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Md. Salim SHEKH (✉ da_salim@rediff.com)

S. P. M. Science and Gilani Art's commerce college, Ghatanji-445301, India

K Ghaderi

Islamic Azad University, Tehran

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Radiating isotropic homogeneous $f(R, G)$ gravity model with quark and strange quark matter

S. H. Shekh¹ *, K. Ghaderi[†],

* *Department of Mathematics, S.P.M. Science and Gilani Arts and Commerce College, Ghatanji, Yavatmal, Maharashtra-445301, India.*

† *Department of Physics, Marivan Branch, Islamic Azad University, Marivan, Iran.* and

Corrospoding author: S. H. Shekh¹

Abstract: Present analysis devoted to the dynamical investigation of radiating quark and strange quark matter in the framework of $f(R, G)$ theory of gravity towards spatially homogeneous isotropic Friedman-Robertson-Walker line element filled with isotropic fluid. We govern the features of the derived cosmological model by considering the well known relation between Hubble parameter H and scale factor. Some kinematical and physical parameters for quark and strange quark matter towards isotropic model is discussed in detail.

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Keywords: Isotropic FRW model; quark and strange quark matter; $f(R, G)$ gravity; cosmology.

1. INTRODUCTION

The present universe is spatially flat and at late-time it has an accelerated expansion (the strength of this acceleration is a remarkable question in recent year) confirmed by the recent observations like SNe-Ia Supernova, Cosmic Microwave Background Radiation (CMBR), Large Scale Structure (LSS) and Wilkinson Microwave Anisotropy Probe (WMAP). The recommendations that have been put forward to explain this observed phenomenon can be classified into one either an exotic component with negative pressure so called dark energy which introduce into General Theory of Relativity (GTR) or second the change in the gravity law through the modification of action in GTR called Modified Gravity (MG). The MG has become one of the most popular candidates to understand the idea of dark energy. Various modification in the action of GTR is present, out of which one where the Hilbert-Einstein action is generalized to a general function of the Ricci scalar R called $f(R)$ theory of gravity, these theory not only are able to simulator the behavior of the cosmological constant but also can reproduce the complete cosmological history (see Ref. Nojiri and Odintsov (2006); Elizalde and Sez-Gmez (2009); Faraoni (2008); Cruz-Dombriz and Dobado (2006)). Appleby et al. (2010) studied $f(R)$ MG models by showing how the problem of a weak curvature singularity with can get up generically in viable $f(R)$ model of present dark energy indicating an internal incompleteness be cured by adding a quadratic correction with a sufficiently small coefficient to the $f(R)$ function at large curvatures. Felice and Tsujikawa (2010) study the $f(R)$ theory to cosmology and gravity such as inflation, local gravity constraints, dark energy, cosmological perturbations and spherically symmetric solutions in weak and strong gravitational backgrounds. Bamba et al. (2012) presented the Λ CDM cosmology, Little Rip and Pseudo-Rip universes, the quintessence cosmolo-

* Email: da.salim@rediff.com

† Email: k.ghaderi.60@gmail.com

gies and phantom with Type I, II, III and IV finite-time future singularities and non-singular dark energy universes as well as equivalent class of dark energy models which includes $f(R)$ gravity. Azadi et al. (2008), Miranda et al. (2009), Sharif and Yousaf (2014), Bhoyar et al. (2016), Chirde and Shekh (2016; 2017) along with many authors have discussed some features of $f(R)$ gravity using different space-time.

Another is $f(R, T)$ gravity presented by Harko et al. (2011) with several aspects of this theory including FRW dust universe. Sharif and Zubair (2012), Chaubey and Shukla (2013), Sahoo et al. (2014), Chirde and Shekh (2015, 2016a), Bhoyar et al. (2015) are some of the authors who have analyzed the same gravity for different context of use.. Next, the gravity which is obtained by adding a function of Gauss Bonnet invariant $G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$ ($R_{\mu\nu}$ is the Ricci tensor and $R_{\mu\nu\sigma\rho}$ is the Riemann tensor) in the Einstein-Hilbert action, the gravity so called $f(G)$ gravity (Nojiri et al. (2006)). The motivation for this theory comes from string theory by low energy effective scale which is consistent with the global universe evolution and local gravity restrictions (Cognola et al. 2007; Felice and Tsujikawa (2009)). The detailed analysis of $f(G)$ gravity is presented in ref. Easson (2005), Nojiri et al. (2005, 2007); Carter and Neupane (2006); Koivisto and Mota (2007); Sadjadi (2014); Sharif and Fatima (2014), Sadeghi et al. (2009). Another modification of GTR is $f(R, G)$ gravity which is a more general class of modied gravity includes $f(R)$ gravity and $f(G)$ gravity in which Bamba et al. (2010) have studied all four types of finite-time future singularities emerging in late-time accelerating (quintessence/phantom) era from $f(R, G)$ gravity also the authors Saltas and Kunz (2011), Felice et al. (2010), Dombriz and Gmez (2012), Makarenko et al. (2013), Atazadeh and Darabi (2014), Laurentis et al. (2015), Benetti et al. (2018) investigated the same gravity in order to classify modified gravity models according to their physical properties.

In 1984, Witten (1984) validated that at a critical temperature $T_c = 100 - 200$ MeV which could have led to the establishment of quarks at larger density than normal nuclear matter density. Strange quark matter is settled with an equation of state (EoS) $p = \frac{(\rho - 4B_c)}{3}$, based on the phenomenological bag model of quark matter where B_c is known as Bag constant. Nowadays, the study of quark and strange quark matter is an exciting area of exploration. Quark matter is considered in strange stars (Drake et al. (2002)), even as small pieces of strange matter (Weber (2005)) or to exist at the centre of neutron stars (Perez-Garcia (2010)).

This paper is organized as follows: In sec. 2 some basic concepts of $f(R, G)$ gravity with the source containing quark and strange quark matter. Metric, components of field equation of $f(R, G)$ gravity with their solution is presented in sec. 3. Sec. 4 contains some physical and kinematical parameters of quark and strange quark matter using matter dominated power law expansion. Finally, Sec. 5 deals with some concluding remarks.

2. BASICS OF $f(R, G)$ GRAVITY WITH STRANGE QUARK MATTER

The most general action for $f(R, G)$ gravity is given as (Dombriz and Gmez 2013),

$$S = \frac{1}{2k} \int d^4x \sqrt{-g} (R + f(G)) + S_M(g^{ij}, \varphi), \quad (1)$$

where $S_M(g^{ij}, \varphi)$ is the matter action, R is Ricci scalar and G is Gauss-Bonnet invariant defined by,

$$G = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\sigma\nu}R^{\alpha\beta\sigma\nu}, \quad (2)$$

where, the notations $R_{\alpha\beta}$ and $R_{\alpha\beta\sigma\nu}$ are occupied for the Ricci and Riemann tensors respectively.

Variation of the standard action (1) with respect to the metric gives us the following gravitational field equation,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = kT_{\mu\nu}^{mat} + \Sigma_{\mu\nu}, \quad (3)$$

where,

$$\begin{aligned} \Sigma_{\mu\nu} = & \nabla_\mu \nabla_\nu f_R - g_{\mu\nu} \square f_R + 2R \nabla_\mu \nabla_\nu f_G - 2g_{\mu\nu} \square f f_G - 4R_\mu^\lambda \nabla_\lambda \nabla_\nu f_G - 4R_\nu^\lambda \nabla_\lambda \nabla_\mu f_G + 4\square_{\mu\nu} f_G + 4g_{\mu\nu} R^{\alpha\beta} \nabla_\alpha \nabla_\beta f_G + \\ & 4R_{\mu\alpha\beta\nu} \nabla^\alpha \nabla^\beta f_G - \frac{1}{2} g_{\mu\nu} (f_R R + f_G G - f(R, G)) + (1 - f_R) (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R). \end{aligned} \quad (4)$$

Here, ∇_μ represents the covariant derivative and $f_R \equiv \frac{\partial f(R, G)}{\partial R}$ and $f_G \equiv \frac{\partial f(R, G)}{\partial G}$, gives the partial derivatives of $f(R, G)$ with respect to R and G respectively.

In this work, we obtain the solution of field equations and the behavior of the universe using some kinematical and physical quantities for the $f(R, G)$ gravity model i.e.

$$f(R, G) = f_0 R^m G^{1-m}, \quad (5)$$

where $f_0 > 0$ be any constant.

Without loss of generality take in to account that $f_0 = 1$, above $f(R, G)$ gravity model becomes,

$$f(R, G) = R^m G^{1-m}, \quad (6)$$

For the values of constant m , two types of gravity models are recovered:

- i) $f(R)$ gravity model corresponding to $m = 1$ while
- ii) $f(G)$ gravity model corresponding to $m = 0$.

Recently, Shekh et al. (2020) investigated relativistic hydrodynamics with some thermodynamical characteristics in $f(R, G)$ gravity towards spatially homogeneous isotropic cosmological model filled with isotropic fluid by considering the power-law in inflation towards scale factor.

Let us consider the matter content energy-momentum tensor T_μ^ν for strange quark matter is of the form,

$$T_\mu^\nu = (p + \rho) u^\nu u_\mu - p g_\mu^\nu = \text{diag}(\rho, -p, -p, -p), \quad (7)$$

For strange quark matter, the linear equation of state (EoS) is given by,

$$p = \omega (\rho - \rho_0), \quad (8)$$

here ω is a constant and ρ_0 is the energy density at zero pressure. If the model is radiating towards EoS $\omega = 1/3$ and $\rho_0 = 4B_c$ the above linear equation of state is reduced to the following EoS for strange quark matter in the bag model.

$$p = \frac{(\rho - 4B_c)}{3}, \quad (9)$$

where B_c is the bag constant. Recently, Sahoo et al. (2018) studied homogeneous and anisotropic locally rotationally symmetric Bianchi type-I model with magnetized strange quark matter together with cosmological constant (Λ) in $f(R, T)$ gravity by considering two specific forms of bilinear deceleration parameter and obtained in the early universe magnetic flux has more effects and it reduces gradually in the later stage while behavior of strange quark matter along with magnetic epoch gives an idea of accelerated expansion of the universe as per the observations of the type-Ia Supernovae. Chirde and Shekh (2018) investigated plane symmetric cosmological model with quark and strange quark matter in the deformations of the Einsteins theory of general relativity where a proper deformation of general relativity in the ultraviolet regime could play the role of describing the transition between general relativity and quantum gravity and revealed that the strange quark matter gives an idea of existence of dark energy in the universe and supports the observations of the type-Ia Supernovae.

3. METRIC, FIELD EQUATION AND THEIR SOLUTIONS

We consider the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) line element in the form,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right], \quad (10)$$

where a be the scale factor of the universe.

The angle θ and ϕ are the usual azimuthal and polar angles of spherical coordinates, with $0 \leq \theta \leq \pi$ and $0 \leq \phi \leq 2\pi$. In this work, we deliberate on the flat universe taken after $k = 0$ with infinite radius. In the co-moving co-ordinate system, the equation of motion (4) for the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) line element (10) with the fluid of stress-energy tensor (9) can be written as,

$$\ddot{f}_R + 2\dot{f}_R \frac{\dot{a}}{a} + 4\frac{\dot{a}}{a} \left(\frac{\dot{a}}{a} \ddot{f}_G + \frac{2\ddot{a}}{a} \dot{f}_G \right) + f_R \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) - \frac{1}{2}(Rf_R + Gf_G - f) = -p. \quad (11)$$

$$2f_R \frac{\dot{a}^2}{a^2} + 3\dot{f}_R \frac{\dot{a}}{a} + 12\dot{f}_G \frac{\dot{a}^3}{a^3} - \frac{1}{2}(Rf_R + Gf_G - f) = \rho, \quad (12)$$

The overhead dot represents the differentiation with respect to cosmic time t measure in Gyr.

Now, we consider some of the kinematical parameters for the FRW cosmological model that are important in cosmological observations.

The spatial volume,

$$V = a^3, \quad (13)$$

The generalized mean Hubble parameter,

$$H = \frac{\dot{a}}{a}. \quad (14)$$

The expansion scalar,

$$\theta = u^\mu_{;\mu} = 3H = 3\frac{\dot{a}}{a}, \quad (15)$$

Deceleration parameter,

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \quad (16)$$

4. MATTER DOMINATED POWER LAW EXPANSION

After the discovery of the late time acceleration of the universe, many authors who have used constant deceleration parameter to obtain cosmological models in the context of different fluids as well as dark energy in general theory of relativity and also in modified theories of gravitation. The sign of deceleration parameter (q) indicates whether the model accelerate or decelerate. The negative sign of q indicates acceleration whereas positive value stands for deceleration. Also, recent observations of SNe-Ia expose that the present universe is accelerating for $H > 0$ and $q < 0$. In this article, we use the well known relation between Hubble parameter H and scale factor a (sing et al. (2008a, 2008b)) as,

$$H = ba^{-\eta}, \quad (17)$$

where $b > 0$ and $\eta \geq 0$ are constants. The above relation yields a constant value of deceleration parameter. Using the definition of Hubble parameter (14), we can get,

$$\dot{a} = ba^{-\eta+1}, \quad (18)$$

$$\ddot{a} = -b^2(\eta - 1)a^{-2\eta+1}. \quad (19)$$

Integration of equation (19) yields,

$$a = (\eta bt + b_1)^{\frac{1}{\eta}} = (a_1 t + b_1)^\gamma, \quad (20)$$

provided $\gamma = \frac{1}{\eta} = \frac{1}{1+q}$ where $q \neq -1$ and $a_1 = \eta b \neq 0$, b_1 are constants of integration.

From the equation (20), it is observed that the scale factor of the model is the functions of cosmic time, which increase with time at $q > -1$, decreases with time at $q < -1$, and does not exist at $q = -1$.

Kinematical parameters of isotropic FRW line element

Expression of deceleration parameter given in equation (16) together with the value of scale factor provided in equation (20). The value of deceleration parameter is obtained as,

$$q = -1 + \eta. \quad (21)$$

Equation (21) designates that the deceleration parameter comes out to be independent of cosmic time. The sign of q indicates whether the universe accelerate or not. The recent observations like CMB and WMAP, as well as with the high redshifts of type-Ia supernova expected the sign of q become negative some where in range $-1 \leq q \leq 0$ ($q = -0.45$) which correspond to the standard accelerating behavior of the universe. Hence, our model ressembles with the value got from recent observations for $\eta = 0.55$ or $\gamma = 2.2$.

The spatial volume and scale factor both are obtained as,

$$V = a^3 = (a_1 t + b_1)^{3\gamma}, \quad (22)$$

We observed that the spatial volume along with scale factor of the model starts with constant value (big-bang) at $t \rightarrow 0$ and with the increase of cosmic time it always expands. Thus, inflation is possible in this model. This shows that the universe starts evolving with zero volume and expands with cosmic time t . The behavior of spatial volume of the model versus cosmic time with an appropriate choice of constant is clearly shown in Fig.1

The Hubble parameter is obtained as,

$$H = \frac{\gamma a_1}{a_1 t + b_1}. \quad (23)$$

The scalar expansion is obtained as,

$$\theta = 3H = \frac{3\gamma a_1}{a_1 t + b_1}. \quad (24)$$

The behavior of Hubble parameter and scalar expansion of the model versus cosmic time with an appropriate choice of constant is clearly shown in Fig.2. The Hubble parameter and scalar expansion are the functions of cosmic time and decreases as cosmic time increases and approaches to zero at later cosmic time. At an initial epoch $t \rightarrow 0$, the

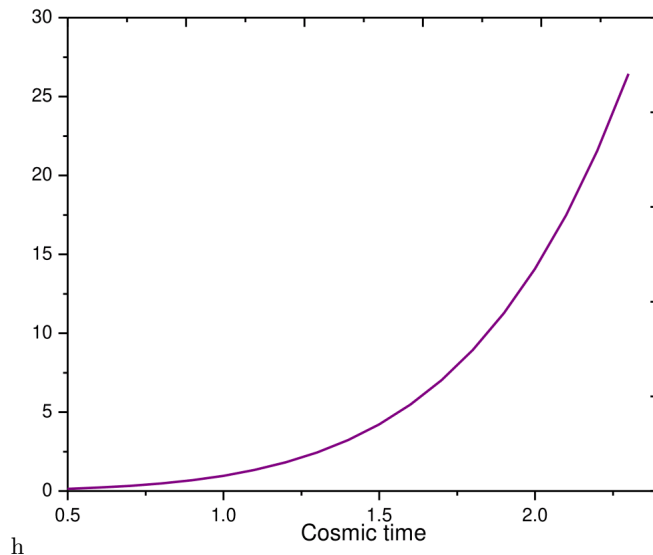


FIG. 1: Behavior of spatial volume of the model versus cosmic time with an appropriate choice of constants $a_1=0.1$, $b_1=0.3$, $\gamma=2.2$.

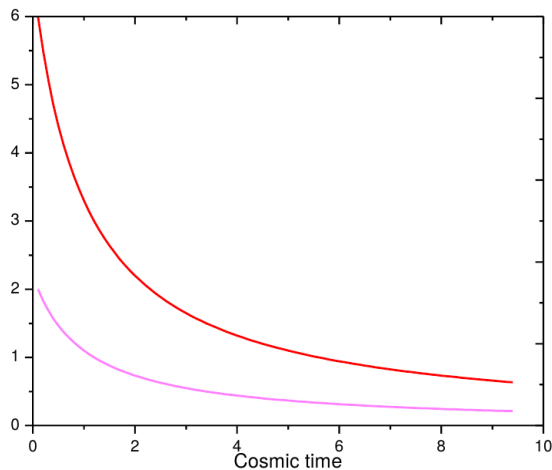


FIG. 2: Behavior of Hubble parameter and expansion scalar of the model versus cosmic time with an appropriate choice of constants $a_1=0.1$, $b_1=0.3$, $\gamma=2.2$.

Hubble parameter and scalar expansion both are constant. These are diverse when the model is in acceleration phase and approaches to zero monotonically at $t \rightarrow \infty$. This suggested that at initial stage of the universe, the expansion of the model is much more faster and then slow down for later cosmic time this shows that the evolution of the universe starts with finite constant rate and with the expansion it declines.

Physical parameters of isotropic FRW line element

In this section we discussed the physical parameters such as pressure for strange quark matter, energy density strange for quark matter, pressure for quark matter and energy density for quark matter.

Strange quark matter in isotropic FRW line element:

From the equations (11) and (12) by applying scale factor provided in equation (20) with the help of linear EoS (9) for $\omega = 1/3$, pressure and the energy density for strange quark matter can be extracted.

Pressure for strange quark matter is defined as,

$$p = \left(\frac{(2\gamma - 1)}{4\gamma^2(\gamma - 1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{ma_1^2 B [2(m - 1)(3 - 2m) + 4(1 - m)\gamma]}{A(a_1 t + b_1)^{4-2m}} - \frac{8\gamma^2 m(m - 1)a_1^4 [1 + 3m - 2m^2 - 2\gamma]}{(a_1 t + b_1)^{4-2m}} \right. \\ \left. - \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{m\gamma a_1^2 [3\gamma - 2]}{(a_1 t + b_1)^{4-2m}} \right\}. \quad (25)$$

By considering $V = (a_1 t + b_1)^{3\gamma}$ and rewriting the above equation we can obtain the equation of state $P - V$ for strange quark matter as,

$$p = \left(\frac{(2\gamma - 1)}{4\gamma^2(\gamma - 1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{ma_1^2 B [2(m - 1)(3 - 2m) + 4(1 - m)\gamma]}{AV^{\frac{4-2m}{3\gamma}}} - \frac{8\gamma^2 m(m - 1)a_1^4 [1 + 3m - 2m^2 - 2\gamma]}{V^{\frac{4-2m}{3\gamma}}} \right. \\ \left. - \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{m\gamma a_1^2 [3\gamma - 2]}{V^{\frac{4-2m}{3\gamma}}} \right\}. \quad (26)$$

Energy density for strange quark matter is given by,

$$\rho = \left(\frac{(2\gamma - 1)}{4\gamma^2(\gamma - 1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{2m\gamma^2 a_1^2}{(a_1 t + b_1)^{4-2m}} + \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{6m(m - 1)\gamma 2a_1^2}{(a_1 t + b_1)^{4-2m}} - \frac{24m(m - 1)\gamma^3 a_1^4}{(a_1 t + b_1)^{4-2m}} \right\}. \quad (27)$$

Equations (26) and (27) represents strange quark pressure and the energy density of the model. We conclude that pressure and the energy density for strange quark matter both are function of cosmic time with inverse relation. Initially when the model start to expand at $t = 0$ both are approaches to some specific constant value but with the expansion when $t > 0$ these are decreases and at infinitely large expansion when $t \rightarrow \infty$, then $p, \rho \rightarrow 0$ as shown in the plot of Fig. 3 and Fig.4. Hence, pressure and the energy density for strange quark matter initially observed in constant amount while at infinite expansion of the model it declines and diverge to zero.

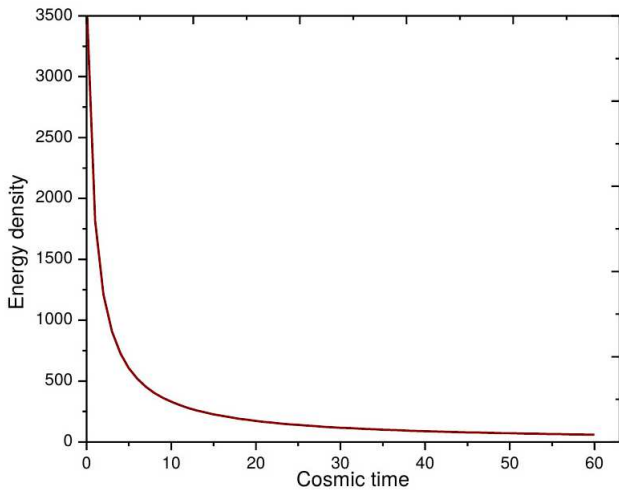


FIG. 3: Behavior of energy density for strange quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

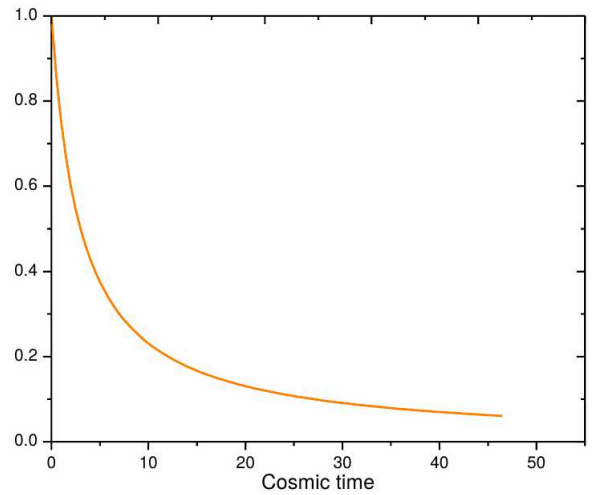


FIG. 4: Behavior of pressure of for strange quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

Quark matter in isotropic FRW line element:

Pressure for quark matter is obtained as,

$$p_q = \left(\frac{(2\gamma - 1)}{4\gamma^2(\gamma - 1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma - 1)a_1^2}{(2\gamma - 1)} \right) \frac{ma_1^2 B [2(m - 1)(3 - 2m) + 4(1 - m)\gamma]}{A(a_1 t + b_1)^{4-2m}} - \frac{8\gamma^2 m(m - 1)a_1^4 [1 + 3m - 2m^2 - 2\gamma]}{(a_1 t + b_1)^{4-2m}} \right\}$$

$$- \left(\frac{4\gamma^2(\gamma-1)a_1^2}{(2\gamma-1)} \right) \frac{m\gamma a_1^2 [3\gamma-2]}{(a_1 t + b_1)^{4-2m}} \} - B_c. \quad (28)$$

By considering $V = (a_1 t + b_1)^{3\gamma}$ and rewriting the above equation we can obtain the equation of state $P - V$ for quark matter as,

$$p_q = \left(\frac{(2\gamma-1)}{4\gamma^2(\gamma-1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma-1)a_1^2}{(2\gamma-1)} \right) \frac{ma_1^2 B [2(m-1)(3-2m) + 4(1-m)\gamma]}{AV^{\frac{4-2m}{3\gamma}}} - \frac{8\gamma^2 m(m-1)a_1^4 [1+3m-2m^2-2\gamma]}{V^{\frac{4-2m}{3\gamma}}} \right. \\ \left. - \left(\frac{4\gamma^2(\gamma-1)a_1^2}{(2\gamma-1)} \right) \frac{m\gamma a_1^2 [3\gamma-2]}{V^{\frac{4-2m}{3\gamma}}} \right\} - B_c. \quad (29)$$

Energy density for quark matter can be expressed as,

$$\rho_q = \left(\frac{(2\gamma-1)}{4\gamma^2(\gamma-1)a_1^2} \right)^m \left\{ \left(\frac{4\gamma^2(\gamma-1)a_1^2}{(2\gamma-1)} \right) \frac{2m\gamma^2 a_1^2}{(a_1 t + b_1)^{4-2m}} + \left(\frac{4\gamma^2(\gamma-1)a_1^2}{(2\gamma-1)} \right) \frac{6m(m-1)\gamma 2a_1^2}{(a_1 t + b_1)^{4-2m}} - \frac{24m(m-1)\gamma^3 a_1^4}{(a_1 t + b_1)^{4-2m}} \right\} + B_c. \quad (30)$$

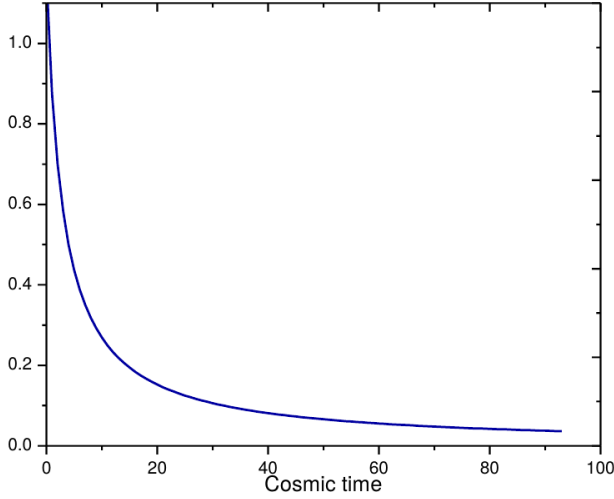


FIG. 5: Behavior of energy density for quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

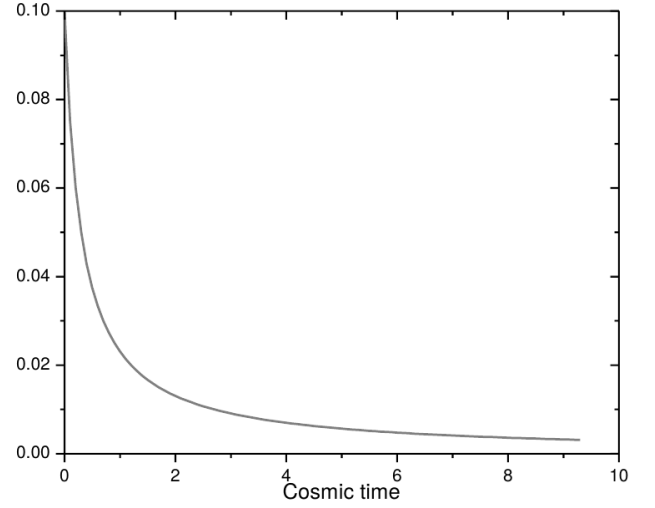


FIG. 6: Behavior of pressure for quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

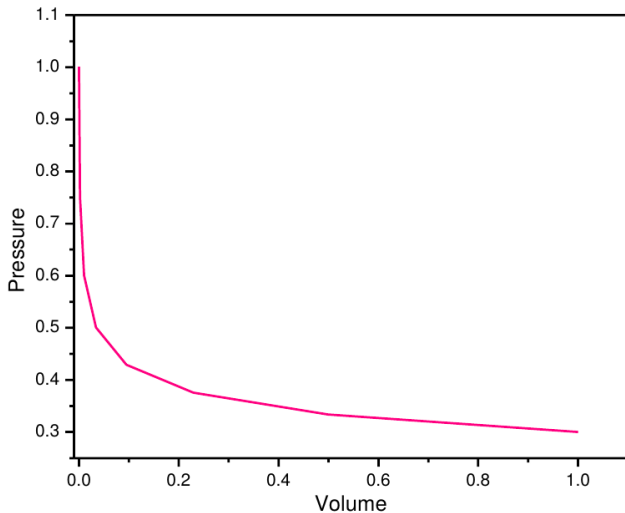


FIG. 7: Behavior of pressure versus volume for strange quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

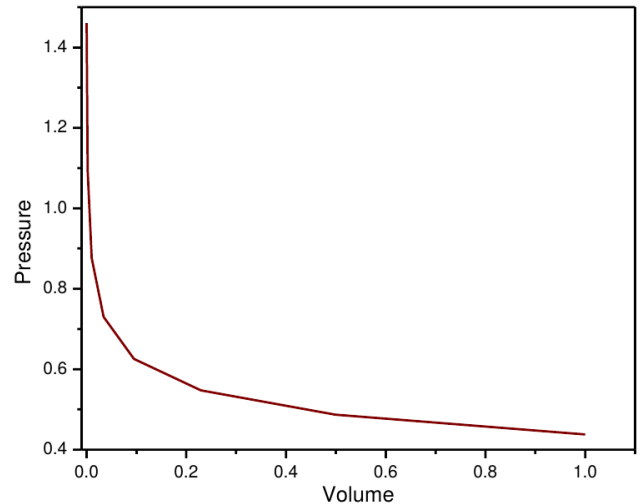


FIG. 8: Behavior of pressure versus volume for quark matter of the model versus cosmic time t with $a_1 = 0.9$, $b_1 = 4$, $\gamma = 2.2$.

Equation (28) and (30) gives the expression of pressure and energy density for quark matter. The behavior of pressure and energy density for quark matter is clearly depicted in Fig.5 and Fig.6 respectively with the appropriate choice of constants and observed that both are same as that of strange quark matter, except that it is shifted by the bag constant as shown in the right side of the equations (28) and (30). The right side view of energy density and pressure for quark and strange quark matter are identical except for the additional bag constant. For energy density of strange quark we add the bag constant whereas for pressure we take away (subtract) it. From equations (28) and (30), one can notice that, $\rho_q \rightarrow B_c$ and $p_q \rightarrow -B_c$ when $t \rightarrow \infty$. Hence our results resembles with the work investigated by those of Sahoo et al. (2018), Chirde and Shekh (2018).

Using equations (26) and (29), the $P - V$ diagrams for the strange quark and quark matter are drawn in Fig.7 and Fig.8. These figures show that in both cases, as isotherms diagrams of an ideal gas, the pressure decreases with increasing volume.

5. CONCLUDING REMARK

The spatial volume and scale factor of the model both are starts with constant value, with the increase of cosmic time also increases and always expands which indicates that the inflation is possible in this our derived model. This shows that the universe starts evolving with constant volume and expands with cosmic time. The Hubble parameter and scalar expansion in the derived model both are the functions of cosmic time and decreases as cosmic time increases and approaches to zero at later infinite cosmic time. At an initial epoch when the model start to expand, the Hubble parameter and scalar expansion both are constant and these are diverse when the model is in acceleration phase and approaches to zero monotonically at infinite expansion. This suggested that at initial stage of the universe, the expansion of the model is much more faster and then slow down for later cosmic time this shows that the evolution of the universe starts with finite constant rate and with the expansion it declines.

The pressure and energy density for quark and strange quark matter both are same as except that of strange quark matter shifted by the bag constant and observed that quark pressure and quark energy density both are approaches to Bag constant with positive and negative sign respectively i.e. $\rho_q \rightarrow B_c$ and $p_q \rightarrow -B_c$ when $t \rightarrow \infty$. The value of

deceleration parameter obtained in the model is always negative and for particular choice of constant $\gamma = 2.2$ it is -0.45 which represents a model is purely accelerating.

We also plotted the $P - V$ diagrams for the strange quark and quark matter and showed that in both cases, as isotherms diagrams of an ideal gas, the pressure decreases with increasing volume.

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Figures

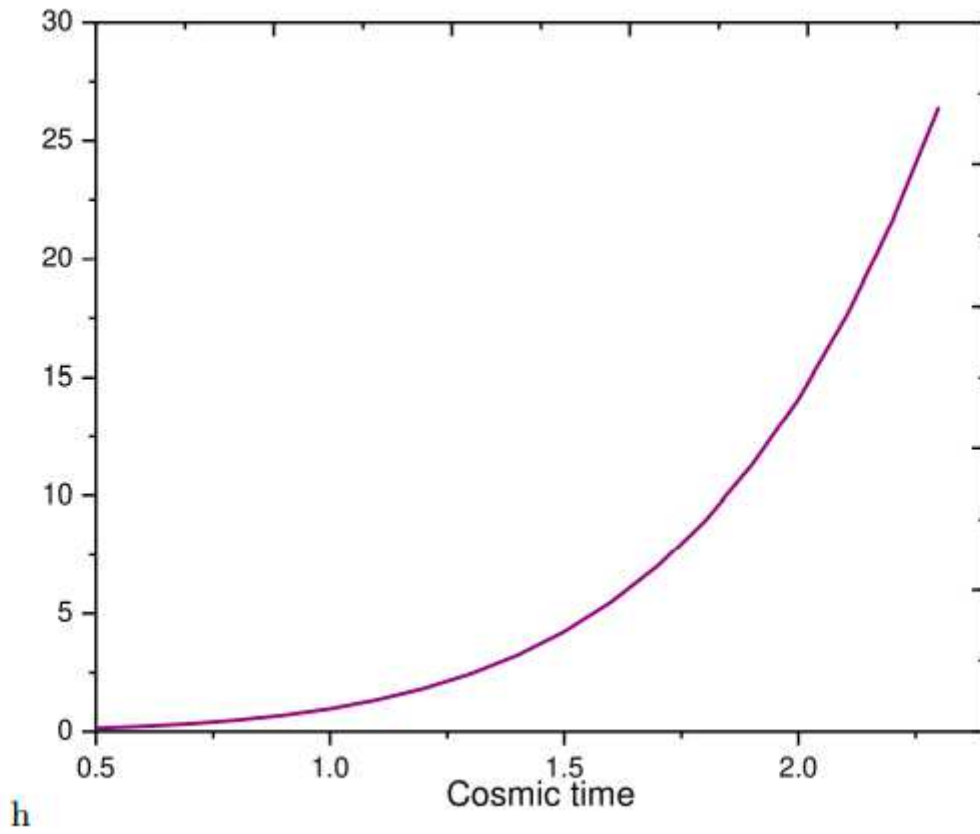


Figure 1

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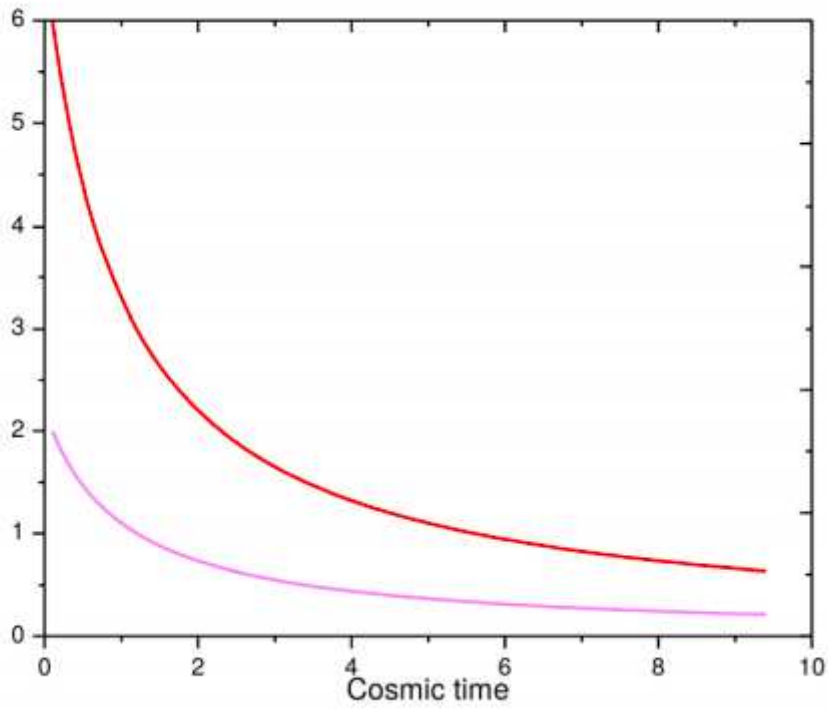


Figure 2

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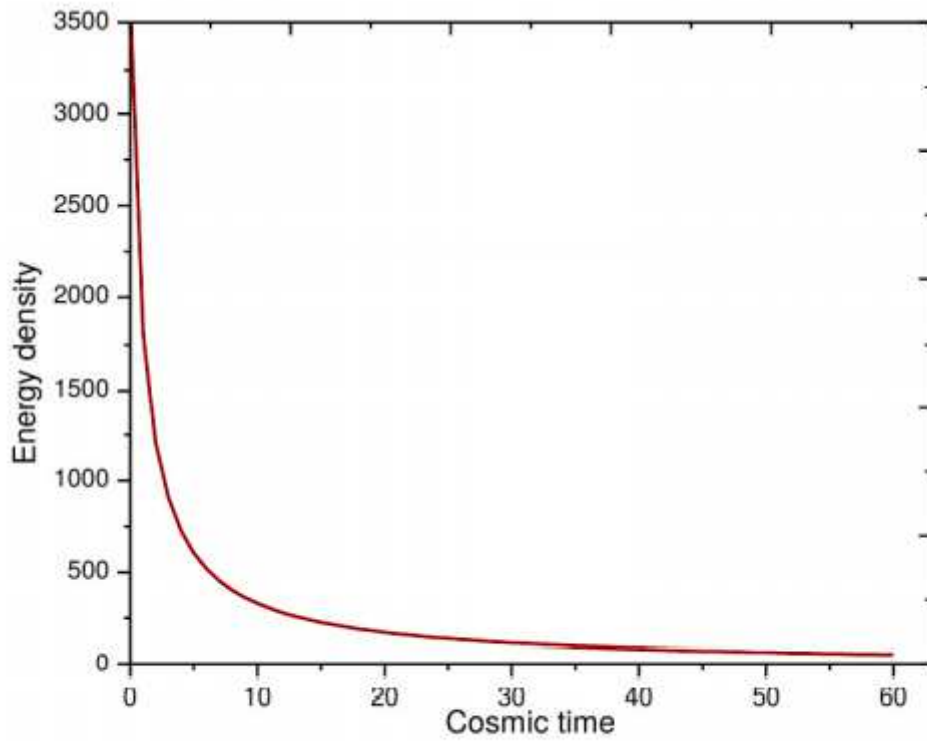


Figure 3

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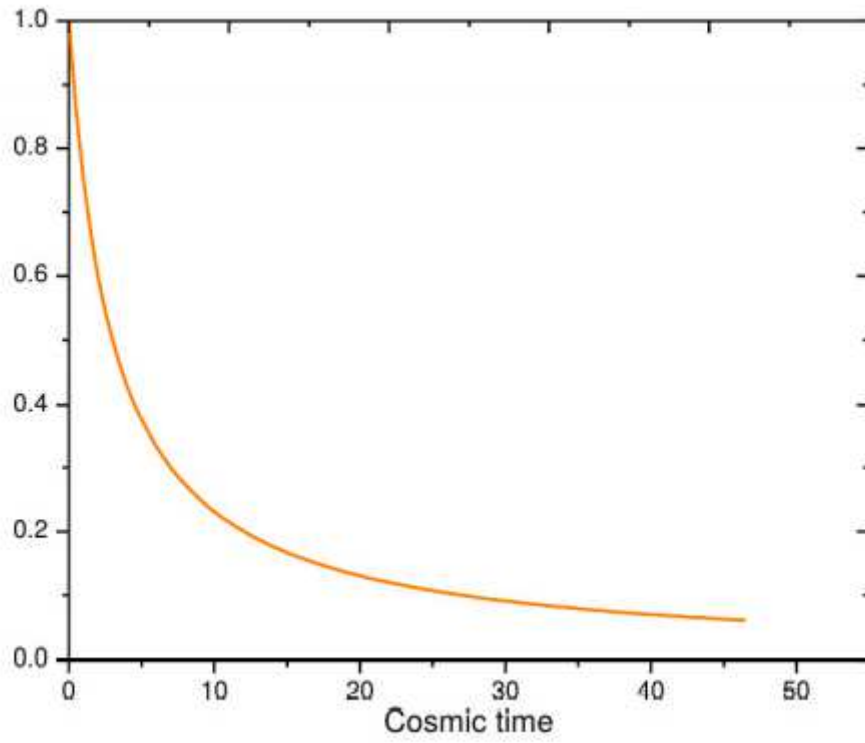


Figure 4

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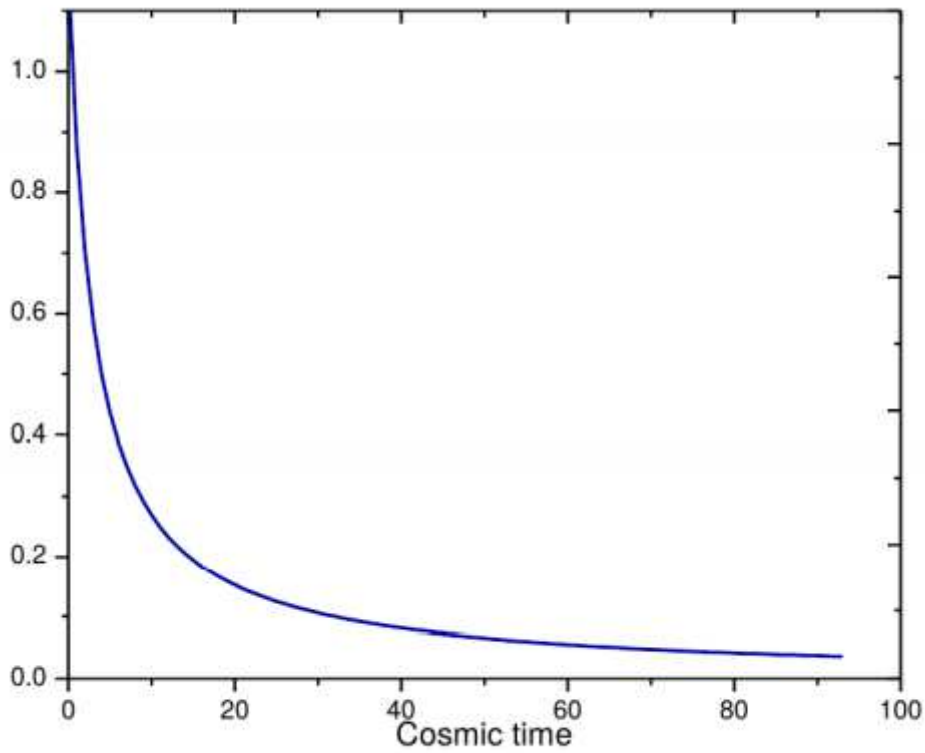


Figure 5

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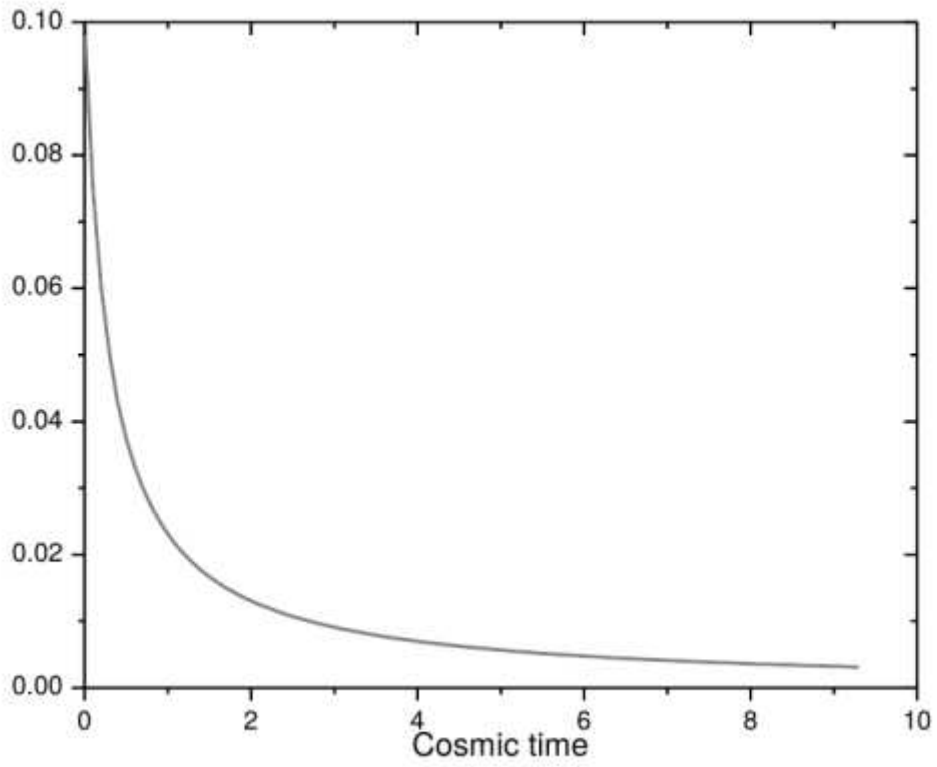


Figure 6

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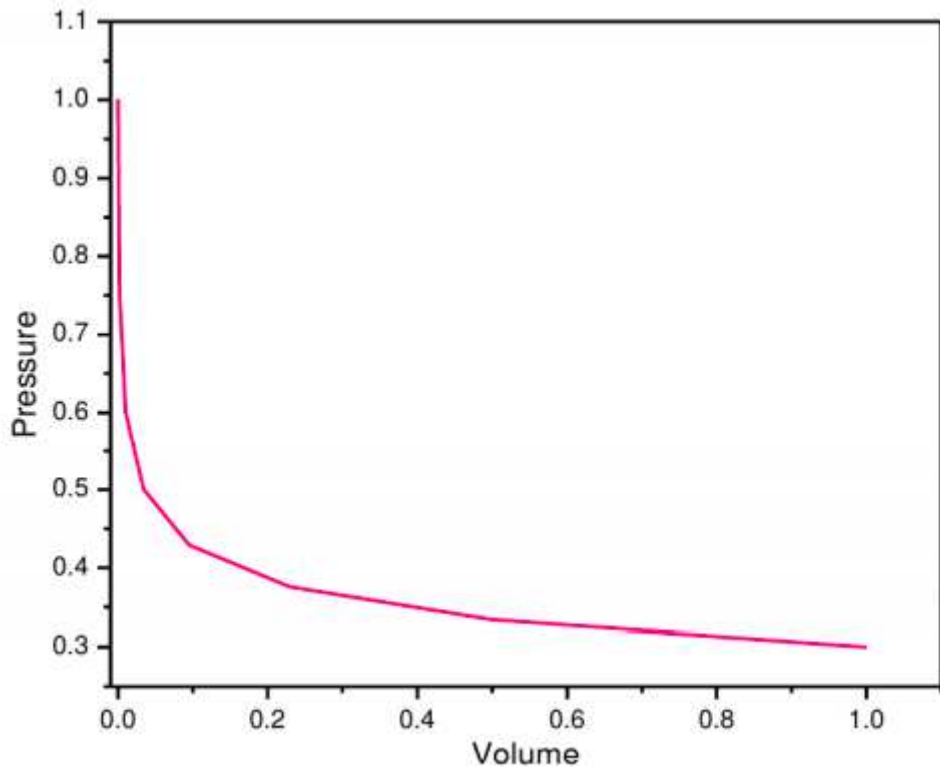


Figure 7

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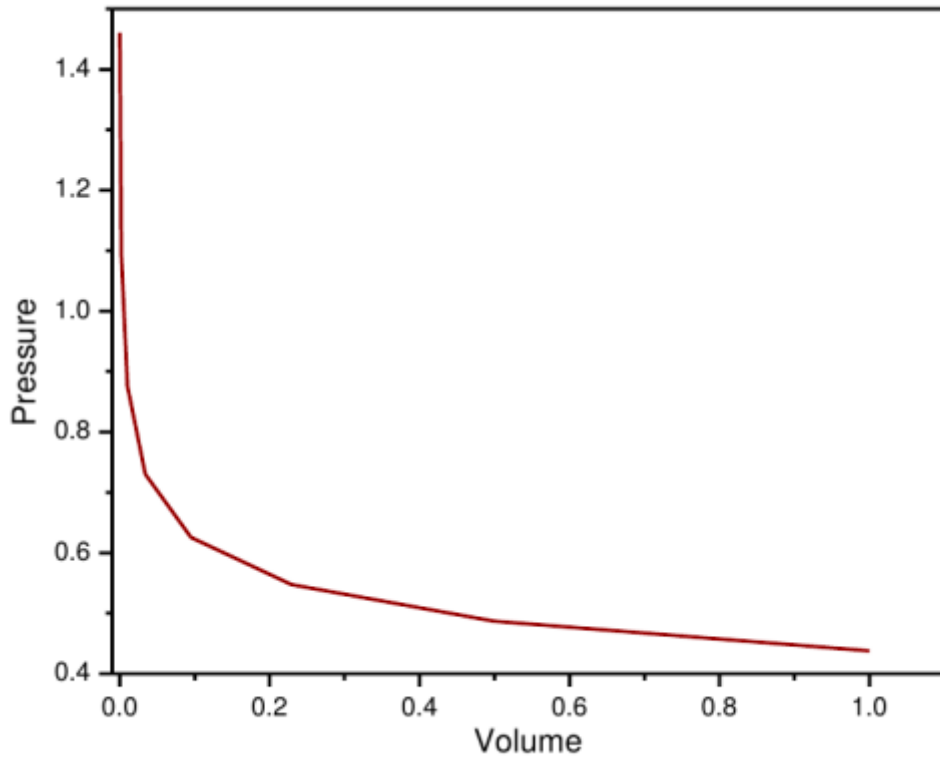


Figure 8

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