Supplementary Information for

No or diffuse phase-transition with temperature in one-dimensional Ising model?

Li Zhang1,2, Li-Li Zhang1, and Yi-Neng Huang1,2\*

1National Laboratory of Solid State Microstructures, School of Physics, Nanjing University, China.

2Xinjiang Laboratory of Phase Transitions and Microstructures in Condensed Matters, College of Physical Science and Technology, Yili Normal University, China.

**\***Corresponding author. Email: ynhuang@nju.edu.cn

Calculation of global-spontaneous-magnetization in 1D-IM

At present, the method to calculate the global-spontaneous-magnetization of Ising model 2, 23 is as follows: (i) Applying a static external field () to the model system; (ii) Constructing the spin orientation ensemble corresponding to the model; (iii) Calculating the global magnetization of the ensemble based on Boltzmann principle; (iv) Let , and the obtained magnetization is the global-spontaneous-magnetization of the system.

Fig. S1. Diagrammatic sketch of ID-IM system and construction of corresponding spin orientation ensemble.

According to this method, the Hamiltonian () of 1D-IM with is,

where is the ith spin and , the interaction energy constant between nearest-neighbor spins, the magnetic moment of a spin, and the number of spins in the model system.

The spin orientation ensemble corresponding to 1D-IM is constructed as shown in Fig. S1, i.e. the whole chain of spins is divided into sub-chains (subsystems) of spins with different spin orientation configurations.

The Hamiltonian () of the subsystem with (the endpoint effect of the subsystems can be ignored for ) is,

and based on Boltzmann principle, the partition function () of the ensemble is,

here , , is temperature, and Boltzmann constant.

From Eq. S3, the global magnetization () per spin in 1D-IM is,

According to and , we get,

in which , , and,

Defining,

Obviously, and .

From and , we obtain,

and,

The eigenvalues of the square matrix is determined by the following characteristic equation,

resulting in,

i.e. the two values ( and ) of are, respectively,

From Eq. S9 and S12, we get,

where and are the eigenvectors corresponding to and , and .

Therefore,

Based on Eq. S4, S12 and S14, we obtain,

According to this equation, it is easy to find for nonzero temperature. vs at series is shown in Fig. S2.



Fig. S2. Global Magnetization () per spin in 1D-IM vs external magnetic field () at series temperature ().

When is present, the susceptibility () per spin in 1D-IM is,

From Eq. S15 and S16, we get,

here is reduced temperature, and reduced magnetic field.

From Eq. S18 and S19, the average internal energy ( ) per spin in 1D-IM is,

and the average specific heat per spin () is,