1 Additional file 1 — Equilibrium

There are four possible equilibrium $(L_a^*,\ A_a^*,\ L_t^*,\ A_t^*)$ for our system of equations as described below.

1 Infection free equilibrium

$$(0, 0, 0, 0).$$

2 Competitive exclusion with only *Ae. triseriatus*

$$\left(0, 0, \hat{L}_t, q_d\hat{L}_t\right),$$

where

$$\begin{aligned}
\hat{L}_t &= K\left(1 - \frac{1}{R_t}\right), \\
R_t &= \frac{1}{\delta_t\left(\frac{1}{\delta_t} + \mu_{L_t}\right)}\ \frac{\beta_t\rho_t}{\mu_{A_t}}, \\
q_d &= \frac{1}{\delta_t\mu_{A_t}}.
\end{aligned}$$

3 Competitive exclusion with only *Ae. albopictus*

$$\left(\hat{L}_a, w_d\hat{L}_a, 0, 0\right),$$

where

$$\begin{aligned}
\hat{L}_a &= K\left(1 - \frac{1}{R_a}\right), \\
R_a &= \frac{1}{\gamma_d\delta_a\left(\frac{1}{\gamma_d\delta_a} + \gamma_m\mu_{L_a}\right)}\ \frac{\beta_a\rho_a/\gamma_a}{\mu_{A_a}}, \\
w_d &= \frac{1}{\gamma_d\delta_a\mu_{A_a}}.
\end{aligned}$$

4 Coexistence of both species

$$\left(\tilde{L}_a, w_d\tilde{L}_a, \tilde{L}_t, q_d\tilde{L}_t\right),$$

where

$$\tilde{L}_a = \frac{K}{1 - \alpha_t\alpha_a}\left(1 - \alpha_t + \frac{\alpha_t}{R_t} - \frac{1}{R_a}\right),$$

$$\tilde{L}_t \quad = \quad \frac{K}{1-\alpha_t\alpha_a}\left(1-\alpha_a+\frac{\alpha_a}{R_a}-\frac{1}{R_t}\right).$$

The main text gives a description of the biological meaning of the population reproduction numbers R_t and R_a.