

# Sensitivity Matrices as Keys to Local Structural System Properties of Large-Scale Nonlinear Systems.

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## Research Article

**Keywords:** Local controllability, local strong accessibility, local observability, local structural identifiability, large-scale nonlinear systems, sensitivity matrix, singular value decomposition, Lie algebraic rank condition (LARC), sensitivity rank condition (SERC).

**Posted Date:** May 27th, 2021

**DOI:** <https://doi.org/10.21203/rs.3.rs-534770/v1>

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**Version of Record:** A version of this preprint was published at Nonlinear Dynamics on January 29th, 2022. See the published version at <https://doi.org/10.1007/s11071-021-07125-4>.

# Abstract

Sensitivities are shown to play a key role in a very efficient algorithm, presented in this paper, to establish three fundamental structural system properties: local structural identifiability, local observability and local strong accessibility. Sensitivities have the advantageous property to be governed by linear dynamics, also if the system itself is nonlinear . By integrating their linear dynamics over a short time period, and by sampling the result, a sensitivity matrix is obtained. If this sensitivity matrix satisfies a rank condition, then the local structural system property under investigation holds. This rank condition will be referred to in this paper as the sensitivity rank condition (SERC). Applying a singular value decomposition (SVD) to the sensitivity matrix not only determines its rank but also pinpoints exactly the system components causing a possible failure to satisfy the local structural system property. The algorithm is very efficient because integration of linear systems over short time-periods and computation of an SVD are computationally cheap. Therefore, it allows for the handling of large-scale systems in the order of seconds, as opposed to conventional algorithms that mostly rely on Lie series expansions and a corresponding Lie algebraic rank condition (LARC). We extensively discuss the (dis)advantages of both algorithms and to which extent their results coincide. A series of examples is presented to illustrate these results.

# Full Text

Due to technical limitations, full-text HTML conversion of this manuscript could not be completed. However, the manuscript can be downloaded and accessed as a PDF.

# Figures

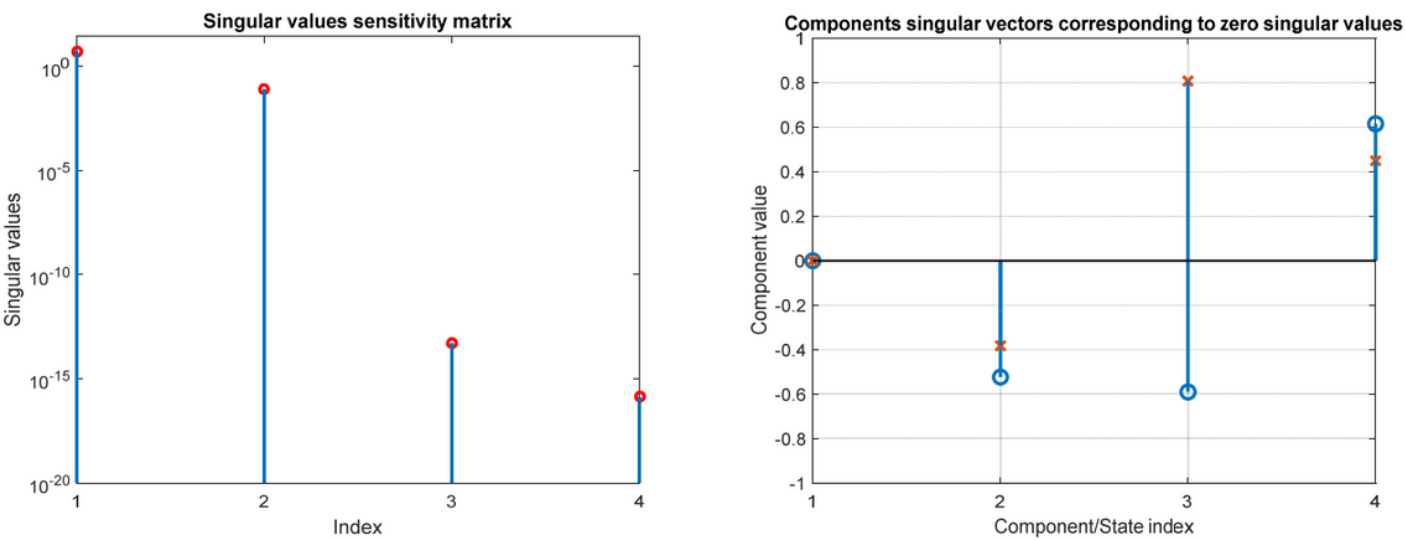


Figure 1

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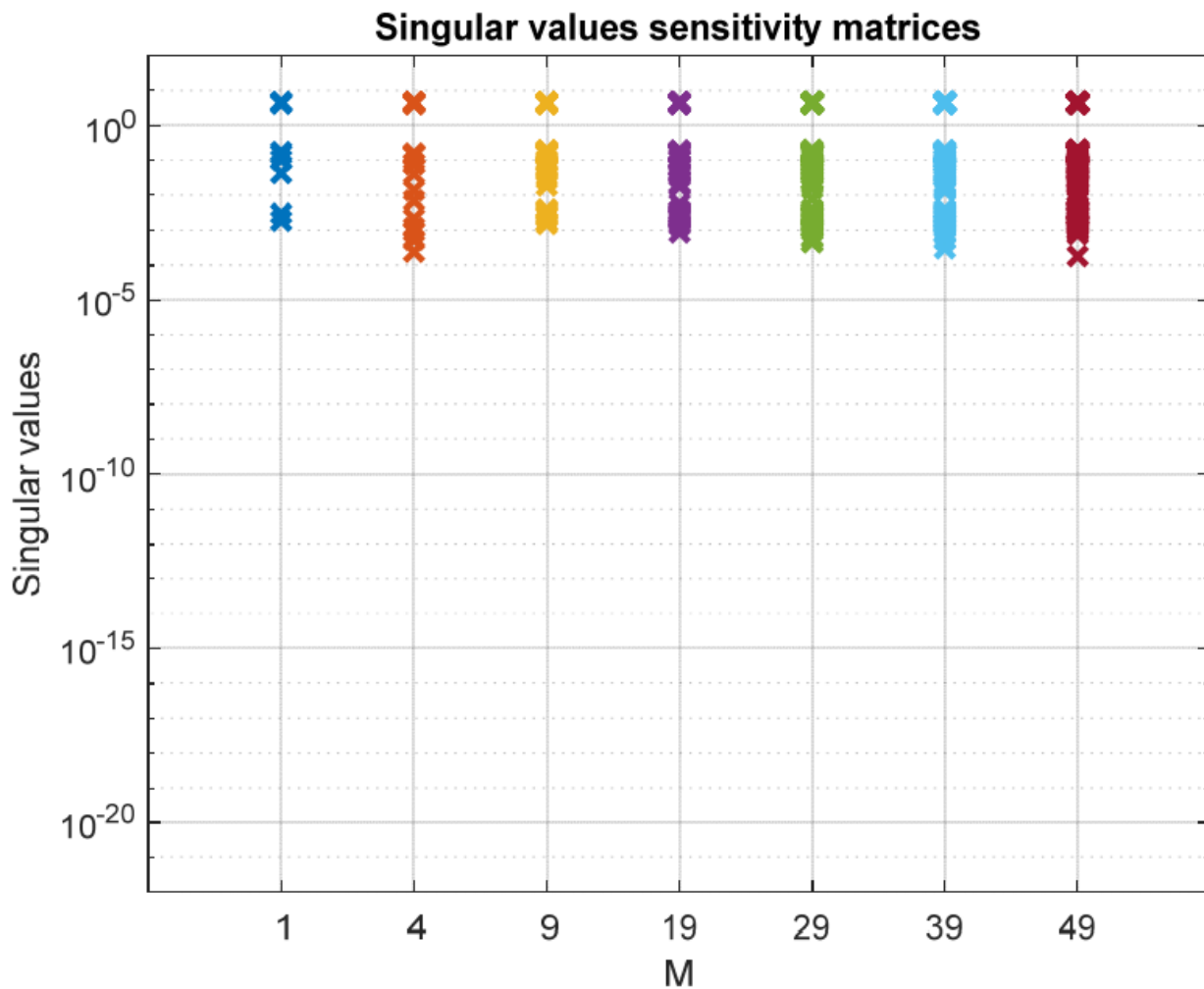


Figure 2

Singular values for  $M = 1, 4, 9, 19, 39, 49$  confirming local strong accessibility in each case because no gaps and associated numerically zero singular values occur.

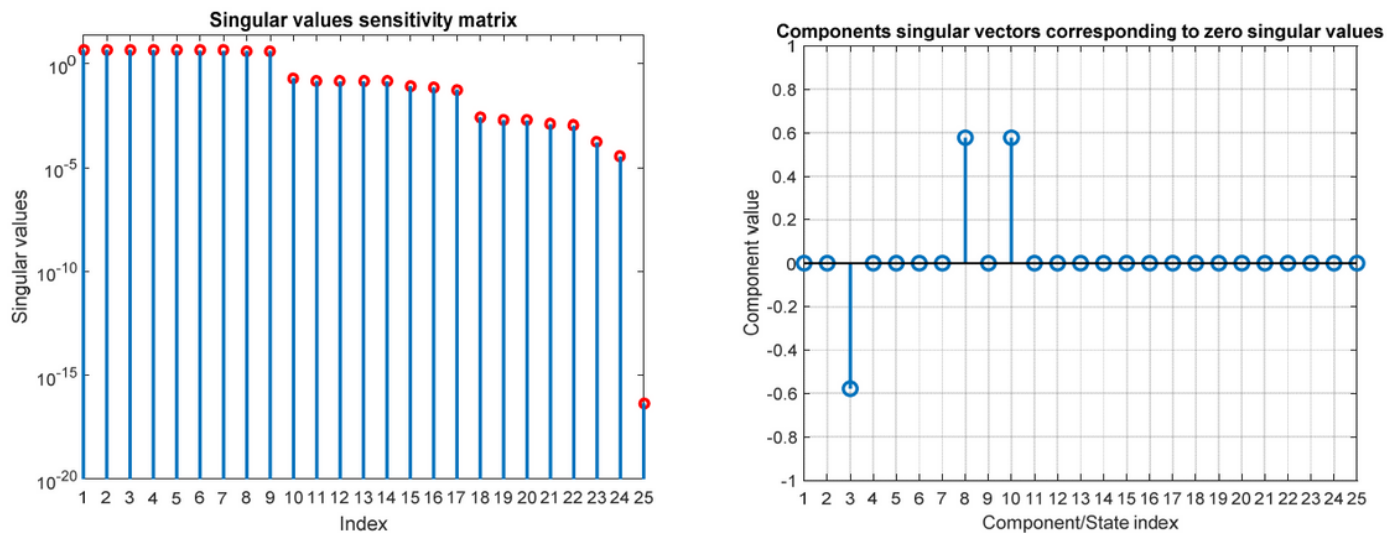


Figure 3

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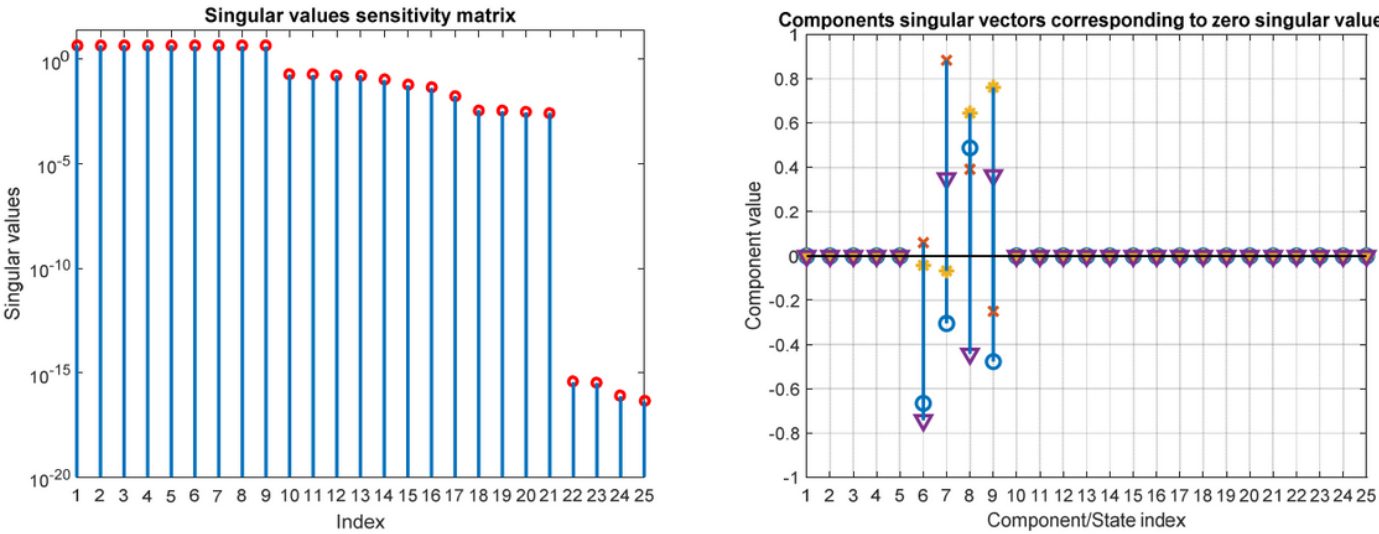


Figure 4

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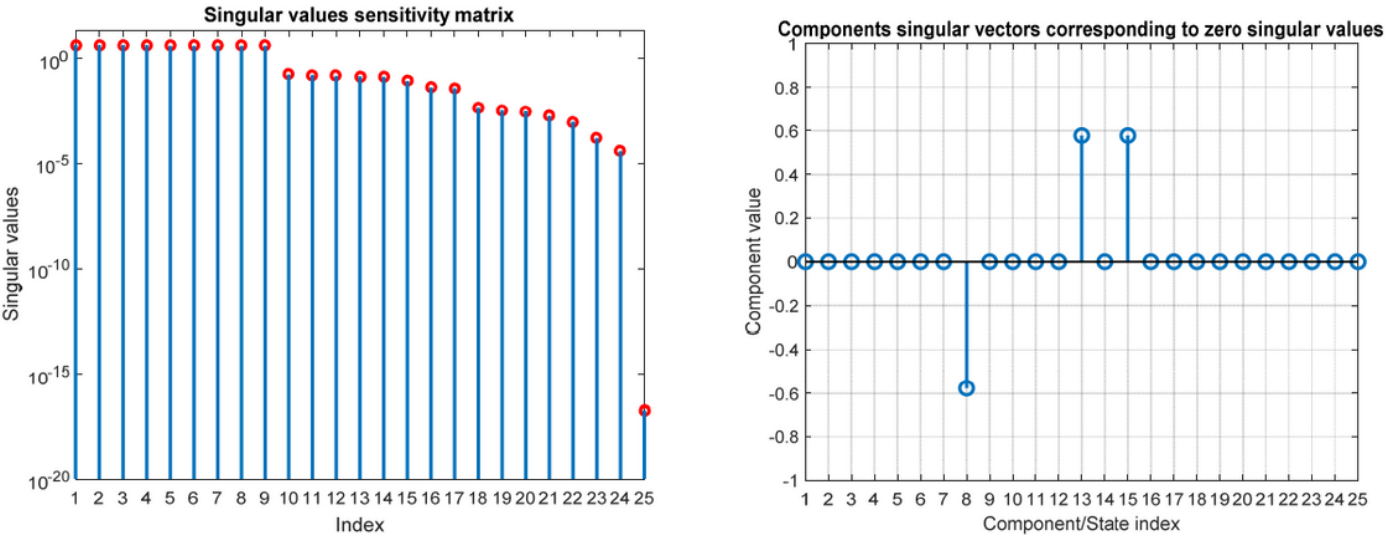
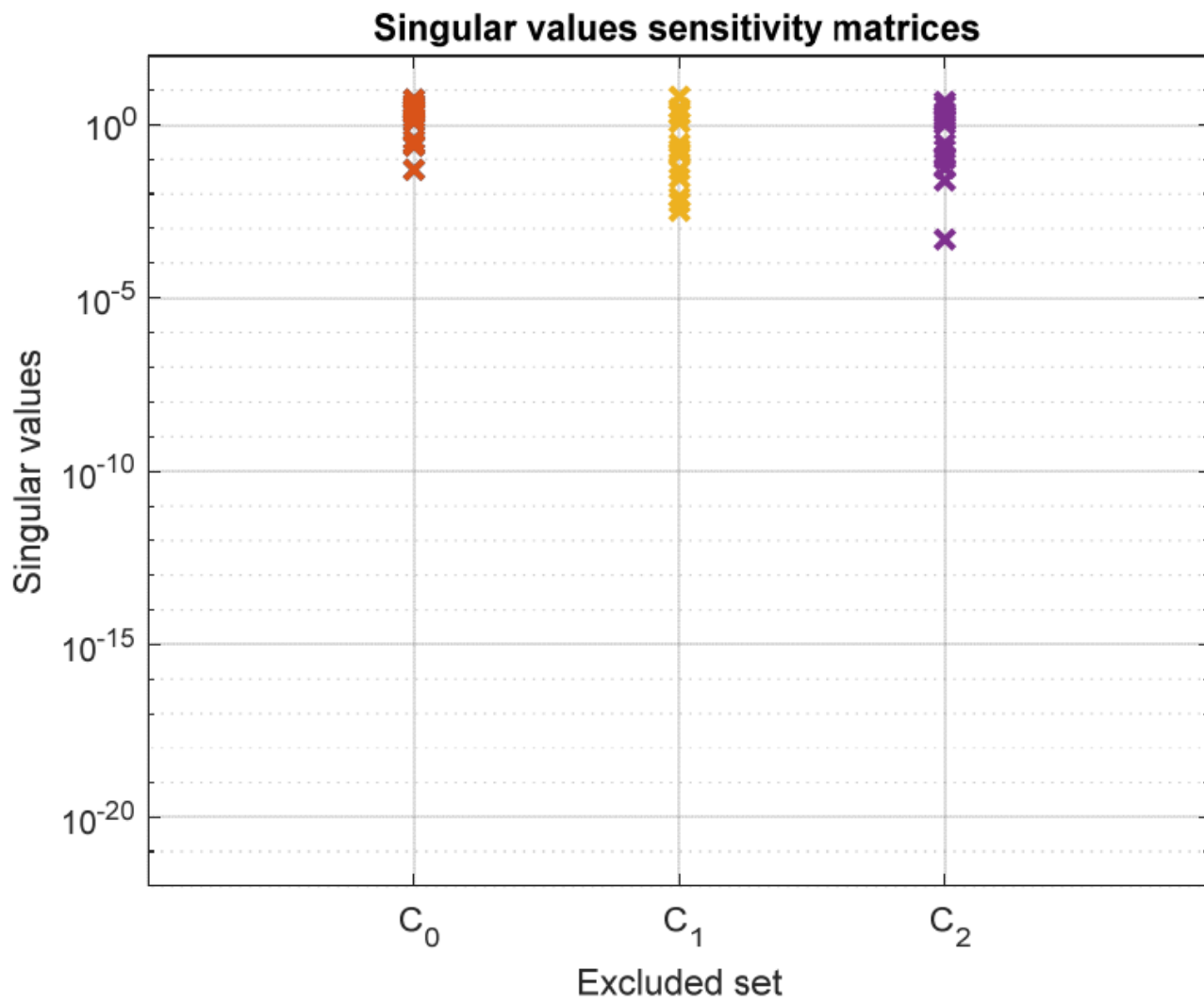


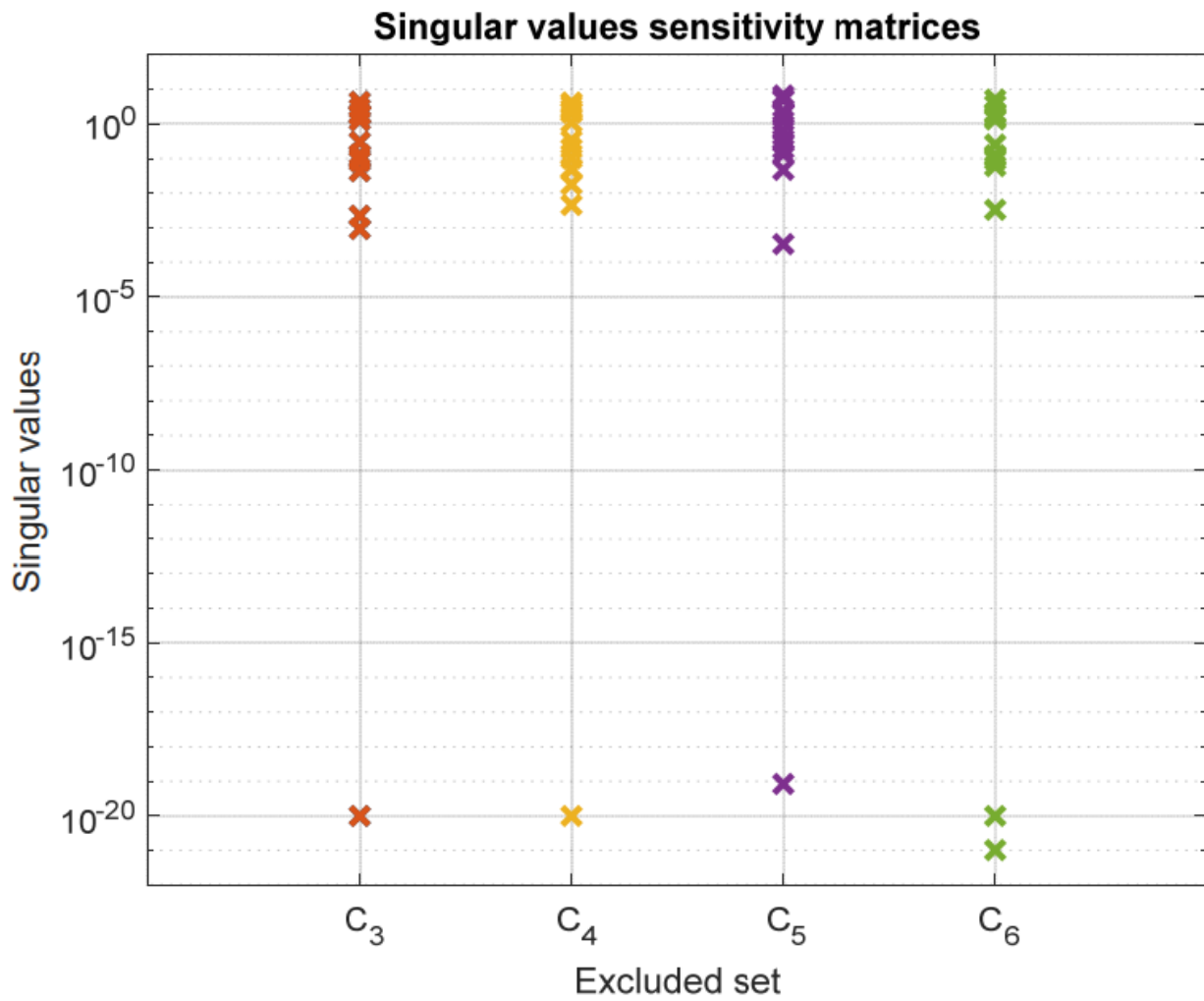
Figure 5

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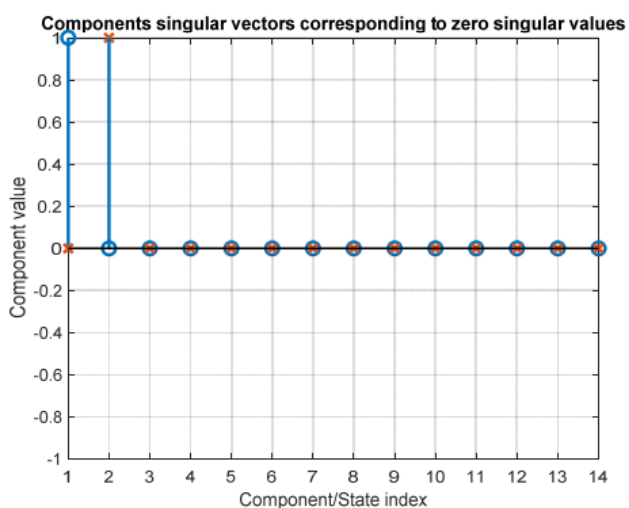
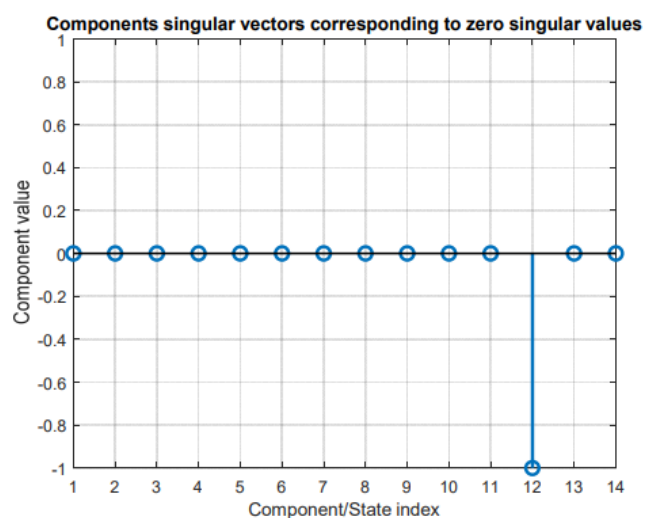
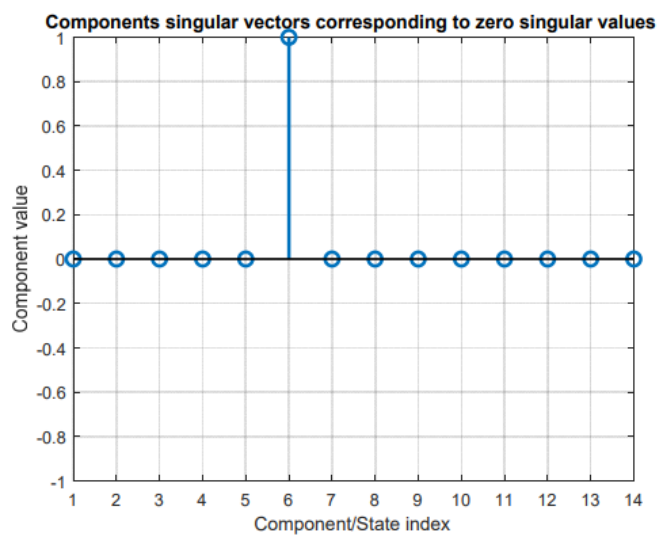
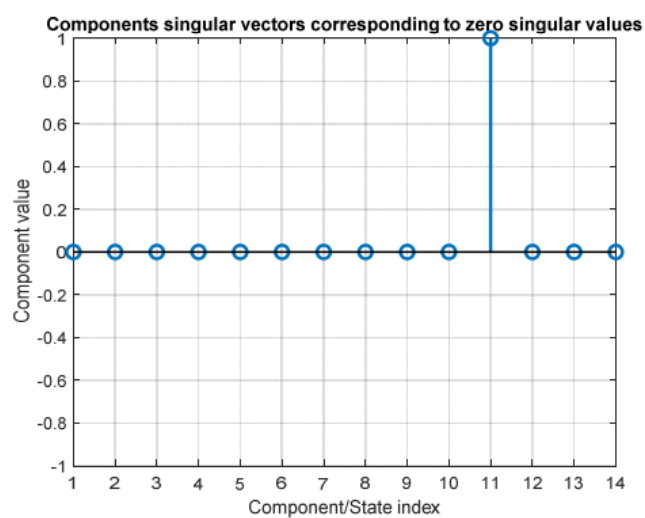
**Figure 6**

Singular values obtained for 3 excluded sets of inputs. Since all of them are non-zero numerically this confirms local strong accessibility in each case.



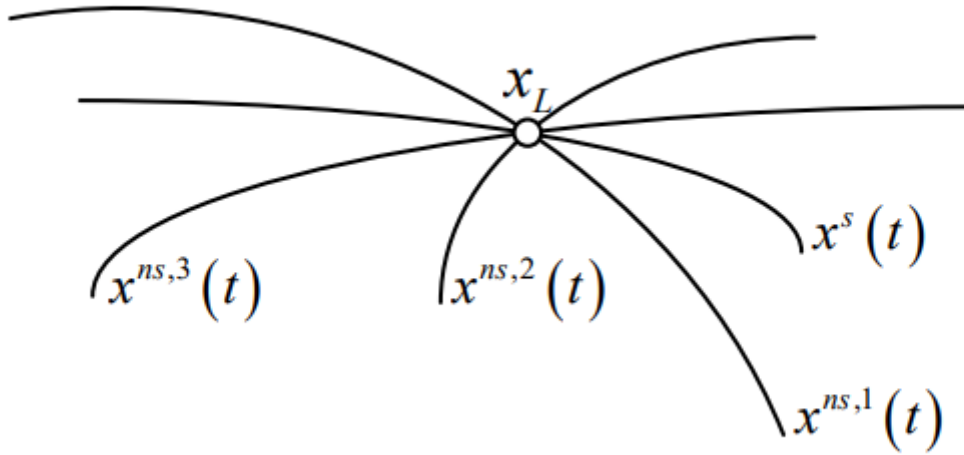
**Figure 7**

Singular values obtained for 4 excluded sets of inputs. In each case at least 1 numerically zero singular value occurs implying local strong accessibility does not hold.



**Figure 8**

Please see the Manuscript PDF file for the complete figure caption.



**Figure 9**

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