Supplementary Information

Rectifying effect in ballistic diode bridge with high performance for thermal energy harvesting
Dry transfer process:

*Figure S1.* Schematic diagram of dry transfer process. (a) Preparations: cover glass coated with PMMA/PVA, cleaned SiO$_2$/Si wafer, and a sticky tape. A multilayer of PMMA/PVA coated on a cover glass and cleaned wafer were prepared to exfoliate hBN and graphene, respectively. (b) Two hBNs and graphene were exfoliated on glass substrates and silicon wafer, respectively. The polymer film with top hBN was peeled from the glass and attached to a PDMS stamp. (c) Graphene and bottom hBN were picked up by sequential stampings, and the encapsulated graphene was transferred onto the wafer with large electrodes.
Figure S2. Micrographic images during dry transfer process. (a) A large thin hBN flake on polymers film was selected. (b) Monolayer graphene on SiO$_2$/Si wafer was found. (c) Graphene was raised by the hBN. (d) The second hBN flake was raised by the graphene. The scale bars represent 20 μm.
Figure S3. (a-b) Optical images of two encapsulated graphene samples on a silicon wafer. While no bubbles are found on the graphene site, bubbles appear only on hBN without graphene. The red scale bar represents 10 μm. (c) The AFM topography scanned on the area indicated by a rectangle in (b) shows flat surface without bubble. (d) Raman spectrum data measured after the encapsulation show clear features of phonon modes of graphene depending on the number of layers.
**Measurement configurations:**

Johnson noise voltage $E$ within the frequency range $dv$ is given as

$$E^2dv = 4RkTd\nu$$

(1)

To detect the Johnson noise signal, the input terminal should provide broad bandwidth. On the other hand, for the low frequency AC measurement, the high frequency noise should be suppressed by using a low pass filter.

**Figure S4.** Measurement configuration. The DC current was applied and measured simultaneously by a source meter (Keithley 2450). A low pass filter with $f_{cut} = 7.2$ kHz was inserted between the output terminals, and the rectified voltage was measured by a multimeter (Keithley DMM 6500) with internal impedance of 10 GΩ.
**Numerical simulation model**

The transmission probability $P_{ij}$ for a charge passing through a channel from terminal $i$ to terminal $j$ is given by integrating angle distribution function $G(\alpha)$ of the incident electron. In this model, incident carriers located within the mean free path, $\lambda$, from the aperture are included for the integration range, so the resultant $P_{ij}$ depends on $\lambda$. The carriers passing after specular reflections from the edges are included as well as the carrier passing without reflection. The transmission probability $P$ through a channel is calculated numerically, as

$$P = \frac{\iint_{D_s} dx dy \int_{\alpha_i}^{\alpha_f} G(\alpha) d\alpha}{M}.$$  \hspace{1cm} (2.1)

![Diagram](image)

**Figure S5.** (a) Transmission probability $P$ through a channel is calculated numerically by integrating angle distribution function $G(\alpha)$ within the range of $\alpha_i \leq \alpha \leq \alpha_f$, where the integration range ($D_s$) is confined by the circular area with a radius equal to $\lambda$. (b) Integration interval of angle $\alpha$ includes one reflection region as well as no reflection.

Here, the carrier with angle $\alpha$ within the range of $\alpha_i \leq \alpha \leq \alpha_f$ can pass through the aperture at the position $(x, y)$ over the integration region $D_s$. The integration range ($D_s$) is confined by the circular area with a radius equal to $\lambda$. M is number of possible trajectories.
defined by number of cells within $D_s$, that is, $M = \iint_{D_s} dxdy$. The angular distribution function is assumed to be given by a Gaussian distribution,

$$G(\alpha) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\alpha}{2\sigma^2}\right),$$  \hspace{1cm} (2.2)

where $\sigma$ is a standard deviation of distribution, and the mean angle is supposed to be zero as the applied potential is parallel to the longitudinal channel direction.

(a) T-shape geometry

For T-shape geometry, the probability of a charge passing through a straight channel or being deflected to a perpendicular channel is calculated for all possible trajectories as shown in Fig. S6. The probability is presented by a function, $P_{ij}$, for transmission ($i \rightarrow j$) for a charge carrier. $\alpha_i$ ($\alpha_f$) is the angle between vectors of $\overrightarrow{MA}$ ($\overrightarrow{MB}$) and unit vector in x-axis $\hat{x}$. Thus,

$$\alpha_i = \arccos\left(\frac{\frac{d}{2} - x}{\sqrt{\left(\frac{d}{2} - x\right)^2 + y^2}}\right) \quad \ldots \quad (2.3a)$$

$$\alpha_f = \arccos\left(\frac{\frac{d}{2} - x}{\sqrt{\left(-\frac{d}{2} - x\right)^2 + y^2}}\right) \quad \ldots \quad (2.3b)$$
Figure S6. (a) When a bias voltage is applied in x-direction, main current $I_h$ flows in straight direction, while a part of the current is deflected toward the vertical channel ($I_v$). (b) When a voltage is applied to -y direction, current from vertical channel is separated into left ($I_l$) and right ($I_r$) horizontal channels, after reflections at the bottom edge.

The probability ratio, $\delta$, is defined by ratio of current passed through straight channel ($I_h$) and current passed through perpendicular channel ($I_v$), that is equal to ratio of their transmission probabilities\(^2\):

$$\delta = \frac{I_h}{I_v} = \frac{P_h}{P_v},$$

(2.4)

where $P_h$ is the probability to pass through straight channel, and $P_v$ is that through perpendicular channel.
Figure S7. Transmission probability of (a) $P_h$ and (b) $P_v$ versus standard deviation, $\sigma$, of the angular distribution function, where Gaussian distribution function is used. In this analytical model, geometric factor $d = 2$ µm and ballistic parameter $\lambda = 1.5$ µm. (c) Transmission ratio, $\delta$, is plotted as a function of $\sigma$.

(b) Tapered geometry

The integral regions are determined in limitation of mean free path, $\lambda$, the transmission probability of a charge passes through constriction either no reflecion or one reflection at tapered edges. The transmission probability $P_{ij} = P_{ij}(D, d, \varphi, \lambda)$, where $D$, $d$, $\varphi$, are geometric parameters depicted in Fig. S8. In case of no reflection, integration region is spanned by points $P(x, y)$ on the region where the carrier travels through constriction if its angle is distributed in the range between two vectors $\overrightarrow{PA}$ and $\overrightarrow{PB}$. For the actual device, $\lambda$ is less than width $D$. The probability includes no reflection and single reflection cases before pass through constriction.
Figure S8. (a) Design and geometric parameters of tapper shape. (b) Integration region (circle) is taken for current passes through the constriction without reflection, where the width of region was chosen as the mean free path, $\lambda$. (c) Shaded regions show mirror imaging areas corresponding to the place where a reflected charge can be considered as it was started. (d) For the backward current, the integration range is depicted by the shaded semicircle.

In case of single reflection, a specular reflection is assumed for the simplicity of the calculation, i.e., a charge is reflected with the same angle as the incident angle. The total distance from point $P$ to reflecting point and from the reflecting point to the aperture should be equal or less than $\lambda$. To simplify the calculation, the reflective trajectory is converted into a straight line by using mirror imaging point $P'$ as shown on Fig. S8c. The simulation results of the transmission ratio are plotted as a function of standard deviation, $\sigma$, and mean free path, $\lambda$ in Fig S9-11.
**Figure S9.** Transmission probability of tapered geometry: (a) the transmission probability for forward current is plotted as a function of standard deviation, $\sigma$. A Gaussian angular distribution function is used in the analytical model with geometry of $d = 0.5 \, \mu m$, $\varphi = \pi/15$ and $\lambda = 1.5 \, \mu m$. (b) Backward transmission versus $\sigma$. (c) Transmission ratio of forward and backward transmission is plotted as a function of $\sigma$.

**Figure S10.** (a) Transmission ratio for the tapered geometry is plotted as a function of $\sigma$ to show
the angle distribution dependency. To understand geometric dependence, one geometric parameter is varied while the other parameters are fixed: \(d=0.5 \, \mu m, \varphi=\pi/12, \sigma=0.5, \lambda=1.5 \mu m\). (b) Aperture width \(d\), (c) MFP \(\lambda\), and (d) tapering angle \(\varphi\) are considered as variables, respectively. 

![Graph](image1)

**Figure S11.** (a) Forward transmission probability \((P_{FW})\) and (b) ratio \((P_{FW}/P_{BW})\) are plotted as a function of \(\sigma\) for different MFPs with \(d=0.5 \, \mu m, \varphi=\pi/15\).

(c) **Ballistic graphene bridge devices**

To analyze DC current asymmetry for the ballistic graphene bridge device, we consider a combined effect of two geometries: T-shape and tapered shape. The transmission probability for each case was calculated separately in the previous section.

![Graph](image2)

**Figure S12.** Illustration to analyze transmission probability of the ballistic graphene bridge device.
A current $I_1$ applied to terminal 1 is split into $I_{14}$ and $I_{12}$ for two channels. According to Landauer-Büttiker theorem of ballistic transport, current could be defined as:

\[
I_{14} = \frac{2e}{h} (T_{14} \mu_1 - T_{41} \mu_4) \tag{2.5}
\]

\[
I_{12} = \frac{2e}{h} (T_{12} \mu_1 - T_{21} \mu_2) \tag{2.6}
\]

Here, $T_{ij}$ is transmission coefficient from lead i to lead j, and $\mu_i$ is chemical potential at lead i. $T_{14}$ and $T_{41}$ correspond to transmission coefficients for forward and backward directions along channel between lead 1 to lead 4. $T_{12}$ and $T_{21}$ are denoted for forward and backward transmission coefficients along channel between 1 and 2. Because the voltage bias is applied from lead 1, it can be assumed that $T_{14} \mu_1 \gg T_{41} \mu_4$ (as $\mu_4 \cong 0$) and $T_{12} \mu_1 \gg T_{21} \mu_2$ (as $\mu_2 \cong 0$). As $T_{ij} = N_i P_{ij}$, Eq. (2.5) and Eq. (2.6) are approximated as

\[
I_{14} \cong \frac{2e}{h} N_1 P_{14} \mu_1 \tag{2.7}
\]

\[
I_{12} \cong \frac{2e}{h} N_1 P_{12} \mu_1. \tag{2.8}
\]

Rectification efficiency can be estimated as the difference between the forward current ($I_{14}$) and backward current ($I_{12}$) compared with the input current ($I_1$) from terminal 1:

\[
\eta \cong \frac{I_{14} - I_{12}}{I_1}, \tag{2.9}
\]

where $I_1 = \frac{2e}{h} N_1 \mu_1$.

On the other hand, $P_{14}$ corresponds to $P_h P_{fw}$ and $P_{12}$ corresponds to $P_v P_{bw}$ as shown in the previous section. Therefore, from Eq. (2.7) and Eq. (2.8),
\[ \eta \cong P_h P_{fw} - P_v P_{bw} \]  

This numerical simulation results of the rectification efficiency are shown in Figure S13-14.

Figure S13. The simulation result for the rectification efficiency is plotted as a function of standard deviation \( \sigma \). The parameters are \( d=0.5 \mu m, \varphi = \pi/12, \sigma=0.5, \) and \( \lambda=1.5 \mu m \).
Figure S14. The efficiency \( \eta \) was mapped as a function of MFP (\( \lambda \)) and aperture width (\( d \)) with \( \varphi = \pi/15 \), for different standard deviation (a) \( \sigma = \pi/5 \), (b) \( \sigma = \pi/4 \), (c) \( \sigma = \pi/3 \), and (d) \( \sigma = \pi/2 \).

References: