LCE Violation for the Relational to Quantum Transition

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Abstract

Quantization historically was never as much a problem as it was a solution to a problem and the problem was the failure of the classical material evolution statement. Under the axiomatic assumption that quantum theory is founded in Heisenberg's principle and Feynman's evolution we show that the QM path integral exists at the negation of the evolution of local conservation of energy(LCE) which in its presence fails with arbitrarily many interference terms. Along with LCE violation we uncover another GR-QM contradiction between the local arrow of time and the uncertainty principle. Every contradiction $\sim(p)\cap(q)$ is also a transition in p changing to q. The problem is GR is also caught up in an implication trail and cannot go through multiple parallel changes for LCE violation in presence of the QM path integral. To improve the recovery we go to an alternate projection of GR that has a set of independent frame invariant statements with a Lorentz invariant distinction of space and time.

1 Introduction

The Mathematical Ambiguity : The statement we would like to pay attention to is Local Conservation of Energy(LCE). It is the material evolution statement or the mechanics of GR and is written in the following equation,

$$\text{local conservation of energy}: \; D_\mu T^{\mu\nu} = 0$$

The equation in a steady state experiment of flat spacetime can be reduced to a zero 3-divergence which upon using Gauss-Ostrogradsky divergence theorem shifts to a flux integral of the form $\oint(T.n)dS = 0$. T here is the energy flux T^{0i} and now the statement says that the amount of energy going into a volume is the amount of energy coming out of that volume if there is no energy created inside. However, the defining material evolution statement of a theory must be unique and when a quantum evolution is already running the above constraint should not hold because the evolution equation is different. We show that with a destructive interference for photons on the opening face of a volume and a constructive interference on the closing face the equation for the closed surface flux integral moves to,

$$\oint(T.n)dS = \text{(interference terms)}$$

$$\implies \sim(\oint(T.n)dS = 0) \cap \text{(Quantum Interference)} \tag{1}$$

The notation means $\oint(T.n)dS = 0$ does not hold true here and quantum interference is true in its place when there is a quantum evolution statement. The work is centred about finding nature at this state of contradiction and understanding what implications could it hold for the Quantization problem of Gravitation. The reason why we believe that such a result could stand is because the classical material evolution statement already fails in the atom and in every quantum experiment where we need QM to develop a comprehension so nature does acknowledge the uniqueness of the mechanics statement of a certain realm. The transitions amongst them however are non-trivial.

The Underlying Contradiction : GR and QM are the two fundamental pillars of our understanding of nature. While the language, the vision and the reality of the two theories are in stark contradiction we try to understand exactly which two statements run into contradiction as we undergo a realm transition and can realm transitions be understood in terms of the underlying contradictions, this is still an open question. A mathematical contradiction,

$$\sim(p)\cap(q)$$

is where p and q are mathematical statements that are defined to be either true or not true with $\sim(p)\cap(q)$ meaning q is true here and p is not true. The contradiction $\sim(p)\cap(q)$ can also be read as negation of p in favor of q, since we are in physics we can also say violation of p in presence of q. In the context of understanding a transition one could say that p has changed to q and now q is true in place of p. However, things become more complicated when p and q are both known to be true but when q holds true p may not always be true but the truth of p can be recovered from q on an emergent limit. In that sense p covers a subset of everywhere q holds and beyond a certain point p does not hold anymore. The classical to quantum transition is hinged on such a change in nature and is written in,

$$\sim(\delta S(x_i, x_f) = 0) \cap (< x_f|U|x_i >= \int e^{\frac{i}{\hbar}[S(x^\mu(\tau))]_{x_i}^{x_f}} D(x^\mu(\tau))) \tag{2}$$

It is the contradiction of the classical equation of motion and the quantum path integral. When the path integral runs the equation of motion does not explicitly hold but becomes true at the stationary phase approximation. The equation of motion breaking down within the Plank limit to become the path integral and the path integral emerging as the equation of motion beyond the Plank limit is an important aspect of the classical to quantum transition. How nature does this contradiction in terms of the violation of the classical equation in presence of the path integral is what we are studying in this paper. Theoretically a proof of the above statement can be constructed from the QM path integral showing a nonzero amplitude for the particle to be found at x_3 while being found at $\{x_1, x_2\}$ before. The probability will either be 1 or 0 for the strict deterministic trajectory passing through $\{x_1, x_2\}$ solved from the second order differential equation at the stationary phase approximation. We are still working out the technicalities of the proof of such a theorem but for now we are studying this as a phenomena. A very mathematical example is,

$$\sim ((i\hbar\gamma^\mu D_\mu - m)\psi = 0) \cap (\mathcal{L} = \bar{\psi}(i\hbar\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}) \tag{3}$$

When we write the QED Lagrangian and the consequent path integral we do not explicitly assert the Dirac equation together because the zero that holds the Dirac equation to be true will remove the matter field information in the QED Lagrangian and because the zero is attained only at the stationary phase approximation with the Dirac equation as an Euler Lagrange equation. A very physical example would be in how Newton's laws are held at the Euler Lagrange equations on the stationary phase approximation which do not hold as you move to the QM path integral and the Schrodinger equation. Notice how the reality of the atom emerges from violation to the classical material evolution statement. The material evolution statement of GR is LCE and we believe underneath in the quantum realm the statement will not stand. Quantization of the mechanics in terms of $\sim$ (LCE) $\cap$ (QM) for the material evolution of GR is an entirely unexplored realm for Gravitation to exist in with many new possibilities and is a way of exploring the intersection of GR and QM. If done right this would be a major advance on the problem.

2 The Central Implication Trail

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 8\pi G T^{\mu\nu} \tag{i}$$

if $x = y$ then $D_\mu x = D_\mu y$

$$\implies 0 = D_\mu T^{\mu\nu} \tag{ii}$$

for a point particle,

$$\implies 0 = D_\mu v^\nu \tag{iii}$$

$$\text{If } v^\mu = t^\mu \tag{iv}$$

$$\implies 0 = D_\mu t^\nu \tag{v}$$

(i) is Einstein's equation, (ii) is local conservation of energy, (iii) is geodesic motion, (iv) is material imposition on frames and (v) is falling of frames. This is the most important implication trail in GR because this is where all the fundamental material evolution statements of the theory is implied and is the sequence of assertions that gives rise to the reality of GR in falling of frames in 1+3 spacetime. GR as a theory is very different from all other theories in this sense because of how all its fundamental laws are connected to one another in an implication trail, part of the reason why the theory was always so immovable because if you try to generalise anything anywhere you will lose the rest of the statements. The violation to (i) is the cosmological constant is an interesting possibility for a project in future.

The idea is to not just look at the equation as the equation but also in terms of everything that the equation implies. Pay attention to the right side of the equation, one can see that matter is coupled to the manifold with the gravitational constant G and if $G = 0$ the presence of matter would have no effect on the equation. The fact that matter does distort the manifold means there is a non-trivial interaction between the two which is interesting to note down because this hasn't been paid enough attention, the original intent of GR was to naturalize gravitation beyond an interaction. If you study how the earth falls around the sun however, you can argue that the earth falling along a geodesic has nothing to do with earth at all because anything in its place would have fallen the same way. However, the fall happens the way it does because the sun has distorted the manifold in the first place and gave it a curvature. The extend to which matter can distort the manifold with G is an important aspect of gravitation. Pay attention to how the fall of gravity in (iii) happens from the right side. The matterless equation $R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = 0$ will not be able to do geodesic motion because there is no way to go to (iii) from here. In order to check whether there is a fall in a matterless universe you would have to introduce a material particle in it which would naturally take the equation back to (i). Einstein's equation is magical on the right side in how it can pick up an ϵ test particle and take it into geodesic motion which again is independent of ϵ but it also says that falling is material in origin. These were some natural observations based on the contradictions between the equation with matter and without matter. The matter side is specifically important for us to pursue because all the questions that can be asked from QM are on the right side of Einstein's theory. All the QFT Lagrangians have QM built into them so QM is crucial.

3 Violation of Local Conservation of Energy with Quantum Interference

Theorem 1 The QM Path Integral exists at $\sim$ (ii).

Proof: The conflict of the material evolution statements between LCE in (ii) of GR and the QM path integral can be established if one works from the quantum double slit experiment for a 2-paths case of the QM path integral.

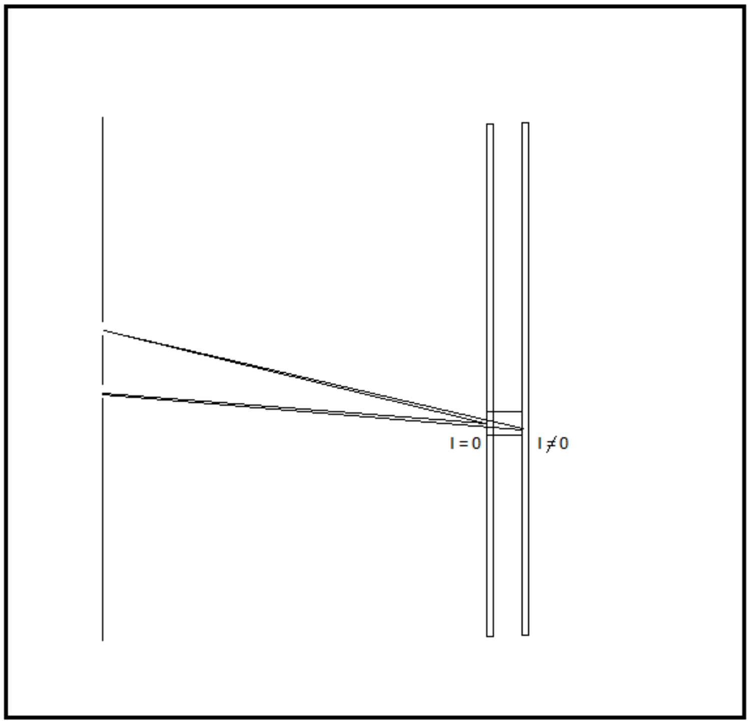

Figure I - The screen is being moved to create a volume from the region of destructive interference in QDSE for photons.

$$D_\mu T^{\mu\nu} = 0 \tag{ii}$$

This in flat spacetime,

$$\implies \partial_\mu T^{\mu\nu} = 0$$

$$\implies \int (\partial_\mu T^{\mu\nu}) d^4x = 0$$

For a steady state experiment with $\nu = 0$ for the energy flux,

$$\implies \int (\partial_i T^{i0}) d^3x = 0$$

By Gauss' divergence theorem,

$$\implies \oint (n_i T^{i0}) dS = 0 \tag{ii'}$$

The closed surface flux integral is nonzero for the volume marked in Figure I since the incoming flux is zero due to destructive interference and the outgoing flux is nonzero with no temporal derivative inside. This proves the theorem.

One could also work from Feynman's discussion[1] on the quantum double slit experiment at the introduction of QM. The underlying contradiction between the classical and the quantum versions of the two slit experiment is,

$$\sim (I = I_1 + I_2) \cap (I = |\psi_1 + \psi_2|^2) \tag{4}$$

The statement that should hold for classical particles is $I_1 + I_2 = I$ whereas the quantum statement that does hold for photons or electrons is $I = |\psi_1 + \psi_2|^2$ where $I_1 = |\psi_1|^2$ and $I_2 = |\psi_2|^2$. What is important here is that we see how the negation holds. $I = |\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1 = I_1 + I_2 + \psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1$ which further implies $I - I_1 - I_2 = \psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1$. Thus, the equality of the statement $I = I_1 + I_2$ is being violated by the interference terms $\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1$. One could construct a proof from here because GR will approach the classical statement from (ii), that for I_1 and I_2 photons entering the volume which shares a plane with the screen I photons will come out with $\oint (T^{0i} n_i) dS = (I - I_1 - I_2)A$, this upon asserting (ii) gives $I = I_1 + I_2$. Equation (4) then completes the proof.

The quantum double slit experiment(QDSE) is a cornerstone in scientific theory. The understanding of QDSE is in acknowledging that uncertainties of indistinguishable alternatives will always interfere. Theorem 1 plays on this contradiction between a singular trajectory and indistinguishable trajectories and speaks of how the truth of both these things are different and that if the paths are indeed indistinguishable then they are not singular and LCE is a deterministic evolution that implies singular trajectories. A singular trajectory in classical theory can be written in,

$$\ddot{x}^{\mu} + (x,p) = x^{\mu}(\tau) \tag{5}$$

Equation (5) says that the acceleration statement $\ddot{x}^{\mu}$ asks for the overdeterminate basis (x, p) to give rise to the strict deterministic trajectory $x^{\mu}(\tau)$ as a solution. The overdeterminate basis is in violation of Heisenberg's uncertainty principle and the acceleration statement is the equation of motion which is an emergence upon stationary phase approximation and that is why the singular reality of the strict deterministic trajectory that is a product of the two elements goes away in the underlying quantum realm when the actions are within the Plank limit. The understanding that nature is not able to hold on to strict deterministic trajectories fundamentally in the quantum realm in order to keep the uncertainty principle, that all the paths should indeed be indistinguishable comes from here.

In Figure I there are no photons entering that volume since the intensity detected on the opening face is zero but there are still photons coming out from the other side with a non-zero flux integral $\oint_S (T.n)dS$ and there by violating LCE because the only way you can create a non-zero flux integral across that 3-volume while maintaining LCE is if you place an active source inside the 3-volume with a temporal derivative. The classical interpretation is placed at $I_1 + I_2 = I$ so regions of destructive interference are beyond classical comprehension. Once the flux integral fails the divergence term inside the volume fails too and this happens because the mechanics or the material evolution equation here is different, thus one can see that (ii) does not hold anymore. $\oint_S (T.n)dS$ is $(\psi_1^{\dagger}\psi_2 + \psi_2^{\dagger}\psi_1)A$ which is deterministically held at 0 by (ii), thus (ii) will not hold in QM for QM is a material evolution of its own. It is the material evolution statements QM and LCE that are in direct contradiction which is the key observation here. However, conservation of energy still holds in QDSE, energy is conserved globally across the entire screen rather than locally across every volume. The nature of equations of the two is different because global conservation of energy is a constraint and local conservation of energy is a dynamics. The dynamics fails on the wake of QM but the constraint still holds. In QM conservation of energy finds its expression in the commutation of the Hamiltonian with itself and that is why the energy states are stationary unless they are interacted or operated on. The second half of the proof of Theorem 1 also shines light on how LCE is strongly reliant on additive energies and energies add like scalars in the thermodynamic regime. If you imagine pumping I_1 and I_2 heat from two sides into a gas chamber the internal energy of the gas has to increase by $I_1 + I_2$ according to LCE. Energies in Einstein's theory are additive too because local energy density is a Lorentz scalar. While most of the matter in the universe is thermodynamic beyond a certain scale and LCE is an important presumption to give rise to geodesic motion, matter does come in many forms and LCE will not stand as a fundamental material evolution statement from our analysis. We would like to call upon our experimentalist colleagues to see if $\sim$ (ii) can be verified in labs, it is going to be more challenging than just introducing a solid volume before the two slits because the question is not the amplitude of finding the photon on the closing face given that it is or is not found on the opening face, rather the question is whether LCE is the same evolution as the QM path integral and for that you have to let both the paths interfere on both the faces to see an undisturbed QM path integral violating LCE. (i) implies (ii) from the Bianchi identity so violations should occur to them together because nature has to be mathematically consistent. The work develops from the equation,

$$I - I_1 - I_2 = \psi_1^{\dagger}\psi_2 + \psi_2^{\dagger}\psi_1 \tag{6}$$

This is much more general than one would imagine because what is going on is a classical equation of motion that implies the constraint $\oint (T.n)dS = 0$ is getting violated by interference terms in presence of its path integral on the question of material evolution. This is going to be difficult for us to show on the geometry side of the theory because we would need the presence of quantum geometry. The problem of LCE violation with quantum interference naturally makes Quantization of GR mathematically necessary and this is an important position to drive the problem to in order for us to succeed at the end with gravitational path integrals. While QDSE is beyond GR understanding its basics illuminates a certain aspect of Quantization of GR, that at some point we will need the following change.

$$\sim (\text{LCE}) \cap (\text{QM Path Integral}) \tag{7}$$

When we speak of Quantization of GR we speak of Quantization of all aspects of GR, that means the geometry, the language, the vision, the symmetries and the matter. When we observe equation (7) we see how it speaks of the negation of a classical equation of motion in favor of a path integral for the material evolution of GR which indeed is like the change in a Quantization. However, this is only a build up for the much awaited Feynman sum over all possible Lorentzian manifolds. In the next sections we concentrate on the change that is asked for by (7) and frame the most natural problems that occurs on this line to see how they could carry the Feynman sums of GR in response.

4 Alternate Projections and Problems on the Line of $\sim$ (ii)

Statements, Bijections and Projections : Theories are written with mathematical statements that are defined to be either true or not true. While a singular statement can be an assertion that is axiomatically true everywhere in the universe it can also be an implication from another deeper statement that holds a more probable axiomatic position instead. Regardless a theory can be defined as a set of statements and its understanding relies on finding a closed set of axioms that when expands helps its implications to expand too. In GR the central implication trail is,

$$
\begin{aligned}
& R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T^{\mu\nu} && \text{(i)} \\
& \text{if } x = y \text{ then } D_\mu x = D_\mu y && \\
& \implies 0 = D_\mu T^{\mu\nu} && \text{(ii)} \\
& \text{for a point particle,} && \\
& \implies 0 = D_\mu v^\nu && \text{(iii)} \\
& \text{If } v^\mu = t^\mu && \text{(iv)} \\
& \implies 0 = D_\mu t^\nu && \text{(v)}
\end{aligned}
$$

(i) is Einstein's equation, (ii) is local conservation of energy, (iii) is geodesic motion, (iv) is material imposition on frames and (v) is falling of frames. While each of these is an individual statement the entire set can be considered a theory. Once two statements come together in an equation you get different fractions of a bijection. For example, an equation like (ii) = (iii) will naturally develop into an if and only if statement because (ii)=0 if and only if (iii)=0. Even if there is not a complete bijection but a mathematical relation would exist between the two, for example (ii) implies (iii). With three statements what we have is a projection. We find a new projection of GR that is also true,

$$
\text{(ii)} = \text{(i)}.\text{(iii)} \ , \ \text{(iii)} = 0 \ , \ \text{(i)} = \text{nontrivial and nonzero} \tag{8}
$$

GR exists at (ii) = (iii) = 0 and (i) = cosmological constant. In that sense the above projection is an improvement on the implication trail. We show that this projection develops into a set of independent assertions that are frame invariant and carry a Lorentz invariant distinction of space and time which can be easily generalized into QM and QFT.

The Implication Problem : LCE violation in presence of the QM path integral is the change that we are studying and how it could naturally be realized into a complete quantization. In terms of the implication trail it is,

$$
\sim \text{(ii)} \cap \text{(Indistinguishable evolution)} \tag{9}
$$

We have seen before that as indistinguishable evolution takes over (ii) in the implication trail fails with arbitrarily many interference terms. This is the diametrically opposite narrative from QM and If (ii) falls like that then the question is how do we write (i) such that gravity could hold at $\sim$ (ii), that is the question. If you pay attention to the first two lines, the covariant derivative on the right side takes you to a zero from a principle of physics that is LCE while on the left side it reaches zero from the Bianchi identity. GR exists at $D_\mu T^{\mu\nu} = 0$ identically and QM exists at $D_\mu T^{\mu\nu} =$ (ii) so a natural question is how do we write (i) such that (ii) or LCE is not implied and thus is aligned with QM, how do we write the gravitational statement independent of LCE? This is a natural implication problem on this line of $\sim$ (ii) and we show that GR does have a projection that does not explicitly imply LCE and can smoothly carry the gravitational constant into QM which (i) has fundamental difficulties to do because (i) implies (ii).

The Recovery Problem : The classical equation of motion does not explicitly hold when the quantum path integral runs which upon stationary phase approximation recovers the classical equation. If three classical equations get violated together because all of them are caught up in an implication trail, how do we recover all three of them back from the QM path integral which should exist at $\sim$ (iii) for all possible trajectories because the QM path integral will only recover (iii) and geodesic motion. In order to recover the entire theory from QM path integral you would need the entire projection that carries the QM path integral. To find that projection that recovers Einstein's theory at (ii) = (iii) = 0 and (i) = nonzero from the intractable position of (10) is the problem that we try to solve.

$$
G^{\mu\nu} - 8\pi G T^{\mu\nu} = \text{(i)} \ , \ D_\mu T^{\mu\nu} = \text{(ii)} \ , \ D_\mu v^\nu = \text{(iii)} \tag{10}
$$

We would have to make use of independent combinations that are movable from QM and is consistent with the QM path integral recovering (iii) simultaneously beyond the Plank limit. This is the recovery problem of Quantization of GR and we show that a way out of this is through a set of Feynman sums of GR that naturally acknowledges a series of quantizations for the change of equation (9) and solves the problem upto (i) held to an unknown violation.

5 The Relational to Quantum Transition

Theorem 2 The Uncertainty Principle exists at $\sim$ (iv).

Proof: (iv) is $v^\mu = t^\mu$ which is the equivalence of the velocity vector v^μ of the particle and the temporal vector t^μ of the frame attached to the particle. The reason why they are equivalent is because motion in your own frame is always temporal and the particle never moves with respect to itself. However this also means that the material frame always records itself at rest and at the origin of its own spatial projection which is $(x = 0, p = 0)$ and thus, is in violation of the Uncertainty Principle. This completes the proof and establishes the contradiction of,

$$\sim \text{(iv)} \cap \text{(Uncertainty principle)} \tag{11}$$

The Relational - Quantum Conflict : A major conflict between GR and QM is that GR is relational and QM is not. It is important that we frame this conflict. It is between two central equations of science, one is Heisenberg's uncertainty principle which says that a momentum state is translationally invariant in space and therefore is an indistinguishable superposition of all possible position states. Hence, their operators in QM would not commute in,

$$[x, p] = i\hbar$$

While on the other side the equation in relativity that is conflicted to this is the equivalence of the velocity vector of the particle and the temporal vector of the local frame which is attached to the particle. This happens because motion in your own frame is entirely temporal and the particle never moves with respect to itself so v^μ and t^μ are same. This is also called material imposition on frames and is the realization of the local arrow of time in equation,

$$\text{local arrow of time} : v^\mu = t^\mu$$

When you attach a frame on a particle, the particle will see itself at rest and at the origin of its own spatial projection which naturally fixes both the position and the momentum simultaneously at zero. This is also (iv) and is an important presumption in the foundations of relativity to the point that in order to ask 'how does reality seem from a moving car' a frame has to be attached to the car and the car has to be at rest with respect to itself. This is a contradiction by presumption on the local arrow of time and if resolved would change the realm of the theory like in classical to SR and classical to QM transitions. At realm transitions it is the presumptions of the parent theory that breaks down. On classical to QM transition the presumption of determinism breaks down, on classical to SR transition the presumption of absolutism breaks down. The presumption of attached local frames breaks down with,

$$\sim (v^\mu = t^\mu) \cap ([x, p] = i\hbar) \tag{12}$$

$[x, p] = i\hbar$ can be considered the first equation in the implication trail of QM in certain formalisms and $v^\mu = t^\mu$ is the second presumption in the implication trail of relativity after 'let the speed of light be constant' so we know the conflict between the two pillars of our understanding of nature is indeed strong. If you wish to give the conflict of the relational and the quantum an equation then equation (12) is it, the conflict between relativity and QM could not be traced further than this. When you ask a relational question of the form how does particle 1 seem from particle 2 you also ask how does particle 2 seem from particle 2 which runs into the contradiction of equation (12). At the end when they become electrons, to ask how does electron 1 at x_1 seem from electron 2 at x_2 is a question that has no meaning in QM because electron 1 at x_1 and 2 at x_2 is entirely indistinguishable from electron 2 at x_1 and 1 at x_2. When indistinguishability starts playing into the very identity of matter it becomes difficult to sustain relational questions. While SR has been successfully brought together with QM for QFT, SR does not have attached local frames but GR does and it is an important problem from QM because frames cannot be attached to quantum matter.

The Resolve from Feynman Sums : The way you remove the frames from matter for (12) is also the way you solve the problem of how to make GR independent of LCE. By going into the Feynman sum over all possible velocities before going into the Feynman sum over all possible trajectories for $\sim$ (ii) in (9). The transition here was,

$$\sim \text{(Overdeterminate basis)} \cap \text{(Complete basis)} \tag{Q_0}$$

The most natural way of doing this in Einstein's theory while preserving Lorentz invariance is to sum the theory over velocities because without spanning all possible velocities we cannot span all possible timelike trajectories in spacetime to sustain a meaningful QM path integral of all timelike paths. This leads to the gravitational statement,

$$G_{\mu\nu} t^\mu t^\nu = 8\pi G T_{\mu\nu} t^\mu t^\nu \text{ for all possible } t^\mu \tag{13}$$

The statement unearths the independence of velocity vectors which is perhaps one of the very few places where relativity and quantum theory are on the same side. While relativity specifies the velocity vector and speaks of how we have the freedom of performing a boost and reality shall transform with it, quantum theory says that once you are in the position basis the canonical momentum cannot be specified which for a falling particle will commute with velocity. This also resolves the conflict of equation (12) because when you sum the theory over velocities you stand at the invariant between the equation and the frame that registers the equation. After that the frame is never invoked anymore while Lorentz invariance is still maintained. The left side of this then becomes the spatial curvature in R_3,

$$R_3 = (g^{\mu\nu} - t^\mu t^\nu)(g^{\rho\sigma} - t^\rho t^\sigma)R_{\mu\nu,\rho\sigma} = R - 2R_{\mu\nu}t^\mu t^\nu = -2t^\mu t^\nu(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}) = -2t^\mu t^\nu G_{\mu\nu} \tag{14}$$

We had to figure out what should be the geometric tensor that goes to zero upon a covariant derivative to equate with the energy momentum tensor because local conservation of energy(LCE) is the natural classical evolution of matter. Since we are pursuing a quantum theory and do not get to make an explicit statement of LCE, the problem of why the Einstein tensor has exactly that structure opens again. The above lines of formalism justifies the structure of the Einstein tensor in terms of two spatial projectors on the Riemann curvature tensor. Einstein' field equation reads $G_{\mu\nu} = 8\pi G T_{\mu\nu}$. Contracting the stress-energy tensor with $t^\mu t^\nu$ should give the local energy density ρ. Hence,

$$R_3 = -16\pi G\rho \tag{1a}$$

This is a frame invariant statement that carries a Lorentz invariant distinction of space and time and speaks of Einstein's equation as the equivalence of spatial curvature and local energy density, both of which are Lorentz scalars. Because we summed the theory over velocities the question needed to raise the conflict from $v^\mu = t^\mu$ cannot be asked anymore. In order to resolve the implication problem we generate the implication trail by a differentiation,

$$\begin{aligned} D(G_{\mu\nu}t^\mu t^\nu) &= 8\pi G D(T_{\mu\nu}t^\mu t^\nu) \\ \implies D(G_{\mu\nu})t^\mu t^\nu + 2G_{\mu\nu}t^\nu D(t^\mu) &= 8\pi G D(T_{\mu\nu})t^\mu t^\nu + 16\pi G T_{\mu\nu}t^\nu D(t^\mu) \\ \implies 8\pi G t^\mu t^\nu D(T_{\mu\nu}) &= (2G_{\mu\nu}t^\nu - 16\pi G T_{\mu\nu}t^\nu)D(t^\mu) \\ \implies \text{(ii)} &= \text{(i).(iii)} \end{aligned} \tag{15}$$

If (i) is held to an unknown violation the most that (1a) can imply is that (ii) is true if and only if (iii) is true. Thus even if the mechanics statement fails in the theory you cannot go on negation to (1a) because it does not imply either (ii) or (iii). This is a way of making the gravitational statement independent of LCE. Thus (1a) cannot be contradicted from Feynman's evolution because it does not imply deterministic trajectories and it cannot be contradicted from Heisenberg's principle because the basis has been halved and we are asking a consistent position basis question from gravity. (1a) has no conflict with QM because it is a gravitational statement that is also a Feynman sum. Feynman sums typically run in such a way that a variable becomes indistinguishable in favor of an interference over all possible alternatives of that variable. When you half the basis (x,p) you acknowledge that one is a superposition of all possible alternatives of the other and that is how it is a Feynman sum. The combination (ii)=(i).(iii) is mathematically interesting because there is actually a true statement in nature that implies this and in terms of solving the problem it will help you to keep track of all the three statements together so that you can recover them back at the classical limit. The assertion of (1a) can be made stronger with another frame invariant statement that carries a Lorentz invariant distinction of space and time and is an equivalent statement of (iii)=0 in,

$$\delta\tau(x_i, x_f) = 0 \tag{2a}$$

This is geodesic motion in terms of the proper time distance being stationary during evolution between any two events. Upon a Lorentz transformation the geodesic trajectory might rotate but the law of stationary evolution in proper time distance will hold as is. The combined assertion will get to 0 = (i).0 where (i) is held to an unknown violation. Now (1a) and (2a) can easily go through the Feynman sum over all possible trajectories. Considering LCE will fall we impose global conservation of energy on the spatial surface as a constraint and normalize ρ to $m\psi^\dagger\psi$ in,

$$R_3 = -16\pi G m\psi^\dagger\psi \tag{1b}$$

Local energy density is gravitational mass energy rather than inertial mass energy so we thought it should be treated like a source term. Einstein's equivalence principle distinguished the two in terms of meaning before equating the two in terms of magnitude and we thought the distinction in the meaning should have a natural reflection on how it should be treated quantum mechanically. This is also a response to our earlier observation that local energy density is a Lorentz scalar and we need it to interfere. This is the Born interpretation in parallel with the QM path integral as per the first principles from Feynman's paper[2]. The Feynman sum over all possible timelike trajectories then is,

$$< x_f|U|x_i >= \int e^{\frac{ia}{\sqrt{\hbar G}}\int_{t_i}^{t_f}\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}dt} D(x^\mu(t)) \tag{2b}$$

One of the most beautiful aspects of Feynman's paper[2] is that it establishes the path integral over all possible trajectories as an equivalent statement to the Schrodinger equation. (2b) is a different Schrodinger equation of gravitation in terms of the Feynman sum over all possible timelike paths and it is much closer to the Schrodinger equation of QM because both of them are material evolution statements. (2b) when taken far beyond the Plank limit recovers geodesic motion in (2a). The fall of gravitation is independent of the mass of what is falling according to the equivalence principle so naturally the constant is the Plank length rather than the Plank constant. There is only one free parameter in the theory which is 'a' whose physical realization is unknown. Thus with [(1a),(2a)] breaking down to [(1b),(2b)] underneath and [(1b),(2b)] becoming [(1a),(2a)] beyond the Plank limit gives us Q_1 in,

$$\sim (\delta\tau(x_i, x_f) = 0) \cap (< x_f|U|x_i >= \int e^{\frac{ia}{\sqrt{\hbar G}}\int_{t_i}^{t_f} \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} dt} D(x^\mu(t))) \qquad (Q_1)$$

This is a QM Path integral that acknowledges the change of equation (7), when written in the [(1b),(2b)] combination violates LCE within the Plank limit and recovers LCE beyond the Plank limit because it recovers (iii) at the stationary phase approximation which is connected to (ii) in an if and only if statement by (1a) which is recovered from (1b) at the absence of interference. We return to classical gravity in 0=(i).0 which is a correct equation too, can (i) be recovered as the cosmological constant from quantum equations underneath we do not know yet but the difference $G^{\mu\nu} - 8\pi G T^{\mu\nu}$ is non-trivial in nature and this was a natural way of approaching the problem from QM. We would like to stop here because as of now we do not know if [(1b),(2b)] are true but [(1a),(2a)] are definitely true everywhere in the classical universe. If [(1b),(2b)] do turn out to be true then we can move to the second quantization Q_2 where R_3 gets written as a second order derivative over the spacetime field for an equation of motion of the spacetime field in (1c) while the differential version of (2b) gives rise to an evolution equation of the matter field in (2c). Both (1c) and (2c) when considered the Euler Lagrange equations will go into an underlying Lagrangian (1c)+(2c) for a Feynman sum over all possible field configurations. On an electrodynamic context as a parallel narrative it would be the same thing as getting to the QED Lagrangian post first quantization after acknowledging the indeterminism in the classical matter field of the Dirac equation(While it is debatable whether the Dirac equation has QM built into it for the classical matter field the key observations here involve noticing the Plank constant in its evolution, the Born interpretation in $\bar{\psi}\psi$ and the linear superposition possible with ψ even prior to the field quantization. The Dirac equation thus is a testament to how Quantizations do occur one on top of the other and that the semi classical theories like [(1b),(2b)] are just as important because it is the indeterministic Dirac field coupled to the electromagnetic field that essentially gets further quantized for Quantum Electrodynamics unlike the charged thermodynamic matter in classical electrodynamics.). So this was an example of how the Feynman sums of GR become truly natural upon realizing $\sim$ (iv)$\cap$(Uncertainty principle) and $\sim$ (ii)$\cap$(Indistinguishable evolution).

6 Conclusion

The QM path integral[2] is one of the very few path integrals in nature that is also an equation of motion, becomes an indeterministic equation of motion as a differential version for a field theory path integral underneath, becomes a deterministic classical equation of motion at the stationary phase approximation. It is because of this property that it is an essential cornerstone on any classical to quantum transition. We solve the implication problem and the recovery problem to an extend, an even greater problem was the relational - quantum conflict. While Einstein's theory is written from a frame an acknowledgment of the uncertainty principle in GR in terms of frame invariance can indeed detach us from frames. The Lorentz invariant distinction in space and time in the projection [(1a),(2a),(3a)] (where (3a) is local Lorentz invariance as the geometry of space and time, we discuss this on a subsequent paper) speaks of frame invariant realms to exist beyond the relational-quantum conflict that are relativistically aligned but not relational from a frame or frame dependent like whole universe questions that may seem just as frame invariant.

7 References

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