Remote Health Monitoring System for Bedbound Patients

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1 Signal model and radar cube

In the FMCW radar signal analysis, we take complex samples of each chirp after complex mixing at the receiver(s). So, it can be shown that the received signal from l'th receiver is like the following if there is a single point target in the radar field:

$$x_l(t_f,t_s) = b_l \exp\Bigg[-j\Big(2\pi f_b t_f + 2\frac{vt_s}{\lambda_{max}} + \tau_l + \xi_l + \Delta\psi(t_f,t_s)\Big)\Bigg] + e_l(t_f,t_s) \tag{1}$$

in which t_f and t_s are fast and slow time indexes, and b_l, ξ_l are reflecting the magnitude and phase mismatches of the virtual channels. Also, τ_l is the phase difference of the receivers due to angle of arrivals, which is a function of receivers spacing (see equation 2 in the main text). $\Delta\psi(t_f,t_s)$ is the residual phase noise. In this equation, it is assumed that the target has a radial velocity of v. The last term in the above equation is the spatially and temporally white additive noise. If we stack the received signals of all antennas, either virtual or physical, in a column vector of $\mathbf{x}(t_f,t_s)$, then:

$$\mathbf{x}(t_f,t_s) = \Gamma\mathbf{a}(\theta)y(t_f,t_s) + \mathbf{e}(t_f,t_s) \tag{2}$$

where Γ is the diagonal matrix containing $b_l\exp(-j\xi_l)$ on diagonal, and $\mathbf{a}(\theta)$ is a column vector with $\exp(-j\tau_l)$ for the l'th element. Ideally, $\mathbf{a}(\theta)$ is equal to the steering vector in equation 2 of the main text. In addition, $y(t_f,t_s) = \exp(-j(2\pi f_b t_f + 2vt_s/\lambda_{max} + \Delta\psi(t_f,t_s)))$ is a scalar. Moreover, if there are K targets in general, then $y(t_f,t_s)$ extended as a column vector:

$$\mathbf{x}(t_f,t_s) = \Gamma A(\theta)\mathbf{y}(t_f,t_s) + \mathbf{e}(t_f,t_s) \tag{3}$$

$$y_i(t_f,t_s) = \frac{\rho_i}{\sqrt{M}}\exp\left[-j\left(2\pi f_{b_i}t_f + 2\frac{v_i t_s}{\lambda_{max}} + \Delta\psi_i(t_f,t_s)\right)\right] \tag{4}$$

where $A(\theta)$ has K columns formed by $\mathbf{a}(\theta_i)$ when the i'th target is at θ_i, and θ is a vector of angles associated with each target. Furthermore, $\mathbf{y}$ is a column vector with the i'th element of (4). Basically, the vector $\mathbf{x}$ is a function of t_f and t_s, and if we demonstrate the vector at different time samples, a cube will be constructed with three axes of fast-time, slow-time, and receiver channels. In fact, $\mathbf{x}$ is only one slice of this cube at t_f, t_s.

In order to obtain range information, we take an FFT over the fast-time, and it will be transformed to the range domain. This is equivalent to collecting $\mathbf{y}(t_f,t_s)$ samples in the fast-time and taking FFT over the samples. Then, the signal model will be:

$$\tilde{\mathbf{x}}(f_{b_i},t_s) = \Gamma A(\tilde{\theta})\tilde{\mathbf{y}}(f_{b_i},t_s) + e(f_{b_i},t_s) \tag{5}$$

with f_{b_i} is the i'th target's beat frequency, N and M are the number of samples in the fast-time and the number of receivers, respectively. The factor $\sqrt{N}$ is the result of FFT to maintain the same power in FFT and time domains. ρ_i is related to the power received from the i'th target. In addition, the entries of $\tilde{\mathbf{y}}(f_{b_i},t_s)$ have Doppler of targets at the range corresponding to f_{b_i}. Also note that $\tilde{\theta}$ is different from the radar cube in time since now it contains the target angles at the particular range of f_{b_i}. This radar cube is shown in Fig. 1a where the highlighted slice is $\tilde{\mathbf{x}}(f_{b_i},t_s)$. On the other hand, the dependency of $\tilde{\mathbf{y}}$ on f_{b_i} is in the residual phase noise, thus for simplicity, it can be a noise term. Therefore, $\tilde{\mathbf{y}}$ is a noisy component with the dependency on t_s only.

In order to detect the angle of targets, or more specifically vector θ, one method is to use a second FFT along the receiver antennas; however, it requires to remove Doppler from the signal to properly estimate the angles since $\tilde{\mathbf{y}}$ contains Doppler, which is an interference for the angle estimation. In contrast, by using Capon filter, we can estimate θ such that the effect of Doppler is eliminated automatically in the covariance matrix calculation of $\tilde{\mathbf{x}}(f_{b_i}, t_s)$:

$$\begin{aligned} R_{\tilde{\mathbf{x}}}(f_{b_i}) &= E(\tilde{\mathbf{x}}(f_{b_i}, t_s)\tilde{\mathbf{x}}^H(f_{b_i}, t_s)) \\ &= \Gamma A(\tilde{\theta}) E\left[\tilde{\mathbf{y}}(f_{b_i}, t_s)\tilde{\mathbf{y}}^H(f_{b_i}, t_s)\right] A^H(\tilde{\theta})\Gamma^H \\ &+ R_{\tilde{\mathbf{e}}}(f_{b_i}) \end{aligned} \tag{6}$$

If there is no phase noise, the middle term in the expectation is $\rho_i^2/MN\, I_K$ in which I_K is an identitiy matrix of size K. Therefore,

$$R_{\tilde{\mathbf{x}}}(f_{b_i}) = \frac{\rho_i^2}{MN}\Gamma A(\tilde{\theta})A^H(\tilde{\theta})\Gamma^H + R_{\tilde{\mathbf{e}}}(f_{b_i}) \tag{7}$$

This proves that the angle estimation with Capon filter reduces the effort to compute Dopplers before angle estimation. Figure 1a indicates the integration direction to compute $R_{\tilde{\mathbf{x}}}(\mathbf{f}_b)$ for all ranges, which ultimately gives the range-angle map (range-azimuth or elevation). A sample map is shown on the range-angle face of the radar cube in Fig. 1a. Also, note that each (range, angle) in the map is the polar representation of the points in xy plane as depicted in Fig. 2(g-i) or 4b.

If the angle of targets is known, equation 1 of the main text gives the final signal, which is an input to the estimator. Therefore, we used two individual estimators for the two targets throughout all experiments. However, the target angle detection requires two steps, namely, CFAR and clustering.

2 CFAR

CFAR is responsible to give an initial guess of the target presence in xy plane. In CFAR, the performance of the system highly depends on CFAR. That is the detected points should be related to the targets rather than other possible reflections from other moving objects. Indeed, there are three CFAR parameters managing its performance: 1) gap length, 2) training length, 3) false alarm rate. Based on the policy of determining the noise power around the target, the CFAR strategy can be different. Among many methods, we found smallest-of cell-averaging CFAR (SOCA-CFAR) to be better in finding close targets.

In an ideal system, the detection rate (P_d) should be very high as 99%, and false alarm rate (P_{fa}) should be very low as 10^{-3}. With low P_d, the number of lost points is high leading to missing targets, which should be avoided in practice. On the other hand, with high P_{fa}, the number of false detected points is high, which in turn, becomes annoying. Therefore, there is a tradeoff between P_d and P_{fa}. We always aim to have high P_d, or low miss probability, even though if there are extra points that are not pertained to any targets. Moreover, the higher the SNR is, the lower the P_{fa} is for a particular P_d. Thus, depending on the size and range of targets, P_{fa} should be adjusted, which is now done for the typical targets we have in the Fig. 1a.

3 Optimum Harmonic Estimator Filter

After CFAR and target point selection, the observed signal from a particular (range, azimuth) point in *slow-time* is a noisy mixture of a periodic signal due to chest wall vibration. As a result, it is reasonable to assume that we have a quasi-periodic signal per target in (5). If the signal for a typical target is $x[n]$ after angular matching, the quasi-periodic signal can be expressed as follows:

$$x[n] = \sum_{l=1}^{L} u_l e^{j\omega ln} + e[n] \tag{8}$$

$u_l = \alpha_l e^{j\phi_l}$ is the complex coefficient of the signal component for l'th harmonic and L is the model order. Correspondingly, ω is the fundamental radian frequency and $e(n)$ is the additive noise, which is uncorrelated with the signal. Also note that the complex coefficients are not varying with time, so the signal is wide sense stationary (WSS). In fact, WSS assumption is violated when there is a sudden short-time event in the signal such as a change in the average or the presence of an interference. More specifically, the target motions makes the signal non-stationary, that is the moments when the radar estimates are not consistent with the reference sensor (see Fig. 4c and Fig. 4e).

By taking Q signal samples, we define a vector of Q latest samples, $x[n] = [x[n], x[n-1], \cdots, x[n-Q+1]]^T$. Here, Q should be at least equal to the fundamental period in (8) corresponding to the fundamental frequency. By rearranging and putting variables in (8) into matrices and vectors, $x[n]$ can be expressed as:

$$x[n] = U[n]u + e[n], \tag{9}$$

$$U[n] = \begin{bmatrix} e^{j\omega n} & e^{j2\omega n} & \cdots & e^{j\omega Ln} \\ e^{j\omega(n-1)} & e^{j2\omega(n-1)} & \cdots & e^{j\omega L(n-1)} \\ \vdots & \vdots & \cdots & \vdots \\ e^{j\omega(n-Q+1)} & e^{j2\omega(n-Q+1)} & \cdots & e^{j\omega L(n-Q+1)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\omega} & e^{-j2\omega} & \cdots & e^{-j\omega L} \\ \vdots & \vdots & \cdots & \vdots \\ e^{-j\omega(Q-1)} & e^{-j2\omega(Q-1)} & \cdots & e^{-j\omega L(Q-1)} \end{bmatrix} \begin{bmatrix} e^{j\omega n} & & \mathbf{0} & \\ & e^{j2\omega n} & & \\ & & \ddots & \\ & \mathbf{0} & & e^{j\omega Ln} \end{bmatrix} \tag{10}$$

$$:= ZD^n \tag{11}$$

$$:= Z[n] \tag{12}$$

We split the matrix U in (10) into two matrices one independent of time (Z) and the other is a diagonal matrix containing time information (D^n). Also, Z is *Vondermonde* matrix, which is a full column rank matrix when $Q \geq L$ (has more rows than columns). In (11), matrix D^n only depends on n and by plugging it to (9), either the matrix Z depends on time index n ((12)) or the vector a depends on n.

In order to find an estimate of u and ω, one can design a filter such that its output is the closest to the noiseless component in (8). Since this method depends on u, so it is signal dependent. The idea is similar to designing a matched filter, but here the output power of the filter is maximum when the true values of u, ω are found. Assume, $h[n] = [h[0], h[1], \cdots, h[Q-1]]^H$ are the complex filter coefficients, then we want to minimize the numerical average difference between the output filter samples and the noiseless signal component i.e.:

$$P = \min_{u,\omega,h} \frac{1}{G} \sum_{n=Q-1}^{P-1} \left| y[n] - \hat{y}[n] \right|^2 \tag{13}$$

$$= \min_{u,\omega,h} \frac{1}{G} \sum_{n=Q-1}^{P-1} \left| h^H x[n] - u^T w[n] \right|^2 \tag{14}$$

where $G = P - Q + 1$, and in (14), $w[n]$ is defined as $\mathrm{diag}(D^n)$. Consider that the objective in (14) is minimizing the average difference between the filter output and the ideal noiseless periodic signal. Also, we have assumed that L is known whereas there are methods for estimating the model order. The problem in (14) is quadratic in both u and h. In fact, it has a bounded optimum solution when $W := 1/G \sum_{n=Q-1}^{P-1} w[n] w^H[n]$ is positive semi-definite.

A three-step solution for the problem in (14) is as follows. First, this is solved for u, then h, and ω. The optimum values are:

$$\hat{u} = W^{-1} G_k h \tag{15}$$

$$\hat{h} = \hat{V}^{-1} (Z^H \hat{V}^{-1} Z)^{-1} 1 \tag{16}$$

$$\hat{\omega} = \underset{\omega \in \Omega}{\operatorname{argmax}}\ h^H \hat{R} h \tag{17}$$

$$G := 1/G \sum_{n=Q-1}^{P-1} w[n] x^H[n]$$

$$\hat{V} := \hat{R} - G^H W^{-1} G$$

and Ω is the set of all possible ω's. For instance, breathing rate is in $[0.1, 0.6]\, Hz$. Indeed, G depends on $x[n]$, which makes the solution dependent on data. Thus, our algorithm is both signal and data dependent. One can search within this frequency range by with a frequency resolution of Δf to obtain the filter output power as a function frequency, then $\hat{\omega}$ corresponds to the peak power. We set Δf to 0.01 Hz in order achieve less than 1 bpm breathing rate resolution, and P, Q, L values are also listed in Fig. 1b. Moreover, equation (15) characterizes the respiratory noiseless waveform as they are labeled as *reconstructed waveforms* in Fig. 4d and Fig. 4f for the subjects in Fig. 4a.