New dispersion formula and results of its application

Andrey Chernov (and8591@gmail.com)
none Company/Institution

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«New dispersion formula and results of its application»

Author **Andrey Chernov**

Address: Andrey Chernov, 16 Polenova str, apt.7, Irkutsk, 664007, Russia.

cell phone: +7 964 54 07 298. E mail: and8591@gmail.com

Content

Abstract.

The aim of the study was to obtain a new physical formula for determining the refractive indices of light as a function of wavelength, which can be applied to a wide range of transparent substances. In the process of research on the basis of Einstein's relativistic formula, such a dispersion formula was obtained. Comparison of the obtained indicators with laboratory indicators showed the high accuracy of the new dispersion formula, which was $\pm \mathbf{10^{-7}-10^{-5}}$ in the calculated wavelength ranges **of more than 100 nm.**

The new dispersion formula is obtained on the basis of the mathematical dependence of the speed of propagation of photons in a transparent substance on the energy density of electron clouds of atoms of the substance. Energy is a universal category, therefore, it is possible to apply the basic version of the new formula (where instead of the wavelength there is the energy density of electron clouds) when conducting research in all areas of light generation, manipulation and detection.

And, finally, the very fact of applying the adapted relativistic Einstein's formula to physical processes occurring at the atomic level allows us to look at the nature of the interaction of light and matter from a new angle.

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Introduction.

Currently, there are no physical formulas that can be applied to a wide range of transparent substances. For example, the well-known physical formula of Lorentz-Lorentz, which is based on the dependence of the refractive index of light **on the density of a substance,** is valid only for isotropic media (gases, non-polar liquids, cubic crystals) and is not applicable for most transparent substances. Therefore, in practice, to calculate the refractive indices, empirical dispersion formulas (Cauchy, Hartmann, etc.) are usually used. These formulas are quite accurate, but at the same time they are not physical formulas.

A new physical dispersion formula was obtained on the basis of the assumption that the speed of propagation $\boldsymbol{v}_{\gamma}$ of photons in a transparent substance depends on **the energy density of electron clouds** $\boldsymbol{Q}_{\boldsymbol{e}}$ of atoms of the substance: the higher the density of the electron clouds, the lower the speed of the photons. In this case, the greater the energy of the photons propagating in the substance, the more the electron clouds of the atoms of the substance are "condensed" by the energy. This leads to the fact that different wavelengths in the same transparent substance propagate at different speeds. Thus, there is a physical relationship between the energy density $\boldsymbol{Q}_{\boldsymbol{e}}$ of electron clouds of atoms of matter and the speed of propagation $\boldsymbol{v}_{\boldsymbol{\gamma}}$ of photons in matter. This dependence, as it turned out in the study, is regulated by Einstein's relativistic formula for determining the total energy of a moving body. As a result of the transformation of this formula, the author obtained a new dispersion formula, which showed very good results.

Methods.

Now let us describe in detail the method of obtaining a new dispersion formula. To do this, we first write down Einstein's relativistic formula for determining the total energy of a moving body:

$$\boldsymbol{E_{total}} = \frac{\boldsymbol{m_0\, c^2}}{\sqrt{\boldsymbol{1-\frac{v^2}{c^2}}}} \quad \text{or} \quad \boldsymbol{E_{total}} = \frac{\boldsymbol{E_0}}{\sqrt{\boldsymbol{1-\frac{v^2}{c^2}}}} \quad \textbf{(1-1)}$$

Where $\boldsymbol{E_{total}}$ is the total energy of a moving body.

$\boldsymbol{E_0}$ – energy of a body at rest.

$\boldsymbol{v}$ is the speed of the body.

Let's transform the formula (1-1) and as a result we get:

$$\boldsymbol{v = c\sqrt{1 - \frac{{E_0}^2}{E^2_{\text{полн.}}}}} \quad \text{или} \quad \boldsymbol{v = c\sqrt{1 - Q_b^2}} \quad \textbf{(1-2)}$$

Where $\boldsymbol{Q_b}$ is an indicator of the ratio of the energy of a body at rest to the total energy of a moving body, $\boldsymbol{0 < Q_b} < 1$.

Now we apply formula (1-2) to the speed of propagation of photons in a transparent substance:

$$\boldsymbol{v_\gamma = c\sqrt{1 - Q_e^2}} \quad \textbf{(1-3)}$$

Where $\boldsymbol{v_\gamma}$ is the speed of propagation of photons in the electron clouds of atoms of a transparent substance.

$\boldsymbol{Q_e}$ is a dimensionless indicator of the energy density of electron clouds of a transparent substance, $\mathbf{0} < \boldsymbol{Q_e} < 1$.

Let's transform the formula (1-3) and get:

$$\frac{c}{v_\gamma} = \frac{1}{\sqrt{1-Q_e^2}} \quad \text{or} \quad n = \frac{1}{\sqrt{1-Q_e^2}} \qquad \textbf{(1-4)}$$

Where $\boldsymbol{n}$ is the refractive index of light in the substance ($\boldsymbol{n} = \boldsymbol{c}/\boldsymbol{v_\gamma}$).

Let's reveal the value of $\boldsymbol{Q_e}$ in the formula (1-4):

$$n = \frac{1}{\sqrt{1-(Q_0+\Delta Q_\lambda)^2}} \qquad \textbf{(1-5)}$$

Where $\boldsymbol{Q_0}$ is a dimensionless basic indicator of the energy density of electron clouds of a transparent substance.

$\Delta \boldsymbol{Q_\lambda}$ is a dimensionless indicator of the increase in the energy density of electron clouds of a transparent substance.

The $\boldsymbol{Q_0}$ index is constant (at constant temperature and pressure). The exponent $\Delta \boldsymbol{Q_\lambda}$ is a variable. It depends on the energy $\boldsymbol{e}$ of the electromagnetic wave, where $\boldsymbol{e} = \boldsymbol{h\gamma} = \boldsymbol{h\,c}/\boldsymbol{\lambda}$. From here we get the formula:

$$n = \frac{1}{\sqrt{1-(Q_0+k_\gamma h\,c/\lambda\ )^2}} \qquad \textbf{(1-6)}$$

Where $\boldsymbol{k_\gamma}$ is the coefficient of proportionality, J^{-1}.

Replace $\boldsymbol{k_\gamma}\ \boldsymbol{h}\ \boldsymbol{c}$ with a single coefficient $\boldsymbol{k_\lambda}$ and obtain a new dispersion formula:

$$n = \frac{1}{\sqrt{1-(Q_0+k_\lambda/\lambda)^2}} \qquad \textbf{(1-7)}$$

Where $\boldsymbol{k_\lambda}$ is the coefficient of proportionality, nm.

$\boldsymbol{\lambda}$ – wavelength, nm.

The coefficient $\boldsymbol{k_\lambda}$ is individual for each substance and depends on the absorption of electromagnetic waves by atoms. It is relatively stable in the visible range of the electromagnetic spectrum. But in the ultraviolet and infrared ranges, the coefficient $\boldsymbol{k_\lambda}$ can significantly change its value due to changes in the absorption of electromagnetic waves by matter. For this reason, according to f. (1-7), the value of the refractive index $\boldsymbol{n}$ can change sharply up to the adoption of anomalous values. This circumstance introduces a limitation on the use of formula (1-7) in these wave ranges.

Let us check the accuracy of the new dispersion formula. Table 1 presents 39 laboratory indices of refraction of light in the visible range of the spectrum in five transparent substances. 13 conventionally known indices of refraction of light are highlighted in bold, 26 conventionally unknown indices, which must be determined using a new formula, knowing the wavelength, are highlighted in regular font. (These refractive indices are commonly known and readily available on the Internet. To be able to verify with the table data, they will be sent to the editor in a separate file). The first column of the table contains the basic indicators $\boldsymbol{Q_0}$ (they were determined by solving a system of equations and subsequent selection of the optimal value $\boldsymbol{Q_0}$). The wavelengths are highlighted in bold in the table, where the proportionality coefficients were calculated using the formula $\boldsymbol{k_{\lambda 1,2} = \lambda_n \left(\sqrt{\frac{n^2-1}{n^2}} - Q_0\right)}$ **(1-8)**, which will be used for interpolation. As can be seen from the table, for an inert gas the number of such coefficients was unity for the entire wavelength range, for other substances - 3. (This is due to different amplitudes of fluctuations in the magnitude of the proportionality coefficients in these substances). Then, using the formula $\boldsymbol{k_{\lambda}} = \frac{k_{\lambda 1}(\lambda_n - \lambda_2) + k_{\lambda 2}(\lambda_1 - \lambda_n)}{(\lambda_1 - \lambda_2)}$ **(1-9)**, the coefficients $\boldsymbol{k_{\lambda}}$ (they are presented in the table in regular font) and then the refractive indices of light are determined by the formula (1-7).

After that, the calculated indices were rounded off in accordance with the number of digits after the decimal point in laboratory refractive indices. Therefore, for an inert gas, the refractive indices of light were rounded up to 7 decimal places, for water - up to 5 decimal places, for solids - up to 4 decimal places. It should be noted that in those cases when the rounding of the numbers led to a complete coincidence of the refractive indices, then the accuracy was taken to be one order of magnitude greater than that of the other refractive indices. For example, in glass, after rounding, two refractive indices completely coincided for wavelengths of 670, 8, and 643.8 nm. The accuracy here was taken as $\mathbf{10^{-6}}$. An order of magnitude higher than the rest of the refractive indices in glass, where the accuracy was $\mathbf{10^{-5}}$. The same method was applied to the rest of the indicators in other substances. The author believes that this approach is the most correct, because the known laboratory parameters, after being obtained experimentally, were also rounded to a certain sign. From this it follows that when comparing the refractive indices, the equality of the commas after zero must be observed, because otherwise, the calculation accuracy indices may increase unreasonably or, conversely, decrease.

After rounding off the calculated indices, they were compared with laboratory refractive indices and the discrepancy between them was determined. The results were tabulated.

Results and discussion.

Table 1 shows the 26 calculated refractive indices of light in 5 transparent substances, which were calculated using the new dispersion formula.

Comparison of the indicators calculated by the new physical formula with laboratory indicators showed the following: in an inert gas the discrepancy was $\mathbf{10^{-7}}$**,** in water and solids $\mathbf{\pm 10^{-6} - 10^{-5}}$. In this case, **the calculated range was more than 100 nm.**

For comparison, the very exact empirical formula of Hartmann has four constants and shows an accuracy of $\mathbf{\pm 10^{-6} - 10^{-5}}$ in the sections of the wavelength ranges **that do not exceed several tens of nm.**

If we compare the new formula with the physical Lorentz-Lorentz formula, then the advantage of the new formula is obvious. This is a much wider range of action among transparent substances, which is equal to the range of known empirical formulas.

Conclusions. In this study, on the basis of Einstein's relativistic formula, a new dispersion formula was obtained. This physical formula was used to calculate 26 refractive indices of light in 5 transparent substances in three states of aggregation. The accuracy of the new dispersion formula was **$\pm 10^{-7} - 10^{-5}$** in the calculated wavelength ranges **of more than 100 nm.** This physical formula can be applied, as well as empirical formulas, to almost all transparent substances.

The new formula is obtained on the basis of the mathematical dependence of the speed of propagation of photons in a transparent substance on the energy density of electron clouds of atoms of the substance. Energy is a universal category, therefore, it is possible to apply the basic version of the new formula (where instead of the wavelength there is the energy density of electron clouds) when conducting research in all areas of light generation, manipulation and detection.

And, finally, the very fact of applying the adapted relativistic Einstein's formula to physical processes occurring at the atomic level allows us to look at the nature of the interaction of light and matter from a new angle.

See Table1 on the following page

Table 1

Substance	λ nm	k_λ nm	**Calculated refractive index**	**Laboratory refractive index**	**Divergence**
Krypton Q_0 =**0,0228741**	450,4		1,0002750	1,0002752	**10^{-7}**
	556,4	**0,2573906**	**-**	**1,0002724**	
	565,1		1,0002723	1,0002722	
	587,3		1,0002719	1,0002719	
	605,8		1,0002715	1,0002716	
	645,8		1,0002709	1,0002711	
Water Q_0 = **0,648752** t= 20 $°C$	447,1	7,3594178	1,33931	1,33942	**10^{-6} – 10^{-5}**
	471,3	**7,3504891**	**-**	**1,33793**	
	486,1	7,3450358	1,33716	1,33712	
	501,6	7,3393246	1,33640	1,33635	
	546,1	**7,322928**	-	**1,33447**	
	577,0	7,3469439	1,33342	1,33338	
	587,6	7,3551824	1,33308	1,33304	
	656,3	**7,4085769**	**-**	**1,33115**	
	670,8	7,4198465	1,33080	1,33080	
	706,5	7,447593	1,32999	1,33002	
Sylvin Q_0 = **0,727035** t= 18 $°C$	480,0	8,5850021	1,4989	1,4990	
	486,1	**8,5766026**	-	**1,4983**	**10^{-6} – 10^{-5}**
	508,6	8,5456209	1,4962	1,4961	
	546,1	**8,4939848**	-	**1,4931**	
	589,3	8,5471312	1,4905	1,4904	
	643,8	8,6141794	1,4876	1,4877	
	656,3	**8,6295574**	-	**1,4872**	
	670,8	8,6473959	1,4866	1,4866	
Light crown glass Q_0 =**0,741579** t=15 $°C$	480,0	6,1563048	1,5234	1,5235	
	486,1	**6,1545121**	-	**1,5230**	**10^{-6} – 10^{-5}**
	546,1	6,1372274	1,5192	1,5191	
	589,3	**6,1241824**	-	**1,5170**	
	643,8	6,1806235	1,5149	1,5149	
	656,3	**6,1935687**	-	**1,5145**	
	670,8	6,2085851	1,5140	1,5140	
Rock salt Q_0 = **0,747572** t=18 $°C$	480,0	8,5940181	1,5541	1,5541	
	486,1	**8,5867621**	-	**1,5534**	**10^{-6} – 10^{-5}**
	508,6	8,5599976	1,5510	1,5509	
	546,1	**8,5153919**	-	**1,5475**	
	589,3	8,5653435	1,5445	1,5443	
	643,8	8,6283611	1,5413	1,5412	
	656,3	**8,6428147**	-	**1,5407**	
	670,8	8,6360523	1,5399	1,5400	

Declarations

1. **Availability of data and materials.**

All data obtained and analyzed in the course of this study is included in this article (and file with additional information.)

2. **Competing interests.** Not applicable (there are no competing interests).

3. **Funding.** Not applicable.

4. **Authors' contributions**. Not applicable.

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Andrey Chernov