supplementary Information

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# 1.MATHEMATICA MODEL OF GENERAL ASC

An individual ASC consists of  arcs. First of all, we locate the centers of these arcs at the origin of a global coordinate system as:

 

whereandare the radius and phASC angle of the $j$th arc, respectively. Besides, among four arcs satisfy a recursive relation:

 

where we set , andis the sign of the curvature of .It is worth noting that the signs of  and is opposite. Moreover, is a parameter that indicatesas the trajectory of a point moving from one end of the th arc to another, ranging from zero to, the central angle of the th arc. Then, we translate and combine these arcs from head to tail tangentially by imposing the recursive relation

 

 where is the translated $i$th arc. Finally, we sum these trajectories up and get the curve model as

 

where  ,  .

It is noted that  describes the shape of serpentine. It is worth noting that ,and determine the free-state shape of ASC. The angle between the velocity vector of ASC at and  is . For convenience, we define a standard left-handed ASC whose , and satisfy a group of symmetric conditions

 

For numerical simulation, we change into a discrete form as

 

where is the number of sample nodes locating at the th arc. In order to unify the distances between nodes distributed along the whole curve of ASC, we normalize  as

 

So, the discrete form of equation is updated as

 

where  ,  and .

The geometric parameters of standard ASC are:

 

and  for left-handed and right-handedness, respectively. Consequently, theof standard ASC is zero. We also derived another ASC whose by setting its  () as . The corresponding point object derived from standard ASC and this varied ASC is denoted as and.

# 2.THE VOLUMETRIC EXPANSION FROM ASC TO POINT OBJECT

This operation expands ASC with a cross-section. To avoid lateral buckling and visualize the twisting effect of , we choose rectangle as the shape of cross-section. First, we establish an orthogonal coordinate system  :

 

where  which turns the second real coordinate into a complex one. The directions of the three-axis () control the normal, width and thickness orientation of the local cross-section, respectively. First, we compact the points of discrete ASC in a  matrix  , where  :

 

By implying forward difference, we calculate the normal vector of cross-section on the th point as the th row of matrix :

 

Then, we characterizes the rotation from  , the direction of global  axis, to  as:

 

Which can be compacted the above as a  quaternion matrixwhere

 

Since  represents the average value of the rotation of the coordinate axis at every two adjacent points, we need to solve the rotation at each point by the spherical linear interpolation(Slerp) and get a quaternion matrix  as

 

Finally, we get the width and thickness vector of cross-section and through rotating fromand by , respectively.

 

We set the point on ASC as the origin of each local coordinate system. The vertices locating at each corner can be packed up and calculated as

 

where  packing up the vertices locating at inner top corner, outer top corner, inner bottom corner and outer bottom corner, respectively.and  are the width and thickness of the cross-section, respectively. Finally, we attach skins on these vertices by triangulation algorithm to fulfill a .

We define,a  matrix of , as:

 

In summary, we define as a set whose elements are and  as:

 

# 3.THE WORKFLOW OF GENERATING LINE OBJECT

This procedure generates two kinds of , thus, and . The notation denotes an open line object and consisted of *n* s with alternative handedness. Similarly, the notation  denotes a line object generated and configurated as the same way as  while be closed as a polygon. Without loss of generality, we start from basic configurations of the open resp. closed line object, thus a straight line resp. a regular polygon. From the above sections, we know there are eleven parameters to describe a point object（foundamental part）. For convenience, we choose  and as variables and define a function  as:

 

For a with a shape of straight line:

**Step1**: Generate standards with given number n and alternative handedness as:

 

where ,  ,  and satisfy the standard geometric conditions . We define a left-handedness  whose and verse vice.

**Step2**: Connect the above n s in turn end to end, resulting as a . We translate so that its head vertex coincides with the end vertex of . The th translation vector  is

 

, where is the coordinates of the head vertex of and is the coordinates of the end vertex of which have been iterated in the previous process.  is the number of discrete points of the centerline of . For  , And then the of is updated as

 

For a with a shape of a regular polygon. The numbers of the edges of the polygon is  :

**Step1**: Generate standards with given number n and alternative handedness as:

 

where ,  ,  and satisfy the standard geometric conditions .

**Step2**: Rotate so that its velocity vector at head vertex coincides with the one at the end vertex of . The th rotational quaternion  is

 

where is the conjugate of the quaternion which represents the rotation of the global frame on the head vertices of and is the quaternion which represents the rotation of the global frame on the end vertices of and have been iterated in the previous process. And then the  and  is updated as:

 

where  is the real(vector) part of a quaternion.

**Step3**: Connect the above n s in turn end to end, resulting as a , details are same as the one of deriving .

For common, we define matrix, containing the coordinates of all the point of one , as:

 

We define quaternion , containing the rotations of global frames on all the points of one , as:

 

In summary , we define as a set whose elements are and  as:

 

It is worth noting that the handedness of  in is alternative in order.

# 4.THE WORKFLOW OF GENERATING FACE OBJECT

Here, we introduce the concept of stacking points and stacking operations and then stack several and one  according to a given layout to form an . The stacking point is the point at which the curvature sign changes on the serpentine in each ASC. If the two arcs constituting the serpentine are the same (the radius of curvature and central angle of two arcs are equal), the stacking point is the geometric center point of the serpentine line. Correspondingly, the stack point order is an order in which each stack point of  is generated in sequence. Then, we do the stacking operation, which connects two line objects by aligning their given stacking points. In the generation of , stack operation is adding a cylinder with its axis perpendicular to the two given stacking points.

Besides, the layout design is customized under a constraint condition which should contain a polygon and no less than three open curves connected to it (a straight line is a particular case of an open curve). The layout adopted in this article is the topology of the tetrarchical unit. Hence, the number of open curves and polygon is 4 and 1, respectively. For a left-handedness resp. right-handedness of , the vector angle of the head and tail vector of an open curve closest to the x-axis in the first quadrant, where the coordinate system established with the geometric center of the layout as the origin, is less resp. more than 45 °.

We generate a left-handedness whose layout is the topology of the tetrarchical through two following sub-steps.

**Step1**: Generate four with the shape of a straight line as:

 

where are defined as the one in equation .

**Step2**: Generate one with the shape of a square as:

 

where are defined as the one in equation whose .

**Step3**: Generate four cylinders as the stacking pins connecting  and . The coordinates of these stacking points are:

 

where . resp.  are the total number of the points of the central line of resp. .

Before executing stacking operations, we need to generate four stacking pins. First, we draw a discrete circle at the origin of the global frame as:

 

where . We compact the coordinates of this circle as a  matrix:

 

We make this matrix as coordinates of circular points of the bottom facet of stacking pin. Next, we shift up these points with  (is the gap between two line objects, here we set it as 2mm) along z axis, resulting in the circular points of the top facet of stacking pin whose coordinates can be depicted as:

 

Consequently, we denote the initial position of the centers of top and bottom facets of stacking pin as:

 

We concatenate the above three matrixes as:

 

We repeat this process four times and get four matrixes of stacking pins as . Then we translate these matrixes so that their bottom centers coincide with corresponding stacking points of . So the is updated as

 

And also the coordinates of the stacking points of is updated as:

 

**Step4**: Rotates and stack them upon the top facets of the four stacking pins. The corresponding quaternion array(for left-handedness) is

 

And the resulting point matrix of are updated as:

 

Finally, we define the matrix as:

 

In summary , we define as a set whose elements are  as:

 

where for left- or right-handedness of the face object.

# 5.THE WORKFLOW OF GENERATING VOLUME OBJECT

We combine multiple s to form a polyhedron . The choice of polyhedron determines the boundary shape of each , and then gives the number of and the shape of . There is an adjustable space for specific parameters within the allowable range of the boundary. Moreover, the choice of polyhedron is not unique, without loss of generality, the polyhedron adopted in this article is a cube. The corresponding boundary shape of is a square generated in above step.

To obtain the facets handedness configuration ,which is the configuration of chirality arranged on each facet, which is a  vector, and for left- or right-handedness, we specified the serial number of each face of the cube (as shown in the figure). We set the origin of reference coordinate system at the geometric center of the cube. Each axis of the system is perpendicular to the faces of cube. The faces whose normal vector is the same as the positive direction of the x, y, and z axes are face-1, face-2, and face-3, respectively. Correspondingly, the remaining faces are face-4, face-5, and face-6, respectively. The above facets are noted as .

Each  has four connection points that can be connected to adjacent . In order to ensure the stability of the cubic structure, each needs to be connected to at least two adjacent , and the connecting points are symmetrical about its geometric center, resulting in two valuable schemes.

We generate a  whose shape is a cube through two following sub-steps.

**Step1**: Generate six  as a facets library whose geometric center locates at the origin of the global frame.

 

**Step2**: Rotate and translate the above s to form a cube. Here, we set the geometric center of this cube at the origin of the global frame and the normal vectors of its facets form a matrix as

 

The corresponding quaternion array(for left-handedness) is

 

where  and  . First, we transfer so that its geometric center

  coincidence with the origin. The body matrix is updated as:

 

Next, we apply affine transformation on each of these as:

 

where .

In summary, we define as a set whose elements are  as:

 

# 6.THE WORKFLOW OF GENERATING VARIED OBJECT

To start with, we generate the varied geometry of undergoing simply fixed condition locating at its first stacking point and displacement condition locating at its tail vertex. So, the problem statement of is



*Subject to*



with



where  .The auxiliary conditions are

(i) ;

(ii)  where ,is the radius and circular angle of the ASC of the th arc of the th in  under standard conditions, respectively;

(iii) , the given displacement condition constraining on the end vertex of ;

 (iv) The standard condition aforementioned in above section.

Next, we generate the varied geometry of undergoing four displacement conditions locating at its four tail vertex. The magnitude of the displacement conditions are equal. We divided into four parts, each of them is consisted of one corner of containing two s and one containing three s. The problem statement of is



*Subject to*



with



where  .The subscript  of  represent the ASCs of the five s belonging to and  represent the ASCs of the three s belonging to . The auxiliary conditions are

(i) 

(ii)  where ,is the radius and circular angle of the th arc of the th ASC in and under standard conditions, respectively. The subscript  of and represent the ASCs of the five s belonging to and  represent the ASCs of the three s belonging to .

(iii) , the given displacement constraints on the end vertices of .

(iv) The standard conditions aforementioned in the above section.

Finally, we generate the varied geometry of undergoing volumetric expansion/shrinking. As we have mentioned in the main text, we fixed the shape of all six s and adjusted the shape of the rest s to realize this variational design. We transfer the displacement of the tail vertex of the  of theto the angle of the rotation of the of the which can be simplified one input variable to control the whole shape of . We implemented this idea as following transferring:

 

where  is the position of the tail vertex of in under standard conditions. We implemented a harmonic search algorithmby MATLAB and solved the above problem statements.