Supporting Information for

**[*High obliquity, high angular momentum Earth as Moon’s origin revisited by Advanced Kinematic Model of Earth-Moon System*]**

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**S1. Proposal of Advanced Kinematic Model.**

**S1.1**. **Calculation of Total Angular Momentum(AM) of Earth-Moon System as the vector sum of constituent AMs.**

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**Figure S1.1. Spin-Orbital configuration of Earth-Moon System.**

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**Figure S1.2. Plane of the Ecliptic, the Lunar Equatorial Plane and the Lunar Orbital Plane.** [Credit: 1996 Encyclopaedia Britannica Inc.]

In Figure S1.1.,

J0 = orbital angular momentum (AM) of Earth-Moon (E-M) system.

J1= spin angular momentum of Moon.

J2=spin angular momentum of Earth.

J3 = vector sum of J0 and J1.

J4 = Total AM of E-M system.

J0, J1 and ecliptic normal are coplanar according to Cassini Law 3.

Before Cassini State transition E-M sysem was in Cassini State I when Moon’s spin axis (J1) and Moon’s Orbital plane Normal (J0) were on the same side of Ecliptic Normal. Presently E-M system is in Cassini State II hence J0 vector and J1 are on the two sides of Ecliptic Normal as shown in the Figure S1.1 and in Figure S1.2.

J3, J2 , J4 and Ecliptic normal are coplanar.

But the plane containing J0,J1 and Ecliptic Normal and the plane containing J2, J3, J4 and Ecliptic normal are two separate planes hence J0 and J1 are shown by dotted lines.

Definitions of Earth’s Obliquity(ϕ), Moon’s orbital plane inclination (α) and Moon’s Obliquity (β):

Axial tilt of Earth’s spin axis with respect to (w.r.t.) Ecliptic Normal = ɸ = 23.44⁰ = 0.4091051767 radians;

Axial tilt of Moon’s spin axis w.r.t. Ecliptic Normal = β = 1.54⁰ = 0.02687807 radians;

Angle between Moon’s equatorial plane and ecliptic plane = β ;

Total axial tilt of Moon’s spin axis w.r.t. E-M orbital AM vector = α+β = 6.68⁰ =0.11658 radians.

All these are observational Astronomy data in the current era and illustrated in Figure S1.1 and Figure S1.2.

**S1.2. The total resultant angular momentum vector of Earth-Moon system.**

According to Cassini Law, Moon’s spin axis Normal to the equatorial plane of Moon, Moon’s orbital plane Normal and Ecliptic plane Normal are co-planar hence these three NORMALS can be drawn on the same page but Earth’s spin axis are not co-planar hence Earth’s spin will be kept out while determining the resultant angular momentum J3 = vector sum of J0 and J1 of E-M system.

**Cassini’s laws**, three [empirical](https://www.merriam-webster.com/dictionary/empirical) rules that accurately describe the rotation of the [Moon](https://www.britannica.com/place/Moon), formulated in 1693 by [Gian Domenico Cassini](https://www.britannica.com/biography/Gian-Domenico-Cassini). They are:

(1) the Moon rotates uniformly about its own axis once in the same time that it takes to revolve around the Earth that is the Moon has a synchronous motion showing the same face to Earth;

(2) the Moon’s equator is tilted at a constant angle (β = about 1.54° of arc) to the [ecliptic](https://www.britannica.com/science/ecliptic), the plane of [Earth’s](https://www.britannica.com/place/Earth) [orbit](https://www.britannica.com/science/orbit-astronomy) around the Sun; and

(3) the ascending node of the lunar orbit (i.e., the point where the lunar orbit passes from south to north on the ecliptic) always coincides with the descending node of the lunar equator (i.e., the point where the lunar equator passes from north to south on the ecliptic). As a consequence of the third law, the north pole of the Moon as projected on the sky (point z), the north pole of the ecliptic (point Z), and the north pole of the lunar orbit (point P, inclined at an angle of about = α = 5.14° to the ecliptic) all lie close to one another on a great circle as Shown in Figure S1.2.

Total Angular Momentum Vector of E-M system is determined in two parts.

*In first part*: J0 the orbital angular momentum and J1 Moon’s spin angular momentum and the Ecliptic Normal are taken coplanar and co-processing and J0 and J1 are placed on the two sides of the Ecliptic Normal since for lunar semi-major axis ‘a’ > 33RE E-M system has settled down in Cassini State II. And AKM is valid within the range 45RE < a < 60.33RE hence E-M system being in Cassini State II is a valid assumption.

*In second part:* J2 the spin angular momentum of oblique Earth, the ecliptic plane normal and J3 the vector sum of J0 and J1 will be treated as co-planar and the vector sum :

will be determined.

**S1.3. Determination of J3 vector = J0 vector + J1 vector.**

Here J0 (orbital angular momentum of E-M system and J1(spin angular momentum of Moon) and Ecliptic normal are coplanar hence the vector triangle ABC can be drawn on one plane.

Sum of the interior angles = 180⁰ = π =3.14 radians

Hence ∆ABC the sum of the interior angles : a + b + c = 3.14

But c = π – (α+β) = 3.14 - 0.116588 = 3.025012654 radians

Working out the vector sum of constituent angular momentum vectors , we arrive at the following results:

Here ‘a’ (semi-major axis of Moon’s orbit) = 3.844×108m; mass of Earth M= 5.9723×1024 Kg and mass of our Moon m = 0.07346×1024 Kg, m/(1+m/M) = reduced mass of Moon =7.25674×1022 Kg, TORB orbital period of Moon around Earth(sidereal period) = 27.3217d and e is eccentricity = 0.0549.

As seen in Figure 1 , Moon’s spin axis is tilted w.r.t. Ecliptic normal by 1.54° and tilted w.r.t. orbital normal by 6.68⁰ to the right of the orbital normal because presently we are in Cassini State II

Where

Since Moon is in synchronous orbit:

As seen in Figure S1.1.:

From ∆ABC in Figure S1.1. we obtain:

S1.4. can be simplified to:

Here we define the following Trignometric Identities:

Substituting these identities in (S1.5.) we get:

For modern times values of inclination and lunar obliquity we obtain:

Sunstituimg (S1.8.) and the magnitudes of J0 and J1 we obtain:

Inspecting Figure S1.1, we see that J3 makes an angle θ with respect to the normal of the ecliptic and J3 lies left to the normal.

By Sin Law:

From (S1.10.) the three angles are:

The angle of inclination of J3 w.r.t. ecliptic normal and left to normal = θ =α –b = 0.08970905087 radians = 5.13995⁰ ~ α ;

**S1.4. Determination of J4 vector = J3 vector + J2 vector =Total AM of E-M system.**

For calculating the total resultant angular momentum J4 we have to consider ∆ABD in Figure S1.1..

Applying Cos Law to ABD we get:

For the time span from a = 45RE to a = 60.336RE Moon’s spin AM is several order of magnitudes smaller than orbital AM hence it is valid to assume that angle a’~ (π-ϕ-α) hence total AM J4 is given as follows:

(S1.12.) simplifies to the following:

Substituting the Trignometric identities in (S1.13.) we obtain:

Taking the modern values of terrestrial Obliquity and lunar orbital inclination:

From (S1.9.):

Earth’s spin axis obliquity with respect to (w.r.t.) the ecliptic normal = Φ = 23.44° = 0.40910 radians ;

Substituting the magnitudes of the parameters we get:

Substituting the numerical values in (S1.14.);

Therefore:

To determine the angles b’ and d the Sine Law is used namely:

From (S1.19. ) the three angles are:

The sum of the internal angles comes to be:

Therefore total angular momentum of E=M system is:

In scalar analysis,

JT = total angular momentum of E-M system= 3.43584×1034Kg-m2/s , (eccentricity was zero)

By vector analysis of AM of E-M system very simple picture emerges.

In real world situation, vectorial total angular momentum of E-M system has been constrained to be almost but not exactly normal to ecliptic plane after Laplace Plane Transition and its magnitude has remained invariant at 3.3749 ×1034 (Kg-m2)/s.

So the vector diagram of Figure S1.1 is valid in assuming that total AM J4 has remained invariant for last 1.5Gy and has remained near-coincident with the Ecliptic Normal.

**S1.5. Determination of the LOM/LOD equation.**

Rewrite (S1.7. )

Rewrite (S1.14.) and substitute (S1.7.) in (S1.14. ):

Let us redefine J0, J1, J2 J3

Substituting (S1.12. ), (S1.13.) and (S1.14.) in (S1.11. ) we obtain:

Divide (S1.15.) by (C×Ω)2 and let ω/Ω=X we get:

Simplifying (31)

Let

Substitute (S1.18.) in (S1.17.) we get:

Substituting Kepler’s third law :

We get:

Let

Substituting (S1.21.) in (S1.20.) we get:

In ideal case where Lunar Orbital Inclination(α) , Earth’s obliquity(ϕ) and Moon’s obliquity( β) are zero then

Sin[α] = A=0 and Cos[α] =√(1-A2)=1;

Sin[Φ] = B=0 and Cos[Φ] =√(1-B2)=1;

√(1-A2) √(1-B2)-A.B = 1;

And Sin[β] = D=0 and Cos[β] =√(1-D2)=1

Substituting the above results in (S1.21. 37)

(S1.22) simplifies to classical KM equation:

(S1.23) is the classical form being used by the Author for Kinematic Modelling with assumptions that Moon’s orbital plane Inclination, Earth’s Obliquity and Moon’s Obliquity are zero degree angle.

F and G have been defined in (33) and N has been defined in (36). Substituting the numerical values of the system parameters we get:

**S2. Introduction**

Six set of data on Lunar Orbital Plane inclination, on Lunar Obliquity and Lunar Orbit eccentricity in past geologic epochs are available from Cuk et.al (2016) simulation work. LOM/LOD set of data in past geologic epochs is available from the personal communication of the author. Utilizing these set of data and Equation 37 in the main text , the set of Terrestrial Obliquities are calculated in the corresponding epochs and from this set of Terrestrial Obliquities the spatial function of evolving Terrestrial Obliquity is determined in this supplementary information file. Thus we have four clearly defined single valued spatial functions which give the evolution of Lunar Orbital Plane inclination, Lunar Obliquity , Lunar Orbit eccentricity and Terrestrial Obliquity in the past up to 1.5Gy and in the present and as well in the future upto the final lock-in of E-M at outer geo-synchronous orbit. Using these four spatial functions and Eq.(37) in the main text, the Transit Time into the past upto 1.5Gy and into future up to the final lock in can be calculated. Thus Advanced Kinematic model acts like a Time Machine using which we can study the E-M configuration upto 1.5Gy and upto 97Gy from NOW in the future. In a sequel paper in Earth’s Future (MS # 2020EF001728) the futuristic study of Sun-Earth-moon has been done.

**S2.1. Evolution of inclination of Lunar orbital plane, eccentricity of Lunar orbit and obliquity of Moon’s spin axis based on the information in Cuk et.al.(2016), Evolution of LOM/LOD based on data in personal communication of the Author and determination of the evolution of terrestrial Obliquity from the LOM/LOD equation.**

***S2.1.1.* *Evolution of Moon’s orbital plane inclination angle (α) which currently is 5.14⁰.***

**Table S2.1. Evolution of inclination angle (α) from 30RE to 60.336RE (Cuk et.al. 2016).**

|  |  |  |  |
| --- | --- | --- | --- |
| ‘a’(× RE) | ‘a’(× 108m) | α (°) | α (radians) |
| 30 | 1.9113 | 28 | 0.4887 |
| 35 | 2.22985 | 16 | 0.279 |
| 40 | 2.5484 | 9.2 | 0.16057 |
| 45 | 2.86695 | 8 | 0.1396 |
| 50 | 3.1855 | 7 | 0.122 |
| 55 | 3.50405 | 6 | 0.1047 |
| 60 | 3.8226 |  |  |
| 60.336 | 3.844 | 5.14 | 0.0897 |

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**Figure S2.1. ListPlot of the inclination angles in TableS1.1.**

The approximate FIT to the ListPlot in Figure S2.1.

Plot of (S2.1) is as follows:

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**Figure S2.2. Plot of the FIT function Eq. S1.1.**

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**Fig.S2.3. The superposition of the ListPlot and the FIT Plot.**

Figure S2.3. gives the correspondence between the ListPlot and Eq.(S2.1).

The correspondence is good hence (S2.1) gives the evolutionary history of Moon’s orbital plane inclination angle in radians.

*S2.1.2. Evolution of Moon’s Obliquity angle which currently is β =1.54°.*

Evolution of Moon’s Obliquity angle (β) from 30RE (Cassini State Transition orbit) to 60RE (current lunar orbit) based on Cuk et.al.(2016).Table S1.2. gives the evolution of Moon’s Obliquity angle.

**Table S2.2. Evolution of Moon’s Obliquity angle(β) from 30RE to 60.336RE(Cuk et.al. 2016).**

|  |  |  |  |
| --- | --- | --- | --- |
| ‘a’(× RE) | ‘a’(× 108m) | β(°) | β(radians) |
| 30 | 1.9113 | 70 | 1.22 |
| 35 | 2.22985 | 55 | 0.96 |
| 40 | 2.5484 | 40 | 0.698 |
| 45 | 2.86695 | 29.27 | 0.5109 |
| 50 | 3.1855 | 19.25 | 0.336 |
| 55 | 3.50405 | 9.27 | 0.1618 |
| 60.336 | 3.844 | 1.54 | 0.0269 |

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**Figure S2.4. ListPlot of the Moon’s Obliquity angles given in Table S1.2..**

The approximate FIT to the ListPlot in Figure S2.4. is:

Plot of (S2.2) is as follows:



**Figure S2.5. Plot of the FIT function Eq.(S1.2).**

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**Figure S2.6. Superposition of ListPlot and Fit Plot of Moon’s Obliquity angle.**

Figure S2.6. Superposition of ListPlot and Fit Plot of the Moon’s Obliquity Angle curve Eq. (S2.2).

The correspondence is good hence Eq. (S2.2) gives the evolutionary history of Moon’s Obliquity angle in radians.

*S2.1.3. Evolution of Moon’s orbit’s eccentricity (e).*

About 80% higher angular momentum(AM) E-M system with highly tilted Earth was born after being impacted by Theia. The Moon accreted from the glancing angle impact generated well mixed Earth’s mantle and impactors debris. As fully formed Moon spiraled outward it passed through Laplace Plane Transition (rL) at 17RE. The passage through (rL) in highly oblique Earth’s environment excited high eccentricity in Moon’s orbit and high inclination of Moon’s Orbital Plane. High eccentricity drained the excess AM to heliocentric Earth’s orbit and Moon’s orbit was circularized through Earth and Moon tidal interaction. Hence highly eccentric orbit excited by Laplace Plane transition was eventually circularized and synchronized. Table (S1.3) gives the evolution of Lunar Orbit’s eccentricity (e) from a=30RE to 60.336RE .

**Table S2.3. Evolution of Moon’s orbit eccentricity from 30RE to 60.336RE .(Cuk et.al. 2016)**

|  |  |  |
| --- | --- | --- |
| ‘a’(× RE) | ‘a’(× 108m) | e |
| 30 | 1.9113 | 0.25 |
| 35 | 2.22985 | 0.23 |
| 40 | 2.5484 | 0.21 |
| 45 | 2.86695 | 0.2 |
| 50 | 3.1855 | 0.15 |
| 55 | 3.50405 | 0.1 |
| 60.336 | 3.844 | 0.0549 |

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**Figure S2.7. ListPlot of the Moon’s orbit’s eccentricity given in Table S1.3.**

The approximate FIT to the ListPlot in Figure S1.7 is:

The Plot of Eq. (S1.3) is as follows:

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**Figure S2.8. Plot of Fit Function Eq.(S1.3).**

Superposition of eccentricity ListPlot and Fit Plot is given in Figure S1.9.

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**Figure S2.9. Superposition of ListPlot and Plot of Moon’s orbit eccentricity Eq.(S2.3.)**

The correspondence is good hence Eq.(S2.3) gives the evolutionary history of Moon’s Orbit eccentricity.

*S2.1.4. The Determination of the evolutionary history of Earth’s Obliquity from Advanced Kinematic Model of tidally interacting E-M system.*

From a previous personal communication arXiv: <http://arXiv.org/abs/0805.0100>

LOM/LOD of Earth Moon system is known over the past geologic epochs.

LOM/LOD for various geological epochs are tabulated in Table S1.4.

**Table S2.4. LOM/LOD and Earth’s Obliquity for past geological epochs.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| a(×RE) | a(×108m) | LOM/LOD | Sin[Φ] | Φ (radians) | Φ° |
| 30 | 1.9 | 23.3752 | -0.464076 | unstable | unstable |
| 35 | 2.23 | 26.1194 | -0.216896 | unstable | unstable |
| 40 | 2.5484 | 28.1147 | 0.0213757 | 0.0213773 | 1.22483 |
| 45 | 2.867 | 29.2938 | 0.113547 | 0.113792 | 6.51 |
| 50 | 3.1855 | 29.5965 | 0.218451 | 0.220227 | 12.6 |
| 55 | 3.5 | 28.9877 | 0.309749 | 0.314929 | 18 |
| 60 | 3.82 | 27.4 | 0.388198 | 0.398676 | 22.84 |
| 60.335897 | 3.844 | 27.32 | 0.397788 | 0.409105 | 23.44 |

As Moon’s orbital plane inclination gets damped from 27.54° to 5.14° under the influence of strong lunar obliquity tides, Earth’s obliquity increases from 1.22483° to 23.44° under AM conservation law.

Rewriting (8) from the main Text:

In (S2.5) all constant and all spatial functions are known except the obliquity angle Φ.

For a given lunar orbit , X=LOM/LOD is known. Using this information Sin[Φ] is determined and hence Φ and tabulated in Table S2.4

We have six set of data from a = 30RE to the present day semi-major axis.

We clearly see that at Cassini State Transition i.e. at 33RE , obliquity is indeterminate. From 40RE to 60.336RE obliquity is well behaved and it is increasing. It increases from 1.22483° to 23.44°. This means that during angular momentum conservative phase i.e. from Cassini State Transition to the present epoch, reduction in Moon’s plane inclination is accompanied with increase in obliquity by necessity.

*S2.1.4.1. Evolutionary spatial functions of terrestrial obliquity(Φ) and LOM/LOD*

Evolutionary spatial functions of inclination angle (α), Moon’s obliquity(β) and of eccentricity ‘e’ have been determined in CELE-D-17-00144 and given above. They are as follows:

The LOM/LOD and Earth’s obliquity angles are tabulated in Table S2.4..

*S2.1.4.1.1. Evolutionary function of LOM/LOD*

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**Figure S2.10. ListPlot of LOM/LOD in different geologic epochs as given in Table S2.4. [Courtesy: Author]**

The approximate FIT function to the ListPlot of LOM/LOD in Table S1.14. is:

The plot of (S2.6) is as follows:

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**Figure S2.11. Plot of FIT function given by Eq.(S1.6). [Courtesy: Author]**

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**Figure S2.12. Superposition of LOM/LOD ListPlot and FIT Plot. [Courtesy: Author]**

Superposition of LOM/LOD ListPlot and Fit Plot is given in Figure S2.12.

The correspondence between LISTPLOT and FIT PLOT is good hence (S2.6) gives the evolutionary history of LOM/LOD.

*S2.1.4.1.2. Evolutionary function of Earth’s obliquity.*

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**Figure S2.13. List Plot of Earth’s obliquity (Φ) angle over different geological epochs given in Table S2.4. [Courtesy: Author]**

The approximate FIT function to the ListPlot of Earth’s obliquity in Figure S2.13 is:

The Plot of (S2.7) is as follows:

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**Figure S2.14. Plot of FIT function given by (S2.7). [Courtesy: Author]**

Superposition of ListPlot and Fit function is given in Figure S2.15.

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**Figure S2.15. Superposition of ListPlot of Earth’s obliquity (Φ) and Fit Plot. [Courtesy: Author]**

The correspondence between LISTPLOT and FIT PLOT is good hence (S2.7) gives an accurate evolutionary history of Earth’s obliquity .

We have altogether 5 spatial function (S2.1), (S2.2), (S2.3), (S2.6) and (S2.7) describing the evolution of inclination angle (α), Moon’s obliquity (β), eccentricity(e) of lunar orbit, LOM/LOD and Earth’s obliquity (Φ) respectively through different geologic epochs. These are tabulated in Table S2.5.

**Table S2.5. evolutionary history of ω/Ω (LOM/lOD),α (Inclination angle) , β (lunar obliquity), e (eccentricity) and Φ (terrestrial obliquity).**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| a (×RE) | a (×108) | ω/Ω | α radians | β | e | Φ(rad) | Sin[Φ] |
| 30 | 1.9113 | 23.372 | 0.480685  (27.4°) | 1.21635  (69.69°) | 0.2524 | unstable | -0.464076 |
| 35 | 2.2295 | 26.114 | 0.26478  (15.17°) | 0.952317  (54.56°) | 0.236 | unstable | -0.216896 |
| 40 | 2.5484 | 28.117 | 0.168969  (9.68°) | 0.71512  (40.97°) | 0.214 | 0.021373 | 0.0123757 |
| 45 | 2.86695 | 29.2938 | 0.124631  (7.1408°) | 0.504756  (28.92°) | 0.1849 | 0.113792  (6.51°) | 0.113547 |
| 50 | 3.1855 | 29.5965 | 0.103801  (5.04736°) | 0.321225  (18.4°) | 0.1493 | 0.220227  (12.6°) | 0.218451 |
| 55 | 3.50405 | 28.9877 | 0.0941394  (5.39379°) | 0.164527  (9.4267°) | 0.10714 | 0.314929  (18°) | 0.309749 |
| 60 | 3.8226 | 27.4 | 0.0898729  (5.149°) | 0.03466  (1.986°) | 0.0584 | 0.398676  (22.84°) | 0.388198 |
| 60.336 | 3.844 | 27.32 | 0.08971  (5.14°) | 0.0268  (1.54°) | 0.0549 | 0.409105  (23.44°) | 0.397788 |

LLR measurement of 3.7cm/y was resulting in too short an age of Moon (~ 3Gy) which was contrary to the observed age of the rocks brought from Moon during Apollo Missions from 1969 to 1972 (curation/Lunar-NASA). These missions brought 382Kg of lunar rock, core samples, pebbles, sand and dust from the Moon surface. It is estimated that Moon’s crust formed 4.4by ago. A team of scientist have studied Apollo 14 zircon fragments. They put the age of Moon at 4.51by (Barbanie et.al.2017). Matija Cuk,; Douglas P. Hamilton,; Simon J. Lock,; Sarah T. Stewart (2016) finally have resolved this conundrum. According to this research, from 3RE to 45RE , Moon does not have a smooth monotonic and spiral expansion. Infact it is bumpy. It is chaotic, gets stuck in resonances and comes out of the resonances and gets stalled and resumes its tidal evolution. In fact Moon takes 3.267Gy to spirally expand from 3RE to 45RE in fits and stalled manner. From 45RE to 60.336RE, Moon smoothly coasts in 1.2Gy. This accelerated spiral expansion in the on-going phase results in present day anomalous velocity of recession of 3.7cm/y. As we will see the consistency with LLR results resolves a long standing problem of mismatch between observed LOD curve and theoretical LOD curve. In this new E-M model, a precise match is obtained between the theory and observation..

The series of papers in CELMEC VII and the main text have set the stage for Advanced KM to be established as a well tested tool for further applications in Space Dynamics. I also envisage the application of this model in earth-quake predictions .

**S2.1.5..The algorithm for calculating the Transit Time from an earlier orbit to a later one.**

Sharma 2011 gives the velocity of recession of the secondary in a tidally interacting binary pair launched in super-synchronous spirally expanding orbit:

The value of the constants in Eq.(S1.8) are as follows:

The given value of K (structure constant) and Q(exponent of the structure factor) ensure the modern day recession velocity of Moon as 3.82±0.07cm/y as measured in the current Lunar Laser Ranging Experiments (Dickey et.al.1994).

Eq.(37) in the main text is a quadratic equation in X = LOM/LOD. Eq.(37) is solved and two roots are obtained. One is negative and the other is positive. The positive root is retained. The value of lunar orbital inclination angle (α in radians) given in Eq.(31), the value of Moon’s obliquity angle (β in radians) given in Eq.(32), the value of Moon’s orbit eccentricity described in Eq.(33) ,and function of terrestrial obliquity angle (ɸ in radians) are substituted in the expression of A, D, k and B and Eq.(37)is solved . This form of X is substituted in Eq.(S2.8).

Eq.(S2.8) is used for calculating the transit time from any earlier orbit to the present orbit 3.844×108m.

The transit time is given by the following time integral:

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