The supplementary material of "Quantum restricted Boltzmann machine is universal for quantum computation"

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The numerical simulation results for Eq.(45) (Fidelity)in the manuscript. In the proof of Theorem 4, we indicate that the overlap (fidelity) between 2L-QRBM $|\Psi_v(\boldsymbol{\theta})\rangle$ and the ground state $|\psi_0\rangle$ can be estimated by

$$F(|\Psi_v(\boldsymbol{\theta}^*)\rangle, |\psi_0\rangle) = 1 - \frac{\epsilon}{e^{-2(\widetilde{E_0}-\lambda^*)\tau+2\ln K}+\epsilon}. \tag{1}$$

One may doubt that, if the value $(\widetilde{E_0}-\lambda^*)$ is extremely small, e.g. $(\widetilde{E_0}-\lambda^*)=2^{-D}$, where D is a large constant, it takes exponential time for $F(|\Psi_v(\boldsymbol{\theta}^*)\rangle, |\psi_0\rangle)$ converging towards 1. Here, we propose some numerical simulation results to show that $\tau = \text{poly}(N)$ promises F converging to 1 rapidly. We simulate the scale of the quantum system ranging from $N=10$ to $N=30$, and we set $(\widetilde{E_0}-\lambda^*)=2^{-30}$, $K=2^{-N}$, and simulation results are illustrated as follows.

The simulation results show that parameter $\tau=\mathcal{O}(N)$ guarantees F converging to 1. This outstanding performance comes from the introduced 'phase shift' λ^* and the hidden nodes in 2L-QRBM.

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Author contributions

All authors contributed extensively to the work presented in this paper.

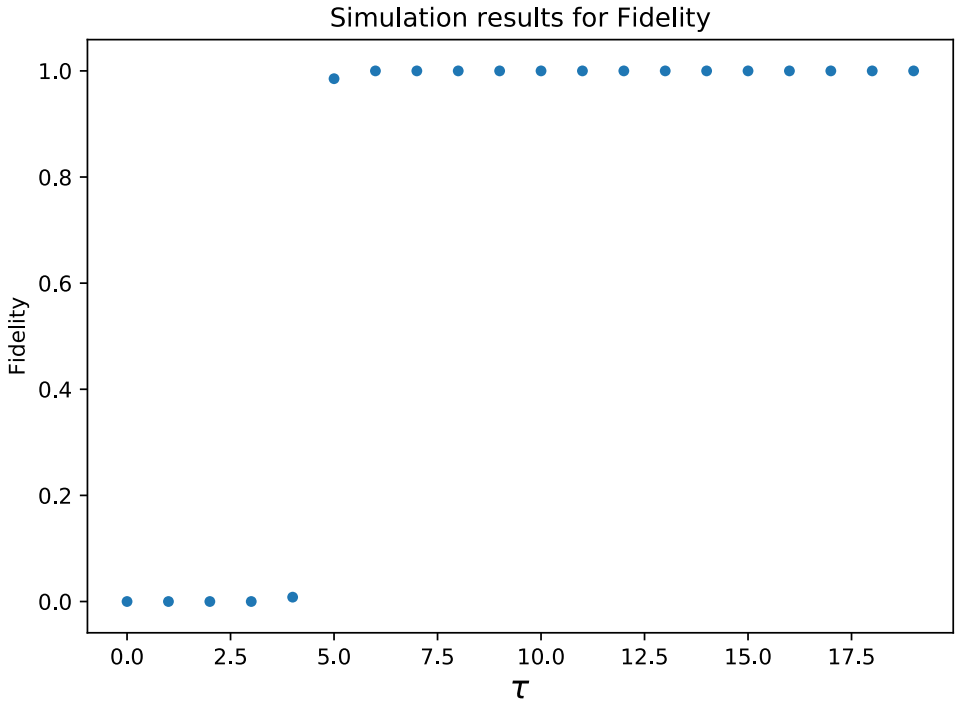

Figure 1: Parameters $N = 10$, $K = 2^{-10}$ and $(\widetilde{E_0} - \lambda^*) = 2^{-30}$.

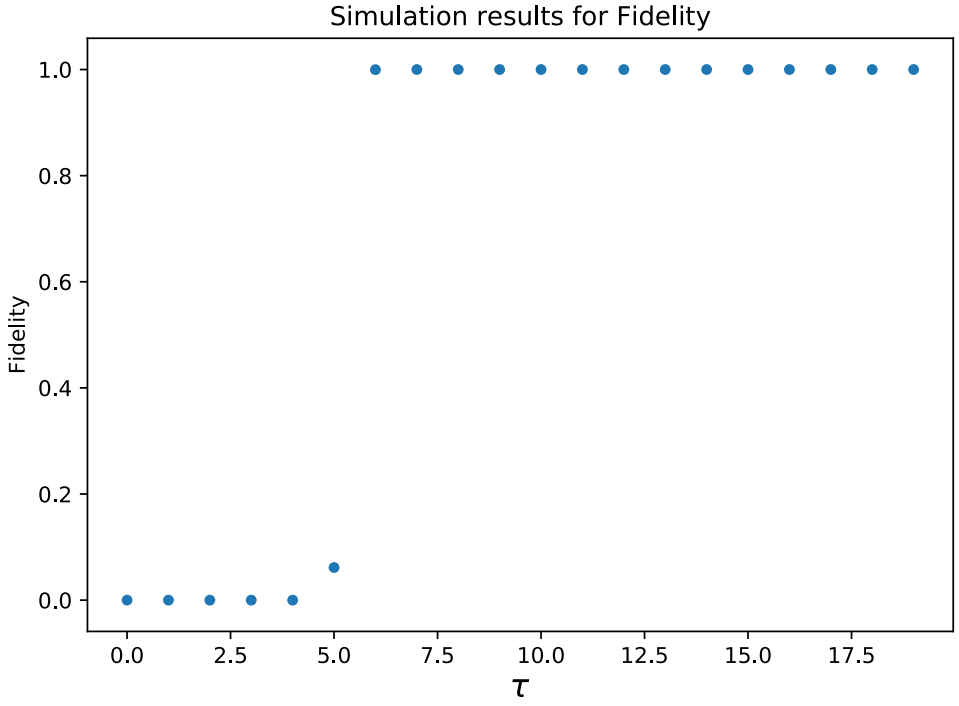

Figure 2: Parameters $N = 15$, $K = 2^{-15}$ and $(\widetilde{E_0} - \lambda^*) = 2^{-30}$.

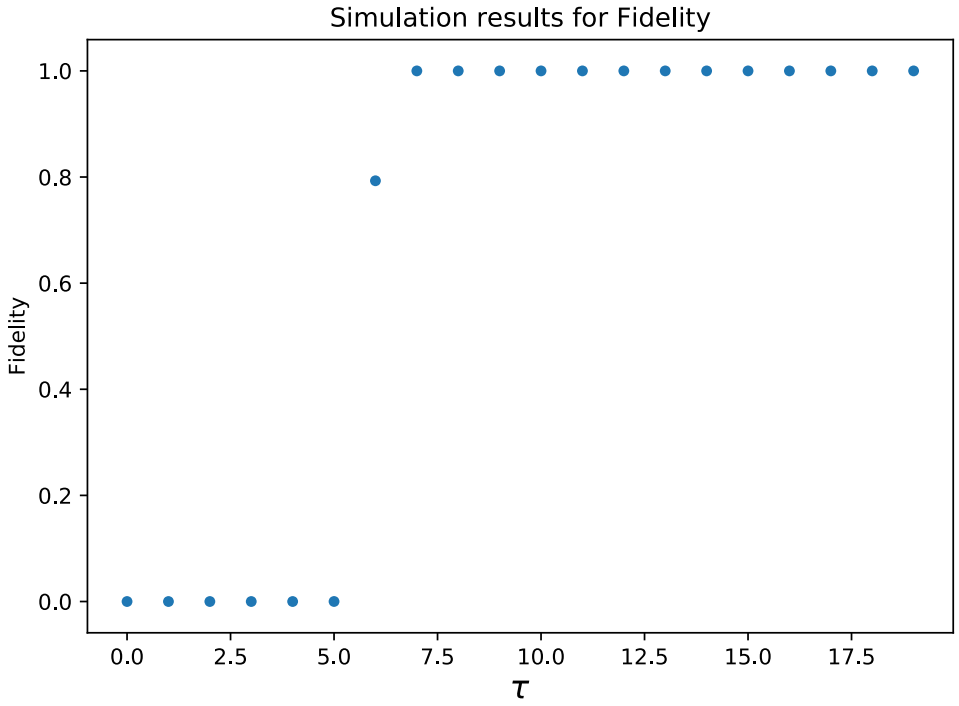

Figure 3: Parameters $N = 20$, $K = 2^{-20}$ and $(\widetilde{E_0} - \lambda^*) = 2^{-30}$.

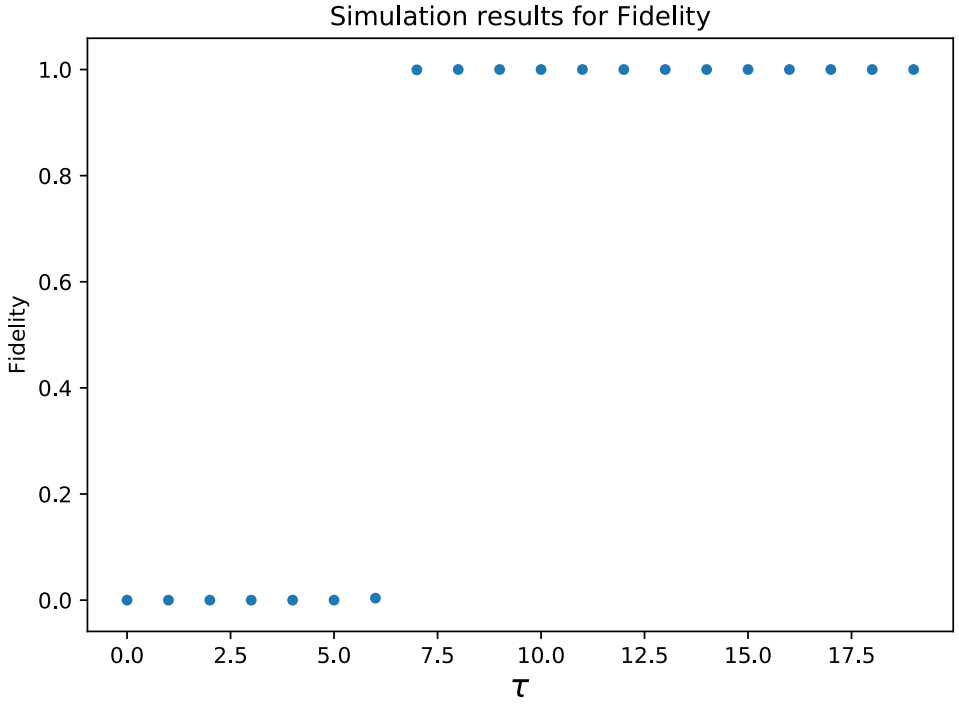

Figure 4: Parameters $N = 25$, $K = 2^{-25}$ and $(\widetilde{E_0} - \lambda^*) = 2^{-30}$.

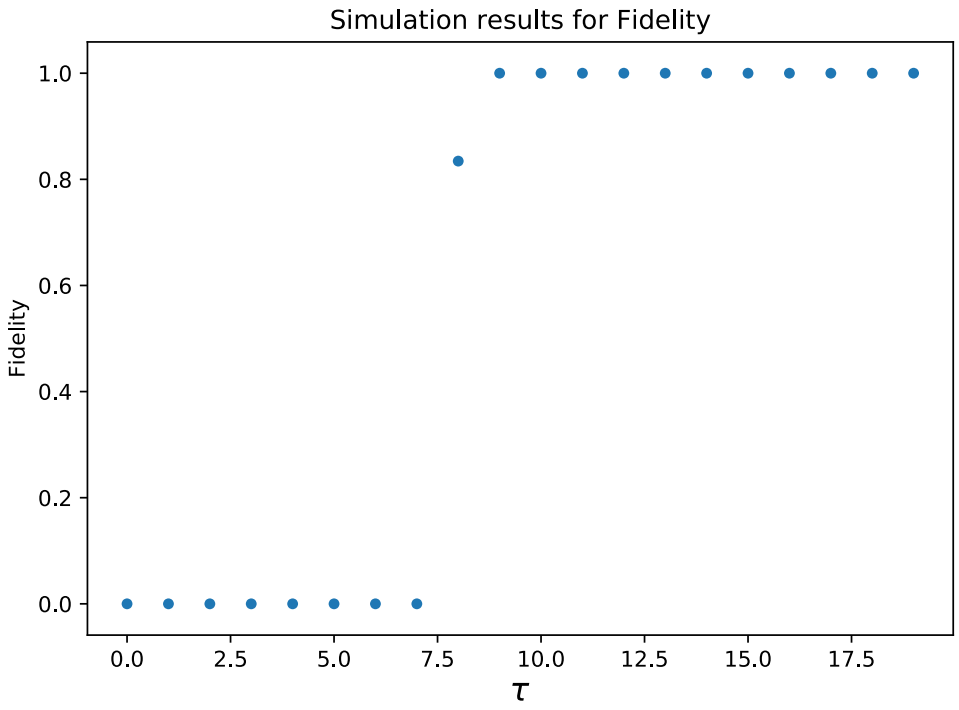

Figure 5: Parameters $N = 30$, $K = 2^{-30}$ and $(\widetilde{E_0} - \lambda^*) = 2^{-30}$.