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Abstract: In this paper Complete Diallel Crosses (CDC) plan is constructed using two Balanced Incomplete Block Designs and the Galois field with the same set of parameters $v = b = 4\lambda + 3, r = k = 2\lambda + 1, \lambda \geq 1$. The designs were isomorphic on each other and the crossing is made between the lines. The analysis of CDC plans to estimate the general combining ability (GCA) effects and specific combining ability (SCA) effects were excluded from the model. The efficiency value of the constructed CDC plan is tends to 1, and universally optimum when v is very large. The construction is also illustrated with the suitable example.

Keywords- Galois Field, Balanced Incomplete Block Design, Complete Diallel Crosses, General Combining Ability, Efficiency and Universal Optimality.

1. Introduction

Diallel crossing is one type of mating design which was introduced by Schmidth (1919) and has been adopted in various situations by Comstock and Robinson (1952).It is useful method for conducting animal and plant breeding experiments, in particular with estimating the effects of the combining ability of lines. Griffing (1956) further contributed to the design of diallel crosses by developing appropriate models and methods of analysis.

Diallel crosses in which all possible distinct crosses in pairs among the available lines are taken is called CDC. Sprague and Tatum (1942) was defined the concept of GCA and SCA and it's uses are explained in the test procedure for diallel crossing. The GCA is the average performance of a line in hybrid combination, while SCA is the interaction between the i^{th} and j^{th} lines.

The concept of diallel crosses plan was introduced by Kempthrone, (1957), Gilbert, (1958), Kempthrone and Curnow, (1961), Das and Sivaram, (1968), Kaushik et.al. (1987), Agarwal and Das, (1990), Divecha and Ghosh, (1994) and Dey and Midha, (1996), Das and Ghosh, (1999), and Sharma and Fanta, (2009) have discussed several methods of construction of complete diallel crosses plan. Balanced Incomplete Block Design introduced by Yates, (1936a) and developed by Fisher and Yates, (1938), Bose, (1939) which were extensively tabled in Fisher and Yates, (1938).

Singh and Hinkelmann, (1990) also discussed some of the plans as optimal diallel crosses plan. Dey and Midha, (1996), constructed CDC plans through triangular type PBIB design with two associate classes. Ghosh and Biswas, (2003) concluded that the CDC plans obtained from two BIB designs with the same parameters are also universal optimal. Prasad, et.al (1999) constructed the universally optimal block designs for diallel crosses by using an application of a Galois field. Das et.al., (1998) were constructed the optimality of nested BIBD and triangular PBIBD using Galois field and Gosh and Divecha has also constructed the two BIB designs with same set of parameters using Galois field to estimate the GCA effects of each line.

In this paper, we construct the CDC plan using two BIB designs with same set of parameters having $v = b = 4\lambda + 3, r = k = 2\lambda + 1, \lambda \geq 1$.The BIBD was developed from the Galois field and the analysis was performed.

To construct the CDC plan, we denote L_1 and L_2 be an isomorphic solutions of two BIB designs based on Galois field GF(v) with $\lambda \geq 1$. Let p be the primitive root of GF(v) and it is prime or prime powers. Then the two initial blocks for the BIB designs L_1 and L_2 are generated by the developing sets of even and odd powers of GF(v) and the designs were superimposed on each other using CDC system IV, crossing is made between the lines.

In the linear model, we estimate only the effects of GCA and SCA effects which has been excluded from the model.

Construction of the method of CDC plan and their analysis, in which the crosses were taken by two BIB designs using the Galois field and the efficiency factor tends to 1 and the universal optimality of CDC plan is discussed in subsequent sections.

2. Method of Construction

Here we discuss the method of construction of CDC plan using L_1 and L_2 be an isomorphic solutions of two BIB designs with same set of parameters $v = b = 4\lambda + 3, r = k = 2\lambda + 1, \lambda = \lambda$ where $\lambda \geq 1$. Let p be the primitive root of GF ($v = 4\lambda + 3$) then the elements of GF(v) can be expressed as $p^0, p^1, p^2, \ldots\ldots\ldots p^{v-1}$. From the initial block the whole design can be constructed.

2.1 Lemma

Let us consider L_1 and L_2 be an isomorphic solutions of two BIB designs based on Galois field GF($v = 4\lambda + 3$) with $\lambda \geq 1$ for constructing CDC plan using GF($v = 4\lambda + 3$) design, we observed the following,

(i) Initial block for the design L_1 and L_2 are generated by the developing sets of even and odd powers
($p^0, p^1, p^3, \ldots\ldots\ldots, p^{v-2}$) reduced mod $v(= 4\lambda + 3)$
($p^2, p^4, p^6, \ldots\ldots\ldots, p^{v-1}$) reduced mod $v(= 4\lambda + 3)$

Let p be the primitive root of GF(v) and it is prime or prime powers.

(ii) Using the two initial blocks, the designs were superimposed on each other and crossing is made between the lines.

(iii) Using CDC plan system IV with $v(v-1)/2$ crosses in v blocks each of size k and each cross occur only once.

(iv) Now, the CDC plan with parameters $v_1= v$, $b_1=b=v$, $r_1=(v-1)=(b-1)$, $k_1=k$, where each pair is repeated r times ('r' denotes each pair occur only once).

For example,

Let $\lambda=1$, The treatment combinations of the design are

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{matrix}$$

Taking these as columns of v x b incidence matrix N, by using lemma, we get the matrix N of order $7x7$ of design as follows,

$$N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}_{(7x7)}$$

Theorem:2.1

Complete Diallel Crosses plan using L_1 and L_2 be an isomorphic solutions of two BIB designs with same set of parameters $v=b=4\lambda+3, r=k=2\lambda+1, \lambda\geq 1$ and the two initial blocks (by using lemma 2.1(i)) were superimposed on each other by taking crosses column wise with $v_1= v$ lines arranged in $b_1=b=v$ blocks each of size k and with total number of crosses are $v(v-1)/2$.

Proof:

Consider the two BIB designs with parameters $v=b=4\lambda+3, r=k=2\lambda+1, \lambda\geq 1$. Since BIBD is symmetric and hence v=b.

By using lemma 2.1(i),

Let p be the primitive element of GF(v) and $p^0, p^1, p^2, ..., p^{v-2}, p^{v-1}$ be the powers of p. Then generate the initial blocks for the design L_1 and L_2 are generated by the developing sets of even and odd powers.

Therefore,

$I_1 = (p^0, p^1, p^3, ..., p^{v-2})$ reduced mod v and

$I_2 = (p^2, p^4, p^6, ..., p^{v-1})$ reduced mod v

where GF(v) = (0, 1, 2, ...v-1)

When L_1 and L_2 are superimposing on each other and the crosses are made between two treatments occur in the two BIB designs. Since both the BIB designs have distinct set of treatment numbers in the i^{th} block (i=1, 2, ...v) not a single cross is repeated and all the possible crosses among (1, 2,.......v) lines are made once.

Therefore, the resulting design is a CDC plan with $\frac{v(v-1)}{2}$ crosses in v blocks each of size k and each line is repeated (v-1) times.

Let N be the incidence matrix of CDC plan. 0,1 are the elements of the incidence matrix N. The incidence matrix of this CDC plan is expressed as,

$$N = \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 & 0 \\ 0 & 1 & 1 & \cdots & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & \cdots & 0 & 1 \end{bmatrix}_{(vxb)}$$

The incidence matrix is binary and equi replicated and hence the concurrence matrix is,

$$NN' = \begin{bmatrix} (v-1) & (v-2) & \dots & \dots & \dots & (v-2) & (v-2) \\ (v-2) & (v-1) & \dots & \dots & \dots & (v-2) & (v-2) \\ (v-2) & (v-2) & \dots & \dots & \dots & (v-2) & (v-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (v-2) & (v-2) & \dots & \dots & \dots & (v-2) & (v-1) \end{bmatrix}_{(vxb)}$$

Since there are $\frac{v(v-1)}{2}$ crosses and each cross occurs only once. Hence the existing plan is CDC plan with parameters $v_1 = v$, $b_1 = b = v$, $r_1 = (v-1) = (b-1)$, $k_1 = k$.

2.1 Numerical Illustrations

Consider two BIB design with parameters $v = b = 7, r = k = 3, \lambda = 1$. The elements of GF(7)={ 0, 1, 2, 3, 4, 5, 6}and the Primitive element of GF(7) is 3. So, the initial blocks for generating BIB designs L_1 and L_2 are respectively.

$I_1 = (3^0, 3^2, 3^4)$ reduced mod 7 = (1, 2, 4)   and

$I_2 = (3^1, 3^3, 3^5)$ reduced mod 7 = (3, 6, 5)

Table 1. The plans of two BIB designs

L_1

1	2	3
1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

L_2

1	2	3
3	6	5
4	7	6
5	1	7
6	2	1
7	3	2
1	4	3
2	5	4

Now we superimpose L_2 on L_1, the following plan of CDC system IV is obtained as,

Table 2: shows the plan of CDC system IV

B_1:	1x3	2x6	4x5
B_2:	2x4	3x7	5x6
B_3:	3x5	1x4	6x7
B_4:	4x6	2x5	1x7
B_5:	5x7	3x6	1x2
B_6:	1x6	4x7	2x3
B_7:	2x7	1x5	3x4

Therefore, L is a plan of CDC system IV of Griffing (1956) with 21 crosses, occurring once. Each of the 7 lines is repeated in 6 crosses.

The incidence matrix of this CDC plan is shown in Table 2,

$$N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}_{(7x7)}$$

The concurrence matrix of the CDC plan is shown as,

$$NN' = \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 6 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 6 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 6 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 \end{bmatrix}_{(7x7)}$$

3. Analysis of Complete Diallel Crosses Plan

Consider the Complete Diallel Crosses plan discussed in theorem 1. Let Y be a nx1 observational vector from a CDC plan, where $\frac{v(v-1)}{2}$ crosses are applied to n plots arranged in b blocks, each of size $k = \frac{v-1}{2}$, as the size of BIB design and number of lines are same as v.

Let the linear model for the CDC plan may be expressed as following,

$$Y = \mu 1_n + \Delta_1 g + \Delta_2 \beta + e \qquad \text{..........(1)}$$

where Y denotes the nx1 observational vector, μ is the general mean, 1_n denotes the nx1 whose all elements are 1, g is the general combining ability (GCA) effects of vector vx1 and β is block effects of vectors of vx1, Δ_1 and Δ_2 are the corresponding design matrices of nxp and n x b respectively. That is, (i, j)th element of Δ_1 is 1 if the ith observation is present in the jth line and is zero otherwise. Similarly, the (i, j)th element of Δ_2 is 1 if the ith observation comes from the jth block and is zero otherwise. e is the random error vector components takes care of specific combining ability (SCA) and is unassignable variation being distributed with mean zero and constant variance σ^2.

Let g_{dij} denotes number of times cross (ixj) occurs in the plan and let s_{di} denotes the number of times the ith line occurs in the plan.

Gupta and Kageyama (1994) obtained the information matrix C as,

$$C_d = G_d - N' k^{-1} N \qquad \text{...........(2)}$$

where $\Delta_1^{'}\Delta_1 = G_{dij}$, $S_{di} = g_{dii}$ and $\Delta_1^{'}\Delta_2 = N_d = (n_{dij})$, where n_{dij} is the number of times ith line occurs in the jth blocks under the model.

∴ The reduced normal equation for GCA effects using design d is,

$$C_d \hat{g} = Q \qquad \text{..............(3)}$$

were $Q = T - v^{-1} N_d B$. Here T is the vector of line totals and B is the vector of block totals.

Now G_d can also be re-written as,

$$G_d = \begin{bmatrix} W_{di} & g_{ii'} \\ g_{ii} & W_{di'} \end{bmatrix} \qquad \text{..............(4)}$$

where W_{di} denotes the diagonal elements as replication number of lines and g_{ii} denotes the off-diagonal elements as replication number of crosses.

Since each line occurs (v-1) times and each cross occurs only one times.

Now equation (4) can be expressed as,

$$G_d = \begin{bmatrix} (v-1) & 1 & \dots & \dots & \dots & 1 & 1 \\ 1 & (v-1) & \dots & \dots & \dots & 1 & 1 \\ 1 & 1 & \dots & \dots & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & \dots & \dots & 1 & (v-1) \end{bmatrix}_{(vxb)}$$

Hence, the coefficient matrix C can be expressed as,

$$C = \begin{bmatrix} (v-1) & 1 & \dots & \dots & \dots & 1 & 1 \\ 1 & (v-1) & \dots & \dots & \dots & 1 & 1 \\ 1 & 1 & \dots & \dots & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \dots & \dots & \dots & 1 & (v-1) \end{bmatrix} - \frac{1}{\frac{(v-1)}{2}} \begin{bmatrix} (v-1) & (v-2) & \dots & \dots & \dots & (v-2) & (v-2) \\ (v-2) & (v-1) & \dots & \dots & \dots & (v-2) & (v-2) \\ (v-2) & (v-2) & \dots & \dots & \dots & (v-2) & (v-2) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (v-2) & (v-2) & \dots & \dots & \dots & (v-2) & (v-1) \end{bmatrix}$$

………………(5)

Now we solve equation (5) by taking diagonal and off-diagonal elements separately.

Diagonal elements of C matrix

The diagonal elements of C-matrix of CDC plan from equation (5) is found as,

$$(v-1) - \frac{(v-1)}{(\frac{(v-1)}{2})} = \frac{(v-1)(\frac{(v-1)}{2}) - (v-1)}{(\frac{(v-1)}{2})}$$

$$= \frac{(v-3)(v-1)}{2(\frac{(v-1)}{2})}$$

Off-Diagonal elements of C matrix

The off-diagonal elements of C-matrix of CDC plan from equation (5) is found as,

$$1-\frac{(v-2)}{(\frac{(v-1)}{2})}=\frac{(\frac{(v-1)}{2})-(v-2)}{(\frac{(v-1)}{2})}$$

$$=\frac{-(v-3)}{2(\frac{(v-1)}{2})}$$

Now using equation (5), the C-matrix can be written as,

$$C=\frac{\begin{bmatrix} \frac{(v-3)(v-1)}{2} & \frac{-(v-3)}{2} & \cdots & \cdots & \cdots & \frac{-(v-3)}{2} & \frac{-(v-3)}{2} \\ \frac{-(v-3)}{2} & \frac{(v-3)(v-1)}{2} & \cdots & \cdots & \cdots & \frac{-(v-3)}{2} & \frac{-(v-3)}{2} \\ \frac{-(v-3)}{2} & \frac{-(v-3)}{2} & \cdots & \cdots & \cdots & \frac{-(v-3)}{2} & \frac{-(v-3)}{2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{-(v-3)}{2} & \frac{-(v-3)}{2} & \cdots & \cdots & \cdots & \cdots & \frac{(v-3)(v-1)}{2} \end{bmatrix}}{(\frac{(v-1)}{2})}$$

Finally, C matrix is reduced to,

$$C=\frac{v(v-3)}{(v-1)}\left[I_v-\frac{1}{v}E_{vv}\right] \qquad \text{..................(6)}$$

Equation (6) can also be expressed as,

$$C=\theta\left[I_v-\frac{1}{v}E_{vv}\right]$$

where $\theta=\frac{v(v-3)}{(v-1)}$ and this is called the non-zero eigen value of C matrix of CDC plan with multiplicity (v-1).

Since the solution of the estimate of GCA line effect is $\frac{1}{\theta}Q_i$, hence

$$\hat{g}_i=\frac{(v-1)}{v(v-3)}Q_i,\ \forall i=1,2.3.........v \qquad \text{...............(7)}$$

Again, the solution of the estimate of the variance of line effect $\hat{g}_i$ and $\hat{g}_j$ is given by $\frac{2}{\theta}\sigma^2$

Hence $v(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2$, where $i \neq j = 1,2,3,.........v$.

$$= \frac{2}{\frac{v(v-3)}{(v-1)}}\sigma^2$$

$$= \frac{2(v-1)}{v(v-3)}\sigma^2 \qquad \text{………….(8)}$$

Now we illustrate the analysis of CDC plan, discussed above with numerical illustration.

Here the C-matrix becomes,

$$C = \frac{14}{3}\left[I_7 - \frac{1}{7}E_{77}\right],$$

Eigen value of the above C matrix is $\theta = \frac{14}{3}$ with multiplicity 6. Now, $\hat{g}_i = \frac{1}{\theta}Q_i = \frac{3}{14}Q_i$ and $v(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2 = \frac{6}{14}\sigma^2$.

4. **Efficiency factor of CDC plan**

Now, we adopt the Randomized Complete Block Design (RCBD), instead of CDC plan with 'r' blocks such that each has $(\frac{(v-2)}{2})$ crosses. So, total numbers of crosses are $\frac{v(v-1)}{2}$, and then the C-matrix of the randomized block design is obtained as,

$$C_R = r(v-2)(I_v - v^{-1}I_v I_v')$$

Hence, the variance of best linear unbiased estimator of any elementary contrast among GCA effects, RBCD is defined as,

$$v(\hat{g}_i - \hat{g}_n) = \frac{\hat{2}}{r(v-2)}\sigma_r^2, \forall i \neq n = 1,2,........v \qquad \text{………….(9)}$$

where v is number of lines, r is number of times each cross occurs in CDC plan and σ_r^2 is error variance.

Now, in case of CDC plan discussed in (8), variance of best linear unbiased estimator of any elementary line contrast among GCA effect for CDC plan is expressed as,

$$v(\hat{g}_i - \hat{g}_j) = \frac{2(v-1)}{v(v-3)}\sigma^2 \qquad \text{……………(10)}$$

Now, efficiency factor of existing CDC plan compare to RBD in r replication is defined as,

$$\therefore \text{Efficiency (E)} = \frac{v(\hat{g}_i - \hat{g}_u)_{RBD}}{v(\hat{g}_i - \hat{g}_j)_{CDC}}$$

$$= \frac{\frac{2}{r(v-2)}\sigma_r^2}{\frac{2(v-1)}{v(v-3)}\sigma^2}$$

[$\because$ here r=1because each cross occurs only once]

$$\therefore E = \frac{v(v-3)}{r(v-2)(v-1)} = 0.93333 \qquad \text{………………(11)}$$

Hence, it is clear that the efficiency factor of proposed CDC plan relative to RBD under the assumption of inter Block variance increases as v increases. Finally, when the efficiency factor tends to 1, provided v is large.

Table 3: variance and efficiency factor

v	***b***	***r***	***K***	**λ**	$v(\hat{g}_i - \hat{g}_u)_{RBD}$	**E**
7	7	3	3	1	0.4	0.93333
11	11	5	5	2	0.2222	0.9779
19	19	9	9	4	0.1176	0.9783
23	23	11	11	5	0.0952	0.9958
31	31	15	15	7	0.0689	0.9968

5. Optimality of Complete Diallel Crosses

Now, we describe the optimality criterion for CDC plan using a BIBD with the parameters in section 2.

Assume that there are v lines and it is desired to find an incomplete block designs involving $\frac{v(v-1)}{2}$ as $b = v$, $r = k$ crosses and n is the total number of experimental unit in d, where n=bk.

From equation (1), the reduced normal equations for estimating linear functions of GCA effects using the CDC plan is, $C_d \hat{g} = Q$ where $Q = T - v^{-1} N_d B$ Here T is the vector of line totals and B is the vector of block totals.

5.1.Lemma

For the CDC plan, the optimality criterion has chosen in the minimization of the average variance of the best linear unbiased estimator of all elementary comparisons between general combining ability effects.

(i) An information matrix C of design is Completely Symmetric.
ie) all diagonal elements of C matrix are equal and again all the off-diagonal elements of C matrix are equal

(ii) The matrix C has maximal trace overall design in the class of computing design.

(iii) For universal optimality Kiefer (1975) proved that, if there is a C-matrix. Such that, trace of C is minimum and it is of the form

$$C = \theta\left[I_v - \frac{1}{v} E_{vv} \right]$$

where I_v denotes identity matrix of order v and θ is the non-zero eigen value of C matrix E_{vv} is the Unit matrix. Then the design is universally optimal.

Theorem 5.1

The complete diallel crosses plan obtained from two BIBD series with same set of parameters satisfying $v_1 = b_1 = 4\lambda_1 + 3 = v_2 = b_2, r_1 = k_1 = 2\lambda_1 + 1 = r_2 = k_2$ and $\lambda_1 = \lambda_2$ are universally optimal and minimize the average variance of the BLUE (best linear unbiased estimator) of all elementary contrast among the GCA effects.

Proof:

From lemma 5.1 (i), using the theorem 2.1, it is obvious that C matrix of CDC plan is completely symmetric, and the design is binary in the sense that $n_{ij} = 0$ (or) 1 $\forall i = 1,2,3,..........v$ and $j = 1,2,3,............v$.

Let d denote the class of connected design for CDC plan. Here $\frac{v(v-1)}{2}$ crosses are arranged in b blocks each of size k and each cross is repeated only once.

$$C = \frac{v(v-3)}{(v-1)}\left[I_v - \frac{1}{v} E_{vv} \right]$$

$$\therefore Tr(C_d) = Tr\left[\frac{v(v-3)}{(v-1)} (I_v - \frac{1}{v} E_{vv}) \right] = \frac{v(v-3)}{(v-1)} Tr(I_v - \frac{1}{v} E_{vv})$$

$$\therefore Tr(C_d) = v(v-3) \qquad ....(12)$$

5.1 Numerical Illustration

Here we illustrate the analysis of CDC plan discussed in the illustration 2.1, For this illustration, the NN' and C matrix are given by,

$$NN' = \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 6 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 6 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 6 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 \end{bmatrix}_{(7x7)}$$

$$C = \frac{1}{3}\begin{bmatrix} 12 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 12 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & 12 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 12 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 12 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 12 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 12 \end{bmatrix}_{(7x7)}$$

So, $C_d = \frac{14}{3}\left[I_7 - \frac{1}{7} E_{77} \right]$

Now from equation (10), we have Trace (C_d) =$v(v$-3)

Put the value of v, we have Trace (C_d) =28

Since for any member d to be universally optimal if Trace (C_d) =n (number of experimental unit).

The equality of C follows for any member of d, the Trace (C_d) is less than the number of experimental unit. Hence the CDC plan is universally optimal over design d.

6. Conclusion

The two BIB designs with same set of parameters developed from the Galois field in CDC plan to estimate the gca effects and efficiency tends to 1, provided v is large, it satisfies the universal optimality and hence the CDC plan is universally optimal.

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