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Research Article

Keywords: Microgrid, Random package loss, Robust H^∞ control, Networked control system

Posted Date: June 2nd, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-463895/v1>

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Robust H_∞ Control of Networked System in Microgrid under Random Packet Loss

Yiwei Feng^{1,*} · Jing Ma¹ · Xin Wang¹

Received: date / Accepted: date

Abstract The desire for sustainable development has led to the advancement of renewable energy generation through microgrids, and the control and stability of microgrids pose serious challenges to the popularization of microgrids. This article focuses on the stability and performance of the microgrid, and due to its limited channel bandwidth and other factors, may result in the loss of random packets from the sensor to the controller and from the controller to the actuator. To solve the problem, this paper designed a feedback controller based on the observer and robust H_∞ control method and has established Bernoulli packet loss model. The proposed controller enables the networked control system robust exponential stability in the sense of mean square. Finally, the correctness and effectiveness of the proposed controller is verified by MATLAB simulation.

Keywords Microgrid · Random package loss · Robust H_∞ control · Networked control system

Declaration

Funding: This study was funded in part by the Key Science and Technology Program of Gansu Province, China [Grant No.:20YF8NA059], in part by the National Natural Science Foundation of China [Grant No.:61863026].

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1 Introduction

Microgrid is a complex system composed of various systems, and it is a trend that cannot be ignored in the distribution network. Nowadays, some hospitals, factories, remote work sites and individual users are equipped with microgrid systems, so the reliable and stable operation of microgrids becomes more and more important. In other words, as the penetration rate of microgrids continues to increase, control issues have become the primary issue of microgrids. Centralized and distributed controllers were kicked out [1] [2]. A fully centralized controller relies on data collected in a dedicated central controller, which performs the required calculations and determines the control actions of all units at a single point, requiring extensive communication between the central controller and the controlled units. On the other hand, in a completely decentralized control, each unit is controlled by its local controller, which only receives local information and does not fully understand other system variables or the actions of other controllers. The [3] proposed a robust decentralized control strategy for the islanding operation of microgrid. The island microgrid is composed of two distributed generation (DG) units. The main DG controls the voltage and frequency of the load, and the slave DG uses the traditional dq current control strategy to adjust its power components. A new type of distributed controller for secondary frequency and voltage control of island microgrid. Inspired by cooperative control technology, the controller uses local information and nearest neighbor communication to jointly perform secondary control actions and was proposed in [4]. The [5] proposes a control architecture that is a compromise between a completely centralized and a completely decentralized control scheme, which can be realized by a hierarchical control scheme composed of three control levels: primary, secondary and tertiary. The main control is related to the voltage, current, virtual impedance and the local layer of the drop loop in DG. The core idea of realizing secondary control is to compensate for the voltage and frequency changes of the microgrid caused by the primary control loop. Contrary to the main control, the secondary control uses a feedback communication system to pass the microgrid parameter value to each DG unit. However, the above documents do not consider the impact of data loss on the system during signal transmission. The transmission of signal data is crucial to the control and stable operation of the microgrid and should be paid attention to.

The networked control system is a communication system that connects devices and controllers over a long distance. Because of its low cost and high reliability, it is introduced as a successful control strategy into the microgrid [6] [7]. However, many studies have focused on the impact of delay on the system or only assume that the packet loss problem only occurs in the channel from the sensor to the controller [8]. And the modeling of the packet loss phenomenon of the networked control system is basically divided into three methods. For example, one approach is to treat packet loss as a binary switching sequence specified by the conditional probability distribution in the signal transmission channel. The binary exchange sequence obeys the white

sequence of Bernoulli distribution, and its values are 0 and 1. The second method is to use a discrete linear system with Markov jump parameters to represent the random packet loss model of the network [9]. The third method is to replace the packet loss with zeros, and then construct an incomplete matrix in the measurement. The [10] considered the dual-channel packet loss in the controller of the networked control system, but did not conduct in-depth research on the uncertainty in the model. However, there are still few documents considering the packet loss problem in the networked control system in the microgrid. This paper applies the designed networked control system controller to the microgrid system. It is assumed that there is interference and network packet loss from the sensor to the controller, and from the controller to the actuator. Using the idea of robust control, the packet loss process is modeled as a Bernoulli packet loss model, and an observer-based feedback controller is designed to make the networked system robust and exponentially stable in the sense of mean square. The Lyapunov function is used to analyze stability, and finally the effectiveness of the proposed LMI method is verified by a simulation example.

This article is organized as follows. The second section gives the basic idea and mathematical model for the establishment of microgrid. The third section expounds the expression of the microgrid networked control problem in the form of state space. The main result of the stability analysis is taken as the LMI condition and an optimization problem is proposed. The simulation parameters and results will be given in Section five. Finally, the work is summarized in Section six.

2 Problem describes

First, we consider a distributed energy resource (DER) unit, which is a DG unit that is electronically coupled to the grid through coupling [11]. The DG device is coupled to the grid by means of electronic coupling. The DG unit consists of a DC voltage source, a voltage source converter, a series filter and a transformer, and the RLC network is used as a local load. Using the state space method to model the system, the state space expression can be obtained as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (1)$$

where

$$A = \begin{bmatrix} -\frac{R_t}{L_t} & \omega_0 & 0 & -\frac{1}{L_t} \\ \omega_0 & -\frac{R_t}{L} & -2\omega_0 & \frac{R_t C \omega_0}{L} - \frac{\omega_0}{R} \\ 0 & \omega_0 & -\frac{R_t}{L} & \frac{1}{L} - \omega_0^2 C \\ \frac{1}{C} & 0 & -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L_t} \\ 0 \\ 0 \\ 0 \end{bmatrix}, C = [0 \ 0 \ 0 \ 1]$$

where, $x = [i_{td} \ i_{tq} \ i_{Ld} \ v_d]^T$, $u = v_{td}$, $y = v_d$ are respectively the state, the control input, the output. v_d is the only output signal which has to be controlled. In order to complete the control of v_d , the input signals v_{td} and v_{tq} are

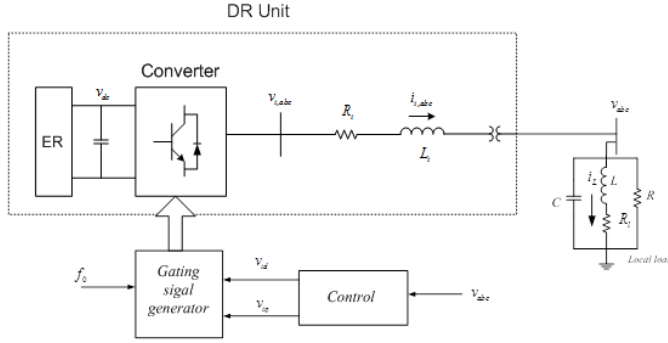


Fig. 1 Schematic diagram of an independent DR connected to the grid

needed. However, since all state variables and load parameters are unknown and uncertain, it is difficult to calculate v_{tq} . Therefore, we assume that v_{tq} is a disturbance variable that can be set to zero. In other words, in order to control v_d , only one of v_{td} and v_{tq} is required to meet the requirements. Discretize model (1) and transform it into the following discrete-time linear steady model

$$\begin{cases} x(k+1) = A_0x(k) + B_0u(k) \\ y(k) = C_0x(k) \end{cases} \quad (2)$$

where $x(k) \in R^n$ and $u(k) \in R^m$ represent the plant states and control input respectively. It is assumed that random data packet loss occurs at the same time during the transmission of data packet data from the sensor to the controller or the controller to the actuator. The sensors, controllers and actuators here are all driven by clocks and data packets will not be out of order. The structure is shown in Figure 2. Suppose the system is also disturbed, denoted by w_k . Since we assume that the data is transmitted from the sensor to the controller or the controller to the actuator will cause packet loss, we add an intermediate link z_k to realize the representation of the two parts. In addition, we also consider the parameter uncertainty in the system, for example, ΔA_0 and ΔC_0 . The disturbed system (2) is as follows

$$\begin{aligned} x_{k+1} &= (A_0 + \Delta A_0)x_k + B_0u_k + Dw_k \\ z_{k+1} &= (C_1 + \Delta C_0)x_k + D_1w_k \\ y_k &= C_2x_k + D_2w_k \end{aligned} \quad (3)$$

where $x_k \in R^n$, $u_k \in R^m$, $z_k \in R^r$ and $y_k \in R^p$ are the state, the control input, the control output and measured output, respectively. $w_k \in R^q$ is the disturbance input belong to $l_2[0, \infty)$. A , B , C_1 , C_2 , D , D_1 , D_2 are known real

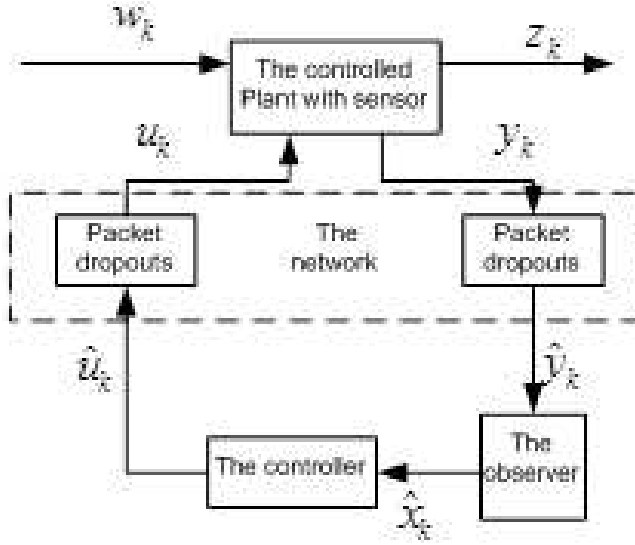


Fig. 2 Networked control system layout diagram

matrices with appropriate dimensions. Parameters uncertain ΔA_0 and ΔC_0 have the following forms

$$\begin{bmatrix} \Delta A_0 \\ \Delta C_0 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_k E \quad (4)$$

where, H_1, H_2 and E are known real constant matrices with appropriate dimensions, and F_k is an unknown real-time variable matrix satisfying $F_k F_k^T \leq I$. Next, this paper establishes a communication random packet loss model

$$\begin{aligned} x_{k+1} &= (A_0 + \Delta A_0) x_k + B u_k + D w_k \\ z_{k+1} &= (C_1 + \Delta C_0) x_k + D_1 w_k \\ y_k &= \alpha_k C_2 x_k + D_2 w_k \end{aligned} \quad (5)$$

$\alpha_k \in \mathbb{R}$ is a random variable representing the random Bernoulli distribution of packet loss from the sensor to the controller. Suppose it is a white noise sequence of linear random variables with the following properties

$$\begin{aligned} \text{Prob}\{\alpha_k = 1\} &= \text{E}\{\alpha_k\} = \bar{\alpha} \\ \text{Prob}\{\alpha_k = 0\} &= 1 - \text{E}\{\alpha_k\} = 1 - \bar{\alpha} \\ \text{Var}\{\alpha_k\} &= \text{E}\{(\alpha_k - \bar{\alpha})^2\} = (1 - \bar{\alpha})\bar{\alpha} = \alpha_1^2 \end{aligned} \quad (6)$$

The dynamic observer-based control method of system (5) is described as

$$\begin{aligned} \hat{x}_{k+1} &= A_0 \hat{x}_k + B_0 u_k + L (y_k - \bar{\alpha} C_2 \hat{x}_k) \\ \hat{u}_k &= -K \hat{x}_k \\ u_k &= \beta_k \hat{u}_k \end{aligned} \quad (7)$$

where $\hat{x}_k \in R^n$ is the state estimate of the system (7), $\hat{u}_k \in R^m$ is the control input with transmission missing, $L \in R^{n \times p}$ is the observer gain and $K \in R^{m \times n}$ is the controller gain. $\beta_k \in \mathbb{R}$ is a random variable representing the random Bernoulli distribution of packet loss from the controller to the actuator. Hence α_k and β_k are independent of each other. Suppose it is a white sequence of linear random variables with the following properties

$$\begin{aligned} \text{Prob}\{\beta_k = 1\} &= \text{E}\{\beta_k\} = \bar{\beta} \\ \text{Prob}\{\beta_k = 0\} &= 1 - \text{E}\{\beta_k\} = 1 - \bar{\beta} \\ \text{Var}\{\beta_k\} &= \text{E}\left\{(\beta_k - \bar{\beta})^2\right\} = (1 - \bar{\beta})\bar{\beta} = \beta_1^2 \end{aligned} \quad (8)$$

Definition e_k is the deviation between the system measured value and the estimated value, the expression is

$$e_k = x_k - \hat{x}_k \quad (9)$$

The closed-loop networked states are given as

$$\begin{cases} x_{k+1} = (A_0 + \Delta A_0 - \bar{\beta}B_0K)x_k + \bar{\beta}B_0Ke_k + (\beta_k - \bar{\beta})B_0Kx_k \\ \quad + (\beta_k - \bar{\beta})B_0Ke_k + Dw_k \\ \hat{x}_{k+1} = A_0\hat{x}_k + B_0u_k + L(y_k - \bar{\alpha}C_2\hat{x}_k) \\ e_{k+1} = \Delta A_0x_k + (A_0 - \bar{\alpha}LC_2)e_k - (\alpha_k - \bar{\alpha})LC_2x_k + (D - LD_2)w_k \end{cases} \quad (10)$$

We will rewrite (10) in compact form

$$\delta_{k+1} = \Psi\delta_k + \eta_k\bar{\Psi}\delta_k + \Gamma w_k \quad (11)$$

where

$$\begin{aligned} \delta_{k+1} &= \begin{bmatrix} x_k \\ e_k \end{bmatrix}, \Psi = \begin{bmatrix} A_0 + \Delta A_0 - \bar{\beta}B_0K & -\bar{\beta}B_0K \\ \Delta A_0 & A_0 - \bar{\alpha}LC_2 \end{bmatrix}, \bar{\Psi} = \begin{bmatrix} B_0K & B_0K \\ LC_2 & 0 \end{bmatrix} \\ \Gamma &= \begin{bmatrix} D \\ D - LD_2 \end{bmatrix}, \eta_k = \begin{bmatrix} (\beta_k - \bar{\beta})I & 0 \\ 0 & (\alpha_k - \bar{\alpha})I \end{bmatrix} \end{aligned}$$

The goal of this paper is to design a controller (7) for the system (5) when the closed-loop system (11) has random packet loss, so that the closed-loop system (11) is exponentially mean square stable and H_∞ performance constraint is satisfied. Under the condition of ensuring that the system is stable in the mean square, it is also necessary to ensure that under the zero initial condition, the control output z of the system satisfies

$$\sum_{k=0}^{\infty} \text{E}\left\{\|z_k\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} \text{E}\left\{\|w_k\|^2\right\} \quad (12)$$

For all non-zero w_k , where $\gamma > 0$ is a prescribed scalar.

3 Main results

In order to prove the convenience and generality of the main results in this section, we will make the following definitions and quote lemmas.

Definition 1: The closed-loop system (11) is said to be exponentially mean-square stable if, with $w_k = 0$, the exist constants $\phi > 0$ and $\tau \in (0, 1)$, so that

$$\sum_{k=0}^{\infty} \mathbb{E} \left\{ \|\delta_k\|^2 \right\} < \phi \sum_{k=0}^{\infty} \mathbb{E} \left\{ \|\delta_k\|^2 \right\} \quad (13)$$

$$\mathbb{E} \left\{ \|\delta_k\|^2 \right\} \leq \phi \tau^k \mathbb{E} \left\{ \|\delta_0\|^2 \right\}, \text{ for all } \delta_0 \in R^n, k \in I^+.$$

Lemma 1 : Let $V(\delta_k)$ be a Lyapunov funtional. If there exist real scalars $\lambda \geq 0, \mu > 0, v > 0$ and $0 < \psi < 1$, so that

$$\mu \|\delta_k\|^2 \leq V(\delta_k) \leq v \|\delta_k\|^2 \quad (14)$$

and

$$\mathbb{E} \{ V(\delta_{k+1}) \mid \delta_k \} - V(\delta_k) \leq \lambda - \psi V(\delta_k) \quad (15)$$

Then the sequence δ_k satisfies

$$\mathbb{E} \left\{ \|\delta_k\|^2 \right\} \leq \frac{v}{\mu} \|\delta_0\|^2 (1 - \psi)^k + \frac{\lambda}{\mu\psi} \quad (16)$$

Next, we give a sufficient condition in the form of the theorem to prove that the closed-loop network system (11) is exponentially mean-square stable and realize the H_∞ performance constraint (12).

Theorem 1: Given communication channel parameters $0 < \bar{\alpha} < 1, 0 < \bar{\beta} < 1$. If there are positive definite matrices $P > 0$ and $Q > 0$, the closed-loop network system (11) has exponential mean square stability, and the H_∞ performance constraint (12) can be achieved for all non-zero w_k , real matrices K and L satisfy the following matrix inequalities,

$$\begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2^T & \Pi_3 \end{bmatrix} < 0 \quad (17)$$

$$\begin{aligned} \Pi_1 = & (A_0 + \Delta A_0 - \bar{\beta} B_0 K)^T P (A_0 + \Delta A_0 - \bar{\beta} B_0 K) + (\Delta A_0)^T Q (\Delta A_0) \\ & + \beta_1^2 (BK)^T P (BK) + \alpha_1^2 (LC_2)^T P (LC_2) - P \end{aligned}$$

$$\begin{aligned} \Pi_2 = & (A_0 + \Delta A_0 - \bar{\beta} B_0 K)^T P (\bar{\beta} B_0 K) + (\Delta A_0)^T Q (A_0 - LC_2) \\ & + \beta_1^2 (B_0 K)^T P (B_0 K) \end{aligned}$$

$$\Pi_3 = (\bar{\beta} B_0 K)^T P (\bar{\beta} B_0 K) + (A_0 - LC_2)^T Q (A_0 - LC_2) - Q$$

where $\alpha_1 = \sqrt{[(1 - \bar{\alpha})\bar{\alpha}]}, \beta_1 = \sqrt{[(1 - \bar{\beta})\bar{\beta}]}$. The form of K and L can refer to Theorem 3 in the [10]. if LMI is feasible, the controller design problem under consideration is solvable.

Proof: The proof in this part will be divided into two parts. The first part is the proof of stability in the case of disturbance is zero in the network control

system. The second part is that the network control system can still satisfy the constraints of H_∞ performance in the presence of disturbances. Define a Lyapunov functional

$$V = x_k^T P x_k + e_k^T Q e_k \quad (18)$$

where P and Q are the positive definite solutions to (17). Then

$$\begin{aligned} \Delta V_k &= E \{V_{k+1} \mid x_k, \dots, x_0, e_k, \dots, e_0\} - V_k \\ &= E \{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k \end{aligned} \quad (19)$$

Using the properties of the expecting value of the Bernoulli distributions α_k and β_k , the following results are obtained. $E \{(\alpha_k - \bar{\alpha})^2\} = (1 - \bar{\alpha})\bar{\alpha} = \alpha_1^2$, $E \{(\beta_k - \bar{\beta})^2\} = (1 - \bar{\beta})\bar{\beta} = \beta_1^2$. Assuming $w_k = 0$, we can obtain

$$\begin{aligned} \Delta V_k &= E \{V_{k+1} \mid x_k, \dots, x_0, e_k, \dots, e_0\} - V_k \\ &= [(A_0 + \Delta A_0 - \bar{\beta} B_0 K)x_k + \bar{\beta} B_0 K e_k]^T P [(A_0 + \Delta A_0 - \bar{\beta} B_0 K)x_k + \bar{\beta} B_0 K e_k] \\ &\quad + [(\Delta A_0)x_k + (A_0 - \bar{\alpha} L C_2)e_k]^T Q [(\Delta A_0)x_k + (A_0 - \bar{\alpha} L C_2)e_k] \\ &\quad + (1 - \bar{\beta})\bar{\beta} [B_0 K x_k + B_2 K e_k]^T P [B_0 K x_k + B_2 K e_k] \\ &\quad + (1 - \bar{\alpha})\bar{\alpha} [L C_2 x_k]^T Q [L C_2 x_k] - x_k^T P x_k - e_k^T Q e_k \\ &= \delta_k^T \Omega \delta_k \end{aligned} \quad (20)$$

$$\Omega = \begin{bmatrix} \Pi_1 & \Pi_2 \\ \Pi_2^T & \Pi_3 \end{bmatrix} < 0 \quad (21)$$

From (4.17) we know that $\Omega < 0$, hence

$$\begin{aligned} \Delta V_k &= E \{V_{k+1} \mid x_k, \dots, x_0, e_k, \dots, e_0\} - V_k \\ &= \delta_k^T \Omega \delta_k \leq -\lambda_{\min}(-\Omega) \delta_k^T \delta_k < -\alpha \delta_k^T \delta_k \end{aligned} \quad (22)$$

where $0 < \alpha < \min \{-\lambda_{\min}(-\Omega), \sigma\}$, $\sigma = \max \{\lambda_{\max}(P), \lambda_{\max}(Q)\}$. It will lead to the following

$$\Delta V_k = E \{V_{k+1} \mid x_k, \dots, x_0, e_k, \dots, e_0\} - V_k < -\alpha \delta_k^T \delta_k < -\frac{\alpha}{\sigma} V_k := -\psi V_k \quad (23)$$

Therefore, from Definition 1 and Lemma 1, it can be proved that the closed-loop system (11) is exponentially mean-square stable when the disturbance is zero. Next, we will prove the existence of disturbance.

For non-zero w_k and output is z_k , the following equation holds

$$\begin{aligned} &E \{V_{k+1}\} - E \{V_k\} + E \{z_k^T z_k\} - \gamma^2 E \{w_k^T w_k\} \\ &= E \{x_{k+1}^T P x_{k+1} + e_{k+1}^T Q e_{k+1}\} - x_k^T P x_k - e_k^T Q e_k \\ &\quad + [C_1 x_k + D_1 w_k]^T [C_1 x_k + D_1 w_k] - \gamma^2 E \{w_k^T w_k\} \\ &= \xi_k^T \Gamma \xi_k \end{aligned}$$

where $\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ \Gamma_{12}^T & \Gamma_{22} & \Gamma_{23} \\ \Gamma_{13}^T & \Gamma_{23}^T & \Gamma_{33} \end{bmatrix}$ and $\xi_k = [x_k \ e_k \ w_k]^T$

$$\begin{aligned}
\Gamma_{11} &= -P + (A_0 + \Delta A_0 - \bar{\beta}B_0K)^T P (A_0 + \Delta A_0 - \bar{\beta}B_0K) \\
&\quad + (\Delta A_0)^T Q (\Delta A_0) + \beta_1^2 (BK)^T P (BK) + \alpha_1^2 (LC_2)^T Q (LC_2) \\
&\quad \quad \quad + (C_0 + \Delta C_0)^T (C_0 + \Delta C_0) \\
\Gamma_{12} &= (A_0 + \Delta A_0 - \bar{\beta}B_0K)^T P \bar{\beta}B_0K + \Delta A_0 Q (A_0 - \bar{\alpha}LC_2) \\
&\quad \quad \quad + \beta_1^2 (B_0K)^T P (B_0K) \\
\Gamma_{13} &= (A_0 + \Delta A_0 - \bar{\beta}B_0K)^T P B_1 + \Delta A_0 Q (D - LD_2) \\
&\quad \quad \quad + (C_0 + \Delta C_0)^T D_1 \\
\Gamma_{22} &= (\bar{\beta}B_0K)^T P (\bar{\beta}B_0K) + (A_0 - \bar{\alpha}LC_2)^T Q (A_0 - \bar{\alpha}LC_2) \\
\Gamma_{23} &= (\bar{\beta}B_0K)^T P D + (A_0 - \bar{\alpha}LC_2)^T Q (D - LD_2) \\
\Gamma_{33} &= D^T P D + (D - LD_2)^T Q (D - LD_2) + D_1^T D_1 - \gamma^2 I
\end{aligned} \tag{24}$$

By using Shure's complement, Γ can be rewritten as

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^T & \Phi_{22} \end{bmatrix} < 0 \tag{25}$$

where $\Phi_{11} = \begin{bmatrix} -P & * & * \\ 0 & -Q & * \\ 0 & 0 & -\gamma^2 I \end{bmatrix}$, $\Phi_{12} = \begin{bmatrix} -P & * & * & * & * \\ 0 & -Q & * & * & * \\ 0 & 0 & -I & * & * \\ 0 & 0 & 0 & -P & * \\ 0 & 0 & 0 & 0 & -Q \end{bmatrix}$,

$$\Phi_{22} = \begin{bmatrix} P(A_0 + \Delta A_0 - \bar{\beta}B_0K) & \bar{\beta}PB_0K & PD \\ Q\Delta A_0 & Q(A - \bar{\alpha}LC_2) & Q(D - LD_2) \\ C_0 + \Delta C_0 & 0 & D_1 \\ \beta_1 PB_0K & \beta_1 PB_0K & 0 \\ \alpha_1 QLC_2 & 0 & 0 \end{bmatrix}$$

It can be concluded from (24) that

$$E\{V_{k+1}\} - E\{V_k\} + E\{z_k^T z_k\} - \gamma^2 E\{w_k^T w_k\} < 0 \tag{26}$$

Summing up (26) from 0 to ∞ . With respect to k yield

$$\sum_{k=0}^{\infty} E\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} E\{w_k^T w_k\} - E\{V_0\} - E\{V_{\infty}\} \tag{27}$$

Since $\xi_0 = 0$ and the system (11) is exponentially mean square stable, it is easy to conclude that

$$\sum_{k=0}^{\infty} E\{z_k^T z_k\} < \gamma^2 \sum_{k=0}^{\infty} E\{w_k^T w_k\} \tag{28}$$

That is, the specified H_∞ norm constraint is realized, and the proof is complete.

The matrix inequality given in (25) is bilinear on P , Q , K and L . let $M = P^{-1}$, $N = Q^{-1}$ and pre and post multiplied by the congruential transformation matrix $\text{diag} [I \ I \ I \ P^{-1} \ -Q^{-1} \ I \ P^{-1} \ Q^{-1}]$. We can get

$$\begin{bmatrix} \bar{\Xi}_{11} & \bar{\Xi}_{12} \\ \bar{\Xi}_{12}^T & \bar{\Xi}_{22} \end{bmatrix} < 0 \quad (29)$$

$$\bar{\Xi}_1 = \bar{\Phi}_{11}, \bar{\Xi}_{12} = \begin{bmatrix} A_0 + \Delta A_0 - \bar{\beta} B_0 K & \bar{\beta} B_0 K & D \\ \Delta A_0 & A_0 - \bar{\alpha} L C_2 & D - L D_2 \\ C_0 + \Delta C_0 & 0 & D_1 \\ \beta_1 B_0 K & \beta_1 B_0 K & 0 \\ \alpha_1 L C_2 & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{22} = \begin{bmatrix} -M & * & * & * & * \\ 0 & -N & * & * & * \\ 0 & 0 & -I & * & * \\ 0 & 0 & 0 & -M & * \\ 0 & 0 & 0 & 0 & -N \end{bmatrix}$$

The uniform transformation used in this method separates the controller gains K and L from the system parameters.

Observer-based controller (7) for the network control system (5) under the H_∞ performance constraint (12) with the minimum γ_{\min} . The following optimization problems need to be solved.

Under the H_∞ performance constraint (12) with the minimum γ_{\min} , the observer-based feedback controller (7) for the network control system (11). The following optimization problems need to be solved.

$$\min_{O_1 : P > 0, Q > 0, M > 0, N > 0, K, L} \gamma \text{ subject to (29)} \quad (30)$$

4 Simulation Example

The penetration rate of microgrid in daily life is getting higher and higher, and ensuring the stable operation of microgrid has become the primary issue. This paper uses system (5) for simulation, and the microgrid parameters are shown in Table (1). Our goal is to control the inverter in the presence of load interference with strong robustness to keep the output AC voltage at the desired value without distortion. The control signal is transmitted through a network cable, but due to limited network bandwidth or other factors, signal transmission may experience signal loss (packet loss), which affects the performance of the network system. Therefore, in the presence of random packet loss, we

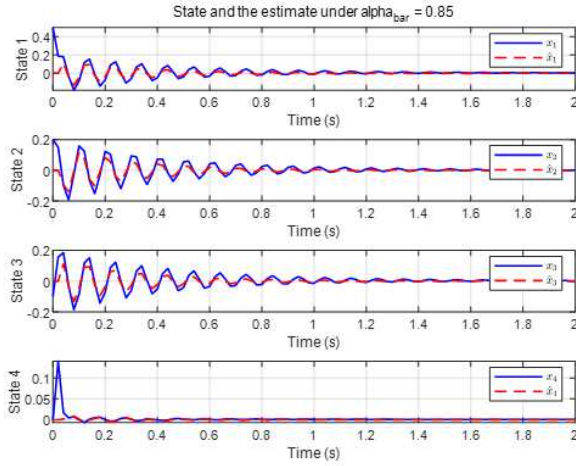


Fig. 3 state and the estimate under $\bar{\alpha} = \bar{\beta} = 0.85$.

aim to design an observer-based feedback controller to make the network system robust and exponentially stable in the mean square sense and achieve the specified H_∞ performance.

First, the observer result is shown in figure 3, and the initial condition is set to $x_0 = [0.5 \ 0.2 \ -0.1 \ 0]^T$, $\hat{x} = [0 \ 0 \ 0 \ 0]^T$ controller to actuator packet loss rate to be the same, that is, $\bar{\alpha} = \bar{\beta}$, which is also the same setting in the simulation. It can be seen that the observer is effective and able to observe the state of the system. The observer shown in the figure is under the condi-

Table 1 Microgrid parameters

Parameters	Value
R_t	1.5 m Ω
L_t	300 μ H
R	76 Ω
L	111.9mH
C	62.855 μ F
w_0	377rad/s
R_s	1 Ω
L_s	10mH

tion of $\bar{\alpha} = \bar{\beta} = 0.85$. Its observer gain and controller gain are respectively $K = [-0.5312 \ 0.0634 \ 0.5349 \ -0.0294]$, $L = [0.7873 \ -0.6682 \ 0.8127 \ 0.0122]^T$. Second, we design a robust controller with Bernoulli distribution of random packet loss, which minimizes the H_∞ performance index. This random packet loss process exists between the sensor to the controller and the controller to the actuator. In other words, we deal with the problem using the LMI toolbox

to solve the optimization problem O_1 , and obtain the performance indicators of the H_∞ controller under different packet loss rates. As shown in table (2). We can see that under different packet loss rates, the control effect is differ-

Table 2 Performance index

$\bar{\alpha} = \bar{\beta}$	γ_{\min}
1.0	0.198
0.95	0.334
0.9	0.352
0.85	0.357

ent. Under its different packet loss rates, the minimum H_∞ performance index $\gamma_{\min} > 0$ of the H_∞ controller we designed. Through simulation, the effectiveness of the controller is proved. It can be seen from Fig. 4 , Fig. 5 and Fig.6

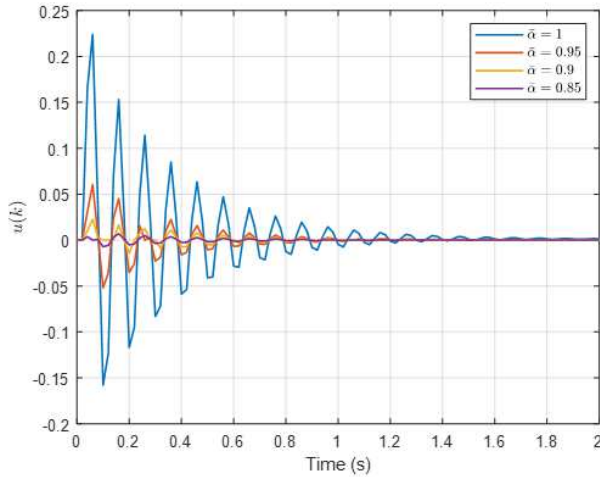


Fig. 4 Controller input under different packet loss rates

that when the packet loss rate is smaller, such as $\bar{\alpha} = \bar{\beta} = 0.95$, the output of the controller is closer to the output of the system controller without packet loss. When the packet loss rate gradually increases, the deviation between the output of the controller and the output of the controller without packet loss becomes larger, and the performance of the robust controller becomes worse, such as γ_{\min} increases.

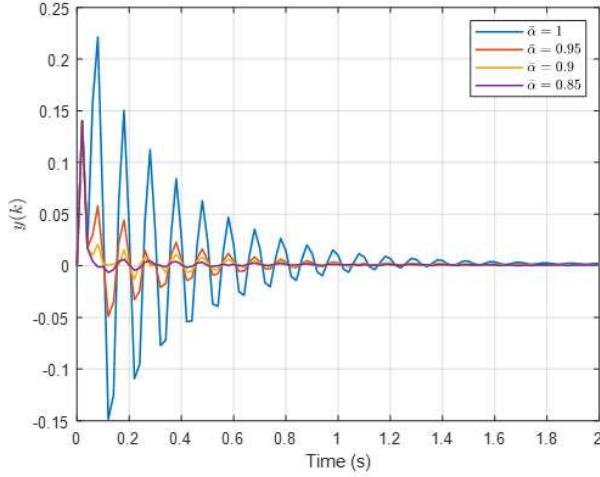


Fig. 5 measured output under different packet loss rates

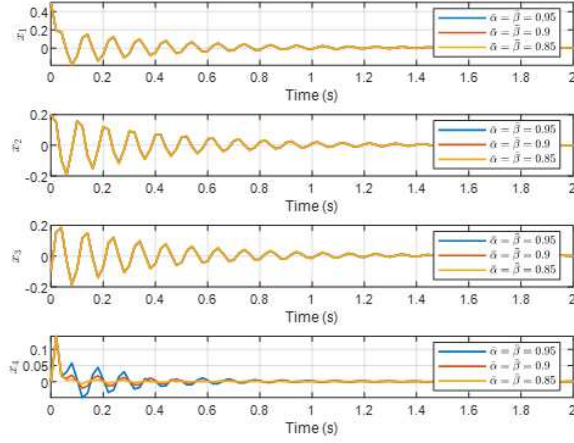


Fig. 6 State under different packet loss rates

5 Conclusions

This paper studies the robust H_∞ problem of networked control systems with random communication packet loss in microgrids, and proposes an observer-based dynamic feedback controller for microgrid networked control based on a random packet loss phenomenon, and proves the controller has robust exponential stability in the mean square sense. The effectiveness of the controller design is verified by simulation and comparison of controller performance under different packet loss situations. In the future work, we will focus on the

stability of the microgrid networked control system due to the simultaneous existence of delay and packet loss, and the severe test of the microgrid communication system brought about by the interconnection of multiple microgrids.

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Figures

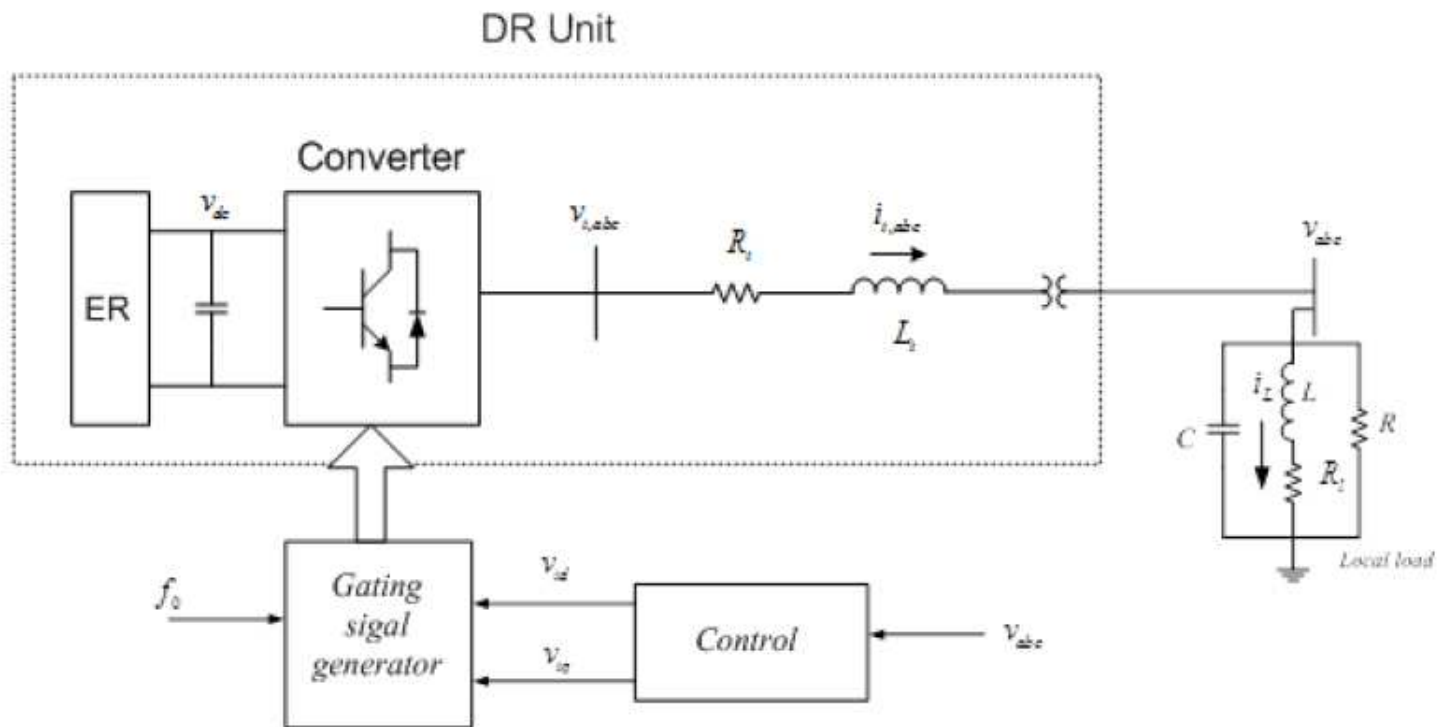


Figure 1

Schematic diagram of an independent DR connected to the grid

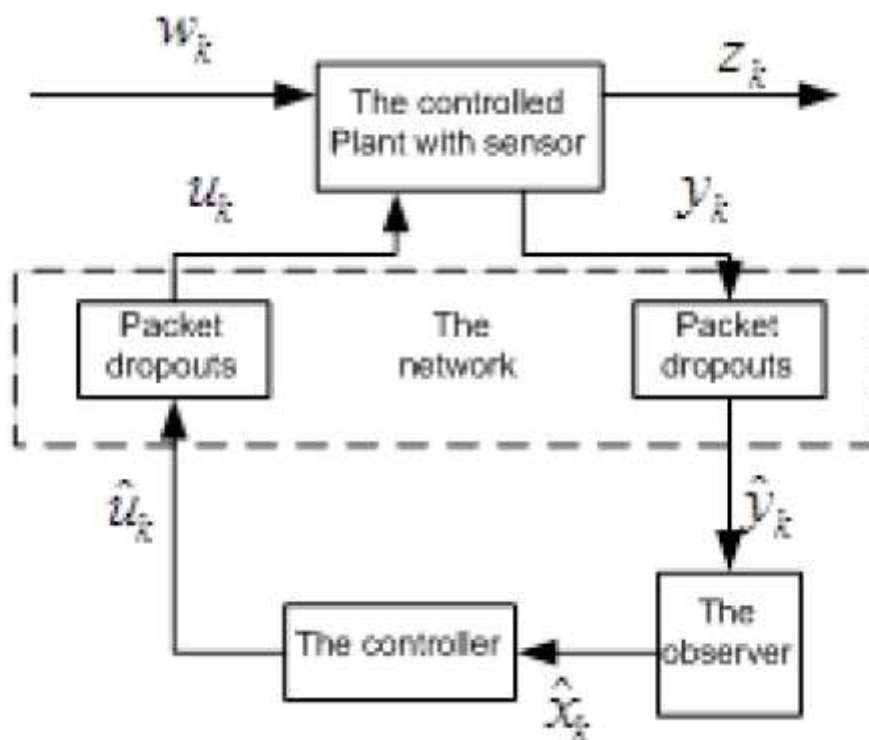


Figure 2

Networked control system layout diagram

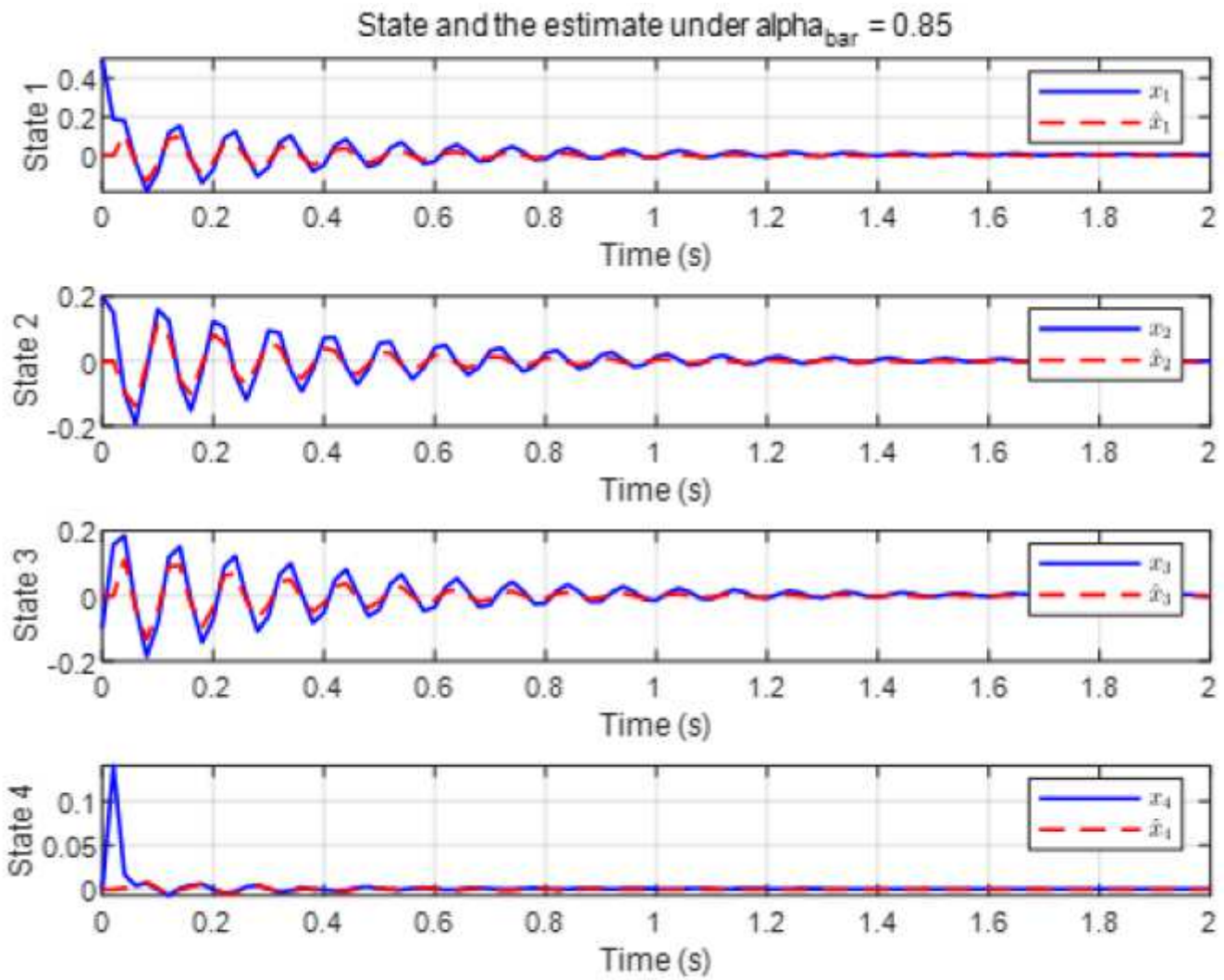


Figure 3

State and the estimate under $\alpha = \beta = 0.85$.

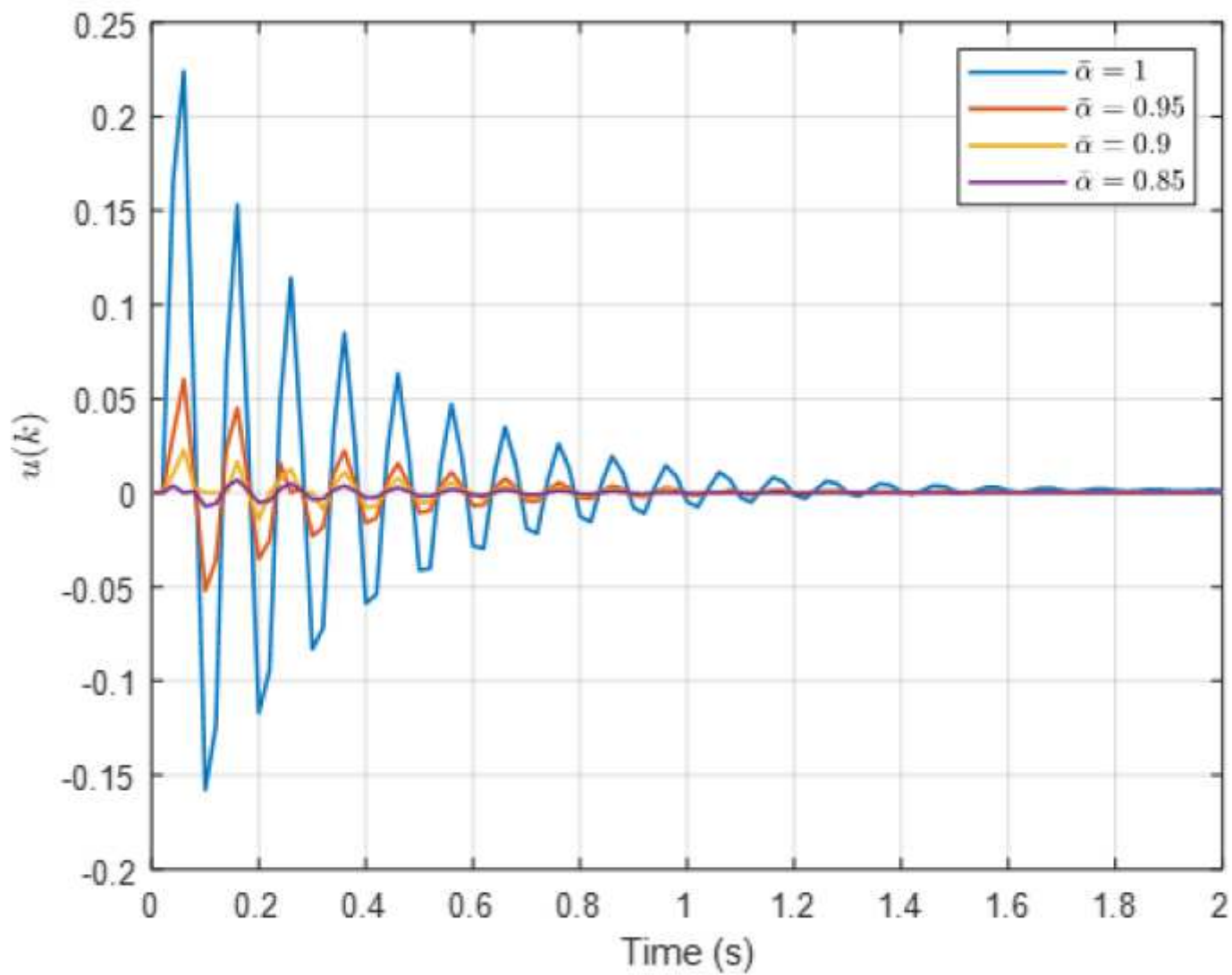


Figure 4

Controller input under different packet loss rates

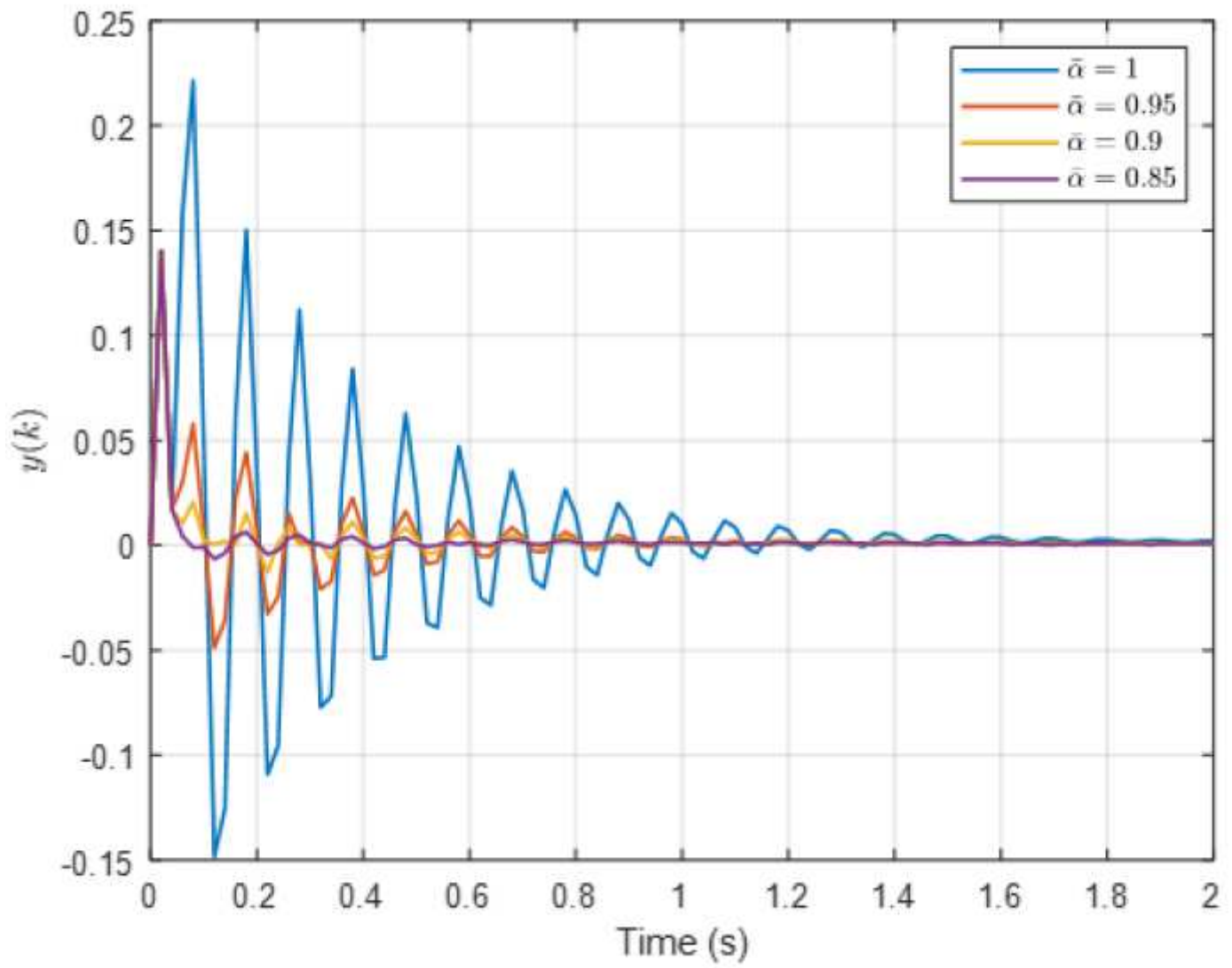


Figure 5

Measured output under different packet loss rates

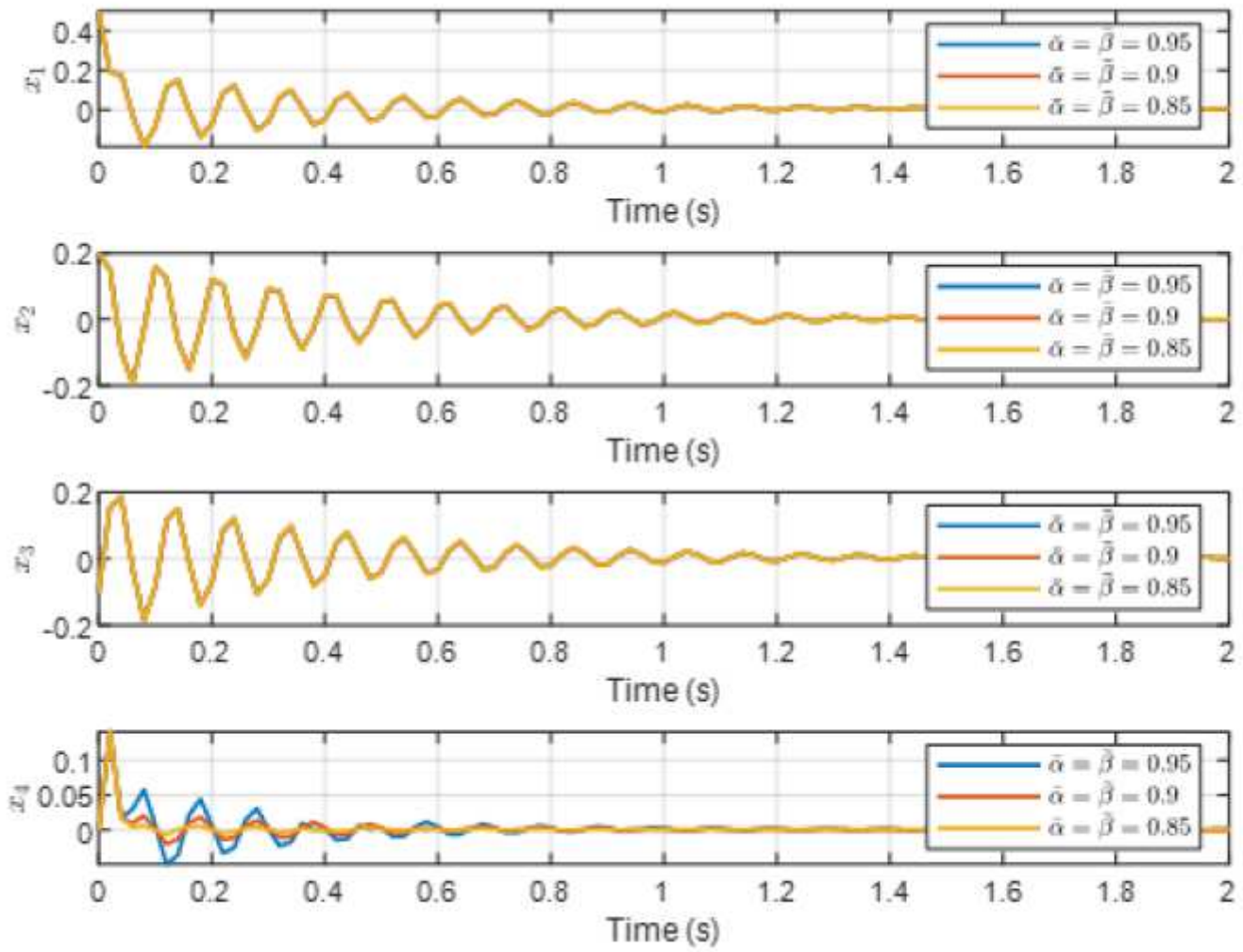


Figure 6

State under different packet loss rates