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Remy Maxime Mbala
The University of Ngaoundere https://orcid.org/0000-0003-1054-8312
David Jaures Fotsa-Mbogne (jauresfotsa@gmail.com)
The University of Ngaoundere https://orcid.org/0000-0002-9090-0429
Jean Michel NLONG
The University of Ngaoundere https://orcid.org/0000-0001-9094-5558
Olivier Menoukeu-Pamen
The University of Liverpool https://orcid.org/0000-0001-9306-3155
Jean Robert Kala-Kamdjoug
Catholic University of Central Africa https://orcid.org/0000-0002-5734-5902

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Optimization of Wi-Fi Direct average time to discovery: A Global Channel Randomization Approach

Rémy Maxime Mbala[a,b], David Jaurès Fotsa-Mbogne[b,c,d*], Jean Michel Nlong[a,b], Olivier Menoukeu-Pamen[e,f], Jean-Robert Kala-Kamdjoug[g]

[a]Département de Mathématiques Informatique, FS , Université de Ngaoundéré,

[b]Département de Mathématiques Informatique, ENSAI , Université de Ngaoundéré,

[c]Laboratoire de Mathématiques, Informatique et Applications, FS, Université de Ngaoundéré,

[d]Laboratoire d'Electricité, Signal, Image et Automatique, ENSAI, Université de Ngaoundéré,

[e]African Institute for Mathematcal Sciences, Ghana,

[f]Department of Mathematical Sciences, Institute for Financial and Actuarial Mathematics, University of Liverpool,

[g]GRIAGES, Catholic University of Central Africa, Cameroon

Abstract

The main objective of this work is to propose concrete time reduction strategies for discovery of Wi-Fi Direct in Android. To achieve our goals, we perform a fairly general mathematical modeling of the discovery of devices using Poisson processes. Subsequently, under asymptotic invariance hypotheses of certain distributions, we derive formulas for the expected time to discovery. We provide sufficient condition for fast convergence to an invariant distribution and determine key decision parameters (jumps intensities) that minimize the average time to discovery. We also propose a predictive model for rapid evaluation of these optimal discovery parameters. Experimental tests in an emulator are also conducted to validate the theoretical results obtained. A comparative performance study is done with some optimization approaches from literature. Compared with existing methods, the improvement of the average time discovery we obtained with the proposed method is above 98.34%.

Keywords : Wi-Fi Direct, time to discovery, semi-Markovian modeling, asymptotic optimization.

AMS Classification : 05C82, 90B15, 90B36, 90C30, 68M20.

1 Introduction

Wi-Fi Direct, also called Peer-to-Peer (P2P) Wi-Fi is an extension of Wi-Fi that allows connecting devices directly without going through a fixed access point. It has been standardized by the Wi-Fi Alliance in 2010 and has several advantages over other existing communication technologies such as Bluetooth or Tunneled Direct Link Setup (TDLS). Bluetooth has a data transfer rate of around $3Mbps$. This flow is significantly lower than that of Wi-Fi technologies whose speeds go up to $54Mbps$. In addition, the communication distance in Bluetooth is only about 10 meters while Wi-Fi Direct has a range of up to 200 meters outside[1] [12]. The 802.11z Wi-Fi standard known as Tunneled Direct Link Setup (TDLS) allows direct discoveries between devices, but by requiring that these devices be connected before hand to the same access point [9]. Wi-Fi Direct is present in all Android phones (since the Ice Cream Sandwich version, API 14), and other devices such as cameras, printers, TV sets. Several file transfer applications (SuperBeam,

*Corresponding author Address: email: jauresfotsa@gmail.com, P.O. Box 455, ENSAI, The University of Ngaoundere

[1]https://www.wi-fi.org/knowledge-center/faq/how-far-does-a-wi-fi-direct-connection-travel

Wi-Fi Shoot or HitcherNet) and screen duplication from one phone to another (Miracast) already use Wi-Fi Direct as communication technology. However, the Ad hoc Wi-Fi has not been able to impose itself on the market since it has several drawbacks in view of current requirements (lack of efficient energy management or quality of service as noted for example in [8]).

According to its technical specification [1], Wi-Fi Direct uses the principle of communication grouping where one of the device plays the role of owner and the others are customers and are all connected to him. The formation of a communication group takes place in five steps: (1) the discovery of device, (2) the discovery of services, (3) the negotiation of the role of owner, (4) the setting up of security parameters and (5) the configuration of IP addresses. The implementation of Wi-Fi Direct in the Android operating system unfortunately presents several limitations, in particular in terms of discovering of device. In fact, mainly empirical studies carried out to evaluate the Wi-Fi Direct technology on Android, show that the discovery of devices step takes more time in the group formation process presented above [3, 7, 8]. A long time to discovery and high latency have an impact on the quality of service, specifically those that have time transmission constraints. Connection cuts in mobile networks can lead to high latencies. Thus, reducing the time to discovery of devices and maintaining low latency during an ongoing session are critical problems to be resolved in order to optimize the group formation process of Wi-Fi Direct, and thus improving the quality of service.

Several works have been carried out on the study of the discovery process in technologies 802.11 (known as Wi-Fi) in order to optimize the scanning times to discover access points to which the device can connect. Solutions and strategies have been proposed to reduce the overall latency of the scan when a device goes from one point of access to another (handover). Shin et al. suggest in [15] the use of the binary mask channels to decide which channel to scan. This mask is updated when the device passes from one access point to another. The mask is initialized to 1 for all channels, to show that all channels are to be scanned. During a passage, the device builds a new mask for the next step. This mask contains the value 1 for non-overlapping channels and those on which the response probes were received. It contains 0 for channels on which there is no activity during the previous scan. So only the channels marked with 1 are scanned if a response probe has not been received in these channels. This technique reduces time to discovery of about 43% relative to the method described in the specification. It however has a drawback because the the masks are built incrementally whenever the number of passages between access points increases. Other work has focused on reducing values of the minimum and maximum time of reception of the probes on a channel during the scan. The authors in [17] set values for these times. Unfortunately, this technique does not guarantee that the process of discovery will always unfold successfully. This can work in certain scenarios and lead to discovery failures if the set of values does not allow the handshake. In [4], the authors propose an adaptive technique which suggests to randomly change the channels following two sequences (the first for non-overlapping channels and the second for other channels), and to adapt the times on a channel according to the information collected on the previous channel. Clearly, one increases the scanning time of the next channel when an access point was not found on the current channel, and the scanning time is reduced otherwise. The limits of this work come mainly from the fact that the results obtained depend on the deployment of the access points and the adaptation function used.

Most of the works done on Wi-Fi Direct are aimed at improving the training procedure groups in order to allow the formation of real Ad hoc mobile networks [2, 5, 6, 11, 12, 14]. To our knowledge, there is limited literature on models that reduce time to discovery in Wi-Fi Direct. Sun et al. have analyzed the discovery process between two Wi-Fi Direct devices [16]. They proposed and validated via simulations on NS-3, a model of the discovery process based on Markov chains. Sun et al. in addition performs a 72% reduction in Wi-Fi Direct time to discovery through the Listen approach Channel Randomization (LCR) consisting of randomly selecting the listening channel from the three non-overlapping social channels. The LCR approach focuses optimization only on listening and not on the scan. In addition, it does not take into account the implementation of Wi-Fi Direct on Android and does not give any specific recommendations for implementation. The main objective of this work is to reduce latency (to prevent reconnections) and the time to discovery of Wi-Fi Direct in Android, in order to meet the time requirements of real-time applications. In order to reach our objectives, we proceed to a Semi-Markovian modeling of the discovery process via Poisson processes whose parameters are then optimized.

The rest of the work is organized into four main sections. In section 2, we review the principles and standards for discovery using Wi-Fi Direct. In sections 3.1 and 3.2, we propose different mathematical models for the discovery process whereas Section 3.3 deals with the optimization of discovery parameters through an explicit process. The parameters are calculated for some examples and simulations studies are carried on in Section 4. Section 5 is devoted to the conclusion and some immediate perspectives.

2 Literature review on discovery in Wi-Fi Direct Android

2.1 Discovery mechanism in Wi-Fi Direct Android

The Wi-Fi Direct device discovery procedure consists of scanning all traditional communication channels of Wi-Fi ($IEEE$ 802.11) and the listening phase. Formally, a device that wants to discover devices in its vicinity must first scan all channels according to the algorithm used in 802.11 standards to identify peer-to-peer groups already formed [9]. If groups are found, the device will try to integrate one of the groups chosen by logging on to the owner. Otherwise, he will look for other Wi-Fi Direct devices that are in the discovery phase. In this phase, the device alternates between listening and searching states [1]. In the search (or scan) state, the device randomly chooses one of the following three types of scan:

- The P2P SCAN SOCIAL where only the three social channels (1, 6 and 11) are scanned;
- The P2P SCAN SPECIFIC for which the scan is done on a single channel chosen randomly on all channels;
- The P2P SCAN SOCIAL PLUS ONE where the scan is done on all three social channels plus an extra channel chosen at random from all the channels.

Based on the type of search, the device will scan each channel sequentially by transmitting a request probe, and waiting for to receive response probes. This waiting time depends on the Wi-Fi pilot used by the device. It can vary depending on the channels (MadWiFi[2]) or can be fixed for all channels (ath5k[3]). In the listening state, the device listens for a random time during which it can receive a request probe and send the corresponding response probe. The listening channel is chosen at random from the three social channels (1, 6 and 11) at the start of the discovery phase and remains unchanged until the end of this phase. The duration of each listening state is given by the formula:

$$T_l = (\alpha + R\ modulo\,((\alpha - \beta) + 1)) \times 100\ Time\ Units^{4} \tag{1}$$

with $R \in \mathbb{N}$, $\alpha = 3$ and $\beta = 1$ respectively denoting the maximum and minimum length of the interval of times to discovery. According to the values of α and β, T_l should be 100, 200 and 300 Time Units (TU) independently of the value of R. Two devices can be with discovered if and only if they are on an appropriate channel with one in the listening state and the other in the searching state for a time long enough to allow the "handshake". Discovery can take a relatively long time for several reasons:

- The progressive scan mode can prolong the search time and prevent any synchronization;
- Devices can choose busy channel or channels with poor quality and hence making it difficult to receive probes;
- The Random choice of listening and searching times.

[2] http://madwifi-project.org
[3] http://wireless.kernel.org
[4] $1\ Time\ Unit = 1024\mu s$

2.2 Standardized discovery parameters

In Wi-Fi, there are two modes of access to medium: the DCF mode (Distributed Coordination Function) which allows equitable access to the radio channel without any centralization of the management of access, and the PCF mode (Point Coordination Function) in which access to the media is managed by a base station (the access point). The PCF mode is made for wireless infrastructure networks while DCF mode is used in wireless networks without a fixed infrastructure (also known as ad hoc networks). In the DCF mode (which interests us), the standard defines temporal variables called IFS (Inter Frame Space) which characterizes the time elapsing between sending frames. We distinguish the Short Inter Frame Space (SIFS) designating the time between the reception of data and sending the corresponding acknowledgment (T_{SIFS}), and the Distributed Inter Frame Space (DIFS) which is the waiting time of a station wanting to start a new transmission (T_{DIFS}). Since the radio channel (air) is shared, the technology implements the Carrier Sense Multiple Access with Collision Avoidance (CSMA / CA) algorithm [9], which aims to avoid frame collisions in the channel. In addition to the two time values DIFS and SIFS, mobiles that want to transmit choose a backoff randomly expressed in number of time slots (T_{Slot}), the unit of which is $20\mu s$. The number of time slots (N_{CW}) is chosen according to a uniform law in an interval called Contention Window (CW) which is by default $[0, 31]$. So the backoff time ($T_{Backoff}$) is therefore a random number drawn between 0 and $620\mu s$, on average $310\mu s$. Once this draw is made, as long as the channel remains free, a mobile decrements at each time slot its T_{Slot} backoff times up to 0. The first to reach a zero backoff emits. As soon as the others detect an activity on the channel, they stop the decrementing of their backoffs to resume it only in case of channel release (deferring period). According to the CSMA/CA algorithm, the average time to handshake in an ideal environment (without interference and without noise) is given by the following relation:

$$\tau_0 = 2T_{DIFS} + 2T_{Backoff} + T_{REQ} + T_{RESP} + T_{SIFS} + T_{ACK} \tag{2}$$

with $T_{Backoff} = N_{CW}T_{Slot}$ and

- T_{DIFS} is time of Distributed Inter Frame Space (DIFS);
- T_{SIFS} is the time of Short Inter Frame Space (SIFS);
- $T_{Backoff}$ is the Backoff time between 0 and $620\mu s$;
- T_{Slot} is the time slot fixed at $20\mu s$;
- N_{CW} is a random number for the backoff count contained in the interval $[0, 31]$;
- T_{REQ} is the travel time of the request probe in the radio channel;
- T_{RESP} is the travel time of the response probe in the radio channel;
- T_{ACK} is the acknowledgment journey time in the radio channel.

The values of T_{DIFS}, T_{SIFS}, T_{Slot} and N_{CW} depend on the mode of access distributed to the media while the values of T_{REQ}, T_{RESP} and T_{ACK} depend on the quality of the channel, the speed of the standard Wi-Fi used, and the size of the request, response and acknowledgment packets respectively. Wi-Fi Direct relies on MAC, Physical, and device standards for accessing the media. If the device is certified by several standards, it is the most recent (Wi-Fi g) which will be used. In general, the sizes of the request, response and acknowledgment probes are respectively between 37 and 76 Bytes, between 37 and 46 Bytes, and 14 Bytes. The following Table 1 summarizes the different values mentioned above according to the most present Wi-Fi standards in device.

Table 1: Norm specifications

Norm	802.11a,g (Wi-Fi a,g)	802.11b (Wi-Fi b)
Flow rate ($Mbps$)	$6, 9, 12, 18, 24, 36, 48, 54$	$1, 2, 5.5, 11$
Backoff time (ms)	3.1×10^{-1}	3.1×10^{-1}
Time of DIFS (ms)	3.4×10^{-2}	5×10^{-2}
Time of SIFS (ms)	1.6×10^{-2}	10^{-2}

3 The Global Channel Randomization (GCR) discovery

The problem is that of the discovery of two devices communicating by direct Wi-Fi. Each device has to randomly choose a listening or search channel among n channels and stay in one of these two modes for a certain (random) time. The times to discovery on the channels are subject according to their quality of these, random laws to be specified. There is discovery if one of the devices is listening and the other is in search mode, all on the same channel for a fairly "long" time according to the quality of the channel. A state of the system is then an $n+5$-uplet $(s_1, s_2, c_1, c_2, \tau_0, \tau_1, \ldots, \tau_n) \in \{0,1\}^2 \times \{1, \ldots, n\}^2 \times \mathbb{R}_+^{n+1}$ where

- $s_i,\ i = 1, 2$ denotes a boolean specifying whether the device i is listening or not;
- $c_i,\ i = 1, 2$ indicates the channel chosen by the device i;
- τ_0 denotes the time spent under any given configuration (s_1, s_2, c_1, c_2);
- $\tau_i,\ i = 1, \ldots, n$ denotes the time required to discover each other on channel i (ie the time until first device is in listening mode whereas device two is in search mode all devices being on channel i).

In a simple way, the transition from one state to another can be modeled by a semi-Markovian process $(S_1, S_2, C_1, C_2, T_0, T_1, \ldots, T_n)$ where T_0 is the random variable corresponding to the time spent under the configuration (S_1, S_2, C_1, C_2) while each other T_i denotes the random variable corresponding to the time to discovery on channel i. The Markov process (S_1, S_2, C_1, C_2) is piecewise constant and its transition matrix P has zeros at diagonal and is time inhomogeneous. We assume that the two devices operate with independent and identically distributed choices (iid) then, the sub-processes (S_1, C_1) and (S_2, C_2) are also iid. In addition, the jump times of S_i and C_i can be considered independent conditionally to the time of the last jump of (S_i, C_i). The purpose of this section is to model the discovery process in different scenarios, and in particular to express the average times to discovery based on certain decision parameters. These estimates are essentially made under asymptotic invariance assumptions of certain distributions.

3.1 Modeling of jump times

In this section, we assume that the discovery on a channel i requires in addition to the constant minimal time to handshake $\tau_{0,i}$, a random time $\delta T_i \sim \mathscr{E}(\lambda_{\tau_i})$ due to the quality of the channel (ie $T_i = \tau_{0,i} + \delta T_i,\ i = 1, \ldots, n$). This case corresponds to the general situation mentioned in the section III.C of [16] except that we consider a continuous space of time. A continuous space of time is less restrictive and more realistic. Depending on whether the minimum (or average) time to discovery[5] ("time to handshake") on a channel i is $\tau_{0,i}$, we could intuitively envisage that the processes (S_j, C_j) jump after a random time T_{s_j,c_j} bounded below by τ_{0,c_j} (ie $T_{s_j,c_j} \geq \tau_{0,c_j}$). We will consider the relatively simple case, where the time spent in a mode (listening or searching) is a random variable $T_s := \tau + E_s$, where E_s is an exponentially distributed random variable with parameter λ_s to be specified ($\mathscr{E}(\lambda_s)$) and $\tau \geq 0$ is deterministic constant to be chosen suitably. Similarly, it is assumed that the time of change of channel is a random variable $T_c := \tau + E_c$, where E_c is an exponentially distributed random variable with parameter λ_c to be specified ($\mathscr{E}(\lambda_c)$). Therefore,

[5] See [16].

we know (see Lemma 3.1) that the time to the change in the state process (S_i, C_i) is a random variable $T_{s,c} := \tau + E_{s,c}$, where $E_{s,c} \sim \mathscr{E}(\lambda_s + \lambda_c)$. Similarly, cut off from $n\tau$, the time that elapses between the initial time and that of the n-th jump of $(S_i, C_i) \sim \Gamma(n, \lambda_s + \lambda_c)$, where $\Gamma(n, \lambda_s + \lambda_c)$ is the Erlang law.

It is interesting to find the law of the jump times of the process (S_1, S_2, C_1, C_2). This task is easier if τ is 0, and the sought law is $\mathscr{E}(2(\lambda_s + \lambda_c))$. However, when $\tau > 0$ it turns out to be more complex. A naive reasoning would lead to choose $\tau > \tau_{0,i}$ to increase the probability of discovery in the event of a good configuration (S_1, S_2, C_1, C_2). However, it is unnecessary to remain in an inadequate configuration for discovery when it occurs. Unfortunately, such configurations are possible. So, instead of fixing a strictly positive lower value for τ, we opt for choosing appropriate values for λ_s and λ_c, knowing that on average the residence time in a state will be non-zero.

Let us recall the following known results that can be easily proved.

Lemma 3.1 *Let $a, b \in \mathbb{R}_+$, $U \rightsquigarrow \mathscr{E}(\lambda_1)$ and $V \rightsquigarrow \mathscr{E}(\lambda_2)$. Assume that U and V are independent. Then,*

(i) $\min(U, V) \rightsquigarrow \mathscr{E}(\lambda_1 + \lambda_2)$;

(ii) $\mathbb{P}(U \leq -a + V) = \mathbb{E}\left(\mathbb{1}_{\{U \leq -a+V\}}\right) = \frac{\lambda_1}{\lambda_1+\lambda_2}$;

(iii) $\mathbb{P}(U \leq a + V) = \mathbb{E}\left(\mathbb{1}_{\{U \leq a+V\}}\right) = 1 - \frac{\lambda_2 e^{-a\lambda_1}}{\lambda_1+\lambda_2}$;

(iv) $\mathbb{P}(U \leq a + V \leq a + b) = \mathbb{E}\left(\mathbb{1}_{\{U \leq a+V \leq a+b\}}\right) = 1 - e^{-b\lambda_2} - \frac{\lambda_2 e^{-a\lambda_1}\left(1-e^{-b(\lambda_1+\lambda_2)}\right)}{\lambda_1+\lambda_2}$;

(v) $\mathbb{P}(a + V \leq U \leq a + b) = \mathbb{E}\left(\mathbb{1}_{\{a+V \leq U \leq a+b\}}\right) = \left(1 - e^{-b\lambda_1} - \frac{\lambda_1\left(1-e^{-b(\lambda_1+\lambda_2)}\right)}{\lambda_1+\lambda_2}\right) e^{-a\lambda_1}$.

3.2 Modeling the time to discovery

In this section, we are interested in the process $(S_1, S_2, C_1, C_2, T_0, \ T_1, \ldots, T_n)$, in particular the transitions of the process (S_1, S_2, C_1, C_2) which has $4n^2$ states. Even for the three social channels, the representation of the transition matrix is quite difficult (36 states). However, thanks to the assumption $\tau = 0$, the systems $\{S_1, S_2, C_1, C_2\}$ and $\{(S_1, S_2), (C_1, C_2)\}$ are independent. Almost surely, the jump times of (S_1, S_2) and (C_1, C_2) do not coincide. According to Lemma 3.1 the probability that a jump of (S_1, S_2, C_1, C_2) coincides with that of (S_1, S_2) is equal to $\frac{\lambda_s}{\lambda_s+\lambda_c}$.

We start by studying the transition matrix P^s of (S_1, S_2). For ordered states $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$ we have

$$P^s = \begin{bmatrix} \frac{\lambda_c}{\lambda_s+\lambda_c} & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & 0 \\ \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & \frac{\lambda_c}{\lambda_s+\lambda_c} & 0 & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} \\ \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & 0 & \frac{\lambda_c}{\lambda_s+\lambda_c} & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} \\ 0 & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & \frac{\lambda_s}{2(\lambda_s+\lambda_c)} & \frac{\lambda_c}{\lambda_s+\lambda_c} \end{bmatrix}.$$

Again focusing on the two ordered events $S_1 = S_2$ and $S_1 \neq S_2$, the transition matrix scaled down is

$$P^{s,r} = \begin{bmatrix} \frac{\lambda_c}{\lambda_s+\lambda_c} & \frac{\lambda_s}{\lambda_s+\lambda_c} \\ \frac{\lambda_s}{\lambda_s+\lambda_c} & \frac{\lambda_c}{\lambda_s+\lambda_c} \end{bmatrix}$$

and it has as invariant distribution $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ which does not depend neither on λ_s, nor on λ_c. To speed up the convergence towards the invariant distribution, one can look at the eigenvalues of $P^{s,r}$ that are 1 and $\frac{\lambda_c-\lambda_s}{2(\lambda_c+\lambda_s)}$. The acceleration of convergence mentioned above consists in minimizing $\frac{|\lambda_c-\lambda_s|}{2(\lambda_c+\lambda_s)}$. Thus, an optimal relationship between λ_c and λ_s is given by

$$\lambda_s = \lambda_c. \tag{3}$$

Now let us study the process (C_1, C_2) which has n^2 states which can be classified in the lexicographic order defined by $(a,b) \leq (c,d)$ if $a < c$ or $a = c$ and $b \leq d$. The transition matrix P^c of (C_1, C_2) is of dimension $n^2 \times n^2$, but it can be reduced by simply checking whether or not $C_1 = C_2$. If $\pi_{c,r}$ denotes the distribution of the initial choice of channel by each device, then the initial probability of having $C_1 = C_2$ is $\sum_{i=1}^{n} \pi_{c,r}^2(i)$. Thus, we restrict ourselves to 2×2 transition matrix (corresponding in order to the states $C_1 = C_2$ and $C_1 \neq C_2$) given by

$$P^{c,r} = \begin{bmatrix} 1 - \frac{\lambda_c\left(u_1 \overline{P}^c u_0^T\right)}{n(\lambda_s + \lambda_c)} & \frac{\lambda_c\left(u_1 \overline{P}^c u_0^T\right)}{n(\lambda_s + \lambda_c)} \\ \frac{\lambda_c\left(u_0 \overline{P}^c u_1^T\right)}{n(n-1)(\lambda_s + \lambda_c)} & 1 - \frac{\lambda_c\left(u_0 \overline{P}^c u_1^T\right)}{n(n-1)(\lambda_s + \lambda_c)} \end{bmatrix},$$

where

- $\overline{P}^c$ which denotes an $n^2 \times n^2$ stochastic matrix satisfying

$$\overline{P}^c_{i,j} = \begin{cases} \frac{1}{2} P^{c,n}_{\lfloor \frac{i-1}{n} \rfloor + 1, \lfloor \frac{j-1}{n} \rfloor + 1}, \text{ if } i \neq j \text{ and } (i-j)\, mod n = 0, \\ \\ \frac{1}{2} P^{c,n}_{((i-1) mod n)+1,((j-1) mod n)+1}, \text{ if } i \neq j \text{ and} \\ i - j = (i-1)\, mod n - (j-1)\, mod n, \\ \\ 0, \text{ otherwise.} \end{cases}$$

  with $P^{c,n}$ a zero diagonal $n \times n$ stochastic matrix describing the transitions of a process $C_i, i = 1, 2$ (when the process S_i is constant);

- u_0 denotes a line vector $1 \times n^2$ such that

$$u_0(i) = \begin{cases} 0, \text{ if } i = 1 + (n+1)\left((i-1)\, mod n\right) \\ 1, \text{ otherwise.} \end{cases},$$

- u_1 denotes a line vector $1 \times n^2$ such that

$$u_1(i) = \begin{cases} 1, \text{ if } i = 1 + (n+1)\left((i-1)\, mod n\right) \\ 0, \text{ otherwise.} \end{cases}.$$

The invariant distribution of $P^{c,r}$ is $\left[\frac{u_0 \overline{P}^c u_1^T}{u_0 \overline{P}^c u_1^T + (n-1) u_1 \overline{P}^c u_0^T} \quad \frac{(n-1) u_1 \overline{P}^c u_0^T}{u_0 \overline{P}^c u_1^T + (n-1) u_1 \overline{P}^c u_0^T}\right]$. By observing the structures of u_0, $\overline{P}^c$ and u_1, we notice that $u_0 \overline{P}^c u_1^T = 1 = u_1 \overline{P}^c u_0^T$. So the invariant distribution of $P^{c,r}$ is $\left[\frac{1}{n} \quad \frac{n-1}{n}\right]$. An analysis of $P^{c,r}$ shows that it has two eigenvalues, namely 1, and $1 - \frac{\lambda_c}{(n-1)(\lambda_s + \lambda_c)}$. Thus, if we wish to accelerate convergence towards the invariant distribution $\left[\frac{1}{n} \quad \frac{n-1}{n}\right]$, we must have n as small as possible (keeping in mind that $n \geq 2$) or take λ_c as large as possible. Indeed, the needed convergence is as faster as the absolute value of the eigenvalue $1 - \frac{\lambda_c}{(n-1)(\lambda_s + \lambda_c)}$ is smaller.

Proposition 3.1 *Let us assume that at the initial time the probabilities of having $S_1 \neq S_2$ and $C_1 = C_2$ are $\frac{1}{2}$ and $\frac{1}{n}$, respectively. Assume further that the initial distribution of the choice of channel for each device is the invariant distribution π_c associated with the matrix $P^{c,n}$. The number of jumps N necessary for a discovery after initialization follows a geometrical law*

$$\mathscr{G}\left(\frac{1}{2n} \sum_{i=1}^{n} \frac{\pi_c(i)\, \lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c + \lambda_s)}}{\lambda_{\tau_i} + 2(\lambda_c + \lambda_s)}\right) \textit{ and thus}$$

$$\mathbb{E}[N] = \frac{2n - \sum_{i=1}^{n} \frac{\pi_c(i) \lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c + \lambda_s)}}{\lambda_{\tau_i} + 2(\lambda_c + \lambda_s)}}{\sum_{i=1}^{n} \frac{\pi_c(i) \lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c + \lambda_s)}}{\lambda_{\tau_i} + 2(\lambda_c + \lambda_s)}}.$$

Proof. The result is essentially based on the invariance of the probabilities of having $S_1 \neq S_2$, having $C_1 = C_2$ and the probability of choosing a channel. ■

Proposition 3.2 *We consider the hypotheses of Proposition 3.1. Then the following statements hold.*

(*i*) *The conditional law of the time T that separates two jumps knowing there is no discovery is given by the density*

$$f_e : t \mapsto f_e(t) = \frac{4n(\lambda_c+\lambda_s)e^{-2(\lambda_c+\lambda_s)t}}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} - \frac{\sum_{i=1}^{n}\left(f_{1,i}(t)+f_{2,i}(t)+f_{3,i}(t)+f_{4,i}(t)\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}, \text{ where}$$

$$\begin{aligned}
f_{1,i}(t) =& 2(\lambda_c+\lambda_s)e^{-2(\lambda_c+\lambda_s)t}\mathbb{1}_{[\tau_{0,i},+\infty[}(t) \\
f_{2,i}(t) =& \frac{2n\lambda_{\tau_i}^2}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)} e^{-\lambda_{\tau_i}t+\tau_{0,i}\left(\lambda_{\tau_i}-2(\lambda_c+\lambda_s)\right)}\mathbb{1}_{[\tau_{0,i},+\infty[}(t) \\
f_{3,i}(t) =& 2n\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)e^{-\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)t+\lambda_{\tau_i}\tau_{0,i}}\mathbb{1}_{[\tau_{0,i},+\infty[}(t) \\
f_{4,i}(t) =& 2(2n-1)(\lambda_c+\lambda_s)e^{-\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)t+\lambda_{\tau_i}\tau_{0,i}}\mathbb{1}_{[\tau_{0,i},+\infty[}(t)
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}[T|E] =& \frac{\frac{n}{\lambda_c+\lambda_s} - \sum_{i=1}^{n}\pi_c(i)e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}\left(\tau_{0,i}+\frac{1}{2(\lambda_c+\lambda_s)}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} \\
&+ \frac{\sum_{i=1}^{n}\pi_c(i)e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}\left(\frac{2\tau_{0,i}(\lambda_c+\lambda_s)}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} \\
&+ \frac{\sum_{i=1}^{n}\pi_c(i)e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}\left(\frac{2(\lambda_c+\lambda_s)(1-2n)}{\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)^2}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}
\end{aligned}$$

(*ii*) *Conditionally to a success of discovery between two jumps, the time T which separates them follows a law having density*

$$f_s : t \mapsto f_s(t) = 2(\lambda_c+\lambda_s)\frac{\sum_{i=1}^{n}\pi_c(i)\left(1-e^{-\left(t-\tau_{0,i}\right)\lambda_{\tau_i}}\right)\mathbb{1}_{\left[\tau_{0,i},+\infty\right[}(t)}{\sum_{i=1}^{n}\frac{\pi_c(i)\lambda_{\tau_i}e^{2\left(t-\tau_{0,i}\right)(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}$$

and

$$\begin{aligned}
\mathbb{E}\left[T|S\right] &= \int_0^{+\infty} t f_s\left(t\right) dt \\
&= \frac{\sum_{i=1}^{n} \frac{\tau_{0,i}\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} \\
&+ \frac{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)^2}}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} + \frac{1}{2\left(\lambda_c+\lambda_s\right)}.
\end{aligned}$$

Proof. Let

$$\begin{aligned}
\pi_{\tau_i}\left(t\right) &= 2n + \left(2n-1\right) e^{-2\tau_{0,i}(\lambda_c+\lambda_s)} \mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right) \\
&- \frac{e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2\left(\lambda_c+\lambda_s\right)} \left(2n\lambda_{\tau_i} + 2\left(2n-1\right)\left(\lambda_c+\lambda_s\right)\right) \mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right)
\end{aligned}$$

and

$$\begin{aligned}
\overline{\pi}_{\tau_i}\left(t\right) &= -2ne^{-2(\lambda_c+\lambda_s)t} + e^{-2(\lambda_c+\lambda_s)t}\mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right) \\
&- 2ne^{-\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)t+\lambda_{\tau_i}\tau_{0,i}}\mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right) \\
&+ \frac{2n\lambda_{\tau_i}}{\lambda_{\tau_i}+2\left(\lambda_c+\lambda_s\right)} e^{-\lambda_{\tau_i}t+\tau_{0,i}\left(\lambda_{\tau_i}-2(\lambda_c+\lambda_s)\right)}\mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right) \\
&+ \frac{2\left(2n-1\right)\left(\lambda_c+\lambda_s\right)}{\lambda_{\tau_i}+2\left(\lambda_c+\lambda_s\right)} e^{-\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)t+\lambda_{\tau_i}\tau_{0,i}}\mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right).
\end{aligned}$$

(i) By applying Bayes rule and knowing that there is a failure of discovery with probability 1 when $t < \tau_0 + \delta T$, we have

$$\begin{aligned}
\mathbb{P}\left(\left\{T \le t\right\}|E\right) &= \frac{\mathbb{P}\left(\left\{T \le t\right\} \cap E\right)}{\mathbb{P}\left(E\right)} \\
&= \frac{\sum_{i=1}^{n} \pi_c\left(i\right) \mathbb{P}\left(\left\{T \le \min\left(t, \tau_{0,i} + \delta T_i\right)\right\}\right)}{\mathbb{P}\left(E\right)} \\
&+ \sum_{i=1}^{n} \frac{\pi_c\left(i\right) \mathbb{P}\left(\left\{\tau_{0,i} + \delta T_i \le T \le t\right\}\right)}{\mathbb{P}\left(E\right)} \\
&\times \mathbb{P}\left(E|\left\{\tau_{0,i} + \delta T_i \le T \le t\right\}\right)\mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right) \\
&= \frac{\sum_{i=1}^{n} \pi_c\left(i\right)\left(\pi_{\tau_i}\left(t\right) + \overline{\pi}_{\tau_i}\left(t\right)\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}.
\end{aligned}$$

It follows that the probability density is

$$\begin{aligned}
f_e : t \mapsto f_e\left(t\right) &= \frac{4n\left(\lambda_c+\lambda_s\right)e^{-2(\lambda_c+\lambda_s)t}}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} \\
&- \frac{\sum_{i=1}^{n}\left(f_{1,i}\left(t\right) + f_{2,i}\left(t\right) + f_{3,i}\left(t\right) + f_{4,i}\left(t\right)\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i}e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}, \text{ and}
\end{aligned}$$

$$\mathbb{E}\left[T|E\right] = \int_0^{+\infty} t f_e\left(t\right) dt$$
$$= \frac{\frac{n}{\lambda_c+\lambda_s} + \sum_{i=1}^{n} \pi_c\left(i\right) e^{-2\tau_{0,i}(\lambda_c+\lambda_s)} \left(-\tau_{0,i} - \frac{1}{2(\lambda_c+\lambda_s)}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}$$
$$+ \frac{\sum_{i=1}^{n} \pi_c\left(i\right) e^{-2\tau_{0,i}(\lambda_c+\lambda_s)} \left(\frac{2\tau_{0,i}(\lambda_c+\lambda_s)}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}$$
$$+ \frac{\sum_{i=1}^{n} \pi_c\left(i\right) e^{-2\tau_{0,i}(\lambda_c+\lambda_s)} \left(\frac{2(\lambda_c+\lambda_s)(1-2n)}{\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)^2}\right)}{2n - \sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}.$$

(ii) Applying Bayes rule once more and using the fact that there is a success probability 0 when $t < \tau_0 + \delta T$, we have

$$\mathbb{P}\left(\{T \le t\} | S\right) = \frac{\mathbb{P}\left(\{T \le t\} \cap S\right)}{\mathbb{P}\left(S\right)}$$
$$= \sum_{i=1}^{n} \frac{\pi_c\left(i\right) \mathbb{P}\left(\{\tau_{0,i} + \delta T_i \le T \le t\} \cap S\right) \mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right)}{\mathbb{P}\left(S\right)}$$
$$= \frac{1}{2n\mathbb{P}\left(S\right)} \sum_{i=1}^{n} \pi_c\left(i\right) \pi_{\tau_i}\left(t\right)$$
$$= \frac{\sum_{i=1}^{n} \pi_c\left(i\right) \pi_{\tau_i}\left(t\right)}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}.$$

We deduce the density

$$f_s : t \mapsto f_s\left(t\right) = 2\left(\lambda_c + \lambda_s\right) \frac{\sum_{i=1}^{n} \pi_c\left(i\right) \left(1 - e^{-(t-\tau_{0,i})\lambda_{\tau_i}}\right) \mathbb{1}_{[\tau_{0,i},+\infty[}\left(t\right)}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{2\left(t-\tau_{0,i}\right)(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}.$$

In addition, we have

$$\mathbb{E}\left[T|S\right] = \int_0^{+\infty} t f_s\left(t\right) dt$$
$$= \frac{\sum_{i=1}^{n} \frac{\tau_{0,i}\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}}$$
$$+ \frac{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\left(\lambda_{\tau_i}+2(\lambda_c+\lambda_s)\right)^2}}{\sum_{i=1}^{n} \frac{\pi_c(i)\lambda_{\tau_i} e^{-2\tau_{0,i}(\lambda_c+\lambda_s)}}{\lambda_{\tau_i}+2(\lambda_c+\lambda_s)}} + \frac{1}{2\left(\lambda_c + \lambda_s\right)}.$$

■

Under the conditions of Proposition 3.1, the expected value of the time to discovery is

$$\mathbb{E}\left[TTD\right] = \mathbb{E}\left[T|S\right] + \mathbb{E}\left[N\right] \mathbb{E}\left[T|E\right] - \frac{1}{2\left(\lambda_c + \lambda_s\right)}. \quad (4)$$

Indeed, for the calculation of the expected value of the time to discovery, we consider the average number of failures, the average time between two jumps conditional to failure as well as the average time between two jumps conditional to success. The time actually used for the discovery between two jumps being $\tau_{0,i} + \delta T_i$, we can ignore the possible surplus of time. Thus, we do not take into account the additional time $\frac{1}{2(\lambda_c+\lambda_s)}$ which corresponds to the average time of mode change.

Theorem 3.1 *Assume the conditions of Proposition 3.1 and also suppose that* $\forall i = 1, \ldots, n$, $\tau_{0,i} = \tau_0$, $\lambda_{\tau_i} = \lambda_\tau$, $\lambda_c = \lambda_s$. *Then the expected value of time to discovery is given by* $g(\lambda_s)$, *where* $\forall x \in \mathbb{R}_+^*$,

$$g(x) = \tau_0 + \frac{1}{\lambda_\tau + 4x} + \frac{(\lambda_\tau + 4x)\, e^{4\tau_0 x}}{4x\lambda_\tau} \left(\frac{2ne^{4\tau_0 x}(\lambda_\tau + 4x)}{\lambda_\tau} - 1 \right) \times$$
$$\left(2n - \frac{\left(16x^2(2n-1) + (\lambda_\tau + 4x)(\lambda_\tau + 4x(1 + \tau_0\lambda_\tau))\right) e^{-4\tau_0 x}}{(\lambda_\tau + 4x)^2} \right).$$

In addition, there is at least one value of λ_s *which guarantees the minimum value of the expected value of the time to discover.*

Proof. The first part of theorem follows from Proposition 3.1 and equation (4). Because g is continuous and $\lim\limits_{x \to \{0^+, +\infty\}} g(x) = +\infty$, we can deduce the existence of at least one global minimum of g on $\mathbb{R}_+^*$. ■

The hypotheses of Proposition 3.1 are not generally satisfied at the start of the process, but they are asymptotically. Thus, the rapid convergence of the subprocesses of the choice of mode, and channel selection is an essential criterion for the usability of the results of Theorem 3.1.

3.3 Determination of optimal discovery parameters

We consider the case of the exclusive use of social channels. However, the formulas considered remain general for a possible application to any number n of channels. It has already been established from relation (3) that it is better to have $\lambda_s = \lambda_c$.

One of the first parts of optimization is to maximize the probability that both devices are found on the same channel by choosing in an adequate way the stochastic matrix of individual channel change $P^{c,n}$. Despite the fact that the choice of the matrix $P^{c,n}$ does not influence the asymptotic behavior $P^{c,r}$, taking into account the values $\tau_{0,i}$ $(i = 1, \ldots, n)$, it seems better to adopt

$$P_{ij}^{c,n} = \begin{cases} 0, \text{ if } i = j \\ 1 - \frac{\tau_{0,j}}{\sum_{k \neq i} \tau_{0,k}} \end{cases}, \; i, j = 1, \ldots, n. \tag{5}$$

Indeed, the channel with a shorter time to handshake is more attractive with regards to time for discovery. One obtains π_c by simply solving

$$\begin{cases} \pi_c P_{ij}^{c,n} = \pi_c \\ \sum_{i=1}^n \pi_c(i) = 1 \end{cases}. \tag{6}$$

Once the matrix $P^{c,n}$ has been determined with its invariant law π_c, the next challenge is to minimize the average time to discovery according to (4).

Having no overview of the $\tau_{0,i}$ $(i = 1, \ldots, n)$, we will consider them all equal for the numerical implementation as in Theorem 3.1. We can observe that when all $\tau_{0,i}$ are equal, then π_c is the uniform distribution. The determination of λ_{τ_i} $(i = 1, \ldots, n)$ is done by considering the packet reception rate. The necessary number of trials allowing the reception of packet follows a geometric law with parameter the packets reception rate (PRR). So, if PRR_i denotes the packets reception rate on channel i then

$$\lambda_{\tau_i} = \frac{PRR_i}{\tau_{0,i}(1 - PRR_i)}. \tag{7}$$

In general, when $PRR < 10\%$ the channel is considered of poor quality; it is of average quality if $PRR \in [0.1, 0.9]$, otherwise it is of good quality. As part of our numerical implementation, we consider for illustrative purposes an average quality for all channels, with constant values of PRR taken in the set $\{0.5, 1\}$.

Considering the case of exclusive use of social channels, we have calculated the ranges of values of T_{REQ}, T_{RESP}, T_{ACK}, τ_0 according to the types of Wi-Fi. The results of this calculation are given according to Table 2.

Table 2: Range of time to handshake

Norm	**802.11a,g (Wi-Fi a,g)**	**802.11b (Wi-Fi b)**
$T_{REQ}\ (ms)$	$\left[6.305 \times 10^{-4}, 1.166 \times 10^{-2}\right]$	$\left[3.096 \times 10^{-3}, 6.994 \times 10^{-2}\right]$
$T_{RESP}\ (ms)$	$\left[6.305 \times 10^{-4}, 7.055 \times 10^{-3}\right]$	$\left[3.096 \times 10^{-3}, 4.233 \times 10^{-2}\right]$
$T_{ACK}\ (ms)$	$\left[2.386 \times 10^{-4}, 2.148 \times 10^{-3}\right]$	$\left[1.172 \times 10^{-3}, 1.289 \times 10^{-2}\right]$
$\tau_0\ (ms)$	$\left[7.055 \times 10^{-1}, 7.249 \times 10^{-1}\right]$	$\left[7.374 \times 10^{-1}, 8.552 \times 10^{-1}\right]$

Numerical results of calculations of the ranges of λ_s and associated average theoretical time to discovery (TTD) are contained in Table 3. The calculations are done for Wi-Fi a, b and g according to the data in Table 2. The optimization is done by combination of golden section method and gradient method [10, 13].

Table 3: Range of optimal λ_s $\left(ms^{-1}\right)$ and expected TTD depending on the Wi-Fi norm and quality of channels

	802.11 a, g (Wi-Fi a, g)	
Quality	**Range of** λ_s $\left(ms^{-1}\right)$	**Range of TTD** (ms)
$PRR = 1$	$\left[1.153 \times 10^{-1}, 1.621 \times 10^{-1}\right]$	$[105.631, 108.604]$
$PRR = 0.5$	$\left[9.339 \times 10^{-2}, 9.590 \times 10^{-2}\right]$	$[192.652, 197.949]$
	802.11 b (Wi-Fi b)	
	Range of λ_s $\left(ms^{-1}\right)$	**Range of TTD** (ms)
$PRR = 1$	$\left[1.344 \times 10^{-1}, 1.530 \times 10^{-1}\right]$	$[110.426, 128.045]$
$PRR = 0.5$	$\left[7.903 \times 10^{-2}, 9.181 \times 10^{-2}\right]$	$[201.363, 233.531]$

Figure 1 graphically shows the functional relation between the average time to discovery and λ_s depending on δ_0 and PRR. We can observe the strictly convex nature of that relation which shows the uniqueness of a minimum average time to discovery. That time increases with respect to the time to handshake (δ_0) while it decreases with respect to the quality of the channel (PRR).

Figure 2 illustrates the dependence of the optimal value of λ_s with respect to τ_0. One observes overall that λ_s would be a decreasing function of τ_0. It means that the longer the time to handshake, the higher the frequency of change of channel.

A statistical regression analysis using R software shows with a P-value of the order of 2.689×10^{-10} and adjusted R^2s of around 99.36%, that the relation between λ_s and τ_0 is close to the log-linear form

$$\lambda_s = e^{\frac{a\tau_0}{PPR}+b}. \tag{8}$$

The parameters a and b of (8) are very significant[6].

The interest of an approximate model such as (8) relies on the ease of adaptation of the potential variation of τ_0. Indeed, the explicit computation of λ_s at a high frequency might require time and energy which are costly to devices with limited resources.

[6] Significance codes under R software in terms of P-value : 0 "***" 0.001 "**" 0.01 "*"0.05 "•" 0.1 " " 1

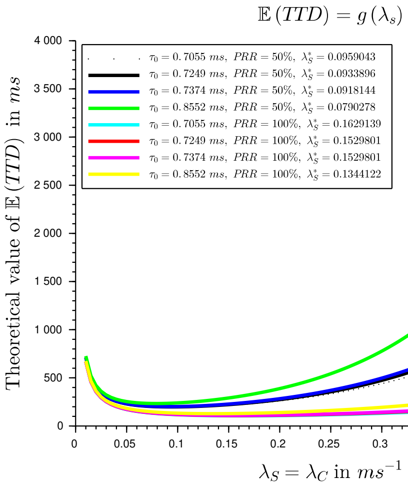

Figure 1: Time to discover as function of λ_s

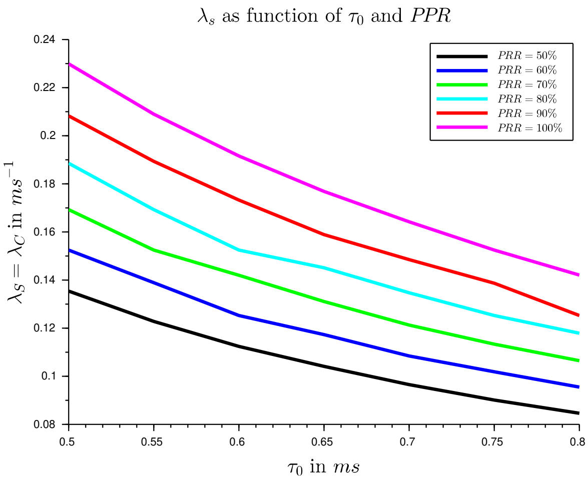

Figure 2: λ_s as function of τ_0 and PRR

Table 4: Parameters a and b of equation (8)

a	**b**
-0.83767 ***	-1.17026 ***

4 Simulation of the discovery process using OMNET ++ emulator

In this section, we perform simulations using the OMNET ++ emulator (Objective Modular Network Testbed in C ++) of the discovery process according to the GCR approach in each of the cases considered, to estimate the empirical means of times to discovery, and to make comparisons with theoretical expected values. It is also about simulating the discovery processes as implemented in the literature , to implement these same processes with the LCR proposal in [16], and to make a global comparative study.

For each of the methods and each of the cases, we performed 1000 simulations. The averages of the times to discovery (TTD) simulations for the different types of Wi-Fi and the different characteristic times to handshake are illustrated in Figure 3. Different scales have been used to easily display the average times to discovery (a unit of $15ms$ for LCR case and a unit of $40ms$ for the specification).

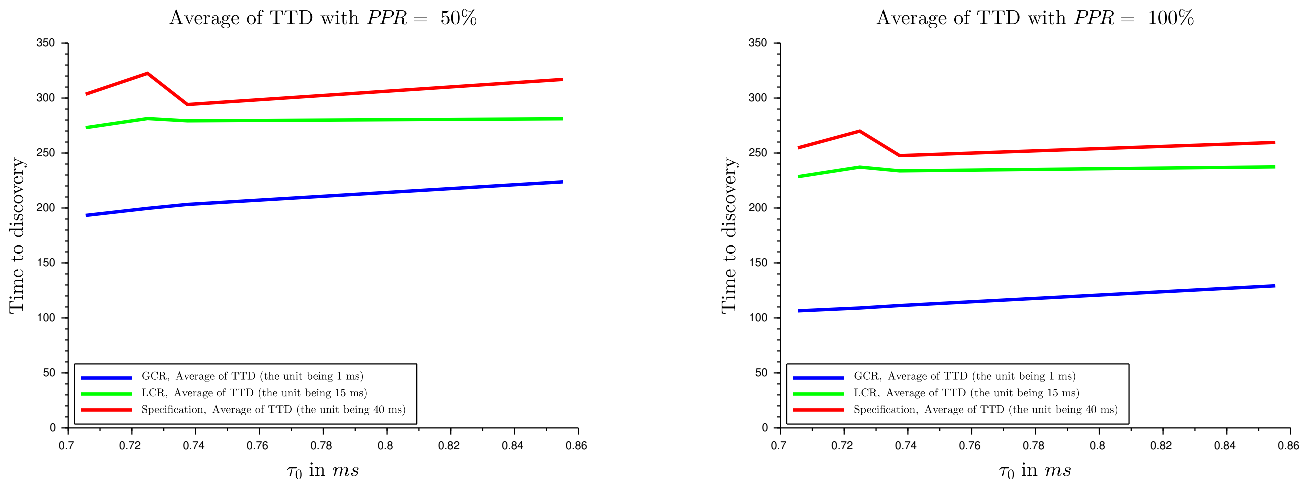

Figure 3: Average time to discovery for each method

As we can observe in Figure 3, the time to discovery increases with the time to handshake (τ_0) and decreases with the quality of the channel (PRR). When $PRR = 1$, the GCR reduces the average discovery time by 96.76% compared to the LRC, and by 98.89% compared to the specification. When $PRR = 0.5$, the GCR reduces the average discovery time by 95.10% compared to the LRC, and by 98.34% compared to the specification. The comparison between the methods is also evaluated by an analysis of variance and a Tukey Honest Significant Test Differences with a 95% confidence level as displayed in Figure 4.

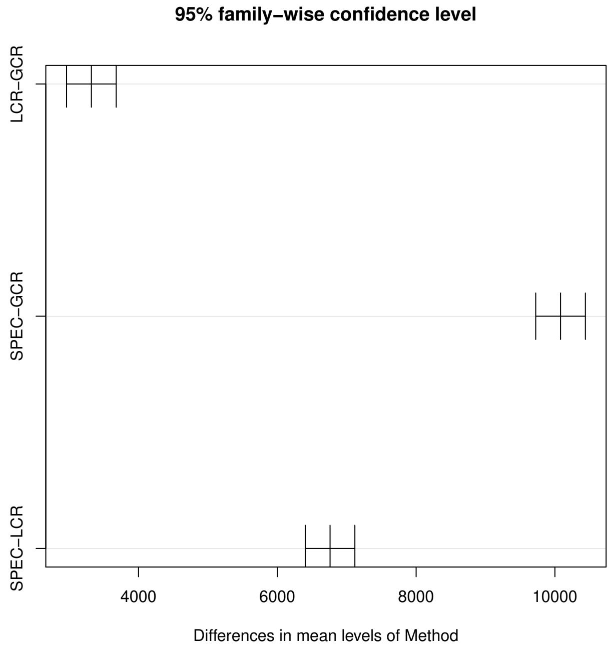

Figure 4: Tukey Honest Significant Test Differences between methods

As Figure 4, no non positive value appears in the confidence intervals of the mean differences of time to discovery. This means those differences are statistically positive. As stated in [16], the Listen Channel Randomization is better than the current specification with a reduction of average discovery time by 65.93%. However, the Global Channel Randomization outperforms the LCR.

5 Conclusion

The problem addressed in this work is that of minimizing time to discovery between two devices wishing to communicate by Wi-Fi Direct. Indeed, the time to discovery in addition to being an indicator of quality of service has an influence on energy consumption. The different Wi-Fi specifications provide a general framework for discovery protocols, but their implementations vary from one manufacturer to another. Since this variability has an impact on the time of discovery, we propose as in [16] a systematic and optimal approach to discovery. The general principle is the switching of channels and the change of mode all in a random fashion. However, subject to the laws of change, we recommended the computation of the optimal parameters of these. In this paper, we adopt the formalism of Poisson processes with constant intensities for channel and mode change. We explicitly determine the expression of the average time to discovery as a function of the time to handshake and the quality of the channels. Thereafter, we showed that there exists an optimal set of parameters for the laws of mode or channel change. In particular, the change of mode must be as frequent as the change of channel. We numerically determined these parameters for some values of time to handshake contained in intervals defined according to Wi-Fi technologies. We also offered a predictive statistical model for rapid evaluation of optimal discovery parameters in the event of a change in context. Finally, a thousand of experimental tests in the OMNET ++ emulator are performed, each with our proposition (GCR), with the LCR approach in [16] and that of the specification. According to the results of the statistical analysis, the GCR approach produces on average shorter discovery times, followed by the LCR method which is itself better than the specification (65.93 % of improvement following [16]). Compared with the specification, the improvement of the average time discovery we obtained with the GCR method is above 98.34 % .

Despite the promising results of this work, there is still a lot to do. Among other things, we should consider the more general case, in which time between successive jumps is bounded from the left by a positive number $\tau > 0$. It would also be interesting to consider the case where the intensities of changes vary over time as a function of previous discovery trials. This could improve the work in [4]. Finally, the laws of choice of changes may be generalized or replaced by other laws in order to see if we obtain better performances.

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Competing interests

The authors declare there is no competing interest.

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