Supplementary Materials to "The effects of pre-response before COVID-19 outbreak on strategic decision making"

The supplement contains the proof about the individual equilibrium. We can show the existence of Nash equilibrium and convergent stability.

Appendix A

Suppose that a proportion ϵ $(0 < \epsilon < 1)$ of the susceptible who acted with probability P and the rest of susceptible $1-\epsilon$ acted with probability Q. Then, the overall proportion of susceptibles who have acted, p, which implies $p = \epsilon P + (1-\epsilon)Q$, and $P \neq Q$. Note that all population is well-mixed homogeneous and individuals play the some strategy P.

Hence, the payoffs to individuals playing P in such a population is

$$E_P(P, Q, \epsilon) = E(P, \epsilon P + (1-\epsilon)Q) = E(P, p),$$

whereas the payoffs to individuals playing Q in such a population is

$$E_Q(P, Q, \epsilon) = E(Q, \epsilon P + (1-\epsilon)Q) = E(Q, p).$$

The payoff gain to an individual playing P strategy against Q strategy in this population is

$$\begin{aligned} \Delta E &= E_P - E_Q \\ &= -Pr - (1-P)\theta\varphi_p + Qr + (1-Q)\theta\varphi_p \\ &= (\theta\varphi_p - r)(P - Q), \end{aligned} \tag{1}$$

where $p = \epsilon P + (1-\epsilon)Q$. The payoff gain ΔE is a measure of the incentives that allow individuals to switch their strategy from Q to P [1]. The probability of infection φ_p that an individual becomes infected when COVID-19 occurs must decrease strictly with a proportion p of game players choose the action strategy.

Existence of Nash Equilibrium. We can show that $\Delta E > 0$ to find the Nash equilibrium in Eq. (1). If $r \geq \varphi_0$, then $r > \varphi_p$ for all $0 < p < 1$, so for any $0 < \epsilon < 1$ of Eq. (1), $\Delta E > 0$ for all $Q \neq P$ if and only if $P = 0$ (such that $P - Q < 0$ for all $0 < Q < 1$). Thus, $P^* = 0$ is the unique Nash equilibrium. If $r \leq \varphi_1$, then $r < \varphi_p$ for all $0 < p < 1$, so for any $0 < \epsilon < 1$ of Eq. (1), $\Delta E > 0$ for all $Q \neq P$ if and only if $P = 1$ (such that $P - Q > 0$ for all $0 < Q < 1$). Thus, $P^* = 1$ is the unique Nash equilibrium. If $\varphi_0 < r < \varphi_1$, there exist only one p^* such that $r = \varphi_{P^*}$. For all $Q < P$, we have $p = \epsilon P + (1 - \epsilon)Q < P$ for any $0 < \epsilon < 1$ and, similarly, for all $Q > P$, we have $p = \epsilon P + (1 - \epsilon)Q > P$ for any $0 < \epsilon < 1$. Hence, for $\varphi_0 < r < \varphi_1$, we always have $\Delta E > 0$ for all $Q \neq P$ if and only if $P = p^*$, so P^* is the unique Nash equilibrium such that $\theta\varphi_{P^*} = r$.

Convergent Stability Through the study of [1], given relative risk r, let P^* denote the associated Nash equilibrium. Suppose a proportion ϵ of the susceptible play strategy P (be closer to P^* than Q) while the rest play Q. That means, mathematically, $Q < P \leq P^*$ or $Q > P \geq P^*$ for $\epsilon \ll 1$. Given φ_p decreases with respect to p, if $Q < P \leq P^*$, $\theta\varphi_{P^*} > r$ for all ϵ, whereas if $Q > P \geq P^*$, $\theta\varphi_{P^*} < r$ for all ϵ. Hence, we have $\Delta E > 0$ and the Nash equilibrium is convergent stable.

When there is such a Nash equilibrium, P^* in the main text is p_{ind}.

References

[1] Bauch CT, Earn DJ. Vaccination and the theory of games. Proceedings of the National Academy of Sciences. 2004;101(36):13391–13394.