Can machine learning really solve the three-body problem?

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Can machine learning really solve the three-body problem?

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Machine learning is becoming one of most rapidly growing technical fields, benefiting tremendous areas in science and industry. Recently, Breen *et al* demonstrated a new study with a multi-layered deep artificial neural network (ANN) on a chaotic three-body system and claimed that it can bring “success in accurately reproducing the results of a chaotic system” with “up to 100 million times faster than numerical integrator”. Here, we use their trained ANN model to predict periodic orbits of the same three-body system, but the detailed comparisons are disappointing. It might be due to the butterfly-effect of chaotic systems, i.e. very sensitive to tiny disturbance, but in practice nearly all ML algorithms derive their solutions statistically and probabilistically and therefore are rarely possible to train them to 100% accuracy. We illustrate here that the current accuracy of the machine learning is not precise enough for correct prediction of periodic orbits of a chaotic three-body system in a long enough duration. Thus, it is still a great challenge for machine learning to solve chaotic systems, such as the famous three-body problem. Without doubt, studies in machine learning on chaotic systems might greatly deepen and enrich our understandings not only on chaos (and the famous three-body problem) but also on machine learning itself.

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I. INTRODUCTION

Machine learning has emerged as one of the most attractive technologies in widespread fields, such as airport facial recognition[1], money laundering detecting[2], computer vision[3], natural language processing[4], advertising[5], and so on. Deep leaning is widely used to identify objects in images, transcribe speech into text, match news items, posts or products with users' interests, and select relevant results of search. Dominating in Go game even made a splash throughout the world by AlphaGo[6]. It has also become one of the most essential parts of addressing scientific and engineering questions[7–12]. For example, a good prediction of fluid dynamic fields has been reported, which has the ability of learning velocity and pressure information from flow snapshots by means of machine learning strategy[12]. The deep neural networks (DNNs) have driven error rates down from 8.4% to 4.9% in voice recognition in five years, and the DNNs for image recognition have helped improve error rates on ImageNet from more than 30% in 2010 to less than 3% today[11]. Note that the 5% threshold is roughly the error rate of humans for image and speech recognition using similar data. Despite a growing amount of literature on the success and efficiency of machine learning in different areas, one would ask one important question: what are the limitations/boundaries of this silver bullet?

The famous three-body problem might be a litmus test to somehow answer this question. The evolution of many bodies in space can be described by Newton's equations of motion, and the gravitational force is the only reason to drive them[13]. This discovery has an essential role in many physical areas, including the famous three-body problem. Numerous distinguished scientists[14,15] dived into this important topic since Newton mentioned it in 1680s. Poincaré[16] discovered that three-body problem is very sensitive to initial conditions, say, it is a chaotic system with the so-called butterfly-effect. That is the reason why this problem remains impenetrable[17] and only three families of periodic solutions were discovered in three hundred years, including the Euler-Lagrange[14,15], the Broucke-Hadjidemetriou-Hénon[18–22] and the figure-eight orbit[23,24]. These years, seeking new periodic solutions of the three-body problem has been paid much attention[25–27]. A big breakthrough was made currently by Li and Liao, who found more than six hundred new periodic solutions of three-body system with three equal masses[28] and 1223 new families of periodic solutions with two equal masses[29]. Besides, they also found 313 new families of collisionless periodic orbits for the free-fall triple

system with different ratios of masses[30] (there were only 4 trajectories reported before[31,32]). To achieve this accomplishment, they used a new numerical strategy, namely the clean numerical simulation (CNS)[33–39], to overcome the failure due to round-off and truncation errors in calculating chaotic issues, because the CNS can give reliable/convergent trajectory of chaotic systems with high precision in a long enough duration.

Recently, Breen *et al*[40] demonstrated a new study with a multi-layered deep artificial neural network (ANN)[3] involved in and claimed that it can bring "success in accurately reproducing the results of a chaotic system" with "up to 100 million times faster than numerical integrator". Breen *et al*[40] showed a good performance of predicting 100 trajectories of free-fall triple system with equal masses by an ANN model trained with 9900 samples. However, it is a pity that they did not illustrate its ability to track periodic orbits, a famous classical problem for three-body system. As mentioned above, seeking periodic solutions of a three-body system has been a challenging work in more than three hundreds year. It, therefore, would be a perfect benchmark to validate this ANN approach.

There are 30 equal-mass cases among the 313 collisionless free-fall periodic orbits found by Li and Liao[30], which are used in this paper as the test dataset to analyze the performance of the trained ANN model given by Breen *et al*[40] for the prediction of periodic orbits of the famous three-body problem. The ANN model uploaded by Breen *et al* was downloaded from GitHub[41].

II. METHOD

Li and Liao[30] directly solved the planar Newtonian three-body problem

$$\ddot{\boldsymbol{r}}'_i = \sum_{j=1, j\neq i}^{3} \frac{G m_j (\boldsymbol{r}'_j - \boldsymbol{r}'_i)}{|\boldsymbol{r}'_i - \boldsymbol{r}'_j|^3}, \quad i = 1, 2, 3, \tag{1}$$

by means of the CNS[33–39], where $\boldsymbol{r}'_i = (x'_i, y'_i)$ and m_i are the position vector and mass of the ith body, G is the gravity coefficient, respectively. Since they are identical mass particles with zero initial velocity for the considered free-fall case, and the dimensionless units are adopted, both mass m_j and gravity G equal to one. The initial positions of the three particles are as shown in Fig. 1, where two of the three particles are at points (0.5, 0) and (-0.5, 0), and the other one is in a shadow region which is constructed by the x' and y' axes and an arc of the unit radius with the dot (-0.5, 0) as the center. This initial

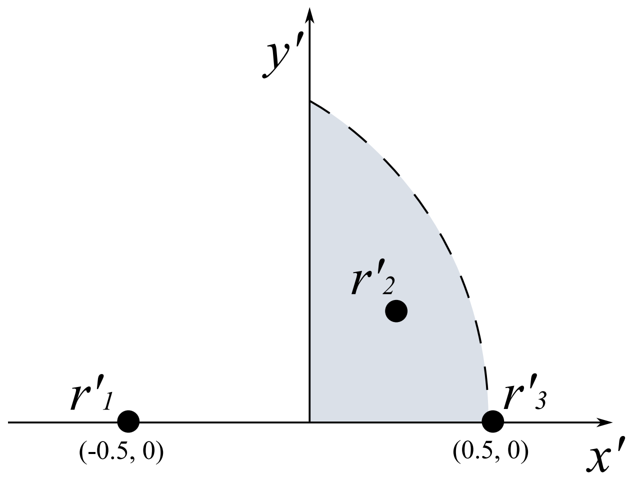

FIG. 1. **The schematic diagram of the initial positions of three particles**.

configuration will cover all planar free-fall three-body system[42]. To guarantee the reliability of the solution, we use the CNS[33–39] to solve Eq. (1). To find the periodic solutions of Eq.(1) for the free-fall triple systems, one has to make sure that

$$\boldsymbol{r}_i'(0) = \boldsymbol{r}_i'(T'), \tag{2}$$

$$\dot{\boldsymbol{r}}_i'(0) = \dot{\boldsymbol{r}}_i'(T') = 0, \tag{3}$$

where T' is the period. For details, please refer to Li and Liao[43].

Note that the training dataset of the machine learning given by Breen *et al*[40] was in another coordinate frame, denoted by $\boldsymbol{r}_i(t)$, where $\boldsymbol{r}_i = (x_i, y_i)$ is the position of the ith body and t is the time, respectively. Their training dataset of the machine learning uses the initial positions $\boldsymbol{r}_1(0) = (1, 0)$, a given $\boldsymbol{r}_2(0)$ and the corresponding $\boldsymbol{r}_3(0) = -\boldsymbol{r}_1(0) - \boldsymbol{r}_2(0)$ with the zero initial velocity $\dot{\boldsymbol{r}}_i(0) = 0$, where $i = 1, 2, 3$. The transformation between the two coordinates $\boldsymbol{r}_i(t)$ and $\boldsymbol{r}_i'(t')$ follows

$$\boldsymbol{r}_i = \alpha \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \left(\begin{bmatrix} -1 & \\ & 1 \end{bmatrix} \boldsymbol{r}_i' - \boldsymbol{r}^{*T}/3 \right), \quad t = \alpha^{3/2} t', \tag{4}$$

where $\boldsymbol{r}^* = (-x_2'(0), y_2'(0))$, the scaling factor

$$\alpha = \frac{1}{\|\boldsymbol{r}_1'(0) - \boldsymbol{r}^*/3\|}, \tag{5}$$

and the rotation angle

$$\theta = \arctan\left(\frac{y_1'(0) - y_2'(0)/3}{x_1'(0) + x_2'(0)/3} \right). \tag{6}$$

First, the 30 periodic orbits of the free-fall triple system given by Li and Liao[43] using the CNS in the frame $\boldsymbol{r}'_i(t')$ are transformed into the frame $\boldsymbol{r}_i(t)$ by means of the above formulas. Then, using the same initial position of each periodic orbits in the frame $\boldsymbol{r}_i(t)$, we can calculate the corresponding orbit by means of the trained ANN model given by Breen *et al*[40]. Both of them are compared with each other in the frame $\boldsymbol{r}_i(t)$.

Breen *et al*[40] used a combination of feedforward deep ANN, which is generally stated here to simplify the following understanding. The input ingredients are time t and one of the three particles initial position $\boldsymbol{r}_2(0) = (x_2(0), y_2(0))$. There are 10 hidden layers, each having 128 nodes, interconnected between the input layer and the output layer. For every node in each layer, there is an activation function $\sigma(x)$ to map the input into a number that is further used as the input of the next layer. Without loss of generality, let us take the ith node of the first hidden layer as an example. The output from this perceptron is

$$o_i = \sigma(\boldsymbol{w}_i \cdot \boldsymbol{r} + b_i), \quad i = 1, 2, ..., 128, \tag{7}$$

where $\boldsymbol{w}_i$ and b_i are so called 'weights' and 'bias', and the main purpose of the supervised learning is to obtain all $\boldsymbol{w}_i$ and b_i for all layers. The output layer is supposed to produce the final guess of $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ at t. After cycles (the utmost epoch is 1000 from figure 3 in the original paper) of iteration, the deviation between the guess and the real positions, so-called loss-function, is minimized to finish the whole training. For the detailed set-up of the ANN, please refer to Breen *et al*[40].

III. COMPARISON OF RESULTS

All of our test data are the free-fall periodic solutions of the three-body problem with equal mass given by Li and Liao[43] using the CNS[33–39]. To precisely estimate the deviation, we define the maximum relative error

$$\delta(t) = \max \frac{||\boldsymbol{o}_i(t) - \boldsymbol{r}_i(t)||}{||\boldsymbol{r}_1(0) - \boldsymbol{r}_3(0)||}, \quad i = 1, 2, 3, \tag{8}$$

where $\boldsymbol{r}_i$ is the i-th periodic orbit given by CNS[43] and $\boldsymbol{o}_i$ is obtained using the ANN model from GitHub[41] uploaded by Breen *et al*[40].

Breen *et al*[40] gave three ANN models with $t \leq 3.9$, $t \leq 7.8$ and $t \leq 10$, respectively. A test using four random periodic orbits indicates that the ANN model uploaded by Breen

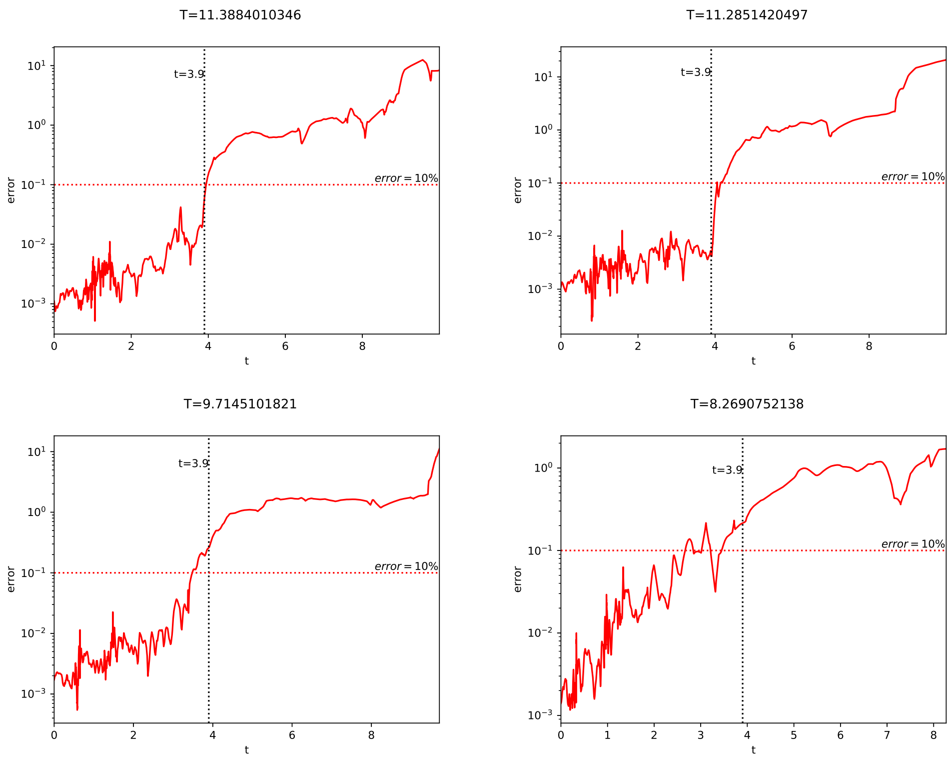

FIG. 2. **Maximum relative error curves of some random tests.**

et al [41] on GitHub was trained from a dataset with $t \leq 3.9$, as shown in Fig.2. Before $t = 3.9$, the ANN model works fine for most of the cases, but errors go through the roof after that. Hence, the validity interval of the ANN model uploaded by Breen *et al*[41] on GitHub is $t \in [0, 3.9]$. This model is the most accurate one, compared with the other two using a dataset with longer time labels, as reported by Breen *et al*[40]. Unfortunately, this ANN model can not offer the required accuracy for the free-fall periodic orbits even in [0,3.9], as mentioned below in details.

Among the 30 equal-mass periodic orbits given by Li and Liao[43] , the smallest period is $T = 3.9038901094$, corresponding to $\boldsymbol{r}_2(0) = (-0.2636343510, 0.1988137050)$. So, this is the only chance to check the prediction accuracy of the ANN model for periodic orbits of the equal-mass free-fall triple system. Unfortunately, the comparison result is rather negative, as shown in Figure 3: the maximum relative-error δ is less than 3% in $0 \leq t < 2.5$ but rapidly grows after $t > 2.5$ and ends up with almost 60%. Note that the ANN model can well predict the trajectory in the first half period ($0 \leq t \leq T/2 \approx 2.0$): one can barely tell the difference until the bodies start to return from the mid-points. However, in the second

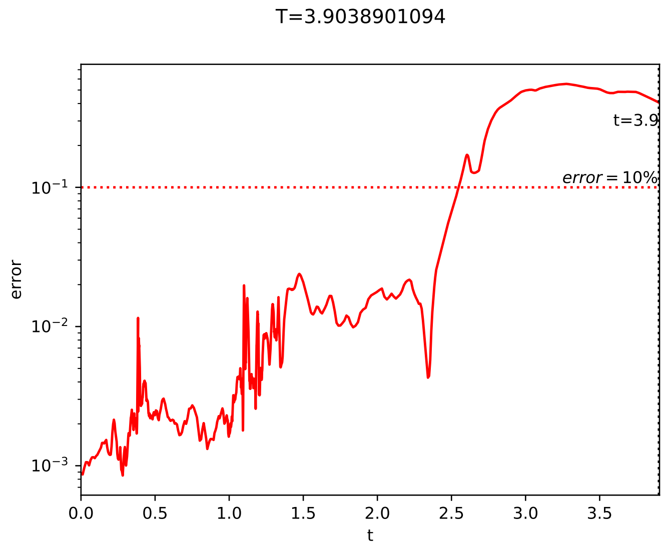

FIG. 3. **The maximum relative-error curve of the periodic orbit** .

half period ($T/2 < t < T$), the deviations of the ANN prediction become rather obvious, as shown in Figure 4.

Serious comparison for the other 29 periodic orbits in one period, i.e. $t \in [0, T]$, is *impossible*, since their periods are much larger than 3.9. Note that in many cases there exist large derivations even in a short duration $0 \leq t \leq 3.9$ that is much smaller than the corresponding periods of these orbits. One example is given in Figure 5 for comparison of a periodic orbit with $T = 12.8337947509$, with the initial condition $\boldsymbol{r}_2(0) = (-0.3762596888, 0.1031140835)$. It is found that the corresponding trajectories of the triple system given by the ANN model (using the same initial condition) have obvious derivations from the CNS results after $t > 1$. More comparisons of the *small parts* (within $t \in [0, 3.9]$) of the periodic orbits are given in Figures 6 to 9: all of them show a distinct deviation sooner or later and thus can *not* predict the corresponding periodic orbits (in a whole period) given by the CNS[43]. Note that the ANN model we used here is the finest one within $t \leq 3.9$ uploaded by Breen *et al.*[40]. If the training dataset in a longer duration is used, a worse performance can be expected.

Note that, in $t \in [0, 3.9]$, cases with a relative-error lower than 10% account for 73.3% of the test dataset. Although the corresponding mean absolute error (MAE) of the test dataset is 0.035 and its counterpart of the training dataset is around 0.02, which seems a good generalization of an ANN model, about one quarter of the ANN predictions have an obvious deviation from the CNS solutions even in the short duration $t \in [0, 3.9]$. Obviously,

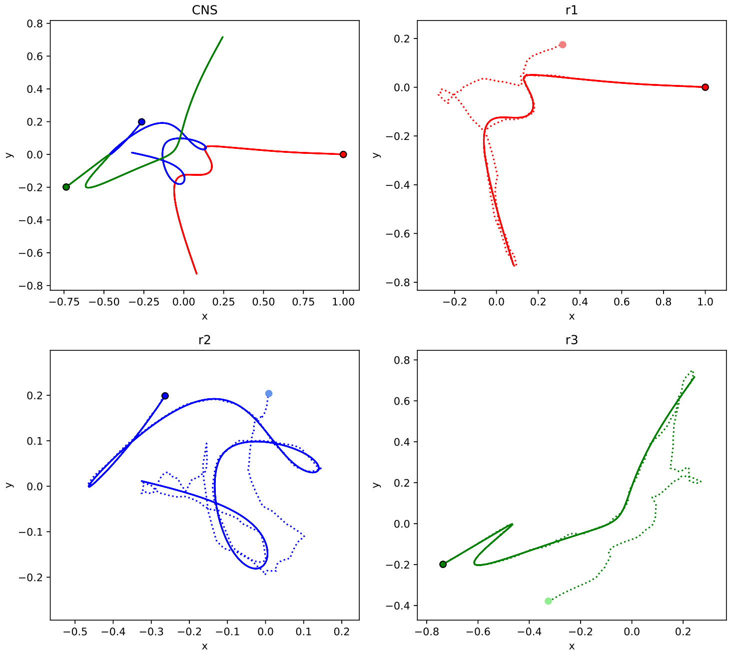

FIG. 4. **Comparisons of periodic trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions in the case of $\boldsymbol{r}_2(0) = (-0.2636343510, 0.1988137050)$. Solid lines: CNS periodic solutions; dashed lines: ANN predictions.

the longer the duration of time, the more the ANN predictions depart from the CNS periodic orbits. However, most of the equal-mass 30 periodic orbits reported by Li and Liao[43] have a period much longer than 3.9. Therefore, it seems that the ANN model[40] could not predict the periodic orbits of the free-fall three-body system, especially for a long period.

IV. DISCUSSION

A trained ANN model in $t \in [0, 3.9]$ for a free-fall triple system with equal masses, which was uploaded on GitHub[41] given by Breen *et al*[40], is used to predict 30 equal-mass periodic orbits, whose initial conditions are exact the same as the 30 periodic orbits of the equal-mass free-fall triple system obtained by Li and Liao[43] using the clean numerical simulation (CNS), a numerical strategy to gain reliable/convergent simulations of chaos in a long enough duration. It is a pity that the detailed comparison between the ANN predictions and the CNS solutions is negative: not only the ANN model can not predict the periodic orbit with the smallest period $T = 3.9038901094$ (the ANN trajectory distinctly departs from the CNS

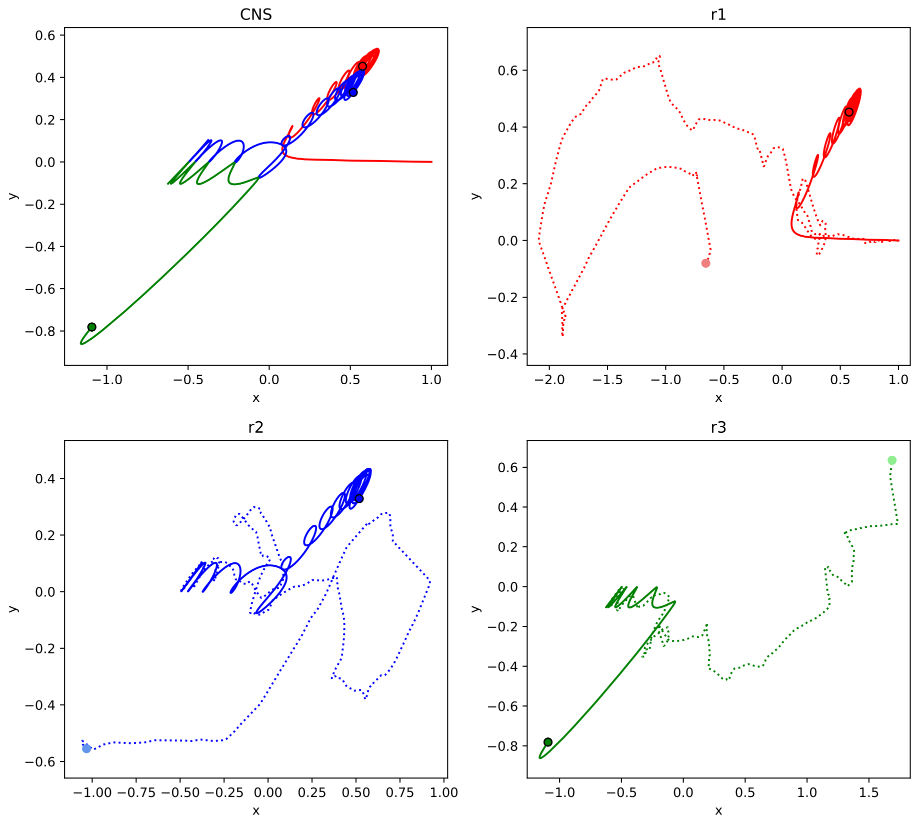

FIG. 5. **Comparisons of trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions in the case of $\boldsymbol{r}_2(0) = (-0.3762596888, 0.1031140835)$. Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

solution at $t \approx 2.5$ and has an obvious deviation at $t = 3.9$), but also many of the ANN predictions have a distinct departure even from a small part (within $t \in [0, 3.9]$) of CNS periodic orbits, as shown in Figures 5 to 9.

Although the deep neural networks (DNNs) have driven error rates down to 4.9% in voice recognition and less than 3% in image recognition today, which are less than the threshold (5%) of human for similar data[11], nearly all ML algorithms derive their solutions statistically and probabilistically and therefore are rarely possible to train them to 100% accuracy. Even the best computer systems of voice and image recognition make errors, as do the best humans. Therefore, we had to be tolerant to the imperfection of machine learning system. However, we illustrate here that the current accuracy of the machine learning is not precise enough for correct prediction of periodic orbits of the famous three-body problem in a long enough duration, which has chaotic characteristic.

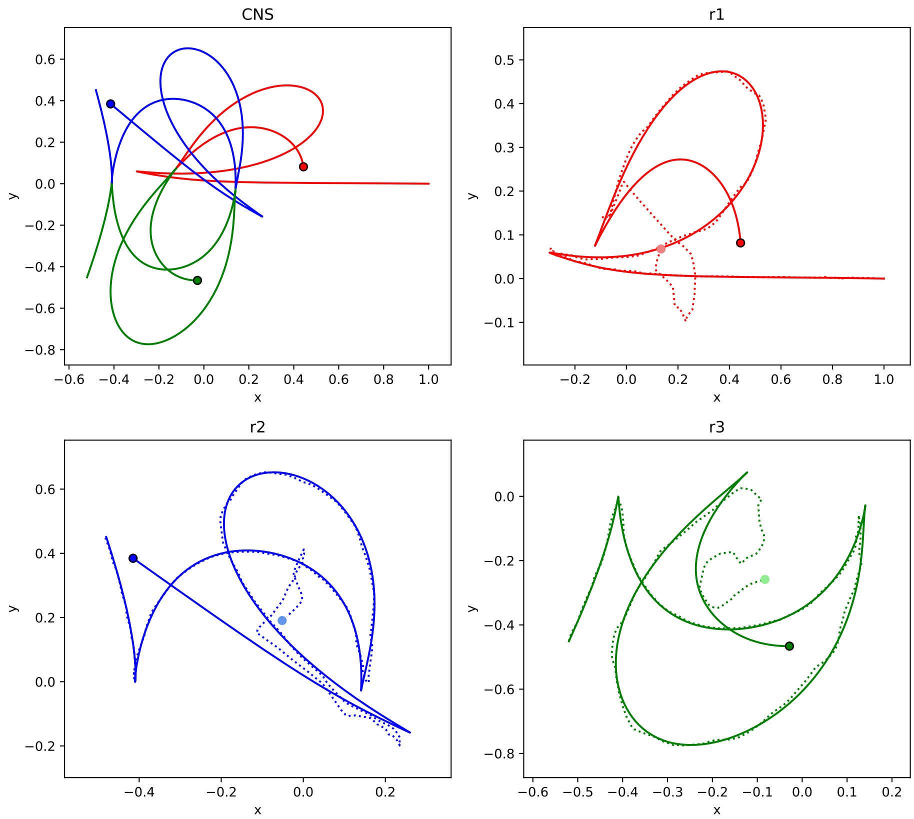

FIG. 6. **Comparisons of trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions in the case of $\boldsymbol{r}_2(0) = (-0.4802896445, 0.4510651575)$. Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

It is well-known that chaotic systems have the so-called butter-fly effect: a tiny difference in initial condition can lead to huge deviation quickly, since error of chaotic systems increases exponentially. Note that, due to the chaotic characteristics, even the micro-level physical uncertainty might lead to macroscopic deviations and randomness of trajectories of three-body systems, as reported by Liao[35] and Li & Liao[37]. Besides, current studies indicate that numerical noises (truncation and round-off errors) might lead to large deviations of numerical simulations of chaotic systems not only quantitatively but also qualitatively even in statistics[38,39]. Similarly, the error of this ANN model for chaotic systems (such as three-body problem) should increase exponentially, too. This might be the reason why the uploaded ANN model[41] is valid in a short interval $t \in [0, 3.9]$. Yes, it is true that we *had* to be tolerant to the imperfection of machine learning system. Unfortunately, numerical noises are unavoidable but chaotic systems are rather sensitive to errors, which might lead to huge

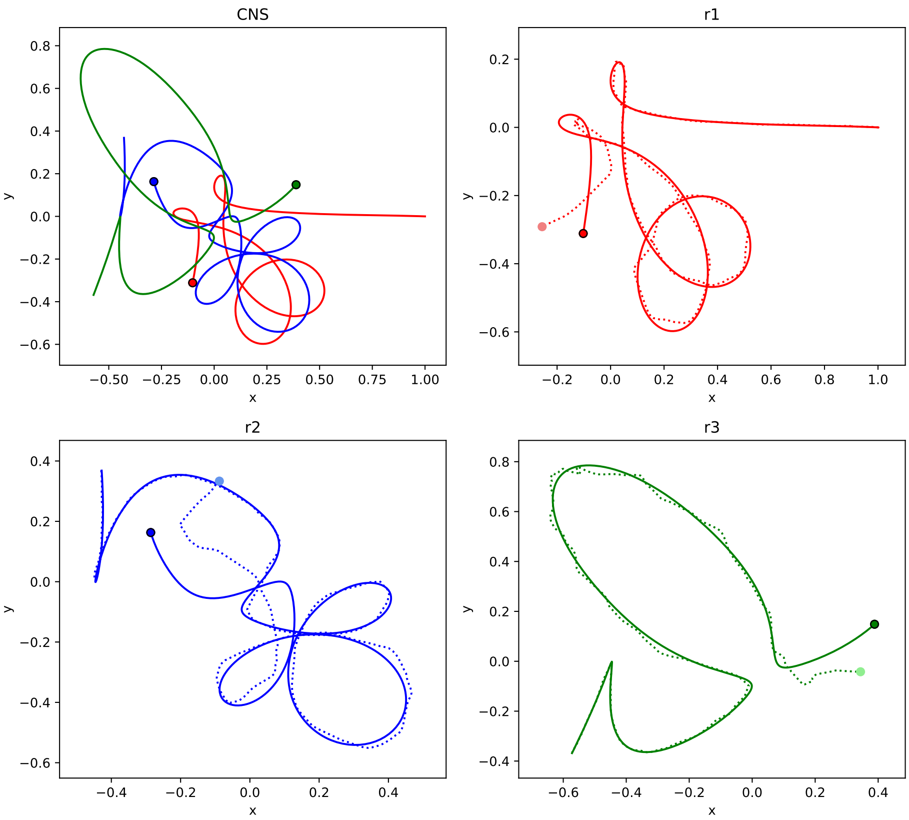

FIG. 7. **Comparisons of trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions in the case of $\boldsymbol{r}_2(0) = (-0.4278347135, 0.3682116774)$. Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

deviations even in *statistics*[38,39].

Possibly, better predictions of periodic orbits might be obtained if one adds periodic orbits in the training data-set, since in general ML algorithms work well only when the distribution of test examples is similar to the distribution of training data. Unfortunately, the periodic orbits of three-body problem is very rare: about six hundreds periodic orbits were obtained from 10 million search computations. So, from statistical viewpoint, ML algorithms would give much larger weight on non-periodic orbits than periodic ones of three-body systems. Thus, it seems that there should be a quite long way to go for machine learning to accurately predict periodic orbits of three-body systems in a long enough duration.

Finally, we should emphasized that the aim of this paper is *not* to discourage further exploration of machine learning, since many successful applications in science and industry strongly support its strength and potential. However, it is better for us to know its boundary:

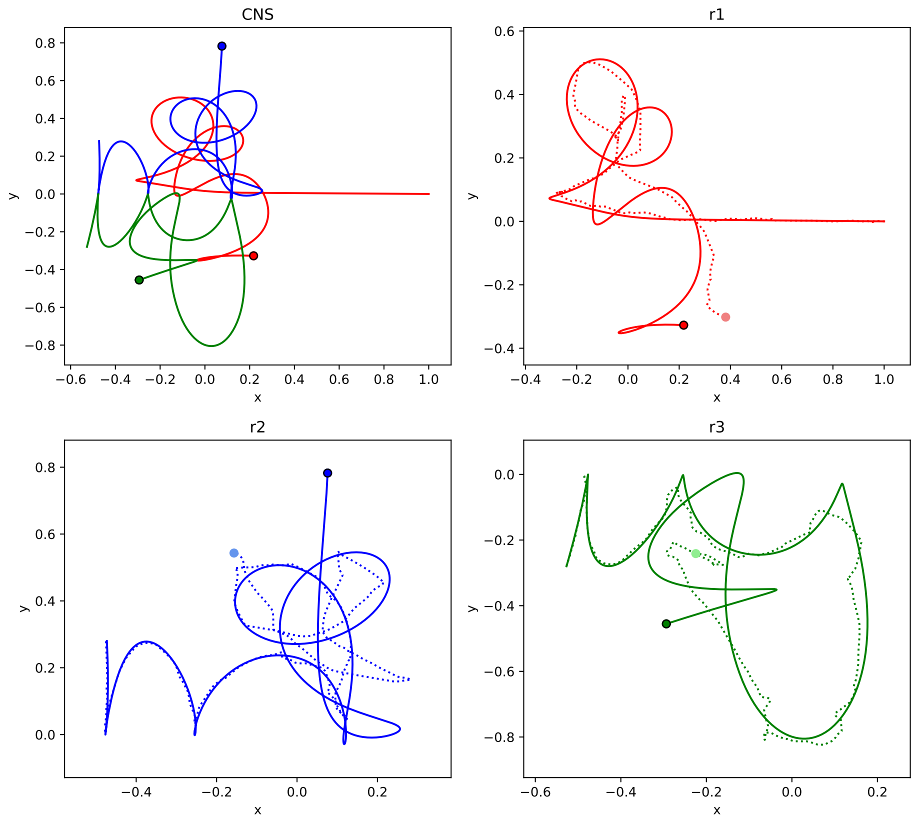

FIG. 8. **Comparisons of trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions in the case of $\boldsymbol{r}_2(0) = (-0.4729359214, 0.2799147266)$. Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

what it can do well and what it might not. Clearly, it is a great challenge for machine leaning to give accurate *enough* prediction of chaotic systems (such as the famous three-body problem) in a long *enough* duration. Three-body problem is very famous in science, which leads to a new field of scientific researches, i.e. chaos that greatly deepens and enriches our understandings about complicated nonlinear dynamic systems. Obviously, studies in machine learning on chaotic systems might greatly deepen and enrich our understandings not only on chaos but also on deep artificial neural network itself.

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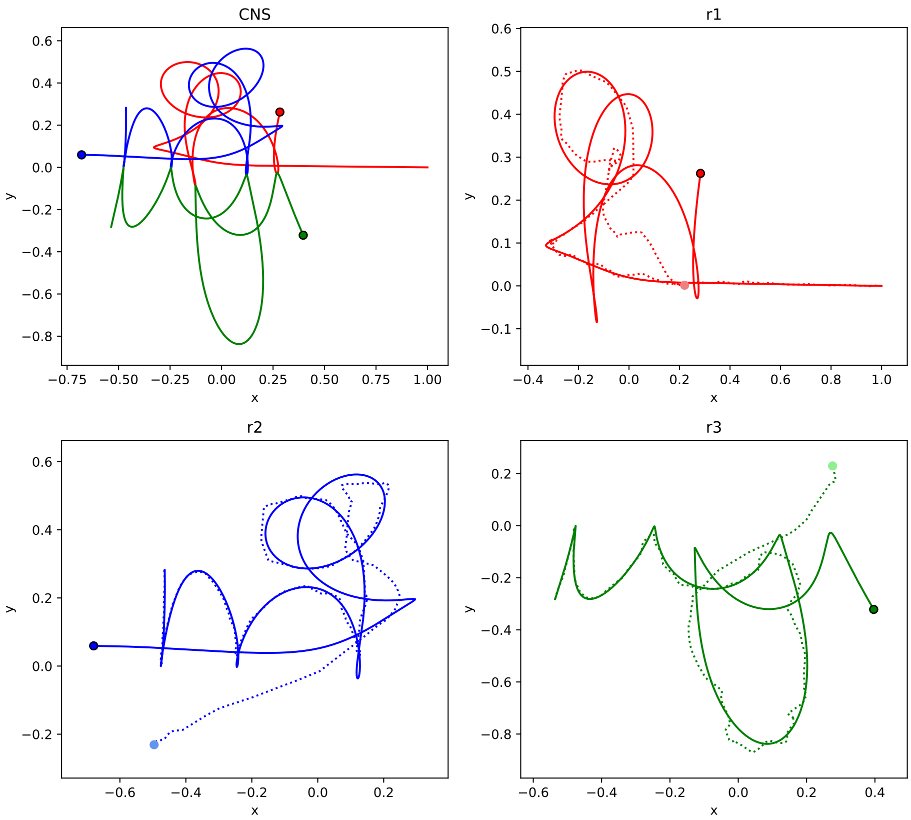

FIG. 9. **Comparisons of trajectory in** $t \in [0, 3.9]$. Bullets indicate the final positions with $r_2(0) = (-0.4640439900, 0.2817630547)$. Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

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Figures

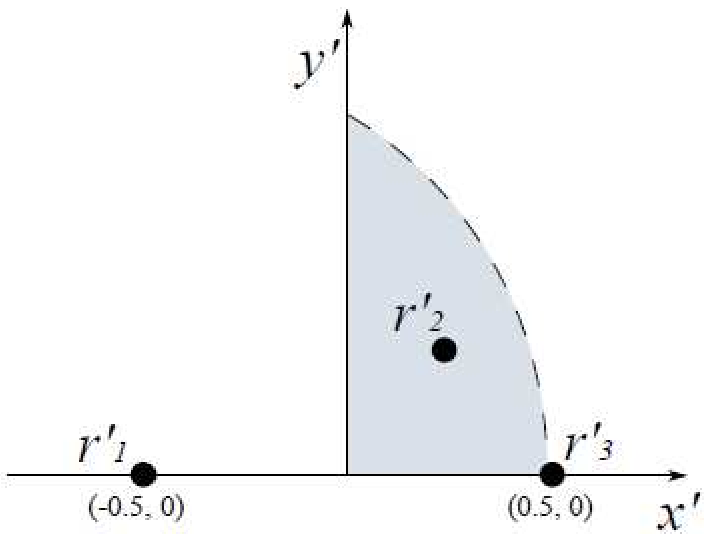

Figure 1

The schematic diagram of the initial positions of three particles.

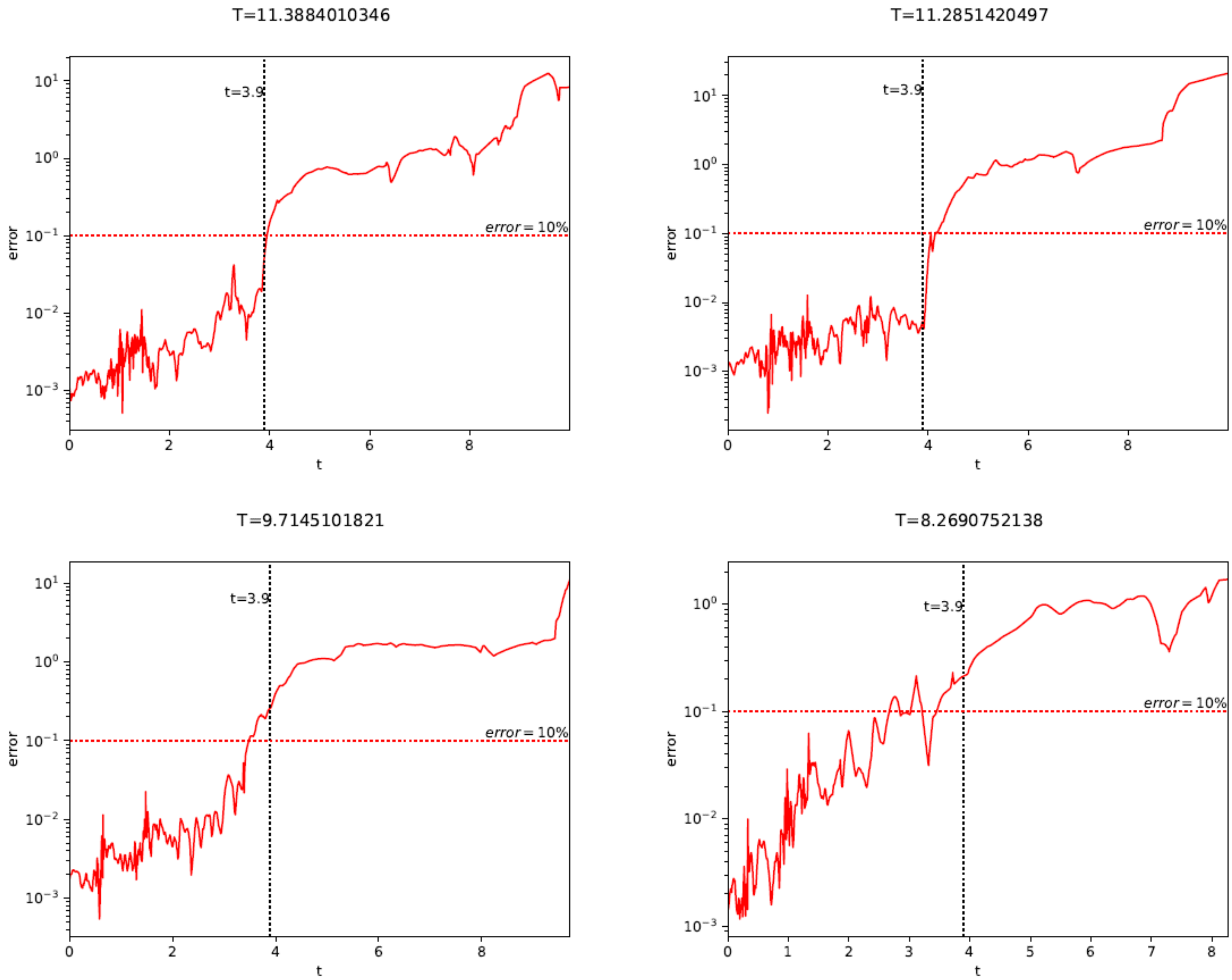

Figure 2

Maximum relative error curves of some random tests.

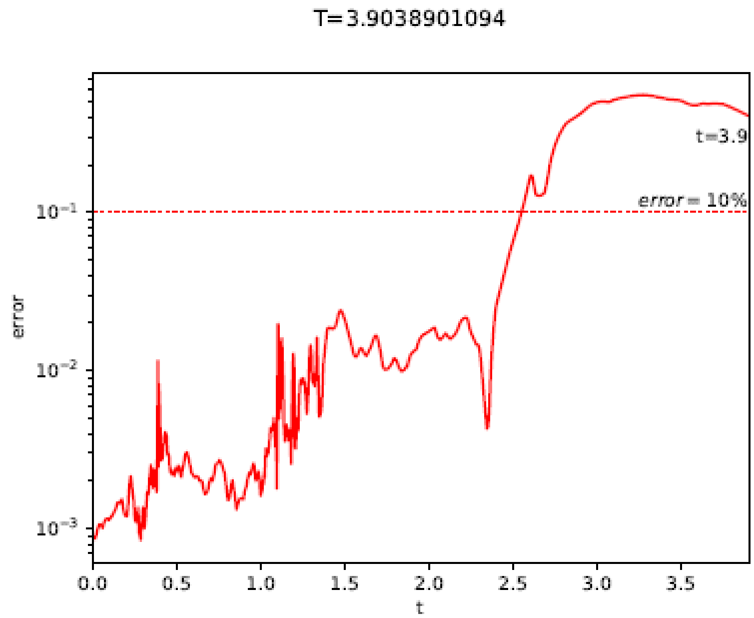

Figure 3

The maximum relative-error curve of the periodic orbit .

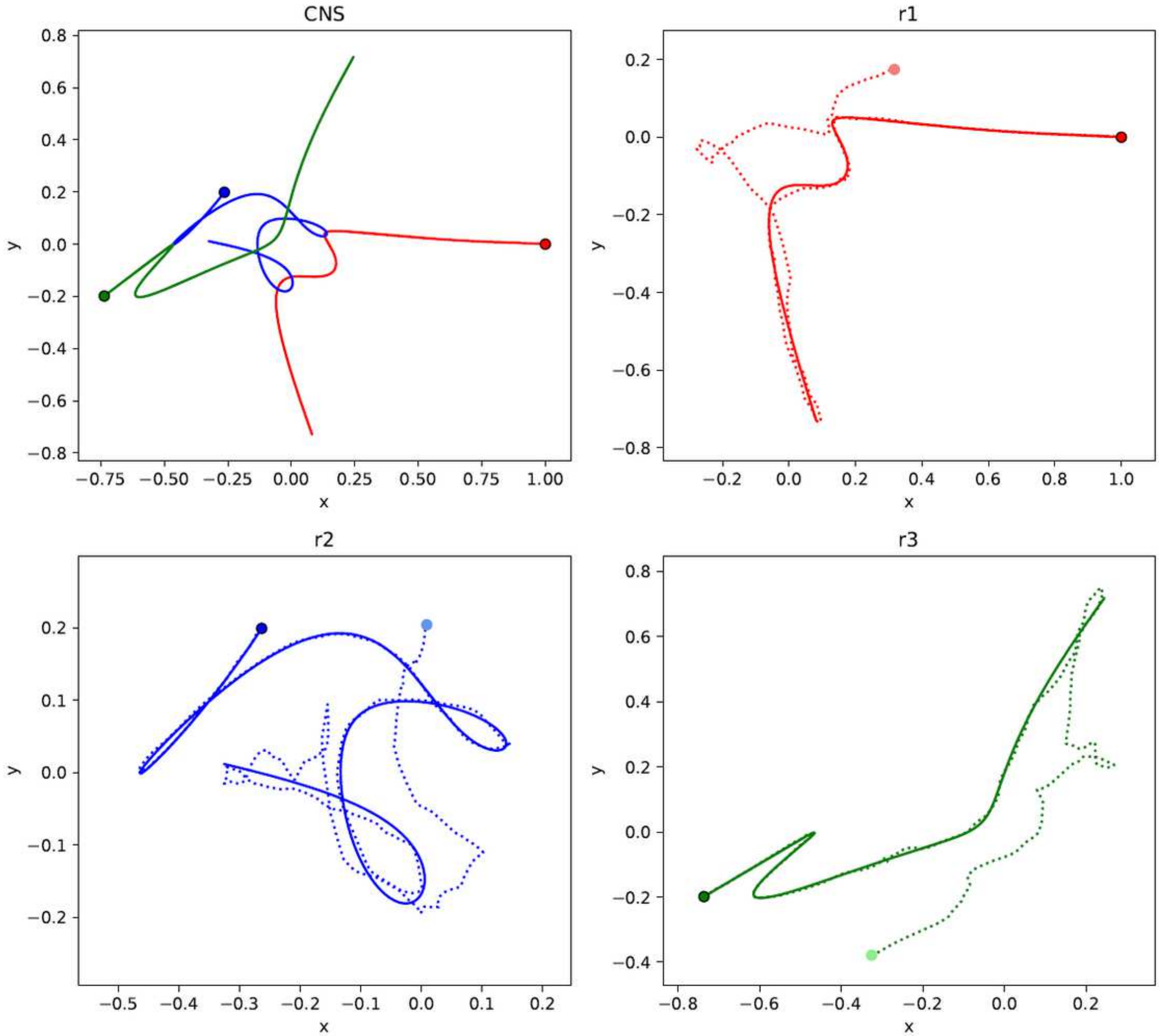

Figure 4

Comparisons of periodic trajectory in t [0; 3:9]. Bullets indicate the nal positions in the case of r2(0) = (-0:2636343510; 0:1988137050). Solid lines: CNS periodic solutions; dashed lines: ANN predictions.

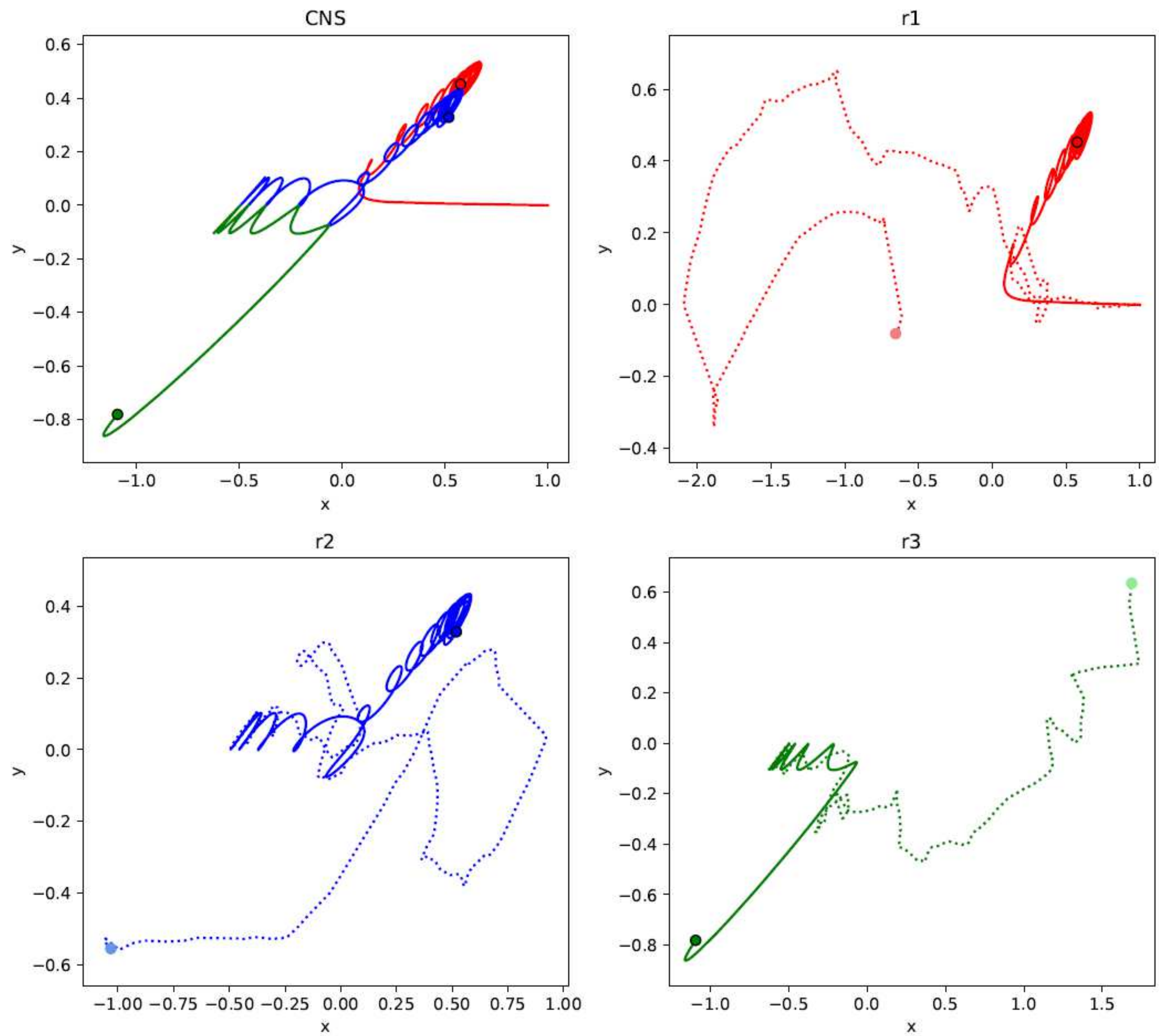

Figure 5

Comparisons of trajectory in t [0; 3:9]. Bullets indicate the nal positions in the case of r2(0) = (-0:3762596888; 0:1031140835). Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

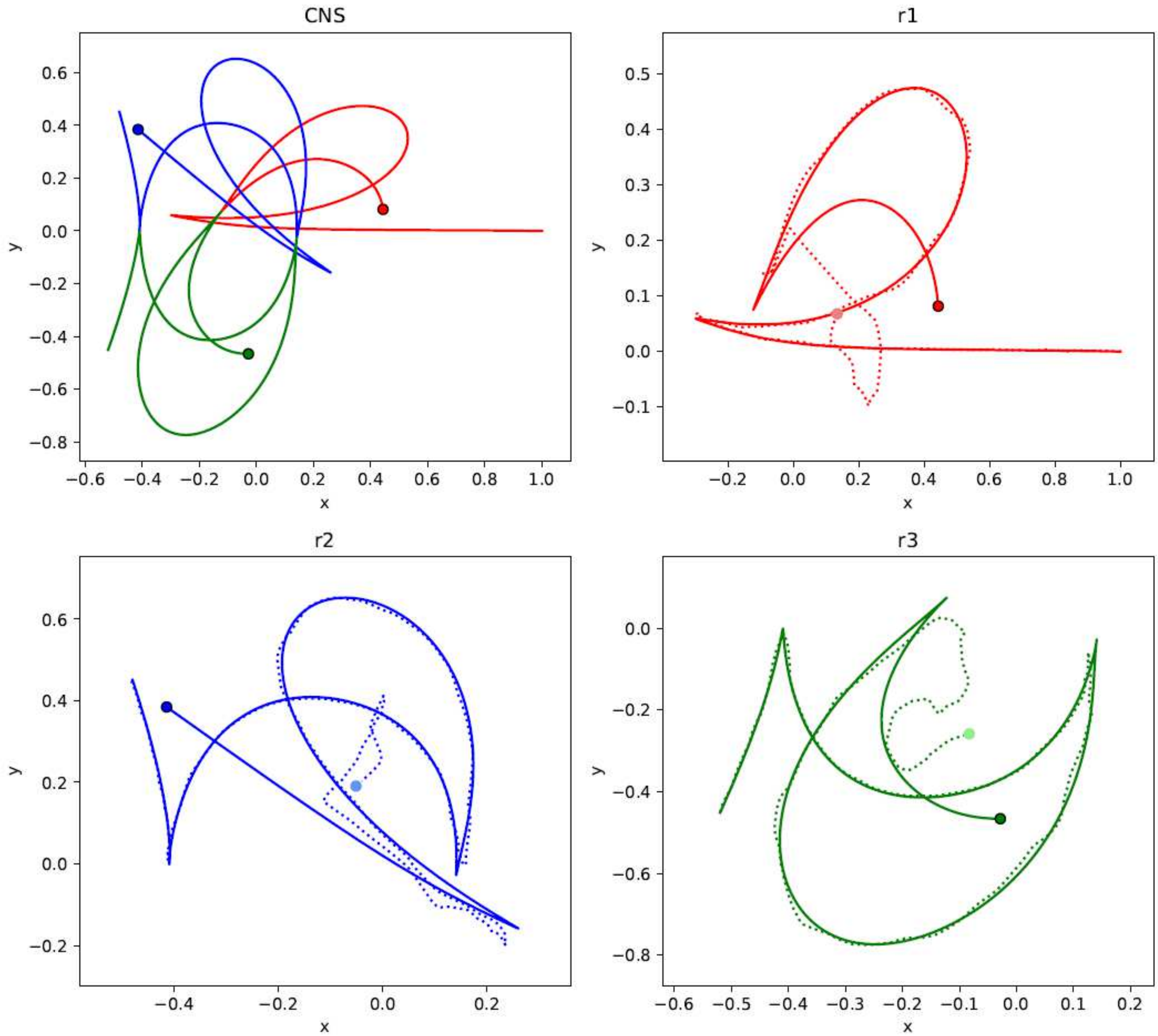

Figure 6

Comparisons of trajectory in t □ [0; 3:9]. Bullets indicate the nal positions in the case of r2(0) = (-0:4802896445; 0:4510651575). Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

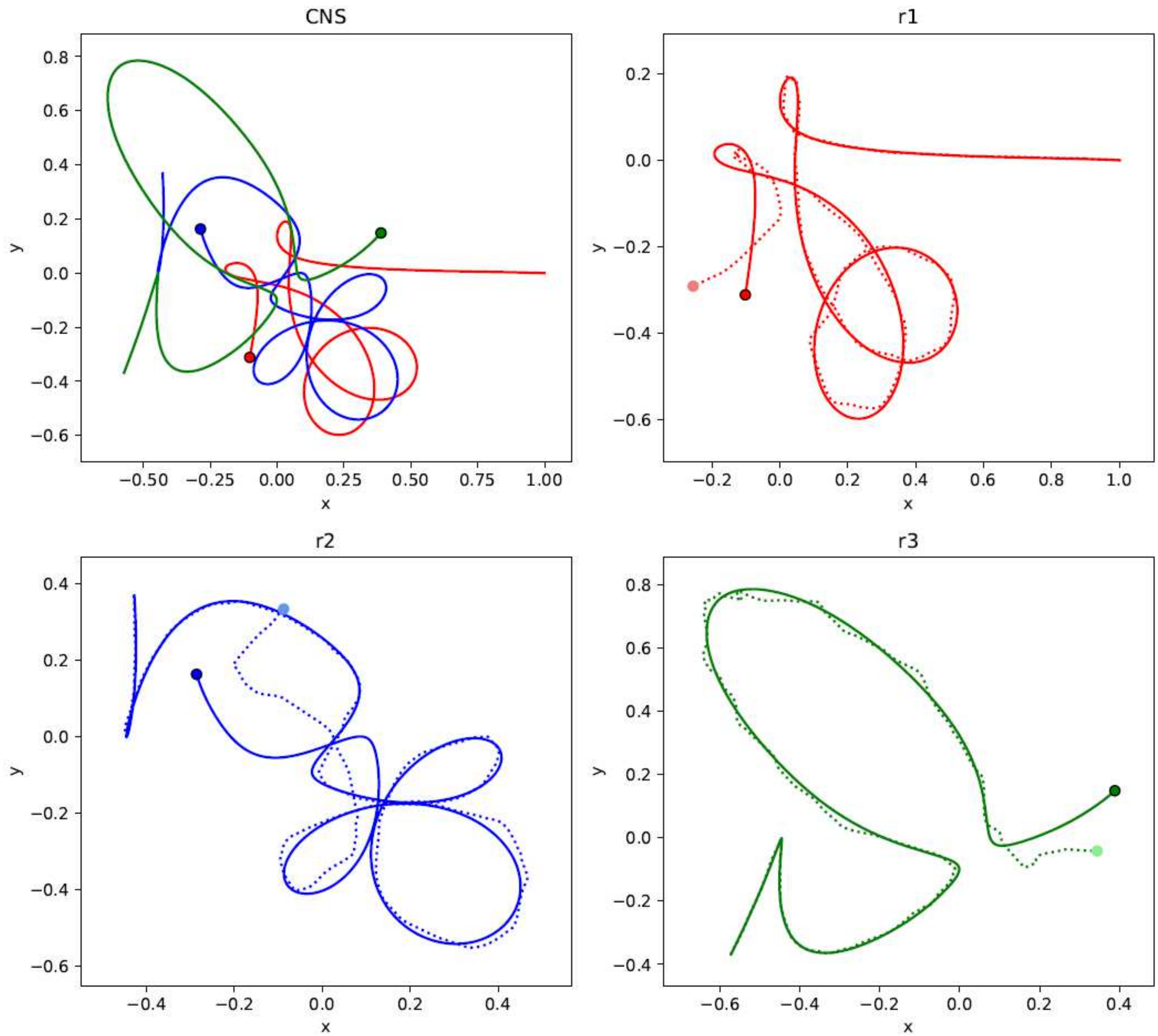

Figure 7

Comparisons of trajectory in t [0; 3:9]. Bullets indicate the nal positions in the case of r2(0) = (-0:4278347135; 0:3682116774). Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

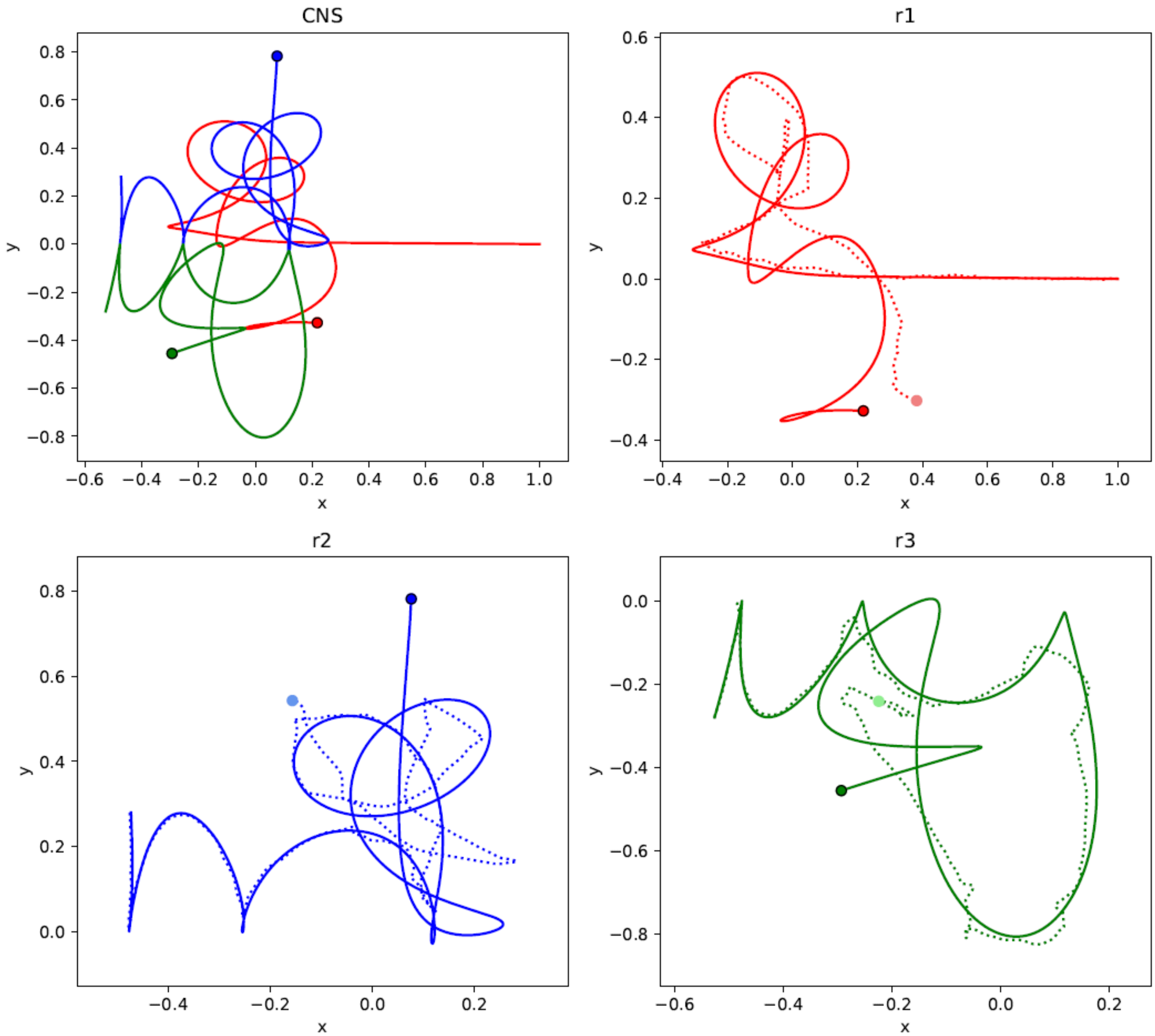

Figure 8

Comparisons of trajectory in t [0; 3:9]. Bullets indicate the nal positions in the case of r2(0) = (-0:4729359214; 0:2799147266). Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

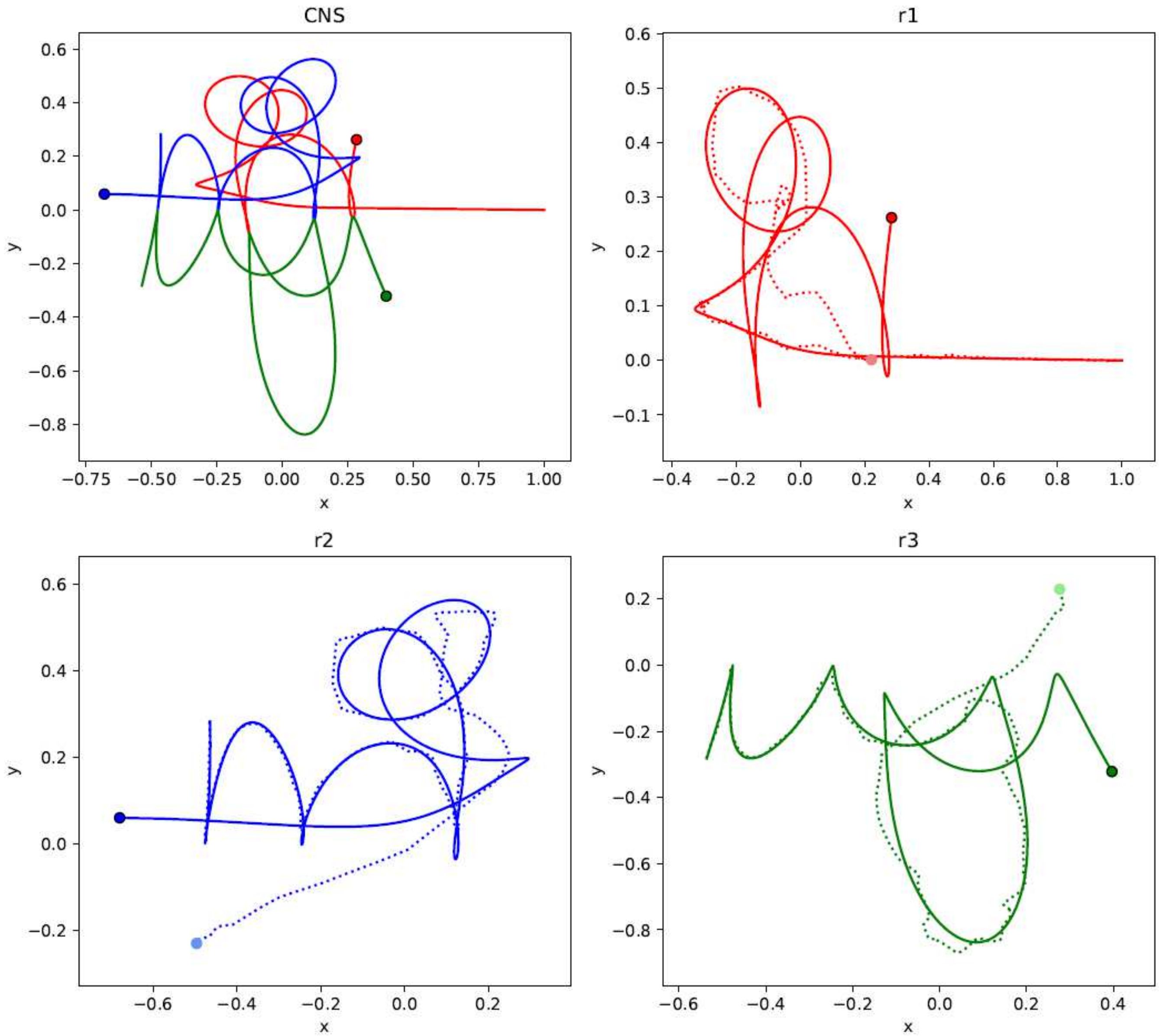

Figure 9

Comparisons of trajectory in t [0; 3:9]. Bullets indicate the nal positions with r2(0) = (-0:4640439900; 0:2817630547). Solid lines: part of CNS periodic solutions; dashed lines: ANN predictions.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- fig5.gif
- fig4.gif