Towards Communication UAV-Based: Improving Throughput By Optimum Trajectory And Power Allocation

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Research

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Towards Communication UAV-based: Improving Throughput by optimum Trajectory and Power Allocation

Sedighe Nasrollahi and Seyed Masoud Mirrezaei*

Abstract

It is predicted that the use of unmanned aerial vehicles (UAVs) in communication systems will be more extensive in future generations of wireless telecommunication networks, due to their facilitating advantages. In this paper, a UAV-based wireless communication system is considered in which a UAV is employed as a relay to connect two ground users. These two disconnected users make a communication pair. Our aim is to maximize the minimum achievable information rate for the communication link between the transmitter and receiver, by cooperatively optimizing UAV trajectory and transmitter and source power allocation. Motivated by the above, we formulate the optimization problem. The solving process is complicated because of the non-convexity of the formulated problem. To overcome this difficulty, we convert the main problem to some sub-problems by fixing some constraints and solve them with iterative algorithms such as successive convex optimization and reach the solution for the main problem. Simulation results show the capability of the proposed algorithm.

Keywords: UAVs; Relay node; Optimization; Power allocation; UAV trajectory

1 Introduction

By increasing mobile users in 5G and also the expansion of internet of things (IoT) applications in future communication systems, communication networks face a high amount of data traffic that is out of available terrestrial base station's capacity. To overcome this challenge and coverage improvement of communication networks, relaying is an effective technique [1, 2]. As most of the communication systems are wired, conventional relaying systems are based on static relaying. Due to cost reduction, device miniaturization, and other advantages, UAVs can be employed as relays in wireless networks. In addition to relaying, UAVs can be used in other applications such as power transfer, data collection, information broadcasting, cargo delivery, traffic monitoring, and emergency situations for example disaster management. These applications benefit from UAV's mobility and on-demand deployment and also the line-of-sight channel between UAV and considered ground transmitter or receiver [3]. According to [4] and 3GPP [5], for the moderate altitude of UAV for example more than 50 meters, the LOS probability is more than 90 percent. So, the channel may be less affected by shadowing, fading, and multipath propagation. Since the horizontal placement of the UAV can be changed, the channel between the UAV and ground nodes has variation. Motivated by this, UAV can get closer to the objective area and achieve a better communication channel with
more gain. As discussed in [6, 7], another parameter of a UAV’s placement is its altitude that influences the coverage probability and system sum rate. UAVs or drones are categorized into two types including fixed-wing and rotary-wing UAVs. The difference between them is that the speed of fixed-wing UAVs is higher and they have weighty cargo, but they must maintain uninterrupted ahead movement to hover overhead, and thus are not appropriate for stationary applications. Unlike this, rotary-wing UAVs, although having limited mobility and payload, are able to move in any direction and stay motionless in the air. Hence, the choice of UAV type entirely depends on the applications [8]. Based on practical applications, various research endeavors have been dedicated to UAV-aided wireless networks in last decade. In [9], a UAV-enabled data collection system was studied where a UAV is sent out to gather a given amount of data from ground terminals (GT) at known fixed locations. In this paper, the propulsion energy of the UAV was considered to be controlled due to its finite onboard energy. UAV-enabled wireless power transfer (WPT) system with trajectory design and energy optimization was investigated in [10]. This research was extended to simultaneous wireless information and power transfer (SWIPT) in cooperative communication systems in [11]. In this paper, the UAV’s transmission ability is powered entirely by radio frequency signal, transmitted from the source via time-sharing mechanism. Energy minimization is another important factor in designing UAV-enabled systems. Though initial attempts for designing energy-efficient UAV communication was appropriated to fixed-wing UAV enabled communication system by maximizing its energy efficiency in bits/Joule [12], authors in [13] investigated energy-efficient communication design for rotary-wing UAVs in a multi-user situation. Another application of UAVs is its role in the field of Internet of Things in which the device’s transmit power is poor so they are not able to communicate over a long area. In such a case, the UAV is a mean to collect the IoT data from the transmitter device and send it to its corresponding receiver [14]. In natural disasters, IoT coverage will be extremely affected, due to the destruction of communications infrastructure. In such cases, having an emergency communication network can be a crucial factor in getting rid of the status quo. UAVs can be applied to implement public safety scenarios to support disaster alleviation measures [15–17]. Coexistence between the UAV and device-to-device (D2D) communication network is another subject that has been addressed in [18, 19]. In these researches, the deployment of UAV as a flying base station for a desired geographical area is analyzed and the most focus is on the coverage probability. Energy harvesting is a new technology for increasing network lifetime. Unlike other devices, an energy harvesting device can receive energy from renewable sources in the environment and provide continuous power supply to wireless devices. In [20], the resource allocation problem for energy harvesting powered D2D communication underlaying UAV-assisted Networks was studied. The security issue of UAV networks is critical to have a protected communication. The authors in [21] considered boosting physical layer security by using mobile relaying system, in which a UAV acts as a mobile relay and flexibly regulates its location in order to enhance the desired wireless communication security. In [22–24], an optimization problem has been formulated to maximize the minimum average secrecy rate over all receivers by jointly optimizing UAV trajectory and transmit
power. Considering UAV as a mobile relay is an extensive issue of researches mentioned in [25, 26]. In [27–30], the scenario of multiple UAVs deployed as areal base stations and relays, respectively, was studied. In this paper, we consider one disconnected communication pair and a UAV ministering as a mobile relay to establish connectivity between the source and destination node. Our goal is to maximize the minimum information rate for our communication pair by cooperatively optimizing UAV trajectory and source and relay transmission power. The formulated optimization problem is difficult to solve due to its non-convexity. To make the problem tractable, we partition the main problem into two sub-problems and solve them by successive convex optimization technique. Then an overall algorithm is produced to solve two sub-problems in an alternate manner. The rest of this paper is organized as follows. In section 2 our desired UAV-relayed system model and the problem formulation are introduced. Section 3 proposes the iterative algorithm based on SCO. In section 4, we present the simulation results to verify the effectiveness of the proposed algorithm. Finally, we conclude this paper in section 5.

2 System Model and Problem Formulation

Consider the scenario that in a disaster (or any reason that destroys the communication infrastructure) area, two disconnected ground nodes that are at a distance of L meters apart, cannot communicate with each other due to long distance or severe blockage. Thus, a UAV relay is deployed to establish communication link between source and destination defined by A and B, respectively. We assume that A and B are located on the ground with known fixed locations and the flight height of the UAV from the ground is constant and equal to H in a period of T seconds. The horizontal coordinates of A and B and UAV are \( (0, 0, 0), (L, 0, 0) \) and \( (x(t), y(t), H) \), respectively. The analytical model of such a system is shown in Fig.1. Note that we assume this scenario for real-time applications such as building interconnection in emergency situations; therefore, the amplify-and-forward (AF) strategy is more suitable than decode-and-forward (DF) due to less complexity. For ease of analysis, the time horizon T is discretized into N equal time slots. The parameter N should be chosen large enough or in other words the elemental slot length be small enough, so that the position of UAV is approximately constant at any time slot. Thus, the trajectory of UAV over T can be rewritten as \( (x[n], y[n]) \) for any \( n = 1, \ldots, N \). But increasing N will bring more computational complexity. In fact, while choosing the value of N, we should consider a tradeoff between the accuracy and complexity [27]. Motivated by AF protocol, the UAV transmits data to B, as soon as received it from A. We partition each time slot into 2 hops. Sending data from source to the UAV happens in the first hop. So, the received signal at UAV in the nth time slot can be expressed as

\[
Y_U[n] = \sqrt{P_A[n]} h_{AU}[n] X[n] + Z_1[n]
\]  

(1)

Where \( P_A[n] \) is the transmission power of A. \( X_A[n] \) is the transmitted signal to UAV and \( Z_1[n] \sim N(0, \sigma^2) \) is the power of additive white Gaussian noise (AWGN) observed at UAV. \( h_{AU}[n] \) represents the LOS channel between UAV and source.
Considering the free-space pass loss model and ignoring the Doppler effect caused by UAV mobility, the channel power gain from source to UAV can be expressed as

\[ h_{AU}[n] = \alpha_0 d_{AU}[n]^2 = \frac{\alpha_0}{x^2[n] + y^2[n] + H^2} \] (2)

where \( \alpha_0 \) illustrates the reference channel power at the distance \( d_0 = 1 \) m. As we see the channel power depends only on the UAV-user distance. For the second hop, the UAV scales the received signal and broadcasts it to the destination with gain \( G[n] \) as follow

\[ G[n] = \sqrt{\frac{P_U[n]}{P_A[n] h_{AU}[n] + \sigma^2}} \] (3)

where \( P_U[n] \) is the transmission power of UAV. Thus the signal received at B can be written as follows

\[ Y_B[n] = G[n] \sqrt{h_{UB}[n]} Y_U[n] + Z_2[n] \] (4)

where \( Z_2[n] \sim N(0, \sigma^2) \) is the power of additive white Gaussian noise at destination. The following equation shows the channel gain of UAV-B link.

\[ h_{UB}[n] = \alpha_0 d_{UB}[n]^2 = \frac{\alpha_0}{(x[n] - L)^2 + y^2[n] + H^2} \] (5)

In the above expressions, \( d_{AU}[n] \) and \( d_{UB}[n] \) are the link distance between source and UAV, and UAV and destination at time slot \( n \). Considering (3) and (4), the...
corresponding signal-to-noise ratio (SNR) at the destination can be written as
\[
\gamma[n] = \frac{P_A[n]P_U[n]h_{AU}[n]h_{UB}[n]}{(P_A[n]h_{AU}[n] + P_U[n]h_{UB}[n] + \sigma^2)\sigma^2}
\]
(6)

The accessible information rate for the source to destination link at nth time slot can be expressed as
\[
R[n] = \frac{1}{2} \log_2(1 + \gamma[n]), \quad n = 1, \ldots, N
\]
(7)

The goal is maximizing the minimum of this rate by optimizing both source/UAV power allocation and UAV trajectory. By defining \( P \triangleq (P_A[n], P_U[n]) \) and \( W \triangleq (x[n], y[n]) \), the optimization problem can be formulated as
\[
(P1) : \max_{P,W} \min_{n=1}^{N} R[n], \quad n = 1, \ldots, N.
\]
(8a)

subject to
\[
\sum_{n=1}^{N} P_A[n] \leq NP_{A}, \quad \sum_{n=1}^{N} P_U[n] \leq NP_{U}
\]
(8b)

\[
P_A[n] \geq 0, \quad P_U[n] \geq 0, \quad n = 1, \ldots, N.
\]
(8c)

\[
(x[n+1] - x[n])^2 + (y[n+1] - y[n])^2 \leq \left(\frac{V_T}{N}\right)^2, \quad n = 1, 2, \ldots, N - 1
\]
(8d)

where \( P_{A} \) and \( P_{U} \) are the average maximum transmission power of source A and UAV. By defining V as the maximum permitted flying speed of UAV, \( \frac{V_T}{N} \) represents the maximum horizontal distance the UAV can fly in each time slot. Constraint (8d) implies that the distance that the UAV travel in one time slot shouldn’t exceed its maximum value. The max-min optimization problem is non-convex because the logarithmic objective function is not convex. In our proposed method we suggest an iterative algorithm to solve this problem.

3 Proposed Method

As we say the main optimization problem is non-convex and incurable to solve. In this part, we introduce two sub-problems and develop an iterative algorithm to solve them alternately to achieve the solution for the main problem. First, we solve the optimization problem with fixed UAV trajectory and obtain the source/relay power allocation and then repeat with fixed power allocation to obtain the optimal trajectory. Finally, the overall algorithm is proposed.

3.1 Power Optimization with Fixed UAV Trajectory

By assuming the UAV trajectory fixed, the constraints reduce to ones that are only on the power. Also, the varying channel is known due to the pre-determined trajectory. So, the main problem can be written as the following form
\[
(P1.1) : \max_{P} \min_{n=1}^{N} R[n], \quad n = 1, \ldots, N
\]
The objective function doesn’t change. So, the problem (P1.1) is still non-convex. To cope with this non-convexity, we utilize iterative approximation helping from successive convex optimization techniques. As mentioned in [31], any convex function is lower-bounded by its first-order Taylor expansion. Motivated by this, we maximize the lower bound of our objective function by optimizing the source and UAV’s power in each iteration. We can write the Taylor expansion of the transmission rate at $\frac{1}{\gamma[n]}$ as follow

$$f(x) \geq f(x_0) + \dot{f}(x_0)(x - x_0), \forall x \rightarrow$$

$$R_{l+1}[n] \geq R_0[n] - \frac{\gamma^2 \log_2 e}{2(\gamma[l][n] + 1)} \left( \frac{1}{\gamma_{l+1}[n]} - \frac{1}{\gamma[l][n]} \right) = R_{lb,l+1}, n = 1, \cdots, N \quad (10)$$

In the above expressions, $l$ and $l+1$ indexes introduce $l$th and $(l+1)$th iterations. From the equation of SNR calculation, it is obvious that $\gamma[n]$ is not convex with respect to $P_A[n]$ and $P_U[n]$. So, we can say that $\frac{1}{\gamma[n]}$ is a convex function of $P_A[n]$ and $P_U[n]$. Therefore, we can rewrite the above problem as

(P1.2): $\max_{P} \min_{W} R_{lb,l+1}[n], n = 1, \cdots, N.$ \quad (11a)

$$\text{s.t.} \sum_{n=1}^{N} P_{A,l+1}[n] \leq N\overline{P_A}, \sum_{n=1}^{N} P_{U,l+1}[n] \leq N\overline{P_U} \quad (11b)$$

$$P_{A,l+1}[n] \geq 0, \quad P_{U,l+1}[n] \geq 0, n = 1, \cdots, N. \quad (11c)$$

(P1.2) is a convex version of (P1.1) and can be efficiently solved by existing standard convex optimization tools such as YALMIP. The optimal solution of (P1.1) is also lower-bounded by the solution of (P1.2).

3.2 Trajectory Optimization with Fixed Power

In this part, the trajectory optimization problem is solved for any desired source or UAV power. This problem can be summarized as

(P1.3): $\max_{W} \min_{W} R[n], n = 1, \cdots, N \quad (12)$

$$\text{s.t.} \quad (8d)$$

Again, we face a non-convex optimization problem because of the non-convex objective function and should utilize successive convex optimization method to find its optimal solution efficiently. We define $\{x_l[n], y_l[n]\}$ and $\{x_{l+1}[n], y_{l+1}[n]\}$, the final location of UAV after $l$th and $(l+1)$th iteration. Since the non-convexity of the objective function is concerning $\{x[n], y[n]\}$, we present two new variables named $s_1[n] = \frac{1}{\overline{h_{AV}[n]}}$ and $s_2[n] = \frac{1}{\overline{h_{UV}[n]}}$. $\gamma[n]$ is convex with respect to $s_1[n]$ and $s_2[n]$ and
first-order Taylor expansion can be used to approximate it. Firstly, we rewrite the $\gamma[n]$ according to new variables as below

$$\gamma[n] = \frac{P_A[n]P_U[n]}{(P_A[n]s_2[n] + P_U[n]s_1[n] + \sigma^2s_1[n]s_2[n])\sigma^2}$$  \hspace{1cm} (13)

Using Taylor expansion, we have

$$\gamma_{l+1}[n] \geq \gamma_l[n] - C_l[n](s_{1,l+1}[n] - s_{1,l}[n]) - D_l[n](s_{2,l+1}[n] - s_{2,l}[n]) = \gamma_{lb,l+1}[n]$$  \hspace{1cm} (14)

where $C_l[n]$ and $D_l[n]$ are the gradients of the $\gamma[n]$ at new variables which can be calculated by

$$C_l[n] = \frac{P_A[n]P_U[n](P_U[n]\sigma^2 + \sigma^4s_2[l][n])}{(P_A[n]s_2[n] + P_U[n]s_1[n] + \sigma^2s_1[n]s_2[n])^2\sigma^4}$$  \hspace{1cm} (15)

$$D_l[n] = \frac{P_A[n]P_U[n](P_A[n]\sigma^2 + \sigma^4s_1[l][n])}{(P_A[n]s_2[n] + P_U[n]s_1[n] + \sigma^2s_1[n]s_2[n])^2\sigma^4}$$  \hspace{1cm} (16)

From $\gamma_{lb}[n]$ we have the lower bound of transmission rate as

$$R_{lb,l+1}[n] = \frac{1}{2}\log_2 1 + \gamma_{lb,l+1}[n]$$  \hspace{1cm} (17)

Now the convex optimization problem for trajectory optimization for given source and trajectory optimization scenario can be summarized as

$$(P1.4): \max \min_{\tilde{W}} R_{lb,l+1}[n], \quad n = 1, \ldots, N$$

$$s.t. \quad (x_{l+1}[n + 1] - x_{l+1}[n])^2 + y_{l+1}[n + 1] - y_{l+1}[n]^2 \leq \left(\frac{VT}{N}\right)^2, \quad n = 1, 2, \ldots, N - 1$$  \hspace{1cm} (18)

The overall algorithm which contains the solving process of two sub-problems can be given by bellow algorithm:

**Algorithm 1 Overall Algorithm**

1: Let $l = 0$ and initialize UAV trajectory and source/UAV power.
2: Repeat
3: Solve (P1.2) and output optimal source/UAV power.
4: Solve (P1.4) using the powers from the last step and output the optimal UAV trajectory.
5: Update $l = l + 1$.
6: Until the increase of the objective value is less than the preordained threshold value or maximum number of iterations has been reached.
7: Output $P_A[n], P_U[n], x[n]$ and $y[n]$ as results.
4 Simulation Results and Discussion

This section provides simulation results to verify the performance of the proposed algorithm. We assume the scenario that the distance between transmitter and receiver is $L=2000$ meters. The altitude of UAV is constant and equal to at 100 meters. The bandwidth of communication channel between source and destination is 20MHz. The noise power spectral density is $-100\text{dBm/Hz}$ and the value of $\alpha_0$ is assumed 30dB. The maximum speed of UAV is $60\text{m/s}$. Another assumption is that the UAV flies from $(0, 0, 100)$ to $(2000, 0, 100)$ in 100 seconds. The maximum average transmission power at source and UAV are the same and equal to 10dBm.

For the first scenario with fixed trajectory, we assume directional trajectory from $(0, 0, 100)$ to $(2000, 0, 100)$ with a constant speed of $20\text{m/s}$. Fig. 2 is the output of power allocation with fixed trajectory. It presents that, when the UAV travels close to the source, it should transmit data with much more power because the link distance to destination is more. In this case, the transmit power of A is less. In other words, the transmit power of A increases as the transmit power of UAV decreases while traveling from source to destination. As we see in Fig. 2 in the middle of the trajectory at time 50s, the power of source and UAV is equal to 10 Mw because of equal link distance.

![Figure 2 Power allocation with fixed trajectory](image)

Fig. 3 shows the achievable signal to noise ratio and its equivalent information rate for the optimized power with fixed trajectory situation. The optimized information rate is about $3.97\text{ bits/s/Hz}$.

In Fig. 4 we show the value of our objective function according to iteration numbers to verify that the maximization procedure of the objective value is satisfied. As it is shown the value of objective function goes from 3.927 to 3.97 which is the optimized information rate.

In the second phase, we check out the case that the power of A and destination are fixed and equal to $\bar{P} = 10\text{mW}$ in the whole time of flying. Fig. 5 shows the optimized x axis of the trajectory in 10 iterations. The UAV flies with its maximum speed to a place near the middle of the trajectory and hovers there for the longest time because the maximum information rate can be achieved there. The convergence of the output can be seen obviously. Both x and y axis of UAV for the last iteration
Figure 3 Optimized SNR and information rate for power allocation with fixed trajectory

Figure 4 Objective function value versus iteration number

are plotted in Fig.6. As we see the optimal y axis is equal to zero. The reason is that it is favorable that link distance reaches minimum and lower power consumption we have.

In the following figures the optimal information rate by the second scenario and the value of objective function for the optimization algorithm which is the lower bound of information rate, are plotted. The value of objective function $R_{th,t+1}[n]$ is equal to 3.485 which is lower than its equivalent rate in Fig.8. In Fig.8, The information rate increases while UAV flying from (0, 0, 0) and gets fixed, as the location of UAV is fixed in Fig.6 from 17s to 83s.

In the above figures, only one parameter has been optimized. In this part, the results of jointly power and trajectory optimization are presented. In Fig.9, the optimized trajectory with x and y axis is plotted. Like the previous part, the optimal y value is equal to zero due to minimizing link distance.
The UAV hovers for a long-time horizon from 17s to 83s in the position of 900.
meters. According to the expression for calculating information rate in (7), the information rate reaches maximum in this place with pre-determined $\bar{r}, \beta_0$ and $\sigma^2$. With this optimized trajectory, the power allocation scheme is like Fig.10. It is explainable that for the time that UAV hovers in the middle of its trajectory, the power of source node A and UAV are almost equal. Before this time period, the UAV transmit with more power because it is close to A and its distance to B is more. From 83 to 100s the transmission power of UAV decreases to 10dBm and the transmission power of A increases to 10dBm due to different link distances.

5 Conclusion
In this paper, a UAV-based relaying system, benefiting the UAV’s mobility is studied. The minimum information rate of considered wireless network is maximized via optimizing both the source/relay power allocation and relay trajectory. To this end, we propose two iterative algorithms for fixed trajectory and fixed power allocation scenarios and find the optimal solution for the lower bound of the maximum rate. According to the results of proposed methods, an overall algorithm is derived which jointly optimizes the power allocations and UAV trajectory.
Figure 10 Power allocation by jointly optimizing power and trajectory

alternately. Simulation results demonstrate that a higher system rate can be achieved by considering mobile relay compared to static relay which is operational for future real wireless networks in temporary situations. For future work, we can intend interference scenarios and also NLOS channel caused by long buildings.

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Abbreviations
UAV: unmanned aerial vehicle
5G: fifth-generation
IoT: internet of things
3GPP: 3rd generation partnership project
LOS: line-of-sight
GT: ground terminals
WPT: wireless power transfer
SWIPT: simultaneous wireless information and power transfer
D2D: device-to-device
SCO: successive convex optimization
AF: amplify-and-forward
DF: decode-and-forward
AWGN: additive white Gaussian noise
SNR: signal-to-noise ratio

Availability of data and materials
Not applicable

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The authors declare that they have no competing interests.

Consent for publication
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Authors’ contributions
All authors contributed in designing the proposed schemes and also writing and reviewing the manuscript. They approved the final manuscript.

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**References**


Figures

Figure 1 The analytical model of the UAV-relayed wireless system

Figure 2 Power allocation with fixed trajectory
Figure 3 Optimized SNR and information rate for power allocation with fixed trajectory

Figure 4 Objective function value versus iteration number

Figure 5 X axis of optimized trajectory with fixed power allocation
Figure 6 X and y axis of optimized trajectory with fixed power allocation

Figure 7 Objective function value versus iteration number

Figure 8 Optimal information rate for the trajectory optimization with fixed power allocation
Figure 9 Optimized trajectory by jointly optimizing power and trajectory

Figure 10 Power allocation by jointly optimizing power and trajectory