

Performance of M-MRC over Generalized- K Fading Channel based on Energy Harvesting Technique

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Abstract: The customary uses of the multi-antenna systems are to reduce the effect of fading in a wireless environment. The maximal ratio combining (MRC) scheme is one of such techniques which provide the optimal performance. This paper analyse the performance measures of an arbitrary branch MRC receiver with energy harvesting techniques over K_G fading channel. We have presented the closed-form expressions for average SNR, outage probability, and average bit error rate (ABER) along with the numerical analysis. Our observations are summarized in the numerical results and discussion section for well understanding of the system. At last, we validate our proposed formulations through the Monte-Carlo simulation.

Keywords: K_G fading channels; MRC; SWIPT; Power Splitter; Outage probability; ABER.

1 Introduction

In wireless communication systems, radio-wave propagation through wireless channels happen to be a complicated phenomenon due to the various effects, such as multipath fading due to reflection, diffraction, and scattering. Several models are presented to depict the statistical behaviour of wireless channels for different situation [1]. In reality the multipath fading and shadowing occur concurrently leads to the composite fading channels. These composite fading channels are superimposed by lognormal shadowing such as Rayleigh-lognormal or Nakagami-lognormal fading channels [2]. However, such lognormal based fading models are complicated and difficult to evaluate the various performance measures of communication systems through mathematical analysis [3]. Consequently, an alternative approach a Gamma distribution is utilized,

which is mathematically more versatile and also accurately describes fading and shadowing phenomena such as Generalized- K (K_G) distribution [4] or K -distribution [5]. The Generalized- K fading model describes the composite multipath/shadowing fading channels and the results obtained using this model is closely matched with the experimentally obtained results. K_G fading model is the composite Nakagami-Gamma distribution which closely approximates Nakagami-Lognormal distribution [6]. It also includes the K -distribution as a special case, and incorporates most of the fading and shadowing effects in wireless communication channels. The main advantage of employing the Generalized K -distribution is that it makes the mathematical analysis much simpler compared to Lognormal-based models. Diversity is a technique which improves the quality of the received signal and exploits the low probability of deep fades simultaneously in different fading paths. Depending upon the combining technique, the performance and the complexity of the diversity receiver varies. Maximal ratio combining (MRC) is the optimal combining technique which gives the best performance compared to other combining receiver [7].

Recently, Energy harvesting (EH) has been considered as a promising technology to extend the lifetime battery of sensor networks [8]. In Wireless Information and Power Transfer (WIPT), the radiated Radio frequency (RF) waves can carry both information and energy simultaneously [9]. For this reason, the concept of Simultaneous WIPT is proposed and quickly becomes an attractive research topic in both industry and academia. In addition, the WIPT technique can simplify the complexity of the wireless communication networks, and reduces the expense in recharging and replacing batteries. In most of the literature, the performance analysis of wireless powered communication (WPC) systems using power splitting (PS) technique over Nakagami- m fading channels has been focused on [10, 11]. However, to the best of our knowledge,

wireless information and power transfer systems over K_G fading channels using fixed power splitter is not available in the literature. In this paper, we have presented the closed form expression for average SNR, outage probability and ABER employing M-MRC receiver in K_G fading channels and using power splitter at the receiver side. Further, the effect of the system and fading parameters on the system performance has been verified through simulation results. In addition to this, we study the effect of average energy harvested at the receiver end with fixed power splitter.

This work is structured as follows: Section 2 presents the system model considered for analysis. Performance analysis of the system is carried out in Section 3. In Section 4, shows the numerical results and discussion. We conclude our work in Section 5.

2 System Model

The channel behavior is considered as a slow and frequency non-selective with the K_G fading model. Figure 1 shows the proposed model that have been considered for evaluation of the system performance.

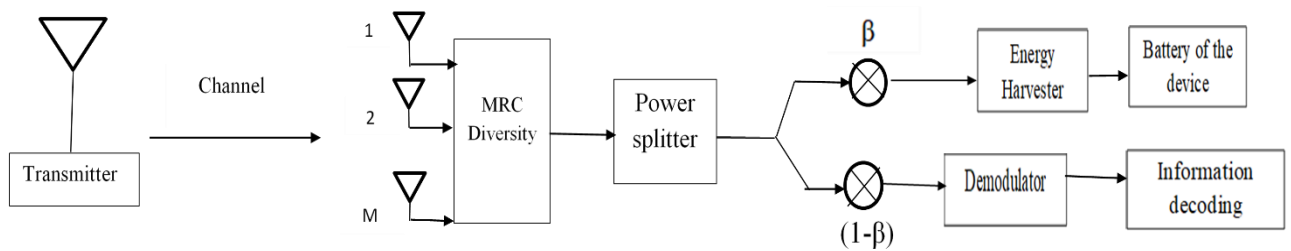


Fig. 1 The System model

The probability density function (PDF) of the signal used for information decoding has been derived as follows. The expression of the received signal from the l_{th} antenna, $l = 1, 2, \dots, M$ in one-bit duration, can be given as

$$r_l(t) = x_l e^{j\phi_l(t)} s(t) + n_l(t), \quad (1)$$

where, $s(t)$ is the transmitted bit signal with energy E_b and $n_l(t)$ is the complex Gaussian noise with zero mean and two sided power spectral density $2N_0$. Random variable (RV) ϕ_l represents the phase and x_l is the K_G distributed fading amplitude having PDF given by [12].

$$p_x(x_l) = \frac{4x_l^{m+k-1}}{\Gamma(m)\Gamma(k)} \left(\frac{m}{\Omega}\right)^{\frac{k+m}{2}} \mathbf{K}_{k-m} \left[2\left(\frac{m}{\Omega}\right)^{\frac{1}{2}} x_l \right], \quad x_l \geq 0 \quad (2)$$

where k and m are the two distribution shaping parameters, $\mathbf{K}_\nu(\cdot)$ is the modified Bessel function of order ν [13,(8.407.1)], $\Gamma(\cdot)$ is the Gamma function. Ω is the mean power which is defined as $\Omega = E\langle X^2 \rangle / k$, with $E\langle \cdot \rangle$ denotes expectation. The instantaneous SNR per branch at output of the diversity branch is $\gamma = X^2 E_b / N_0$ and the average SNR is given as $\bar{\gamma} = k\Omega E_b / N_0$.

In MRC diversity, to maximize the output SNR at the combiner output, the instantaneous SNR of each branch firstly being scaled by a weight factor and all the weighted branches are added up at the combiner. Thus, the instantaneous output SNR of the MRC combiner with M diversity branches is given as [14]

$$\gamma_{MRC} = \sum_{l=1}^M \gamma_l \quad (3)$$

Using Eq. (2) with some rearrangement and by performing RV transformation, the PDF of γ for MRC receiver over K_G fading channel is evaluated as [4]

$$f_\gamma(\gamma) = \frac{2\Xi^{\frac{(Mm+k)}{2}}}{\Gamma(mM)\Gamma(k)} \gamma^{\frac{(Mm+k-2)}{2}} \mathbf{K}_{k-Mm} \left[2\sqrt{\Xi\gamma} \right], \quad \gamma \geq 0 \quad (4)$$

where, $\Xi = \frac{km}{\gamma}$.

The received signal from the output of MRC is then fed into a power splitter, where the signals are split to the information receiver and energy receiver separately. For each fading state, the signal power split to information decoding (ID) is denoted by $(1-\beta)$ with $0 \leq \beta \leq 1$, and that to energy harvesting (EH) as β where β can be adjusted over different fading states.

Applying random variable transformation, $f_s(s) = \frac{1}{\beta} f_\gamma\left(\frac{s}{\beta}\right)$, we can readily obtain the

PDF by inserting Eq. (4) in $f_s(s)$ as

$$f_s(s) = \frac{2\Xi^{\frac{(Mm+k)}{2}}}{\beta^{\frac{(Mm+k)}{2}} \Gamma(mM) \Gamma(k)} s^{\frac{(Mm+k-2)}{2}} \mathbf{K}_{k-Mm} \left[2\sqrt{\frac{\Xi s}{\beta}} \right], \quad (5)$$

3 Performance of MRC Receiver

3.1 Average Output Signal-to-Noise Ratio

Average output SNR $\bar{\gamma}_{out}$ of MRC receiver for K_G fading channels using PS technique

can be given as $\bar{\gamma}_{out} = \int_0^\infty s f_s(s) ds$. Putting $f_s(s)$ from Eq. (5) into $\bar{\gamma}_{out}$, yields

$$\bar{\gamma}_{out} = \int_0^\infty \frac{2\Xi^{\frac{(Mm+k)}{2}}}{(1-\beta)^{\frac{(Mm+k)}{2}} \Gamma(mM) \Gamma(k)} s^{\frac{(Mm+k)}{2}} \mathbf{K}_{k-Mm} \left[2\sqrt{\frac{\Xi s}{1-\beta}} \right] ds \quad (6)$$

Now, solving the integral, changing the variables and rearranging using [13,(6.561.16)],

an expression for $\bar{\gamma}_{out}$ can be obtained as

$$\bar{\gamma}_{out} = \frac{\bar{\gamma}(1-\beta)}{km\Gamma(mM)\Gamma(k)} \Gamma(k+1)\Gamma(Mm+1) \quad (7)$$

3.2 Outage Probability

Outage probability is given as [2], $P_{out} = \int_0^{\gamma_{th}} f_s(s) ds$ where γ_{th} is the threshold value of the output SNR. Putting $f_s(s)$ from Eq. (5), the expression can be obtained as

$$P_{out} = \frac{2\Xi^{\frac{(Mm+k)}{2}}}{\beta^{\frac{(Mm+k)}{2}} \Gamma(mM)\Gamma(k)} \int_0^{\gamma_{th}} s^{\frac{(Mm+k-2)}{2}} \mathbf{K}_{k-Mm} \left[2\sqrt{\frac{\Xi s}{\beta}} \right] ds \quad (8)$$

Integrating Eq.(8) with some manipulations and using [13,(6.561.19)], the outage probability will results

$$P_{out} = \frac{4\sqrt{\pi}}{2^{Mm+k+p+\frac{1}{2}} \Gamma(mM)\Gamma(k)} \sum_{p=0}^{k-Mm-\frac{1}{2}} \frac{\left(k-Mm-\frac{1}{2}+p\right)!}{p! \left(k-Mm-\frac{1}{2}-p\right)!} g\left(Mm+k-p-\frac{1}{2}, 2\sqrt{\frac{\Xi\gamma_{th}}{\beta}}\right) \quad (9)$$

where, $g(\cdot, \cdot)$ is the incomplete gamma function.

3.3 Average Bit Error Rate

The average bit error rate (ABER) can be expressed in terms of the PDF of the output

SNR as $P_e(s) = \int_0^{\infty} p_e(\varepsilon|s) f_s(s) ds$ where $p_e(\varepsilon|s)$ is the conditional BER which depends

on the modulation scheme used.

3.3.1 Binary Coherent Modulations

For binary coherent modulations, the expression of the conditional BER can be given

as $p_{e, ch}(\varepsilon|s) = \frac{1}{2} \operatorname{erfc}(\sqrt{as})$, where $a=0.5$ for coherent frequency shift-keying and $a=1$

for coherent phase shift-keying modulations, respectively [12]. Putting $p_{e, ch}(\varepsilon | s)$ and $f_s(s)$ into $P_e(s)$, solving the integral, the ABER can be obtained in closed form as

$$P_{e, ch} = \frac{\frac{\Xi^{(Mm+k)}}{\beta^{(Mm+k)}}}{\Gamma(mM)\Gamma(k)} \int_0^\infty \text{erfc}(\sqrt{as}) s^{\frac{(Mm+k-2)}{2}} \mathbf{K}_{k-Mm} \left[2\sqrt{\frac{\Xi s}{\beta}} \right] ds \quad (10)$$

Writing the complementary error function $\text{erfc}(\sqrt{as})$ as [15, (06.27.26.0006.01)]

$$\text{erfc}(\sqrt{as}) = \frac{1}{\sqrt{\pi}} G_{1,2}^2 \left(\begin{matrix} 0 \\ as \end{matrix} \middle| \begin{matrix} 1 \\ \frac{1}{2} \end{matrix} \right) \quad (11)$$

where, $G_{p,q}^m \left(z \middle| \begin{matrix} a_1, a_2, \dots, a_p \\ b_1, b_2, \dots, b_q \end{matrix} \right)$ is the Meijer's G-function [13, (9.301)].

Further Eq. (10) can be simplified by changing the variable and solving the integral using [15, (03.04.26.0009.01) and (07.34.21.0011.01)], the expression for coherent BER can be obtained as

$$P_{e, ch} = \frac{\left(\frac{\Xi}{a\beta} \right)^{\frac{(Mm+k)}{2}}}{\sqrt{\pi} \Gamma(mM) \Gamma(k)} G_{2,3}^2 \left(\frac{\Xi}{a\beta} \middle| \begin{matrix} -\frac{(Mm+k-2)}{2} & -\frac{(Mm+k-1)}{2} \\ \frac{k-Mm}{2} & -\left(\frac{k-Mm}{2} \right) & -\frac{(Mm+k)}{2} \end{matrix} \right) \quad (12)$$

3.3.2 Binary Non-Coherent Modulations

In binary non-coherent modulations, the expression for the conditional BER can be obtained as $p_{e, nch}(\varepsilon | s) = \frac{1}{2} \exp(-as)$, where $a=0.5$ for non-coherent frequency shift-keying and $a=1$ for differential phase shift-keying modulations, respectively [12]. By putting $p_{e, nch}(\varepsilon | s)$ and $f_s(s)$ into $P_e(s)$ and solving the integral, an expression for ABER will results

$$P_{e,nch} = \frac{\frac{\Xi^{\frac{(Mm+k)}{2}}}{\beta^{\frac{(Mm+k)}{2}} \Gamma(mM) \Gamma(k)}}{\int_0^\infty s^{\frac{(Mm+k-2)}{2}} \exp(-as) K_{k-Mm} \left[2\sqrt{\frac{\Xi s}{\beta}} \right] ds} \quad (13)$$

After changing the variable and with some manipulations using [13,(6.631.3)], integration in Eq. (13) is solved which results the final expression for ABER as

$$P_{e,nch} = \frac{1}{2} \left(\frac{\Xi}{a\beta} \right)^{\frac{Mm+k-1}{2}} \exp\left(\frac{\Xi}{2a\beta} \right) W_{-\frac{Mm+k-1}{2}, \frac{k-Mm}{2}} \left(\frac{\Xi}{a\beta} \right) \quad (14)$$

where, $W_{\lambda,\mu}(\cdot)$ is the Whittaker function [13, (9.220)].

4 Numerical Results and Discussion

In the present section, the numerical results are shown and validation of the proposed analysis is carried out through the Monte-Carlo simulation. During the numerical evaluation, parameters such as k , m and β are selected suitably, to present the complete analysis of the system. We presented the two most important performance measures outage probability and ABER along with the plot of harvested power vs SNR in dB. It can easily be recognised that with simultaneous power and energy transmission all the performance measures degrades considerably. The outage probability vs average SNR of each branch for $M=3$ and $M=5$ have been plotted in Figure 2 respectively, for different values of m , k and β . From the plot, it can be analysed that, at a higher value of β , m and k , significant improvement in the outage probability is noticed, whereas the performance of the channel degraded as M increases.

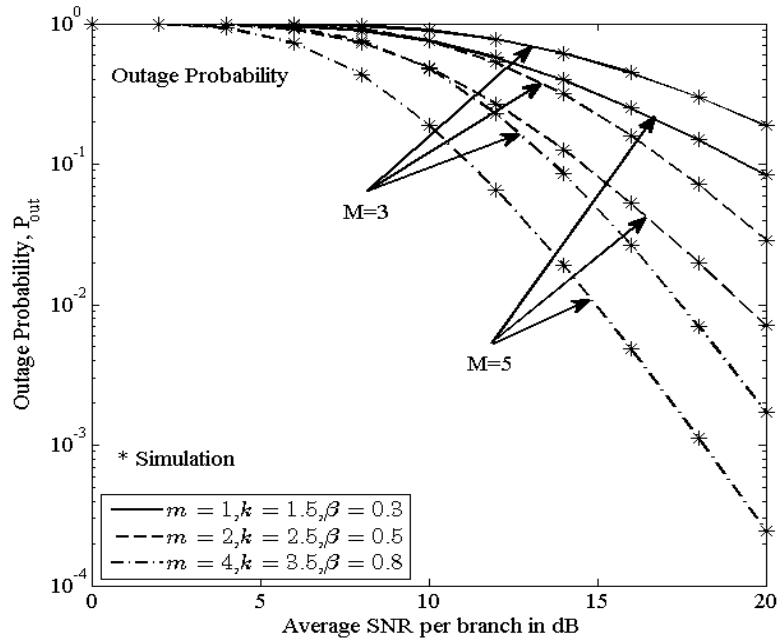


Fig. 2 P_{out} vs SNR for different values of k , β and m

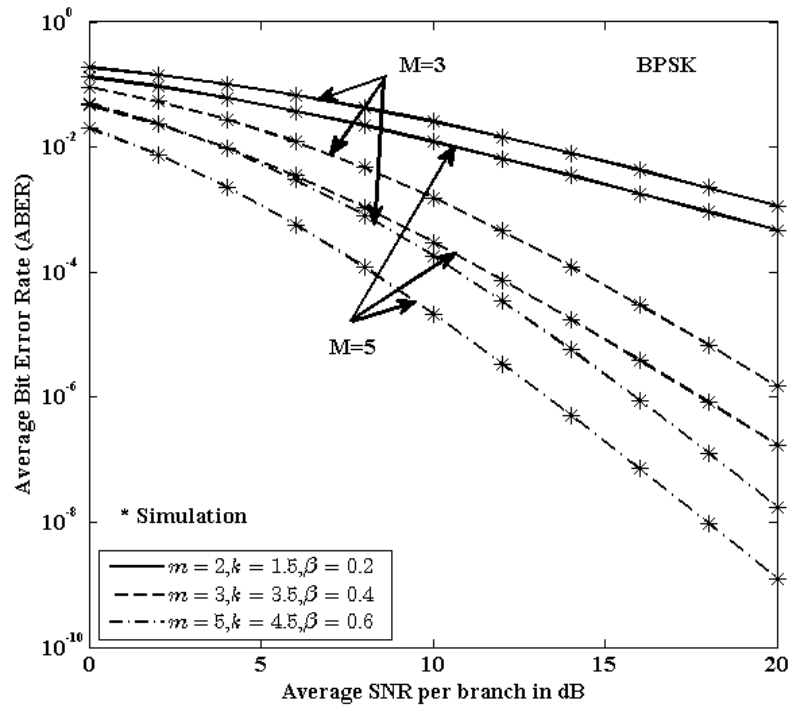


Fig. 3 ABER for Coherent BPSK

In Figures 3 and 4, ABER vs average SNR per branch for coherent BPSK and non-coherent DPSK modulation schemes have been plotted respectively. As expected, the ABER improves with an increase of k , m , and β . Further, we can conclude that BPSK performs better than DPSK and their performance improves with improving fading parameters. Particularly, the numerical results are compared with the Monte-Carlo simulation. It is clear from the figure that the analytical result perfectly matches the simulation result over the entire SNR region.

From the power vs SNR curve as shown in Figure 5, we have plotted for $M=5$, $m=2$ and $\beta=0.8$. It can be observed from the figure that a significant amount of power can be harvested at the receiver above 40 dB SNR, beyond which the harvested power is increased exponentially. Practically, this is not possible to maintain such a high SNR in this link as the transmitter is always supposed to connect to a conventional power source.

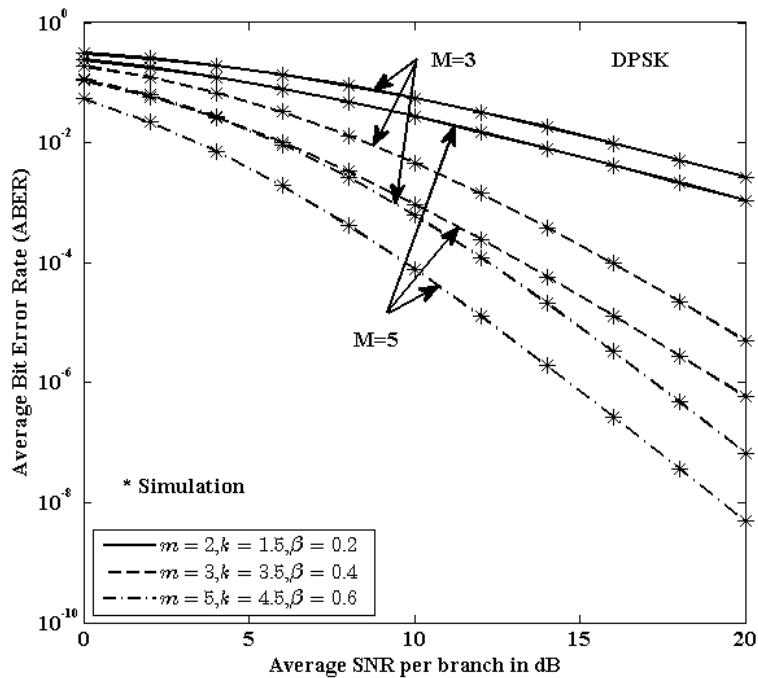


Fig. 4 ABER for Non-Coherent DPSK

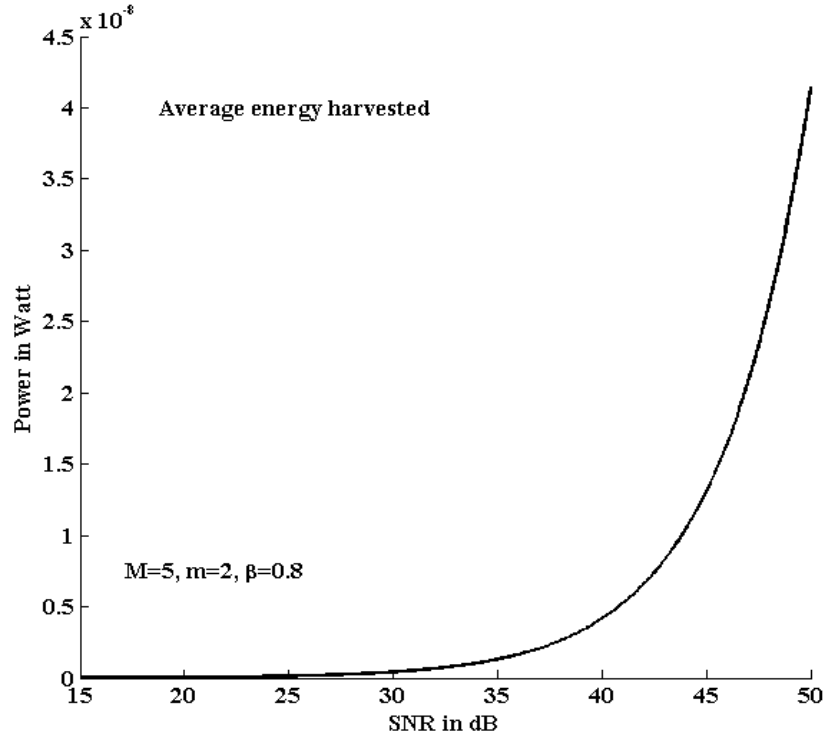


Fig.5 Power vs SNR in dB

From the above plots, we have also observed that the order of diversity has a significant impact on the performance measures such as outage and ABER, which can be easily observed from Figure 2, Figure 3, and Figure 4. However, from the energy harvesting point of view, the improvement is not significant. In this case, the receiver has no fixed power supplies and thus we get the expected power as shown in Figure 5.

5 Conclusion

In this paper, the closed-form mathematical expression including the PDF of output SNR, outage probability and ABER have been derived for M-MRC receiver over K_G fading channels for an arbitrary number of diversity branches and considering the Power splitter factor. The performance expressions are in the form of the Gamma function, Meijer's G-function and Whittaker function, which makes the calculation easier. Numerically evaluated results have been plotted to demonstrate the effect of

fading parameter, power splitter factor on the receiver performance. Finally, the validation of the proposed analytical expressions is carried out by comparing them with Monte-Carlo simulations.

Declarations

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References

1. Stüber, G. L. (2012). *Principles of Mobile Communication* (Vol. 9781461403647). <https://doi.org/10.1007/978-1-4614-0364-7>
2. Simon, M. K., & Alouini, M. S. (2005). *Digital Communication over Fading Channels: Second Edition*. *Digital Communication over Fading Channels: Second Edition*. <https://doi.org/10.1002/0471715220>
3. Tellambura, C. (1997). Analysis of m-ary phase-shift keying with diversity reception for land-mobile satellite channels. *IEEE Transactions on Vehicular Technology*. <https://doi.org/10.1109/25.653065>
4. Shankar, P. M. (2004). Error rates in generalized shadowed fading channels. *Wireless Personal Communications*, 28(3). <https://doi.org/10.1023/B:wire.0000032253.68423.86>
5. Abdi, A., & Kaveh, M. (2000). Comparison of DPSK and MSK Bit Error Rates for K and Rayleigh-Lognormal Fading Distributions. *IEEE Communications Letters*, 4(4). <https://doi.org/10.1109/4234.841317>
6. Aleksic, D., Stefanovic, M., Popovic, Z., Radenkovic, D., & Ristic, J. (2009). On the K and KG fading channels. *Serbian Journal of Electrical Engineering*. <https://doi.org/10.2298/sjee0901187a>
7. Subadar, R., & Sahu, P. R. (2009). Performance analysis of dual MRC receiver in correlated hoya fading channels. *IEEE Communications Letters*, 13(6), 405–407. <https://doi.org/10.1109/LCOMM.2009.090465>
8. Mishra, D., De, S., Jana, S., Basagni, S., Chowdhury, K., & Heinzelman, W. (2015). Smart RF energy harvesting communications: Challenges and opportunities. *IEEE Communications Magazine*. <https://doi.org/10.1109/MCOM.2015.7081078>
9. Zhang, R., & Ho, C. K. (2013). MIMO broadcasting for simultaneous wireless information and power transfer. *IEEE Transactions on Wireless Communications*, 12(5), 1989–2001. <https://doi.org/10.1109/TWC.2013.031813.120224>

10. Van, P. T., Le, H. H. N., Le, M. D. N., & Ha, D. B. (2016). Performance analysis in wireless power transfer system over Nakagami fading channels. In *International Conference on Electronics, Information, and Communications, ICEIC 2016*. <https://doi.org/10.1109/ELINFOCOM.2016.7562971>
11. Zhong, C., Chen, X., Zhang, Z., & Karagiannidis, G. K. (2015). Wireless-Powered Communications: Performance Analysis and Optimization. *IEEE Transactions on Communications*. <https://doi.org/10.1109/TCOMM.2015.2488640>
12. Bithas, P. S., Sagias, N. C., Mathiopoulos, P. T., Karagiannidis, G. K., & Rontogiannis, A. A. (2006). On the performance analysis of digital communications over Generalized-K fading channels. *IEEE Communications Letters*. <https://doi.org/10.1109/LCOMM.2006.1633320>
13. Gradstein, I., & Ryzhik, I. (1994). *Table of Integrals, Series, and Products*. Academic Press. Inc. New York.
14. Tiwari, D., Soni, S., & Chauhan, P. S. (2017). A New Closed-Form Expressions of Channel Capacity with MRC, EGC and SC Over Lognormal Fading Channel. *Wireless Personal Communications*, 97(3), 4183–4197. <https://doi.org/10.1007/s11277-017-4719-9>
15. Wolfram Mathworld <http://functions.wolfram.com>.