Modelling long Range Dependence and Non-linearity in the Covid-19 Mortality Rates

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Abstract

This paper relates to allaying the global fear on the persistence of the mortality rates due to the outbreak of the current COVID-19. This study thus uses the fractional integration techniques to examine the degree of persistence in the COVID-19 mortality rates for the global data and six high-risk countries, namely China, France, Italy, Spain, the United Kingdom and the United States. We find evidence in favour of mean reversion for the global mortality rates in all cases, except Italy and Spain where anti-persistence is observed. The reversion is expected to be faster for the United States followed by France. This means that the effects of the shocks caused by the pandemic on mortality rates will recover automatically by themselves in the countries where mean reversion is noticed. Therefore, we suggest that, in addition to the global strive for medical breakthrough over the virus, other current policies, including social distancing, frequent hand sanitization, appropriate regional lockdowns, etc. introduced by most of these countries are important in drastically reducing the number of contacts with the virus. This will consequently have decreasing effects on mortality rates, and then ensure fast rate of reversion.

1. Introduction

In the very recent time, the threats from the outbreak of the severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), popularly regarded as COVID-19, have resulted into global panic. In fact, even the world-acclaimed developed countries are in awe of the pandemic. The major reason cannot be disassociated from the high infectious rate of the virus, as well as the increasing death rate reported globally due to the present inability to provide a lasting cure. Meanwhile, as a result of this high infectious rate, limited facilities in many countries due to unpreparedness for the outbreak of the virus, and absence of scientifically proved cure, the number of deaths recorded globally and in certain countries in the short period of its occurrence is alarmingly high. Since January 9, 2020 when the first death was recorded in China, it has unexpectedly become 102,151 in just 67 days. Disturbingly, some of the most advanced countries which are well known for high medical professionalism and standard medical facilities have recorded the highest mortality rates. For instance, as at April 4, 2020, the number of deaths in Italy and the United States are respectively 18,849 and 18,594, thus accounting for as high as 18.45% and 18.20% of world total respectively. For a holistic view, five of the world's most developed countries, namely United States, the United Kingdom, Italy, France and Spain have accounted for 75,441 deaths (73.85% of world total) out of the global sum of 102,151. Interestingly, China, from where the virus broke out from, has recorded only 3,339 deaths (3.27%) as at that date.

Undoubtedly, high and unprecedented mortality rates have both economic and socio-economic implications. They are crucial in the evaluation of a country's population dynamics (Yaya et al., 2018) and analysis of economic performance. If mortality is more prominent among the younger generation, life expectancy will be adversely affected (see Yaya and Gil-Alana, 2018), and there will likely be tendency for high dependency ratio in the future. If, on the other hand, it majorly affects prominent people with means of economic resources, economic growth and prosperity may be impaired, at least in the short-run. In addition, increasing occurrence of death may motivate emigration which could eventually result into brain drain. Hence, it is important to examine and understand the degree of persistence of the death rates associated with the outbreak of COVID-19. Will the death rates be persistent or mean reverting? Answering this question is important for the assessment of socio-economic progress of countries and investment in public health (Yaya and Gil-Alana, 2018).

Therefore, this study has the objective of examining long range dependence in the global and country-specific COVID-19 mortality rates using the fractional integration techniques. The uniqueness of this study lies in the consideration of the ongoing global pandemic, as against the focus of previous studies on general year-to-year infant or adult mortality rates (see Caporale and Gil-Alana, 2014; Yaya et al., 2018; Yaya and Gil-Alana, 2018). In addition, we extend our methodology to capture nonlinearities in time (see Cuestas and Gil-Alana, 2016) following the revelation from past studies that mortality rates are often associated with nonlinear trends (see Booth et al., 2002, Shang et al., 2006, Yaya and Gil-Alana, 2018 inter alia). In short, both the linear and nonlinear approaches will help to understand the exact integration order should fractional persistence results. In other words, the dynamic properties of the mortality rates, whether stationary ($d = 0$), non-stationary ($d = 1$) stationary mean reverting ($0 < d < .05$), non-stationary mean reverting ($0.5 \leq d < 1$) or persistent ($d > 1$) can be determined. These possibilities have implications for policy effectiveness. The mean reversion cases imply that the effects of shocks will only be transitory. If persistence is the case, the effects of shocks will be long-lasting except strong policy measures are introduced to achieve initial trends.

The remainder of this paper is organized thus: Section 2 develops the methodology and describes the data. Section 3 presents and discusses the results, while the conclusion is given in Section 4.

2. Methodology And Data

2.1 Methodology

We commence the methodology with the definition of a process that has integration order *d* thus:

$$(1-L)^d x_t = \mu_t, \quad t = 1, 2, ..., T, \qquad (1)$$

where the logged series is represented by x_t. The backward shift operator defined as $Lx_t = x_{t-1}$ is denoted by L, while the error term that is assumed to follow a white noise process is u_t.

Meanwhile, the left hand side of equation (1) is expressed in its binomial form as:

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - ... \qquad (2)$$

such that the extent of the degree of dependence between observations in distant time is informed by the value of *d*. In other words, degree of dependence becomes stronger as *d* becomes higher. If $d = 0$, then the process is strictly stationary. If $0 < d < 0.5$, the process is said to be covariance stationary and mean reverting. However, the process ceases being covariance stationary if $0.5 \leq d < 1$ is the case, although there is still mean reversion. The implication is that the effects of shocks will eventually disappear in the long run, but will be slower compared to the case of covariance stationary. The last case scenario relates to $d \geq 1$ which represents persistence. It indicates that the effects of shocks will likely remain forever without the ability of the series to regain its original trend, unless strong policy measures are formulated.

So, in order to estimate equation (1), we follow the parametric LM test proposed by Robinson (1994), and it follows a regression model with linear time trend t, as given below:

$$y_t = \gamma_0 + \gamma_1 t + x_t; \quad (1-L)^d x_t = \mu_t, \qquad t = 1,2, \ldots,T \qquad (3)$$

where γ_0 is the intercept and γ_1 is the coefficient of the linear time trend. A positive and significant value of the slope coefficient γ_1 suggests that the series is characterized by increasing trend.

Owing to the stand of the literature on the possibility of mortality rates to exhibit nonlinear trend pattern, we further consider an advanced fractional dependence test due to Cuestas and Gil-Alana (2016). It incorporates a nonlinear deterministic function that relies on the Chebyshev polynomials in time. Hence, equation (3) is re-specified in a nonlinear form thus:

$$y_t = f(\varphi,t) + x_t, \qquad t = 1,2,\ldots, \qquad (4)$$

where $f(\cdot)$ is the nonlinear function that relies on the unknown parameter vector φ of order *n* given as:

$$y_t = \sum_{i=0}^{n} \delta_i P_{i,T}(t) + x_t, \qquad t = 1,2,\ldots, \qquad (5)$$

where the sample size is denoted by T. *n* is the Chebyshev polynomial order of the form $P_{i,T}(t)$ which is explicitly expressed as:

$$P_{i,T}(t) = \sqrt{2}\cos\,[\,i\pi(t-0.5)/T], \qquad t = 1,2,\ldots; \quad i = 0,1,\ldots, n \qquad (6)$$

with

$$P_{0,T}(t) = 1 \qquad (7)$$

Nonlinearity increases as *n* rises beyond 1. If $n = 0$, the model can be said to be expressed in terms of constant alone (i.e. δ_0). If $n = 1$, then the model is linear having both constant and trend (i.e. δ_0 and δ_1) equivalent to equation (3). Nonlinearity results if $n > 1$. In particular, as nonlinear order *n* increases, the intensity of nonlinearity increases. Cuestas and Gil-Alana (2016), however, opine that *n=2* and *n=3* are enough to conclude that the series exhibits nonlinear time trend.

2.2 Data source and description

We make use of data on mortality rates associated with the COVID-19. The data are collected for the world aggregate, China from where the virus originated and outside China, particularly the high-risk countries, namely France, Italy, Spain, the United Kingdom and United States. Since the first case of the virus varies across countries, the start dates of the data vary accordingly, but they have uniform end date which is 10/04/2020[1]. The data are all sourced from DataStream.

[1] The start dates for China, France, Italy, Spain, the United Kingdom and United States are respectively 31/12/2019, 24/1/2020, 30/01/2020, 31/01/2020, 31/01/2020 and 22/01/2020. The global data has the same start date with China.

The brief descriptive statistics of the log-transformed series are provided in Table 2. Italy records the highest average mortality rate followed by Spain, while France observes the least mean value. Italy still unfortunately has the highest recorded cases of mortality rates while China ranks the least. Expectedly, the high standard deviation statistics for all the countries, despite logging the data, give credence to the high number of deaths recorded over the few months ahead of the zero death cases at inception of the virus. The Jarque-Bera statistic that combines information from both skewness and kurtosis statistics to give a more accurate result suggests that the null hypothesis of normal distribution cannot be rejected for all the countries with the exemption of China. The graphical illustrations of the series are further provided in Figure 1 to have a foresight of their trends. Expectedly, the unimaginable increase in the mortality rates in all the countries is indicated by their observed steep trends. Only in China does it appear that the mortality rates become stable since around the beginning of April.

Table 1: Statistical description

Series	Mean	Max.	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
World estimate	7.0985	11.5342	3.2203	-0.8614	2.9184	8.3041
Country-specific estimates						
China	6.4038	8.1134	2.6006	-1.4935	3.7986	26.6896
France	4.5134	9.4877	3.4786	-0.0202	1.4810	**3.8484**
Italy	6.9496	9.8442	2.7870	-0.8062	2.4810	**4.3036**
Spain	6.5507	9.6705	3.0653	-0.8130	2.3981	**3.6325**
United Kingdom	5.4501	9.1003	2.8451	-0.4552	2.0234	**2.0034**
United States	5.9538	9.8306	2.6517	0.0041	1.5858	**2.5001**

The bolded values indicate the non-rejection of the null hypothesis of normal distribution of series at 5% significance level.

3. Discussion Of Results

We begin this section with the discussion of the fractional persistence results when the time trend is assumed to follow a linear pattern. As conventional in the literature, the fractional integration model is estimated under three different assumptions for the deterministic terms, namely model with no deterministic terms, model with only a constant (intercept), and model with both constant and a linear trend. As clearly observed in Table 2, the model with constant and linear trend seems to produce the estimates for the global sample and virtually all the countries. The only exemption is China which favours the model with a constant. Table 3 thus re-presents the results of the best models form Table 2. For the global estimate, the unit root hypothesis is resoundingly rejected in favour of I(d>1), thus indicating a high degree of persistence. Turning to the country-specific cases, the same conclusion seems appropriate for China whose fractional differencing coefficient is given as 1.3654. Also, the rejection of the I(1) null hypothesis is ascertained for three other countries, namely France, United Kingdom and United States, although it is in respect of $0.5 \leq d < 1$, signifying non-stationary mean reversion. Only in the cases of Italy and Spain can we say mortality rates exhibit random walk since the *d* estimates are closer to unity.

Table 2: Fractional persistence with linear trend

Series	No regressors	An intercept	A linear time trend
World estimate	1.0415*** (0.0307)	1.3625*** (0.0793)	**1.2582*** (0.0897)**
Country-specific estimates			
China	1.0361*** (0.0328)	**1.3654*** (0.0819)**	1.3302*** (0.0845)
France	1.1072*** (0.0514)	1.2714*** (0.0867)	**0.8881*** (0.1291)**
Italy	0.8463*** (0.0416)	0.9998*** (0.0003)	**0.9751*** (0.0836)**
Spain	0.9490*** (0.0667)	0.9999*** (0.0005)	**0.9673*** (0.1451)**
United Kingdom	0.9928*** (0.0754)	1.0000*** (0.0005)	**0.6795*** (0.1777)**
United States	1.0527*** (0.0557)	1.3545*** (0.1020)	**0.7985*** (0.1901)**

****** *represents significance at 1% critical level. Estimated values in bold correspond to the best model, while those in parentheses are the standard errors.*

Table 3: Results of the best models from Table 2

Series	d	Intercept	Trend
World estimate	**1.2582*** (0.0897)**	-8.1894*** [-11.70]	0.2033*** [2.68]
Country-specific estimates			
China	**1.3654*** (0.0819)**	-6.7053*** [-12.90]	---------
France	**0.8881*** (0.1291)**	-13.6763*** [-2.77]	0.2899*** [8.43]
Italy	**0.9751*** (0.0836)**	54.0793 [0.26]	0.1056** [2.11]
Spain	**0.9673*** (0.1451)**	8.5626 [0.09]	0.2562*** [2.86]
United Kingdom	**0.6795*** (0.1777)**	-12.1119*** [-6.92]	0.3235*** [8.46]
United States	**0.7985*** (0.1901)**	-12.3337*** [-9.19]	0.2826*** [11.60]

*** *represents significance at 1% critical level. Estimated values in parentheses and brackets are respectively the standard errors and t-values.*

However, certain empirical studies have challenged the modeling of the degree of persistence in mortality rates with linear models. In particular, they argue that mortality rates exhibit nonlinear dynamics (see Hill et al., 1999; Booth et al., 2002; Shang et al., 2006, Yaya and Gil-Alana, 2018). Incorporating this tendency, we further use the approach of Cuestas and Gil-Alana (2016) to account for nonlinearities along the time trend of the mortality rates of our sampled cases. The approach is flexible is that it helps to determine different nonlinear orders, such that significance at any polynomial degree from 2 symbolizes nonlinearity along the path of the series. However, we restrict the polynomial degree to 3 as conventional in the literature (see Yaya and Gil-Alana, 2018), as that is enough to infer the presence of nonlinearities. In light of this, we start with the estimation of the nonlinear model when second-order polynomial is assumed. The results are presented in Table 4. The only country whose mortality rates do not follow a nonlinear trend is the United Kingdom. Except for the global, China and France mortality rates whose results are consistent with the linear case, significant changes are observed in the *d* estimates of the remaining countries. While the situation becomes stationary mean reverting for the United States due to its value being lower than 0.5 (i.e. 0.3496), the estimates of Italy and Spain turn negative, implying short memory or anti-persistence.

We extend the nonlinearity focus to polynomial of order 3. This higher order is important it provides a more accurate *d* estimate if the third-order time coefficient is significant. In what seems to affirm this assertion, Table 5 shows that for the cases where the third-order time coefficients and *d* estimates are significant (world, China, Italy and Spain), there are alterations to the results reported above. The findings suggest that at the global scene and in China, mortality rates are non-stationary mean reverting since their estimates are now lower than 1, but above 0.5, but short-memory is evident for Italy and Spain following their negative estimates. As a confirmation, such evidence is also found in Table 4 for Italy and Spain. For the other countries whose *d* estimates are not significant, it means the nonlinearities in the time trend of their mortality rates are only inherent to the second degree, except the United Kingdom, thus making us to rely still on the results with *m=2* in Table 4.

Following from these results, we establish that nonlinearities matter in the time trends of the mortality rates of the global and country-based samples, except the United Kingdom. So, mean reversion is evident for the world, China, France, the United Kingdom and the United States, although it is faster for the United States, followed by France. This implies that the effect of shocks as caused by the outbreak of the COVID-19 is temporary. In other words, the mortality rates in these countries are not induced by factors that warrant intense policies to be formulated to restore normalcy. Even with the present social and economic policies adopted by these countries, mortality rates will still revert back to their original trend in no distant time, regardless of the high number recorded in some of the countries, especially United States. On the other hand, mortality rates in Italy and Spain exhibit short memory, i.e. they are anti-persistent.

Table 4: Non-linear Fractional Persistence based on Chebyshev inequality with *n=2*

Series	$\hat{d}$	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$
World estimate	**1.2558*** (0.0884)**	-1.2427 [-0.50]	-3.2097** [-2.24]	-1.0714* [-1.93]
Country-specific estimates				
China	**1.1430*** (0.0941)**	0.0532 [0.03]	-2.8187*** [-3.21]	-1.6047*** [-4.22]
France	**0.5031*** (0.1441)**	-1.6613*** [-6.36]	-3.7298*** [-15.60]	0.8959*** [5.87]
Italy	**-0.3597** (0.1320)**	-2.7216*** [-73.20]	-4.6614*** [-106.00]	-0.7538*** [-25.90]
Spain	**-1.1014*** (0.2756)**	-6.3405*** [-145.00]	-7.9188*** [-163.00]	-1.2749*** [-46.70]
United Kingdom	0.1212 (0.2283)	-4.5984*** [-7.62]	-5.4600*** [-8.16]	0.1694 [0.58]
United States	**0.3496* (0.2006)**	-0.1110 [-0.22]	-0.3644 [-0.71]	2.3508*** [11.10]

***, ** and * *represent significance at 1%, 5% and 10% critical levels respectively. Values in parentheses and brackets are respectively standard errors and t-values, while the bolded fractional differencing coefficients denote the significance of nonlinear time trend at the second-order polynomial.*

Table 5: Non-linear Fractional Persistence based on Chebyshev inequality with *n=3*

Series	$\hat{d}$	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$	$\hat{\delta}_3$
World estimate	**0.6385*** (0.1233)**	-1.1850*** [-3.29]	-3.8330*** [-26.80]	-1.1378*** [-11.80]	-1.0636*** [-14.2]
Country-specific estimates					
China	**0.8149*** (0.1105)**	-1.8600 [-1.54]	-3.0472*** [-12.00]	-1.7223*** [-11.50]	-0.7019*** [-6.39]
France	0.2673 (0.1626)	-0.9847*** [-4.55]	-2.9736*** [-11.70]	1.4316*** [7.77]	0.3573*** [3.46]
Italy	**-0.5961*** (0.1985)**	-2.8659*** [-38.10]	-4.8457*** [-49.90]	-0.8796*** [-13.10]	-0.0591* [-1.85]
Spain	**-1.4310*** (0.2863)**	-6.7793*** [-42.30]	-8.4562*** [-43.20]	-1.5963*** [-13.70]	-0.1232** [-2.81]
United Kingdom	-0.2077 (0.2608)	-9.7047*** [-5.93]	-11.5026*** [-5.93]	-3.2613*** [-3.01]	-1.0993*** [-3.23]
United States	-0.3020 (0.2768)	-4.3772*** [-6.01]	-5.2811*** [-6.15]	-0.3242 [-0.68]	-0.8008*** [-5.47]

, ** and * ***represent significance at 1%, 5% and 10% critical levels respectively. Values in parentheses and brackets are respectively standard errors and t-values, while the bolded fractional differencing coefficients denote the significance of nonlinear time trend at the third-order polynomial.

4. Conclusion

The COVID-19 pandemic currently plaguing many countries of the world is a significant source of global panic as a result of the high number of deaths recorded on a daily basis. Researchers, governments of countries and the international community are thus concerned with the possibility of the mortality rates due to the pandemic to persist or revert back to its initial trend. On this note, this study uses the fractional integration technique to examine the degree of persistence in the COVID-19 mortality rates for the world and six high-risk countries, namely China, France, Italy, Spain, the United Kingdom and United States. Following empirical discoveries that mortality rates often exhibit nonlinearities along their trend paths, we additionally employ the nonlinear fractional integration model developed by Cuestas and Gil-Alana (2016). Our findings show that mortality rates in China, France, the United Kingdom and United States are mean reverting, implying that the effects of shocks caused by the outbreak of the infectious virus is only transitory. This further implies that even mortality rates in these countries will still come back to their original trends as the effects of shocks will automatically recover by themselves. However, the mean reversion will be faster for the United States, even though her current mortality rate is on the high side. For Italy and Spain, we find evidence in favour of anti-persistence indicating that present mortality rates are unconnected from their past trends.

Therefore, we suggest that, as medical solutions are being searched for the virus across countries, intensifying the enforcement of relevant policies introduced by the government, such as social distancing, frequent hand sanitization, appropriate regional lockdowns, etc. will drastically reduce the number of recorded cases of COVID-19 infected victims. This will consequently reduce mortality rates, and aid their fast recovery processes to their original trends.

Declarations

Compliance with Ethical Standards:

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Figures

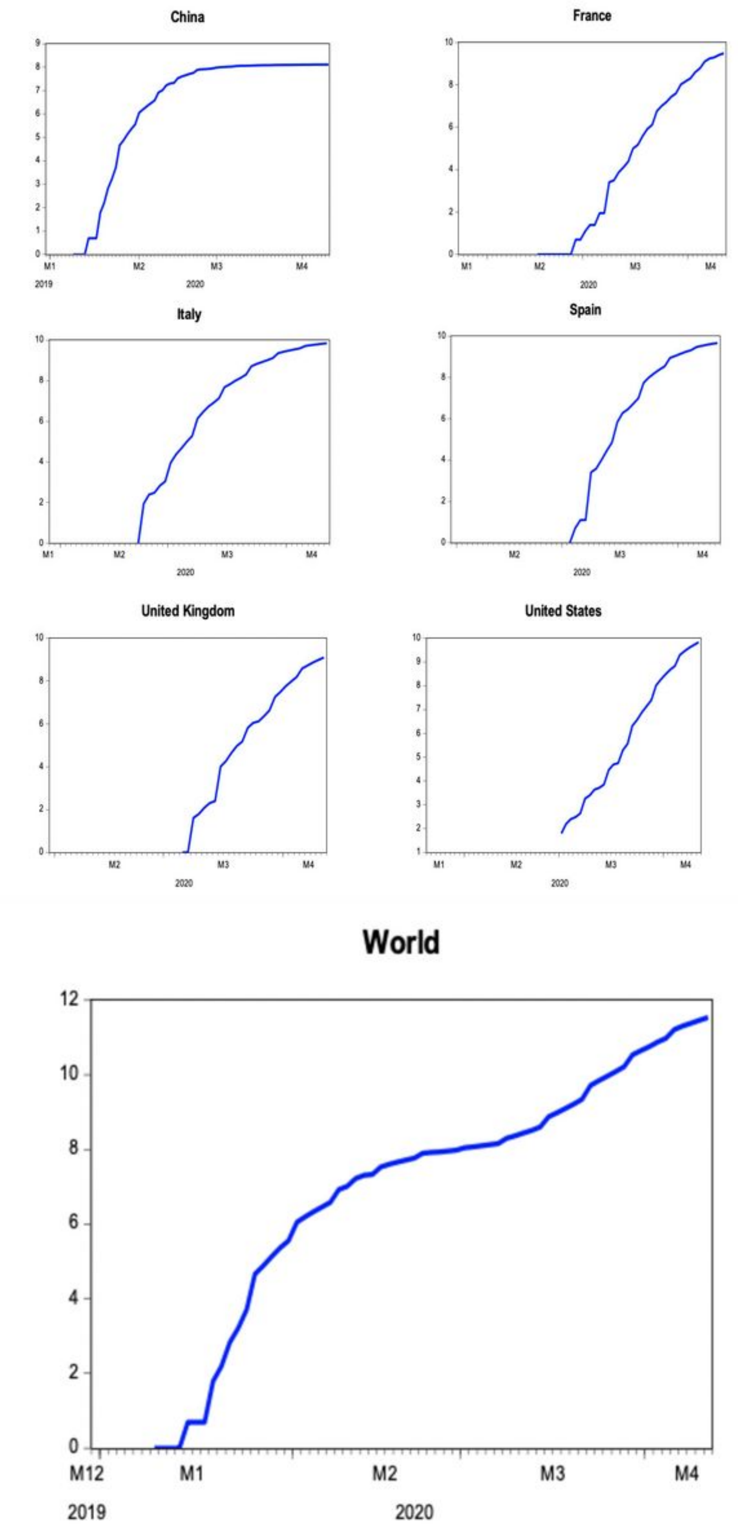

Figure 1

Trends in COVID-19 mortality rates Sources: DataStream, WHO reports – daily updates