# Appendix 1. Acceptance criteria and parallel tempering

The RJMCMC algorithm aims to extract samples from model spaces such that samples follow their posterior probability. Each proposed model needs to be accepted or rejected under the equilibrium condition. For a fixed dimension (i.e., ), this condition can be met by accepting proposed models with the probability given by the Metropolis-Hastings criterion:

Here, and are the last-accepted and proposed models, respectively; and is the probability that a random walk from to occurs. Our setting simplifies Equation because of the symmetricity of a random walk (i.e., ):

Equations and assume that the model space dimension does not change through a random walk. Green (1995) has introduced the Metropolis-Hastings-Green criterion that can account for the dimensional change:

The Jacobian in Equation adjusts unit volume change in the model space. Specific configurations of a random walk are known to cancel out similar terms in Equation (Dosso et al. 2014; Sen and Biswas 2017), leading to

To enhance the efficiency of model sampling, the parallel tempering technique (Geyer and Thompson 1995; Sambridge 2014) introduces an additional parameter representing temperature () to Equations and :

and

Higher temperatures than unity lose the acceptance criterion to accept more models. Such tempered chains can walk through the broader region in the model space consequently. However, this modification by breaks the equilibrium condition, and thus the sampled model does not simulate the posterior probability distribution. The parallel tempering technique remedies this issue by running 100 MCMC chains with different temperature in parallel. At each iteration, two chains are arbitrarily selected, and their temperatures are swapped with a probability of

where superscripts represent the indices for MCMC chains. Equation guarantees that this temperature swap meets the equilibrium condition. The posterior probability can be retrieved by gathering models accepted with non-tempered chains. Our inversion involveschains having a unit temperature (i.e., non-tempered). For the other 80 chains, we randomly set temperatures, which range from 1 to 20.

# Appendix 2. Algebraic expression for sediment reverberations

When a plane, impulsive, unit-amplitude, upgoing, nearly-vertical SV waveform incidents to this structure model, a radial-component elastic response at the sediment top is approximately given by

in the time domain and

in the frequency domain (Yu et al. 2015), where , , and represent a thickness, Vs, and a density of the overriding sediment layer, respectively; and represent Vs and a density of the half space; represents free-surface reflection; represents a two-way travel time of S-wave passing through the sediment layer; and and represent reflection and transmission coefficients at the layer interface, respectively. From equation , we obtain the denominator of deconvolution without damping: