Title page

Stiffness Model of the Armature Assembly in a Jet Pipe Pressure Servo Valve

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Stiffness Model of the Armature Assembly in Jet Pipe Pressure Servo Valve

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Abstract: The armature assembly of the jet pipe pressure servo valve plays an important role in connecting the torque motor and the jet pipe amplifier. A stiffness model of its complex structure is very necessary for analyzing the dynamic/static performance of the jet pipe pressure servo valve. At the present work, the component parts in the armature assembly are simplified into linear elastic beams. The simplified armature assembly is a fourfold statically indeterminate structure under the premise of small deformation. The unknown forces and moments are solved by using the section continuity condition as the additional supplement equation, and the functional relationship between the electromagnetic torque produced from the torque motor and the armature rotation angle /the nozzle displacement is derived based on the Castigliano's Theorem. The finite element model of the armature assembly is also established to calculate the deformation under different loads and different spring tube lengths. The simulated displacements with the finite element method are consistent with the theoretical results. The experimental results of the recovery pressure of the jet pipe valve verified the theoretical model. The proposed stiffness calculation method can be used as a reference for designing and optimizing the armature assembly in the jet pipe pressure servo valve.

Keywords: Jet pipe servo valve • Armature assembly • Energy method • Finite element method • Stiffness • Pressure control valve • Torque motor

1 Introduction

An electro-hydraulic pressure servo valve receives the analog electric signal and outputs the hydraulic pressure signal that varies with the size and the polarity of the electric control signal. At present, the most common pressure servo valve is the two-stage pressure servo valve (TSPSV), which is widely used in electro-hydraulic load simulators, the passive robotic systems [1] and the aircraft anti-skid braking systems [2]. The performance of a TSPSV is directly related to the control accuracy and work reliability of the force servomechanism. Generally, a TSPSV consists of three parts: the electro-mechanical convertor, the pre-stage hydraulic amplifier and the main slide valve. Many studies on TSPSVs focus on the above three parts.

Torque motors are the most concerned electro-mechanical converters. How the magnetic resistances influence the performance of torque motor was studied in [3] and the authors finally derived modified expressions to calculate the electromagnetic torque constant and magnetic spring stiffness more accurately. Jeong [4] proposed a novel torque motor model with the consideration of the magnetic flux leakage and permanent demagnetization effect which had been missed in previously models. Urata analyzed how the inequality of the four air-gaps affects the performance of the torque motor and found that the relative incline could cause its null point shift [5]. Zhang, et al [6] applied novel hybrid-magnetization pole arrays in a torque motor to improve the output torque without increasing its size or mass. In [7], magnetic fluids are filled in the air gap of a torque motor to suppress the vibration and noise. Besides studying the torque motor, scholars have devised some innovative electro-mechanical converters, such as piezoelectric actuator [8], piezoelectric ring benders [9-11], and giant magnetostrictive actuators [12,13] to pursue higher dynamic performance.
The common pre-stage hydraulic amplifiers in TSPSVs are spool valves [14], single nozzle flapper valves [15], double nozzle flapper valves [16], jet pipe valves [17] and deflector jet valves [18-20]. The minimum flow channel size in jet pipe valves is larger than that in nozzle flapper valves so that the jet pipe valve is not easily blocked by the contamination, leading to its high reliability. In [21], the transient turbulent model, large eddy simulation, has been applied to evaluate the turbulent flow in a jet pipe valve under different inlet pressure values and deflection angles. Although CFD simulations can provide the flow details in the jet pipe valve, theoretical analyses are still necessary for the engineering design practice. Based on the flow balance equations of a hydraulic bridge, an improved nonlinear explicit model considering the absorption effect was developed in [22] and was linearized for practice applications.

Original research on the main power amplifier of a TSPSV focuses on the basic pressure-flow characteristic, the fluid visualization, the lubrication characteristic and the anti-wear method. Maré and Attar extended the basic model of the spool valve to be appropriate for the low temperature condition [23]. Mondal, et al studied the leakage flow behaviour of a hydraulic spool valve around the null position under blocked-load by experiments and numerical simulations [24]. Later, Feki, et al [25] and Afatsun, et al [26] each established a mathematical flow model of the spool valve considering the leakage. GAO, et al [27] employed the finite element method (FEM) to simulate 2D flow fields in a spool valve and used the particle image velocimetry apparatus to validate the simulation. In [28], the pressure transient flow force acting on hydraulic spool valves was experimentally studied but further accurate analytical models needed to be set up for estimating this force. Methods for reducing the moving resistance attract many scholars. Numerical analysis was used to explore the influence of the spiral groove on the spool valve’s lubrication characteristic in [29]. In [30], influence of different heat treatments for the spool made up by AISI 1.7225 steel were experimental studied and carburizing surface hardening method was suggested. In [31], ceramic materials were used to be used in the spool-sleeve pair to suppress the wear.

To sum up, few studies focus on the connector between the electro-mechanical convertor and the hydraulic amplifier. In a jet pipe pressure servo valve (JPPSV), the armature assembly connects the torque motor and the jet pipe valve and functions for converting the torque input from the torque motor to displacement of the nozzle in jet pipe valve. Modeling the armature assembly stiffness is significant for predicting the static/dynamic performance of the servo valve. Urrata [32] established the stiffness model of the elastic components in a nozzle flapper servo valve with the beam theory and designed torque-loading tests to verify the theoretical analysis. However, the armature assembly in a JPPSV is more complex than that in a nozzle flapper servo valve. So far, the stiffness of the armature assembly in jet pipe servo valves can only be obtained by the experimental measurement or the time-consuming simulation [33], which is inconvenient for the design or optimization of JPPSVs.

In this research, the mathematical model of the stiffness of the armature assembly in a JPPSV was deduced. The paper is organized as follows. Section 2 presents the principle of a JPPSV and the structure of the armature assembly. Section 3 shows the detailed theoretical model and finite element model of the armature assembly. Section 4 gives the test of the recovery pressure of the jet pipe valve for verifying the proposed stiffness model. Finally, conclusions are drawn in Section 5.

2 Working Principle of the JPPSV and the Structure of the Armature Assembly

Fig. 1 presents the configuration of a JPPSV. This valve consists of a torque motor, a jet pipe valve and a main slide valve. There are four oil ports: the port T, the port L, the port P and the port J, which are respectively the jet pipe valve control port, the oil supply port, the oil return port, and the pressure output port. The control oil from the J-port enters into the jet pipe through the guide pipe and is ejected from the nozzle at a high speed. When the coil is not energized, the jet pipe stays at the middle position, and the flow accepted by the two receiving holes is equivalent, resulting in the same pressure at both ends of the spool. Thus, the spool keeps still at the left position under the action of the spring. And then, T-port and L-port are connected. After the coil is energized, the electromagnetic torque is generated on the armature and the armature rotates counterclockwise to actuate the jet pipe moving towards the left. Then the left receiver hole receives more oil flow causing the pressure of the left receiving hole to become larger than that of the right receiving hole. When the pressure difference between two receiving holes overcomes the spring force, the spool moves towards the right and the P-port and the L-port are connected to supply high pressure oil to the brake disc in an aircraft. The output pressure at the L-port feedbacks to act on the right end of the spool. The spool will stop at the position where the sum of the feedback pressure and the spring force is
balanced by the pressure differential force. Once the output pressure is higher than the desired pressure, the feedback pressure will actuate the spool to move towards the left. Consequently, the flow from the P-port to the L-port decreases to reduce the output pressure to the desired pressure.

Figure 1  Configuration of the JPPSV

The above JPPSV is a typical multi-field coupled system. The torque motor converts the electronic signal to the output torque. The torque can be regarded as the input of the armature assembly. The output of the armature assembly is the displacement of the nozzle and the rotation angle of the armature, which is mainly determined by the structure stiffness characteristics. The armature assembly of the JPPSV is defined as the movable components, which include the zero-adjusting wire, the guide pipe, the jet pipe, the armature, the spring tube and the nozzle (shown in Fig. 2). The zero-adjusting wire is designed for adjust the null position. The guide pipe provides the control oil to the jet pipe. One end of the zero-adjusting wire or the guide pipe is fixed on the torque motor housing, and the other end is welded with the upper end of the jet pipe. The neck of the jet pipe connects with the inner surface of the spring tube with a press fitting. The outer surface at the top of the spring tube is pressed into the hole in the armature. The other end of the spring tube is also fixed on the torque motor housing. The nozzle is connected to the bottom of the jet pipe with thread, and this connection is strengthened by welding. The z-axis in Fig.2 is perpendicular to the paper surface and its direction is outward.

Figure 2  Structure of the armature assembly in the JPPSV

3 Mechanical Model of the Armature Assembly

What we desire is to find out the function relationship between the electromagnetic torque and the output displacement at the concerning location in the armature assembly. This relationship can be obtained by two methods: the theoretical method and the FEM.

3.1 Theoretical Model

3.1.1 Model Hypothesis

In order to facilitate the analysis, the following assumptions are made:

1. Deformations of the structure are much smaller than the geometry size and the material of each part is uniform and isotropic;

2. The residual stress caused in the spot wielding is ignored because an ageing treatment is carried on after the assembly process;

3. Interference fit forces are neglected because they are distributed uniformly along the circumferential direction and have no significant effect on the structure deformation;

4. The bending deformation of the jet tube is limited to the y-z plane.

3.1.2 Mechanical Model

Except for the armature, members in the object structure are slender, which are simplified as variable cross-section beams. The upper end of the spring tube and the armature are simplified as a part of the neck of the jet tube. The simplified structure is depicted in Fig. 3.
One end of the guide pipe/zero-adjusting wire/spring tube are fixed with the torque motor housing, which means that there are three fixed supports in the structure, causing the simplified armature assembly to be a complex statically indeterminate structure. Due to the assumption (4), it is a fourfold statically indeterminate problem and there are four unknown constraint reactions.

As shown in Fig. 4, by cutting the armature assembly at the cross section A and B, the statically indeterminate structure is divided into three simple statically determinate structures (the zero-adjusting wire part, the guide pipe part and the jet pipe part). The zero-adjusting wire part and the guide pipe part are prismatic curved beams, while the jet pipe part is a variable cross-section beam. A–F represents the cross sections. The guide pipe part is divided into three sections: B–H, H–I and I–J, which are numbered 1–3. The zero-adjusting wire part are also divided into three sections: A–K, K–L and L–M, numbered 4–6. Also, the jet pipe part is divided into six sections: A–B, B–C, C–D, D–E, E–G and E–F, numbered from 7 to 12. The coordinates and dimensions necessary for the next analysis is also presented in Fig. 4.

Four unknown external loads appear at the cross section A and B, and they are $F_A$, $T_A$, $F_B$ and $T_B$. Besides, other external loads include the electromagnetic torque $M_a$ and the additional concentrated force $F_G$. $M_a$ acts at the middle symmetry plane of the armature (seen at cross section D in Fig.4) and $F_G$ acts at the end surface of the nozzle. $F_G$ is the virtual force for calculating the nozzle displacement and its value is 0 at the final calculation.

3.1.3 Theoretical Stiffness Model

After the armature assembly is simplified, its stiffness can be solved by force method. The basic procedure for force method are the following steps:
**Step 1**: calculate the internal forces of each section.

For the section numbered \( i \) (\( i = 1, 2, ..., 12 \)), its internal bending moment is \( M_i \) and its internal moment is \( T_i \). \( M_i \) and \( T_i \) of each section can be obtained by using the section method and local equilibrium. The analysis results are listed as the following equations

\[
\begin{align*}
M_1 &= F_A x \\
M_2 &= T_A \sin \theta + F_A R_{KL} \cos \theta + F_A R_{KL} \sin \theta \\
M_3 &= T_A + F_A (R_{KL} + x) \\
M_4 &= F_B x \\
M_5 &= T_B \sin \theta + F_B R_{KL} \cos \theta + F_B R_{KL} \sin \theta \\
M_6 &= T_B + F_B (R_{KL} + x), \quad T_6 = F_B (l_{BH} + R_{HI}) \\
M_7 &= T_A + F_A (l_{AB} + x) + F_B x \\
M_8 &= T_A + T_B + F_A (l_{AB} + x) + F_B (l_{BC} + x) \\
M_9 &= T_A + T_B + F_A (l_{AB} + x) + F_B (l_{BD} + x) - M_a \\
M_{10} &= -F_G x \\
M_{11} &= T_A + T_B + F_A (l_{AE} + x) + F_B (l_{BE} + x) - M_a \\
& \quad - F_G (l_{EG} - x)
\end{align*}
\]

\[(1)\]

\[
\begin{align*}
T_1 &= T_A \\
T_2 &= F_A l_{AB} \sin \theta + F_A R_{KL} \left(1 - \cos \theta\right) - T_A \cos \theta \\
T_3 &= F_A \left(l_{AB} + R_{KL}\right) \\
T_3 &= T_B \\
T_5 &= F_B l_{BH} \sin \theta + F_B R_{HI} \left(1 - \cos \theta\right) - T_B \cos \theta \\
T_6 &= F_B (l_{BH} + R_{HI}) \\
T_7 &= T_B = T_6 = T_{10} = T_{11} = T_{12} = 0
\end{align*}
\]

where \( l_{AB}, R_{KL}, l_{BH}, R_{HI}, l_{AB}, l_{AC}, l_{BC}, l_{AD}, l_{BD}, l_{AE}, l_{BE} \) and \( l_{EG} \) are dimensional chain parameters.

**Step 2**: calculate the strain energy of each section.

The strain energy caused by shear force is excluded. For the elastic deformation, the total strain energy \( U \) of the whole structure is the sum of the strain energy of each section

\[
U = \sum_{i=1}^{12} U_i,
\]

where \( E_i \) is the material elasticity modulus of No. \( i \) section, \( l_i \) is the moment of inertia of No. \( i \) section, \( G_i \) is the shear modulus of the material of No. \( i \) section, \( I_{ew} \) is the polar moments of inertia of No. \( i \) section. When \( i = 2 \), \( l_x = R_{KL} d \theta \). When \( i = 5 \), \( l_x = R_{BD} d \theta \).

**Step 3**: calculate the total strain energy of the whole structure.

**Step 4**: calculate the unknown forces/moments.

Firstly, substitute Eqs. (1), (2) into Eq. (3). Then we can get the total strain energy by substitute Eq. (3) into Eq. (4). However, \( F_A, T_A, F_B \) and \( T_B \) are still unknown. To solve these four unknowns, the additional supplement equation is listed in Eq.5 based on the deformation compatibility conditions.
\[ f_{12} = \frac{l_{AB}^2}{2E_{i_1}} + \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{R_{KL}}{4E_{i_9}} \left( \frac{\pi - 4}{4G_{i_5}^2} \right) + \frac{2l_{AK} + \pi R_{KL}}{4G_{i_5}^2} \]

\[ f_{13} = f_{31} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{14} = f_{41} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{15} = \left[ \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \right] M_{\alpha} - \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{22} = \frac{l_{AB}^2}{2E_{i_1}} + \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{23} = f_{32} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{24} = f_{42} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{25} = -\left( \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \right) M_{\alpha} - \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{33} = \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} \]

\[ + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{34} = f_{43} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{35} = \left[ \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \right] M_{\alpha} - \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{44} = \frac{l_{BC}^2}{2E_{i_8}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{CD}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

\[ f_{45} = \left( \frac{l_{DE}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \right) M_{\alpha} - \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} + \frac{l_{EF}^2}{2E_{i_9}} \]

where \( l_{CD}, l_{DE}, l_{BH}, l_{HM}, l_{MI} \) and \( l_{EF} \) are the dimensional chain parameters, \( l_1 = l_2 = l_3 = l_4 = 2l_5 = 3l_6 = 3l_7 = 4l_8 = 5l_9 = 6l_{i_1} = 7l_{i_5} = 8l_{i_2} = 9l_{i_3} = 10l_{i_4} = 2p_1 = 2p_2 = 2p_3 = 2p_4 \).

Eq. (5) is a quaternion linear equation. Rewrite it as Eq. (6).

\[ Ax = b \]  \hspace{1cm} \text{where the matrix A is composed by} \ f_{ij} (i=1,2,3,4; j=1,2,3,4), \ b = \{-f_{15}; -f_{25}; -f_{35}; -f_{45}\}, \ x = \{F_A, T_A, F_B, T_B\}. \]

The explicit solution of Eq. (6) is very complex.
However, we can easily obtain its solution in MATLAB. Suppose the solution is

\[ \mathbf{x} = \mathbf{A}^{-1} \mathbf{b} = \begin{bmatrix} \mathbf{T}_A; \mathbf{F}_A; \mathbf{F}_B; \mathbf{T}_B \end{bmatrix}, \]  

(7)

**Step 5:** calculate the stiffness.

Substitute Eq. (7) into Eq. (4) again. We can obtain the total strain energy \( U \).

The armature rotation angle \( \varphi_D \) and the nozzle displacement \( \delta_G \) can be computed with the second Castigliano’s theorem [34]

\[ \varphi_D = \frac{\partial U}{\partial M} \bigg|_{F_G=0} = \left( \frac{2I_{AD}l_{DE} + l_{DE}^2}{2E_AI_4} + \frac{2I_{AE}l_{EF} + l_{EF}^2}{E_aI_6} \right) F_A \]

\[ -C_1 T_A - \left( \frac{2I_{BE}l_{EF} + l_{EF}^2}{2E_dI_4} + \frac{2I_{BE}l_{EF} + l_{EF}^2}{E_dI_6} \right) F_B \]  

\[ -C_2 T_B + C_3 M_a \]  

(8)

\[ \delta_G = \frac{\partial U}{\partial F_G} \bigg|_{F_G=0} = \frac{3l_{DE}l_{EF}^2 - 6l_{DE}l_{EF}l_{EC} + 2l_{EC}^3 - 3l_{EC}l_{EF}^2}{6E_aI_6} F_A \]

\[ + C_1 T_A + \frac{3l_{BE}l_{EF}^2 - 6l_{BE}l_{EF}l_{EC} + 2l_{EC}^3 - 3l_{EC}l_{EF}^2}{6E_dI_6} F_B \]  

\[ + C_2 T_B - C_2 M_a \]  

(9)

\[ C_1 = l_{DE}/E_AI_4 + l_{EF}/E_aI_6 \]

\[ C_2 = l_{EF}/(l_{EF}^2 - l_{EC})/E_aI_6 \]

The stiffness coefficients of the armature assembly are key for the analysis of the static/dynamic performance of the JPPSV. Under the action of electromagnetic torque \( M_a \), the armature rotates \( \varphi_D \) and the nozzle displaces \( \delta_G \). Therefore, there are two stiffness constants, \( k_1 \) and \( k_2 \), respectively representing the electromagnetic torque-armature rotation angle stiffness and the electromagnetic torque-nozzle displacement stiffness. They are calculated by Eqs. (10) and (11)

\[ k_1 = \frac{M_a}{\varphi_D}, \]  

(10)

\[ k_2 = \frac{M_a}{\delta_G}, \]  

(11)

So far, the functional relationship between the armature rotation angle/the nozzle displacement and the electromagnetic torque is specified. Equations for calculating the stiffness are listed as Eqs. (1)~(11) and we can solve them by using MATLAB.

### 3.2 Finite Element Model

For the elastic system, FEM is a mature and wide simulation tool to calculate the static mechanical response. Here, we aim to validate the above-mentioned stiffness model by a detailed finite element model.

#### 3.2.1 Solid and Element Model

The FEM analysis is carried out with commercial software ANSYS Workbench 16.0. Table 1 lists the dimensional parameters for the establishment of the 3D solid model. Table 2 lists the material used in the armature assembly. The nozzle is brazed to the bottom of the jet pipe and they are made of the same material. Thus, the nozzle is identified as one part of the jet pipe. The material of the armature and the spring tube differs for that of the jet pipe. So, the solid models of the armature and the spring tube are established separately.

The solid body is automatically divided into the grid model in the software. Fig. 5 shows a detailed finite element model (with 358,408 nodes and 223,658 elements). The element type is the ten-node tetrahedron (SOLID187) by default in the software.

#### Table 1 Dimension parameters of the armature assembly of a JPPSV

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_1)</td>
<td>1.6 mm</td>
<td>(d_2)</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>(d_a)</td>
<td>0.7 mm</td>
<td>(d_i)</td>
<td>1.3 mm</td>
</tr>
<tr>
<td>(d_2)</td>
<td>1.6 mm</td>
<td>(b)</td>
<td>27.4 mm</td>
</tr>
<tr>
<td>(a)</td>
<td>10.4 mm</td>
<td>(d_0)</td>
<td>2.74 mm</td>
</tr>
<tr>
<td>(d_2)</td>
<td>2.6 mm</td>
<td>(l_{AB})</td>
<td>2.3 mm</td>
</tr>
<tr>
<td>(l_{AC})</td>
<td>7.5 mm</td>
<td>(l_{AD})</td>
<td>8.8 mm</td>
</tr>
<tr>
<td>(l_{AE})</td>
<td>10.1 mm</td>
<td>(l_{AG})</td>
<td>27.7 mm</td>
</tr>
<tr>
<td>(l_{BC})</td>
<td>5.2 mm</td>
<td>(l_{BG})</td>
<td>6.5 mm</td>
</tr>
<tr>
<td>(l_{BE})</td>
<td>7.8 mm</td>
<td>(l_{EF})</td>
<td>4.3 mm–12.3 mm</td>
</tr>
<tr>
<td>(l_{IF})</td>
<td>12.3 mm,</td>
<td>(l_{BH})</td>
<td>7.6 mm</td>
</tr>
<tr>
<td>(R_{H})</td>
<td>4.8 mm</td>
<td>(l_{H})</td>
<td>12.3 mm</td>
</tr>
<tr>
<td>(l_{AK})</td>
<td>8.9 mm</td>
<td>(R_{KL})</td>
<td>6 mm</td>
</tr>
<tr>
<td>(l_{LM})</td>
<td>8.3 mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Table 2 Material and mechanical properties of each parts in the armature assembly

<table>
<thead>
<tr>
<th>Part</th>
<th>Material</th>
<th>Elastic modulus at 20 °C/MPa</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-adjusting wire</td>
<td>1Cr18Ni9Ti</td>
<td>193000</td>
<td>0.31</td>
</tr>
<tr>
<td>Guide pipe</td>
<td>1Cr18Ni9Ti</td>
<td>193000</td>
<td>0.31</td>
</tr>
<tr>
<td>Jet pipe</td>
<td>1Cr18Ni9Ti</td>
<td>193000</td>
<td>0.31</td>
</tr>
<tr>
<td>Armature</td>
<td>IJ79</td>
<td>157320</td>
<td>0.30</td>
</tr>
<tr>
<td>Spring tube</td>
<td>C17300</td>
<td>127600</td>
<td>0.29</td>
</tr>
</tbody>
</table>
3.2.2 Boundary Condition

The bottom of the spring tube and the zero-adjusting wire and the guide pipe is brazed to the torque motor housing. So, three fixed constraints are respectively set at the outer cylinder at the lower end of the spring tube, and the bottom end of the zero-adjusting wire and the guide pipe.

As mentioned in the section 3.1.2, there are three interference fitting surfaces in the actual armature assembly. Due to the assumption (3) in the section 3.1.1, these surfaces are set as the bonded contact during the simulation, to obtain the continuous grid distribution.

3.2.3 Loadation Condition

As illustrated in Fig. 5, the electromagnetic torque $M_a$ is converted into two force loads $F$, which acting on both longitudinal end sides of armature, and the conversion relation is

$$F = \frac{M_a}{2b}$$

![Finite element model of the armature assembly](image)

**Figure 5** Finite element model of the armature assembly

3.3 Comparison between the theoretical result and the FEM result

Under the conditions of the geometry size in Table 1 and material properties in Table 2, the static mechanical analysis of the armature assembly is carried in ANSYS16.0.

After inputting the two force loads $F$, we can correspondingly obtain the armature rotation angle and the nozzle displacement. The solution with FEM is taken as the simulation calculation result. In addition, we can calculate the stiffness constants based on Eqs. (1) ~ (11) in MATLAB and also determine the load-displacement relationship, which is regarded as the theoretical analysis result. Fig. 6 and Fig. 7 respectively show the electromagnetic torque-armature rotation angle relationship and the electromagnetic torque-nozzle displacement relationship calculated by both methods, and we tried five different lengths of the thin-wall part of the spring tube.

Fig. 6 and Fig. 7 show that there is a linear relationship between the armature rotation angle/the nozzle displacement and the electromagnetic torque both in theoretical calculation and finite element simulation results. The scatter points in Fig. 6 and Fig. 7 were linear fitted by the least square method. After fitting these data by FEM, the reciprocal of the slope is taken to obtain the simulated values of $k_1$ and $k_2$. The reciprocal of the linear slope in Fig. 6 and Fig. 7 is the theoretical value of $k_1$ and $k_2$. Table 3 and Table 4 list the theoretical and simulation values under different lengths of the spring tube, which can quantitatively express the relative errors of theoretical analysis result and finite element simulation result in calculating the stiffness constants of the armature assembly.

In Table 4 and Table 5, it is observed that the maximum relative error of stiffness calculated by two methods is less than 9%, meeting the requirements of engineering accuracy. When the length of the spring tube increases from 4.3 mm to 12.3 mm, the stiffness constants calculated by two methods both decrease.
4.2 Theoretical relationship between the current and the recovery pressure

During the experiment, the torque motor works at its null position and the theoretical output torque $M_a$ can be calculated by \[ M_a = K_i i + K_m \varphi_0, \] \[ (13) \]
where $K_i$ is the electromagnetic torque constant and $K_m$ is the magnetic spring stiffness.

Under the action of this torque, the armature rotation angle and the nozzle displacement can be calculated by Eq. (10) and Eq. (11).

By solving the simultaneous system of Eq. (10), Eq. (11) and Eq. (13), we can obtain

\[ \delta_G = \frac{K}{K_2 (1 - K_m/k_1)} i = f(i), \] \[ (14) \]
where $f$ expresses the mapping relation between $i$ and $\delta_0$, $k_1$ and $k_2$ are calculated by the stiffness model proposed in Section 3.1.

The relation between $\delta_0$ and $p$ has been derived in [22] and here we rewrite it as

$$p = \frac{A_1^2(\delta_0)C_d\Delta p \cos \theta}{A_1^2(\delta_0)C_d^2 + A_1^2(\delta_0)C_d^2} - \frac{A_2^2(\delta_0)C_d\Delta p \cos \theta}{A_2^2(\delta_0)C_d^2 + A_2^2(\delta_0)C_d^2},$$

$$= g(\delta_0)$$

(15)

$$A_1(\delta_0) = \left[R_1^2 \theta_1 + R_2^2 \theta_2 - \left(R_1^2 \sin \theta_1 + R_2^2 \sin \theta_2\right)\right]/2$$

$$A_2(\delta_0) = A_1(-\delta_0)$$

$$A_3(\delta_0) = \pi R_1^2 - A_1(-\delta_0)$$

$$A_4(\delta_0) = A_3(-\delta_0)$$

$$\theta_1(\delta_0) = 2 \arccos \frac{R_1^2 + (R_2 + 0.5e - \delta_0)^2 - R_2^2}{2R_1(R_2 + 0.5e - \delta_0)}$$

$$\theta_2(\delta_0) = 2 \arccos \frac{R_1^2 + (R_2 + 0.5e - \delta_0)^2 - R_2^2}{2R_1(R_2 + 0.5e - \delta_0)}$$

where $A_1$-$A_4$ are four throttle areas, $\Delta p$ is the differential pressure, $\Delta p = p_s - p_t$, $p_s$ is the supply oil pressure, $p_t$ is the return oil pressure, $C_d$ is the nozzle discharge coefficient, $C_d$ is the orifice discharge coefficient, $\theta_1$ $\theta_2$ is half of the axis angle of two receiver holes, $R_1$ and $R_2$ are respectively the radius of the receiver hole and the jet nozzle, $e$ is the distance between receiver holes. $g$ expresses the mapping relation between $\delta_0$ and $p$.

Substitute Eq. (15) into Eq. (14) and we can get the theoretical relationship between $i$ and $p$

$$p = g\left[f(i)\right],$$

(16)

4.3 Comparison of Theoretical Results and Experimental Results

The experiment was carried out on the test bench as shown in Fig. 9. Detailed test devices of the jet pipe valve are shown in Fig. 10. The working fluid in the testbed was 10# aviation hydraulic oil and its temperature was kept around 25°C. The supply oil pressure was 8 MPa and the return oil pressure was 0.4 MPa. The control current of the servo valve was provided by the servo amplifier. Two coils of the torque motor were connected in parallel. When $i = -12$ mA, $p_L = p_R$, which means that the nozzle locates at the middle location. Thus the zero bias current $i_0$ was determined as $-12$ mA. Take control current $(i - i_0)$ as the abscissa and plot the tested recovery pressure $p$ in Fig. 11.

![Figure 9](image9.png)  
**Figure 9** Photograph of the test bench

![Figure 10](image10.png)  
**Figure 10** Details of the testing apparatus

According to Eq. (16), the theoretical relationship curve
between \(i\) and \(p\) can be obtained with the parameters listed in Table 5. Besides, \(k_1=7.143\ \text{N} \cdot \text{m/rad}\) and \(k_2=0.6366\ \text{N} \cdot \text{mm}\), which have been calculated and listed in Table 3 and Table 4. The calculation results are shown as the dotted line in Fig. 11.

**Table 5** Parameters for Calculating the Theoretical Relationship between the Control Current and the Recovery Pressure

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p_s)</td>
<td>8 MPa</td>
<td>(p_t)</td>
<td>0.4 MPa</td>
</tr>
<tr>
<td>(C_d)</td>
<td>0.62</td>
<td>(C_n)</td>
<td>0.97</td>
</tr>
<tr>
<td>(\theta)</td>
<td>22.5°</td>
<td>(R_r)</td>
<td>0.155 mm</td>
</tr>
<tr>
<td>(R_j)</td>
<td>0.15 mm</td>
<td>(e)</td>
<td>0.01 mm</td>
</tr>
<tr>
<td>(K_i)</td>
<td>0.9 N·m/A</td>
<td>(K_m)</td>
<td>4.4 N·m/rad</td>
</tr>
</tbody>
</table>

**Figure 11** Relationship between the control current and the recovery pressure

Usually, the ferromagnetic material with hysteresis characteristic was used in the torque motor [35], resulting in the hysteresis loop in the experimental curve in Fig. 11. As presented in Fig. 11, the experimental value of the recovery pressure gain of the jet pipe valve is relatively close to the theoretical calculated result and their trend is consistent, indicating that the two results are basically the same. The aforementioned stiffness model is verified.

## 5 Conclusion

In this research, the stiffness model of the armature assembly in a JPPSV is established with energy method. In order to validate the proposed model, the static mechanical response of the armature assembly is simulated by FEM, and then their comparison is discussed. The recovery pressure test experiment of the jet pipe valve controlled by a torque motor is designed to verify the theoretical stiffness calculation method. Conclusions are presented as follows:

1. Two stiffness constants calculated by the theoretical analysis are basically consistent with those simulated by FEM, which suggests that the proposed stiffness model is appropriate for designing the elastic components in a torque motor.
2. The experiment results of the jet pipe valve recovery pressure are close to the theoretical pressure calculations. The correctness of the stiffness analysis method is verified indirectly.
3. In elastic range, the deformation of key locations in the armature assembly varies linearly with the electromagnetic torque. And the length of the spring tube has great influence on the stiffness characteristics.

## 6 Declaration

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**Availability of data and materials**

The datasets supporting the conclusions of this article are included within the article.

**Authors’ contributions**

CH was in charge of the whole trial and wrote the initial manuscript. YY provided guidance for the whole research and revised the manuscript. JL assisted with experiment result analyses and the manuscript revision. All authors read and approved the final manuscript.

**Competing interests**

The authors declare no competing financial interests.

**Consent for publication**

Not applicable

**Ethics approval and consent to participate**

Not applicable

**References**


Stiffness Model of the Armature Assembly in a Jet Pipe Pressure Servo Valve


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