A Decomposition and Dynamic Programming Aggregation Method for the Optimal Water Allocation of Reservoirs in Series

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A Decomposition and Dynamic Programming Aggregation Method for the Optimal Water Allocation of Reservoirs in Series

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Abstract

Research on water allocation of multiple reservoirs with the purpose of reducing water spills and improving the local runoff utilization is a matter of great concern in humid areas with uneven temporal and spatial distributions of water resources. An optimization model for a system of reservoirs in series is developed to minimize water shortages. Several constraints restrict the objective function, including available water, operation rules and water rights for replenishment of the reservoirs with water. The model features multiple dimensions with a single coupling constraint of the large-scale system. A decomposition and dynamic programming aggregation method (DDPA) is proposed; the subsystem models and the aggregation model are both solved with the classical one-dimensional dynamic programming. Compared with the conventional decomposition-coordination method, the proposed method is concise but reliable because it can directly use the results of subsystems to form the one-dimensional dynamic programming aggregation model, avoiding the iterative calculations according to the coordinating function. Compared with the meta-heuristic algorithms, the proposed method is more efficient because it is independent of any algorithm parameter. The proposed method may provide a new reference for solving similar multi-reservoir optimization models.

Keywords

reservoirs in series, joint operation, multidimensional dynamic programming, decomposition, aggregation, meta-heuristic algorithm

0 Introduction

Reservoirs are the main water sources in the hilly regions of southern China and southeast Asia. The mean annual rainfall is over 1,000 mm in these humid regions, but 70-80% of that occurs during the flood season due to the monsoon climate (Quinn et al. 2018). Although the annual total inflows into the local reservoirs are usually more than what is needed to meet demands, a large amount of water is drained due to uneven temporal and spatial distribution of inflows, which results in seasonal water shortages. Because of this shortage, it is necessary to replenish a reservoir from other reservoirs during the dry season (Ming et al. 2017; Reca et al. 2015), thus gradually forming a system of reservoirs in series.

The basic principle of joint operation for a multi reservoir system is to redistribute water resources through the hydraulic connections among connected reservoirs (Ahmad et al. 2014). This setting can maximize the storage capacity of each reservoir and reduce loss of water. Thus, the operation purpose of reservoirs in series is to reduce water spills and improve runoff utilization; the decision variable is the amount of water supply in each period, which is subject to the reservoir's capacities including water storage and water supply (Rani et al. 2020; Chang et al. 2019; Birhanu et al. 2014; Sattari et al. 2009). The number of decision variables and constraints increases with the growing number of reservoirs in the system. Furthermore, hydraulic connections among the reservoirs should be added into constraints, which inevitably complicate the model (Ehteram et al. 2017; Ashrafi and Dariane 2017).

Mathematical programming methods are classical algorithms for solving optimal water allocation models of reservoirs. Dynamic programming based on Bellman's principle (Bellman and Dreyfus 1964) is highly applicable to this type of multi-stage decision-making process. However, the 'curse of dimensionality' is induced when dealing with a large number of reservoirs (Cheng et al. 2017; Chen et al. 2016). Decomposition (Turgeon 1981) is the mainstream idea to dealing with multidimensional optimization problems when using dynamic programming. The decomposition-coordination method (Mahey et al. 2017; Li et al. 2014; Tan et al. 2019) is most commonly used and it achieves the optimal state of the whole system through the successive iterative calculations between the large-scale system and subsystems according to the coordinating variables. Therefore, the convergence performance of the coordination variable has a great impact on the final results. In addition, the Lagrange multiplier method (Jafari and Alipoor 2011) is usually used to deal with the constraints when adopting the conventional decomposition-coordination method. However, it is restricted by the differentiability and convexity of the objective function and it may have difficulty in handling some complex constraints, for instance, the constraint which contains if statements.

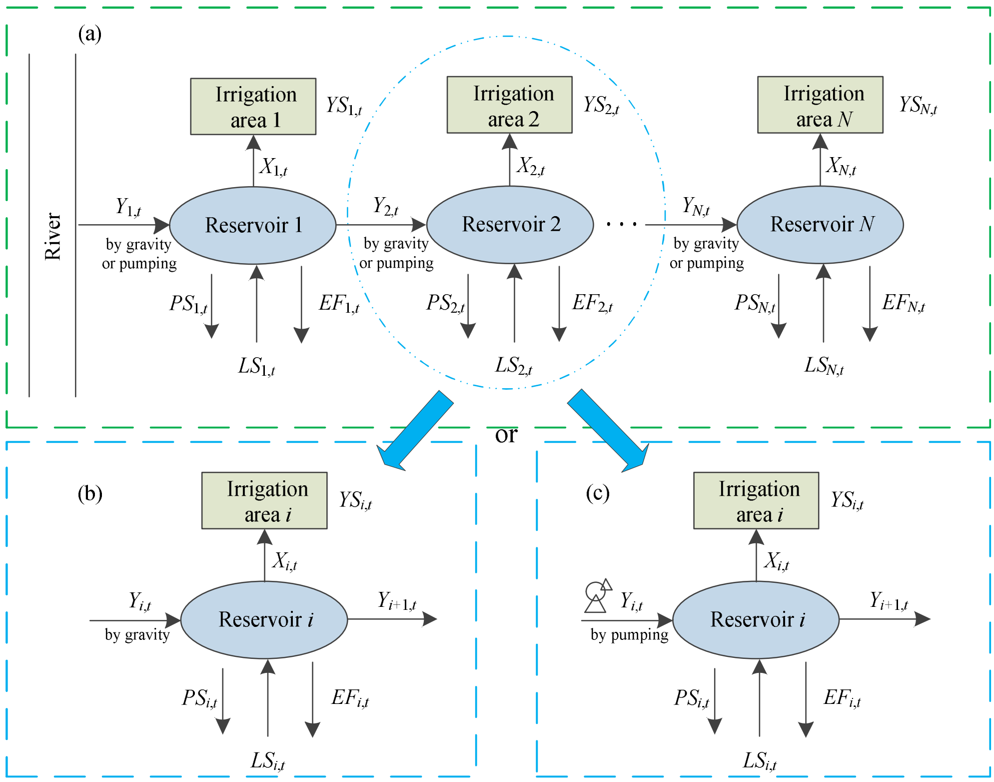
Recently, meta-heuristic algorithms such as the genetic algorithm (Allawi et al. 2018a), the particle swarm optimization algorithm (Chen et al. 2018) and the ant colony algorithm (Moeini et al. 2013) have become the most popular methods for the optimization of reservoir operation models. The greatest advantage of meta-heuristic algorithms is that they can solve the multidimensional model without the decomposition operations. However, the globally optimal final results cannot be guaranteed because decision variables are randomly sampled and updated within the feasible regions according to the specific iteration rules, which is a common limitation among these algorithms (Hossain and El-shafie 2013; Allawi et al. 2018b). Furthermore, when the meta-heuristic algorithms are adopted, a constrained optimization problem usually transforms into an unconstrained one by penalty functions or other methods (Wan et al. 2016; Pina et al., 2017). Nevertheless, the meta-heuristic algorithms may fail to handle models with some complicated constraints. What is worse, the algorithm parameters, such as the crossover and mutation rates in GA and the inertia weight and acceleration coefficients in PSO, cannot be clearly determined despite the fact that these values are important factors which can affect the performances of algorithms (Zhang and Dong 2019). There are only empirical value ranges for most algorithm parameters. Various approaches for setting parameters such as the deterministic strategy (Draa et al. 2015) and the adaptive strategy (Harrison et al. 2018; Cui et al. 2016) are derived but they are not proven to be of universal significance (Karafotias et al. 2015).

In light of the shortcomings of the aforementioned algorithms, a decomposition and dynamic programming aggregation method (DDPA) is proposed for the optimal water allocation of reservoirs in series. The method was applied to a system of dual reservoirs and dual pumping stations in Nanjing, China to verify its performance.

1 Model and method

1.1 Description of the system

A system of reservoirs in series is a common water supply system in hilly regions of southern China. There are several reservoirs with hydraulic connections between each other in the system as shown in Figure 1(a). The reservoirs usually operate jointly but have their own supply objects. During the operation period, each reservoir in the system can be replenished with water from another reservoir or from an outside river by pumping or gravity.



Note: (a) a system of reservoirs in series; (b) a subsystem of a reservoir; (c) a subsystem of a reservoir and a pumping station

Figure 1 A system of reservoirs in series

In Figure 1: $X_{i,t}$ (L^3), $YS_{i,t}$ (L^3), $LS_{i,t}$ (L^3), $Y_{i,t}$ (L^3), $PS_{i,t}$ (L^3) and $EF_{i,t}$ (L^3) are water supply, water demand, local inflow, water replenishment, water spill and evaporation of reservoir i in period t, respectively, N is the total number of reservoirs in the system, i is the sequence number of reservoir, i = 1, 2,…, N, and t is the sequence number of each period.

1.2 Optimization model

1.2.1 Objective function

The operation cycle of the system in this study is one year, so the objective function is to minimize the annual sum of the squared water shortage of each reservoir (Celeste and Billib 2009; Jothiprakash et al. 2011), which can be expressed as Equation (1):

$$\min F = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{i,t} - YS_{i,t}\right)^{2} \tag{1}$$

where F is the annual sum of squared water shortage of each reservoir in each period and T is the total number of periods.

1.2.2 Constraints

① Annual available water of the whole system

The annual available water of the whole system includes the total available water of the reservoirs and the available volume from outside water sources limited by the water rights, which can be expressed as Equation (2):

$$\sum_{i=1}^{N}\sum_{t=1}^{T} X_{i,t} \le SK + BZ \tag{2}$$

where SK is the total annual available water of the reservoirs and BZ is the maximum volume that can be acquired from outside water sources.

② Annual available water of reservoir i

The annual available water of each reservoir is the sum of local inflows and water replenishments from other water sources, which can be expressed as Equation (3):

$$\sum_{t=1}^{T} X_{i,t} = W_i \quad (i=1,\ 2,\ \ldots,\ N) \tag{3}$$

where W_i (L^3) is the annual available water of reservoir i, which should meet the requirement of Equation (4):

$$\sum_{i=1}^{N} W_i \le SK + BZ \tag{4}$$

③ The operation rule of reservoir

The lower and upper bounds of each reservoir's water storage in each period can be expressed as Equation (5):

$$V_{i,t}^{\min} \le V_{i,t} \le V_{i,t}^{\max} \tag{5}$$

where $V_{i,t}$ (L^3) is the water storage of reservoir *i* during period *t* and $V_{i,t}^{\min}$ (L^3) and $V_{i,t}^{\max}$ (L^3) are the lower and upper bounds of storage during period *t*, respectively.

According to the water balance equation, the water storage of reservoir *i* in period *t* can be determined by Equation (6):

$$V_{i,t} = V_{i,t-1} + LS_{i,t} - X_{i,t} - Y_{i+1,t} - EF_{i,t} \tag{6}$$

The water replenishment and water spill of each period can be determined according to the lower and upper bounds of water storage as follows:

a. If $V_{i,t} < V_{i,t}^{\min}$, then the reservoir should be replenished with water. $Y_{i,t}$ and $PS_{i,t}$ can be determined by Equation (7) and Equation (8), respectively.

$$Y_{i,t} = \min(V_{i,t}^{\min} - V_{i,t} \;,\; Y_{i,t}^{\max}) \tag{7}$$

$$PS_{i,t} = 0 \tag{8}$$

b. If $V_{i,t} > V_{i,t}^{\max}$, then excess water should be released. $Y_{i,t}$ and $PS_{i,t}$ can be determined by Equation (9) and Equation (10), respectively.

$$Y_{i,t} = 0 \tag{9}$$

$$PS_{i,t} = V_{i,t} - V_{i,t}^{\max} \tag{10}$$

c. If $V_{i,t}^{\min} \le V_{i,t} \le V_{i,t}^{\max}$, then both of $Y_{i,t}$ and $PS_{i,t}$ should be zero, shown as Equation (11).

$$Y_{i,t} = PS_{i,t} = 0 \tag{11}$$

The correct water storage of each period can be obtained by Equation (12).

$$V_{i,t}{}' = V_{i,t} - PS_{i,t} + Y_{i,t} \tag{12}$$

④ Maximum water supply can be expressed as Equation (13)

$$X_{i,t} \le YS_{i,t} \tag{13}$$

⑤ Maximum water replenishment is restricted by the pumping capacity or water conveyance capability, which can be expressed as Equation (14):

$$Y_{i,t} \le Y_{i,t}^{\max} \tag{14}$$

where $Y_{i,t}^{\max}$ (L^3) is the maximum water replenishment of reservoir *i* in period *t*.

⑥ Initial and boundary conditions

A restriction was imposed to make the final storage of the reservoir equal to the initial storage. If this restriction is not imposed, the solution would tend to empty the reservoir in the final period.

⑦ Non-negative constraints

It is also necessary to introduce non-negativity constraints to avoid negative values, which would be physically impossible.

1.3 Solving methods

In the above model, the constraints from □ to □ can be transformed into the feasible regions of the decision variable $X_{i,t}$. Therefore, it is a dynamic programming problem of N+1 dimensions, which is restricted by the constraints □ and □. A decomposition and dynamic programming aggregation method for this optimal water allocation of reservoirs in series is proposed in this paper, which solves both of the subsystem models and the aggregation model with dynamic programming. In order to evaluate the solving performance, two commonly used meta-heuristic algorithms, the genetic algorithm and particle swarm optimization algorithm, are adopted simultaneously.

1.3.1 Decomposition and dynamic programming aggregation

① Decomposition of the system

The 'curse of dimensionality' will inevitably arise if using dynamic programming (DP) to solve the model directly, hence it is necessary to first reduce the dimensions. The large-scale system will then be decomposed into several subsystems and each subsystem only consists of a reservoir or a reservoir and a pumping station as shown in Figure 1(b) and (c).

② Optimization of subsystems

The optimization model of the subsystem is expressed as Equation (15) and (16), where the objective function is expressed as

$$\min f_i = \sum_{t=1}^{T} (X_{i,t} - YS_{i,t})^2 \tag{15}$$

and the annual available water of subsystem is expressed as

$$\sum_{t=1}^{T} X_{i,t} = W_i \tag{16}$$

where f_i is the annual sum of squared water shortage in each period of subsystem i.

Moreover, the lower and upper bounds of water storage, water balance equation, initial and boundary conditions should also be imposed on each subsystem, which can be considered in the operation rule. The operation rule is integrated into recursive procedures of DP to correct water storage $V_{i,t}$ and obtain water spill $PS_{i,t}$ and water replenishment $Y_{i,t}$ of each period simultaneously. In this manner, one-dimensional dynamic programming can be adopted to solve the subsystem models as shown in Figure 2(a).

③ Aggregation of the system

For subsystem i, the annual available water W_i can be discretized in its feasible region, and the objective function value f_i can be obtained under each W_i. After that, the aggregation model can be formed directly using the optimization results of subsystems $f_i(W_i)$, which can be expressed as Equation (18) and (19).

Objective function:

$$\min F = \sum_{i=1}^{N}\sum_{t=1}^{T}\left(X_{i,t} - YS_{i,t}\right)^2 = \sum_{i=1}^{N} f_i(W_i) \tag{18}$$

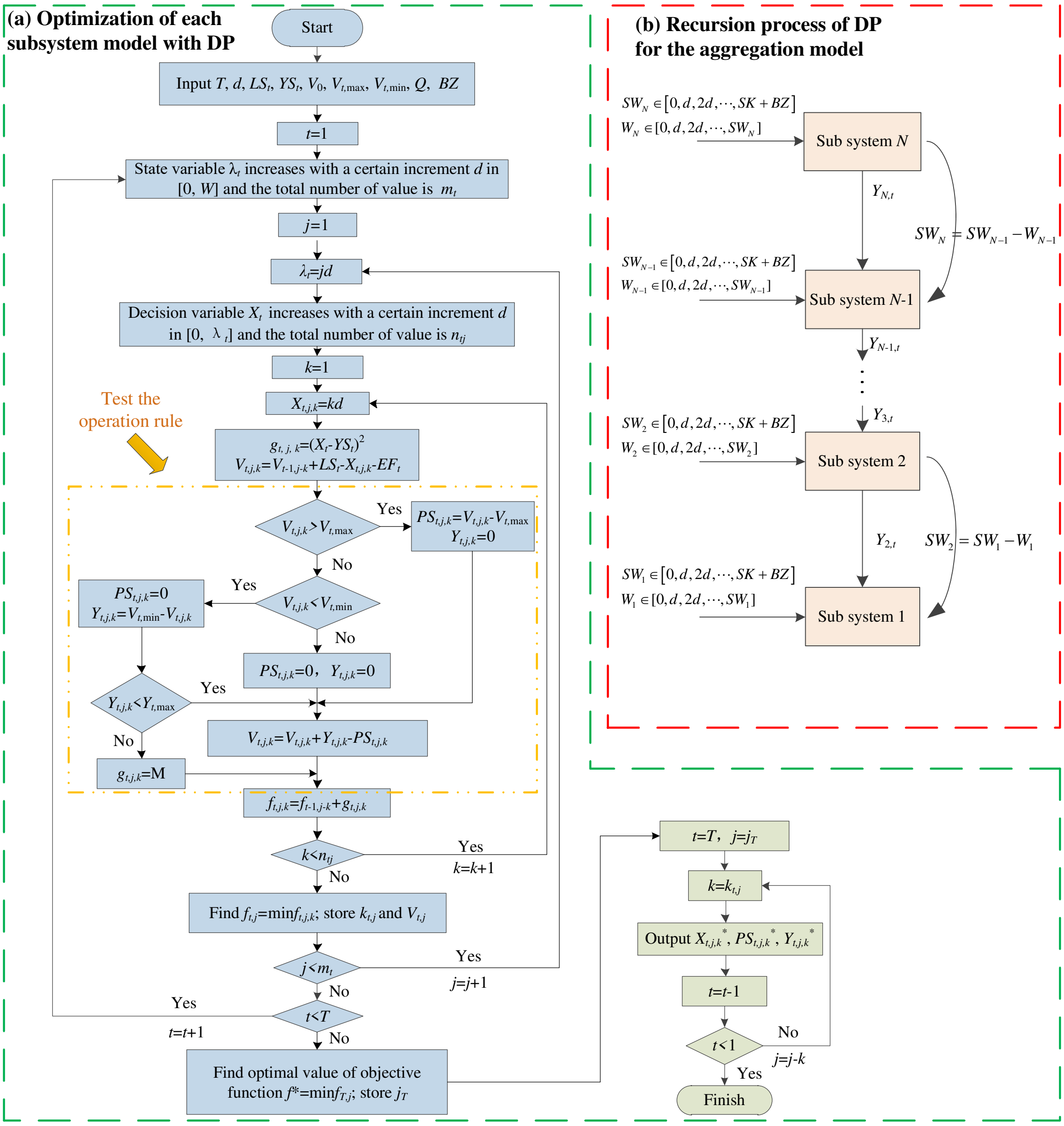
Coupling constraint:

$$\sum_{i=1}^{N}\sum_{t=1}^{T} X_{i,t} = \sum_{i=1}^{N} W_i \le SK + BZ \tag{19}$$

The aggregation model is also a typical one-dimensional dynamic programming model, where the decision variable is the annual available water of subsystem W_i . The inverse recursion of DP should be applied as shown in Figure 2(b), so that the water replenishment from subsystem i-1 to subsystem i can be known when the model of subsystem i-1 is optimized. The state transition equation can be expressed as Equation (20)

$$SW_i = SW_{i-1} - W_{i-1} \quad (i=2,\ 3,\ \ldots,\ N) \tag{20}$$

where SW_i (L^3) is the sum of available water in subsystems from i to N, SW_{i-1} (L^3) is the sum of available water in subsystems from i-1 to N, and W_{i-1} (L^3) is the available water in subsystem i-1.



Note: d is a certain increment; the aggregation model is optimized in reverse order so that Y_{i+1} is a known quantity when V_i is calculated according to Equation (6); M is a positive number, which is large enough.

Figure 2 Flow chart of DDPA

After obtaining the optimal results of the aggregation model F^* and $[W_1{}^*, W_2{}^*, \ldots, W_N{}^*]$, the optimization results $[X_{i,t}, PS_{i,t}, Y_{i,t}]^*$ of each subsystem can be acquired by searching the results of the subsystem models according to $W_i{}^*$, which can finally compose an optimal operation scheme of the whole system.

In conclusion, the essence of the decomposition and dynamic programming aggregation method is to transform the N+1 dimensional dynamic programming into the N+1 iterative calculations of one-dimensional dynamic programming. In this process, both of the subsystem models and the aggregation model are one-dimensional dynamic programming models. Therefore, the ‘curse of dimensionality’ can be avoided and the global optimal results can be acquired based on Bellman’s principle.

1.3.2 Genetic algorithm and particle swarm optimization algorithm

① Genetic algorithm

The genetic algorithm (GA) has been successfully applied to the different kinds of optimal reservoir operation problems (Zhu et al. 2014; Tsai et al. 2019). The optimizing procedure of GA is based on the natural selection: initial chromosomes (solutions) are randomly generated first and the next generations are updated through the selection, crossover and mutation operators until the termination criterion is met. In each iteration, the selection operator is carried out according to the principle of roulette. Equation (21) and Equation (22) are used for the crossover and mutation operators, respectively,

$$X_{i,t}^{\mathrm{child}} = \alpha X_{i,t}^{\mathrm{par1}} + (1-\alpha) X_{i,t}^{\mathrm{par2}} \tag{21}$$

$$X_{i,t}^{new} = \beta Y S_{i,t} \tag{22}$$

where $X_{i,t}^{\mathrm{child}}$ is an individual offspring generated through the crossover operator, $X_{i,t}^{\mathrm{par1}}$ and $X_{i,t}^{\mathrm{par2}}$ are both parent individuals, $X_{i,t}^{\mathrm{new}}$ is a new individual generated through the mutation operator, and α and β are both random values that vary in [0, 1].

② Particle swarm optimization algorithm

The particle swarm optimization algorithm (PSO) has also been demonstrated to be appropriate for coping with this type of problem (Wan et al. 2016; Yousefi et al. 2018). The optimizing procedure of PSO starts with a number of initial particles provided stochastically in the feasible region. Each particle's position vector and velocity vector are updated according to Equations (23) and (24) in each iteration until the termination criterion is met:

$$\upsilon_s(k) = \omega \upsilon_s(k-1) + c_1 r_1 (p_s(k-1) - \theta_s(k-1)) + c_2 r_2 (g(k-1) - \theta_s(k-1)) \tag{23}$$

$$\theta_s(k) = \theta_s(k-1) + \upsilon_s(k) \tag{24}$$

where υ_s is the velocity vector of particle s, θ_s is the position vector of particle s, k is the number of iteration times, ω is the inertia weight, p_s is the best position of particle s, g is the global best solution among all the particles, c_1 and c_2 are acceleration coefficients and r_1 and r_2 are random values that vary in [0, 1].

2 Case study

In order to compare the performances of the three algorithms, the model is applied to a dual-reservoir-and-dual-pumping-station system in Nanjing, China, which consists of the SH reservoir, the HWB reservoir and two pumping stations, HZ and XZ, as shown in Figure 3. The system is located at 32° 27' N, 118° 46' E, in the subtropical monsoon climate zone. The mean annual rainfall is 1002.7 mm in this region but 73.6% of the annual rainfall occurs between June and September during the flood season.

The system provides irrigation water and each reservoir provides water independently to its own irrigation area. The average topography of the HWB reservoir is much higher than that of the SH reservoir. During water shortages, HZ pumping station can replenish the HWB reservoir with water from the SH reservoir, while the SH reservoir can be replenished with water from the BL river through the XZ pumping station.

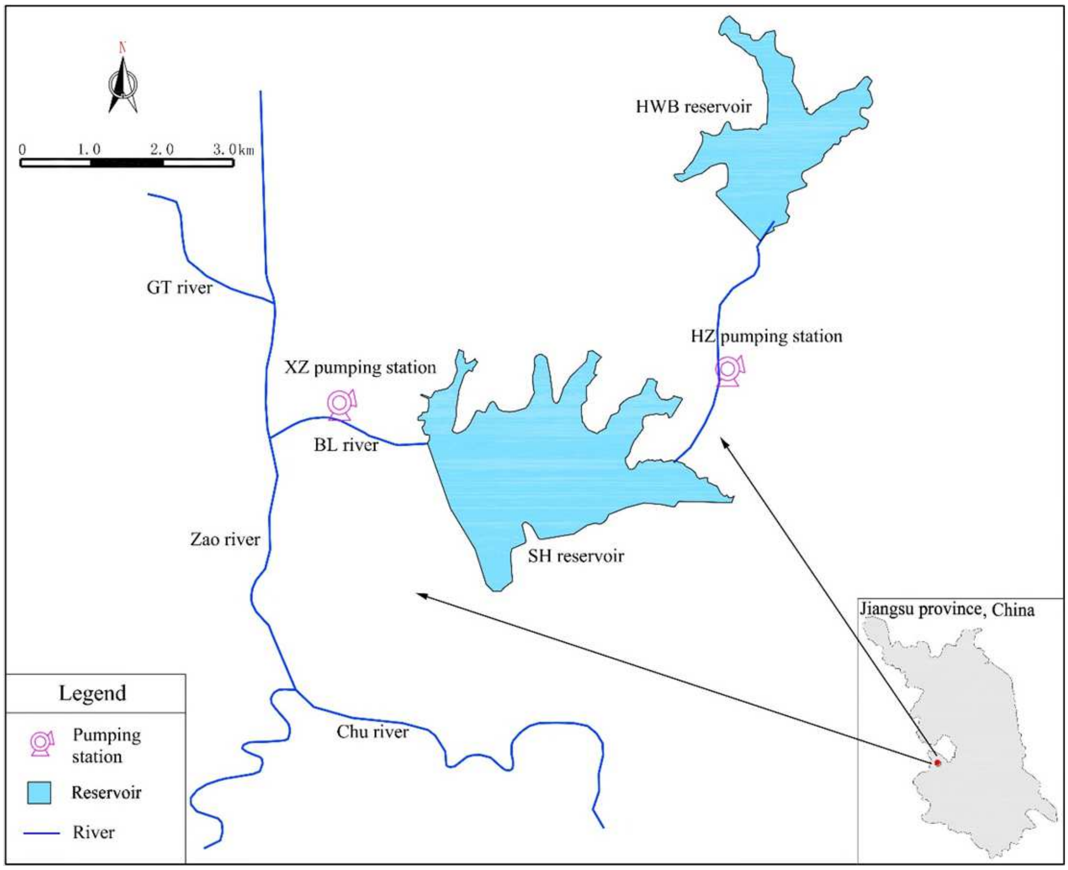


Figure 3 Location of the system

① Reservoir and pumping station

The characteristics of reservoirs and pumping stations are shown in Tables 1 and 2, respectively.

Table 1 Characteristics of reservoirs

Reservoir	Dead storage capacity (10^4 m^3)	Utilizable capacity (10^4 m^3)	Total storage capacity (10^4 m^3)	Limited storage capacity in flood season (10^4 m^3)	Irrigation area (hm^2)
SH	600	1157	2473	1000	1667
HWB	457	936	1891	1393	2067

Table 2 Characteristics of pumping stations

Pumping station	Design discharge (m^3/h)	Design pumping head (m)	Maximum daily operation duration (h)	Water rights (75%) (10^4 m^3)
XZ	10000	19.4	20	360
HZ	7500	15.0	20	/

Notes: The annual water rights of the XZ pumping station, which means the maximum volume that can be pumped from the BL river, are allocated by the local water authority; the HZ pumping station is an internal pumping station in the system and is not defined or limited by any water rights.

② Inflows and water demands

The inflow and water demand in each month at 75% probability of exceedance are shown in Table 3.

Table 3 Monthly inflows and water demands unit: 10^4 m^3

Reservoir	Category	Period												Total
		Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	
SH	Inflows	115	45	11	32	2	28	32	50	445	781	618	320	2479
	Demands	23	15	18	17	23	26	30	31	523	323	159	33	1221
HWB	Inflows	33	55	1	36	10	9	37	56	218	133	157	20	766
	Demands	59	79	17	16	23	27	31	35	550	559	345	77	1817

③ Evaporation

Evaporation loss is a function of evaporation depth and average free water surface of a reservoir in each period, which can be described as Equation (25). The average free water surface of each reservoir in a specific period is determined by its surface-volume relationship, which was provided by Liuhe Water Authority.

$$EF_t = 0.1 \times k_t \times E_t \times (\alpha V_t + \beta) \tag{25}$$

where EF_t (10^4 m^3) is the evaporation loss of a reservoir in period t, E_t (mm) is the evaporation depth of E_{601} evaporator in period t as shown in Table 4, k_t is the correction coefficient for period t as shown in Table 4, V_t (10^4 m^3) is the average water storage in period t and α and β are the reservoir coefficients. For the SH reservoir, α = 1.194×10^{-3}, β = 2.575, and for the HWB reservoir, α = 1.657×10^{-3}, β = 1.862.

Table 4 E_t and k_t of each month

Period	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.
E_t (mm)	56	42	38	23	28	41	58	86	98	115	110	71
k_t	1.04	1.12	1.12	1.05	0.92	0.9	0.88	0.92	0.94	0.94	0.98	1.06

3 Results and analysis

3.1 Optimization Results

The results obtained with different algorithms are shown in Table 5. The conventional operation of the system was conducted in Excel-2010 according to the standard operation policy (SOP), while the other three optimization methods were programmed in Visual Basic language based on a PC with i3 CPU/3.70 GHz/4 GB ram.

Compared with the results using SOP, the water allocation of the system can be optimized by using the optimization methods. By increasing the water diverted from the SH reservoir to the HWB reservoir, the total water spill of the SH reservoir and the total water shortage of the HWB reservoir were reduced. Among the three optimization methods, DDPA minimized the objective function better than the other algorithms. The value of objective function obtained with DDPA was decreased by 7.5% and 5.6% when compared to GA and PSO, respectively.

Although the amount of water replenishment from the BL river was same, the total water spill of the system obtained with DDPA was reduced by 2.0% while the total water shortage was reduced by 5.3% and 10.0% when compared to GA and PSO, respectively.

It is indicated the utilization rate of local runoff in the DDPA results is relatively higher on premise of the same external water rights.

On the other hand, the operating time of DDPA was 3.85 times and 3.10 times than GA and PSO respectively, though the results obtained with DDPA were better. As shown in Table 5, GA and PSO are more efficient in each operation when the algorithm parameters are determined. However, the operating time is quite longer when considering the tests of the appropriate algorithm parameters of GA and PSO, which is discussed in the next section.

Table 5 Solving results of different methods unit: 10^4 m^3

Methods	Reservoir	Water supply	Water spill	Water replenishment	Water shortage	Evaporation	Objective function	Single operation time（s）	Total operation time（s）
SOP	SH	1221	241	360	0	278	/	/	/
	HWB	1663	0	1099	154	202			
	Total	2884	241	1459	154	480			
DDPA	SH	1221	98	356	0	272	234	24.05	24.05
	HWB	1799	0	1243	18	210			
	Total	3020	98	1599	18	482			
GA	SH	1221	100	356	0	272	251	6.24	1682
	HWB	1798	0	1241	19	209			
	Total	3019	100	1597	19	481			
PSO	SH	1221	100	356	0	272	248	7.77	1533
	HWB	1797	0	1241	20	210			
	Total	3018	100	1597	20	482			

Notes: (1) The conventional operation was conducted according to the SOP regardless of the objective function value and operation time; (2) the objective function value of GA is the minimum value among five runs when the crossover rate, mutation rate and population are 0.7, 0.03 and 200, respectively; (3) the objective function value of PSO is the minimum value among five runs when the inertia weight, acceleration coefficients and population are 0.8, 2.1 and 200, respectively; (4) the single operation time is an average value when the algorithm parameters are determined; (5) the total operation time contains the time consumed for testing the appropriate algorithm parameters of GA and PSO.

3.2 Evaluation indexes

The volumetric reliability and vulnerability indexes (Chanda et al. 2014) were selected to evaluate the performances of different algorithms. The volumetric reliability index is expressed as Equation (26), which indicates the ratio of water supply to water demands. A high percentage of this index is desirable (Asefa et al. 2014). The vulnerability index is expressed as Equation (27), which represents the maximum failure rate during the operation. A low percentage of this index is desirable (Asefa et al. 2014).

$$\mathrm{Re}l_i = \frac{\sum_{t=1}^{T}(X_{i,t}/YS_{i,t})}{T} \tag{26}$$

$$Vul_i = \max_{t=1}^{T}\left\{1 - \frac{X_{i,t}}{YS_{i,t}}\right\} \tag{27}$$

where Rel_i (%) is the volumetric reliability index of reservoir i and Vul_i (%) is the vulnerability index of reservoir i.

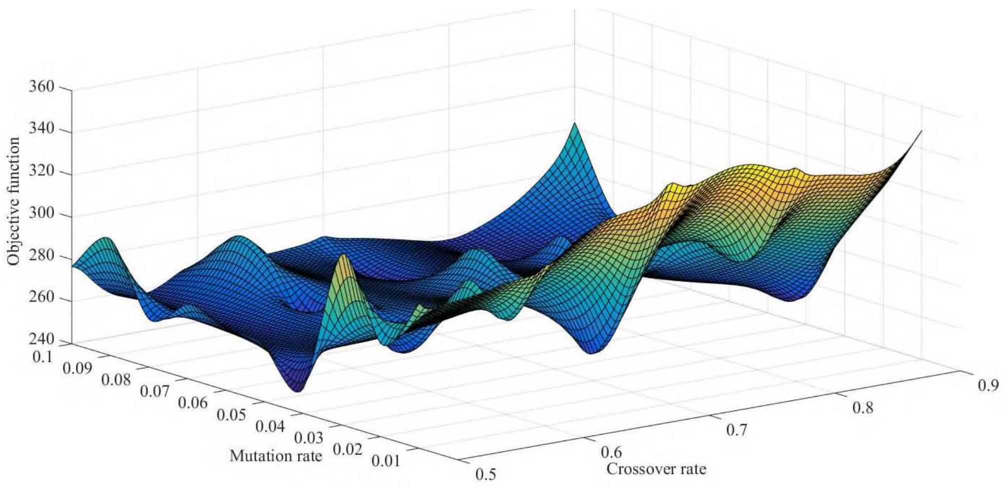
The volumetric reliability and vulnerability indexes of different methods are shown in Table 6. For the SH reservoir, the water demand can be met entirely using the SOP or optimization methods because the available water in the reservoir is much more than the water demand. For the HWB reservoir, the volumetric reliability index of DDPA is 99.51%, which is 1.5%, 0.02% and 0.05% more than that of SOP, GA and PSO, respectively. The vulnerability indexes of DDPA and GA are both 8.20%, which are 20.93% and 0.54% less than that of SOP and PSO, respectively. Thus, the performance of DDPA is relatively better because it can simultaneously achieve a high percentage of volumetric reliability index and a low percentage of vulnerability index.

Table 6 Volumetric reliability & vulnerability

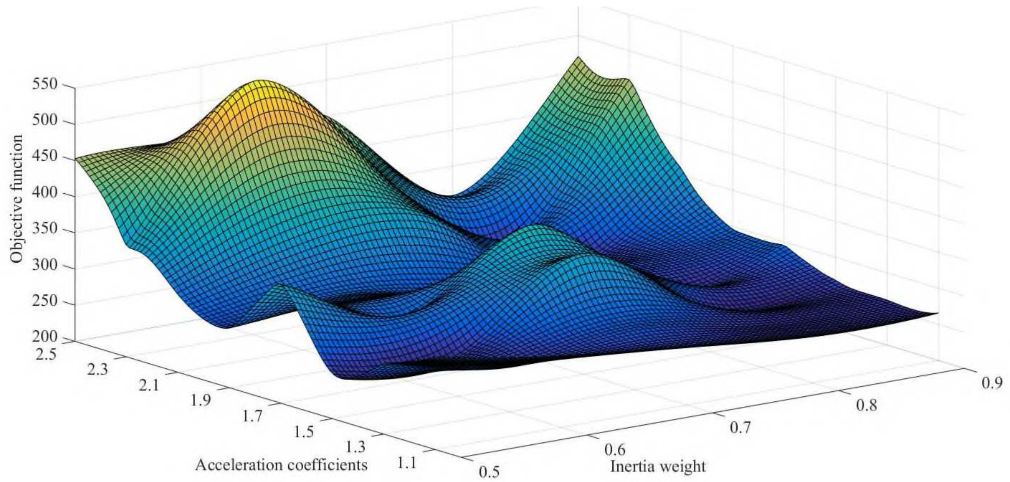
Reservoir	Methods	Volumetric reliability (%)	Vulnerability (%)
SH	SOP	100	0
	DDPA	100	0
	GA	100	0
	PSO	100	0
HWB	SOP	98.01	29.13
	DDPA	99.51	8.20
	GA	99.49	8.20
	PSO	99.46	8.74

4 Comparison and discussion

The random parameters of different meta-heuristic algorithms have a significant influence on the performances of the algorithms. Objective function values obtained with different algorithms under different combinations of parameters are shown in Figure 4. The crossover rate and mutation rate of GA vary from 0.5 to 0.9 and 0.01 to 0.1, respectively, while the inertia weight and acceleration coefficients (c_1=c_2) of PSO vary from 0.5 to 0.9 and 1.1 to 2.5, respectively. The population sizes of the two algorithms are both 200.



(a) Genetic algorithm



(b) Particle swarm optimization algorithm

Notes: Each objective function value under different combinations of parameters is the minimum value among five runs.

Figure 4 The objective function values obtained with GA and PSO

As shown in Figure 4, the most appropriate crossover rate and mutation rate of GA are 0.7 and 0.03, respectively, and the corresponding optimum objective function value is 251. The most appropriate inertia weight and acceleration factors of PSO are 0.8 and 2.1, respectively, and the corresponding optimum objective function value is 248. The optimum objective function value obtained with DDPA is 234 and the difference among the three algorithms is minor, which can verify the reliabilities of each other.

Nevertheless, the results obtained with GA or PSO depend on the algorithm parameters, which can be quite different depending on various factors. On the contrary, there is no additional parameter in DDPA and the dynamic programming based on Bellman’s principle can guarantee the global optimal results. Therefore, the final optimal results achieved by DDPA are always the same while the actual operation time just varies within a minor range. Correspondingly, if considering the time consumed for testing the algorithm parameters, the operation time of GA or PSO would be much longer than that of DDPA as shown in Table 5.

5 Conclusion

This study developed an optimization model for the system of reservoirs in series to minimize water shortage, which is restricted by available water, operation rules and water rights for replenishment with water. A decomposition and dynamic programming aggregation method was proposed to resolve this model. The primary characteristic of the method is that the subsystem models and the aggregation model are both solved with the classical one-dimensional dynamic programming. Therefore, it can reduce the dimension by transforming the N+1 dimensional dynamic programming into the N+1 iterative calculations of one-dimensional dynamic programming.

Compared with the conventional decomposition-coordination method, the proposed method is concise but reliable because it can directly use the results of subsystems to form the one-dimensional dynamic programming aggregation model, avoiding the iterative calculations according to the coordinating function. Compared with the meta-heuristic algorithms, the proposed method is more efficient because it is independent of any algorithm parameter.

Declarations

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Conflicts of interest/Competing interests: none.

Availability of data and material (data transparency): all of data can be found in this paper.

Code availability: the codes for DDPA, GA and PSO and detailed solving results are available at https://github.com/ChengJilin/Codes-in-manuscript-for-WRM-.git.

Ethics approval: not applicable.

Consent to participate: not applicable.

Consent for publication: not applicable.

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Figures

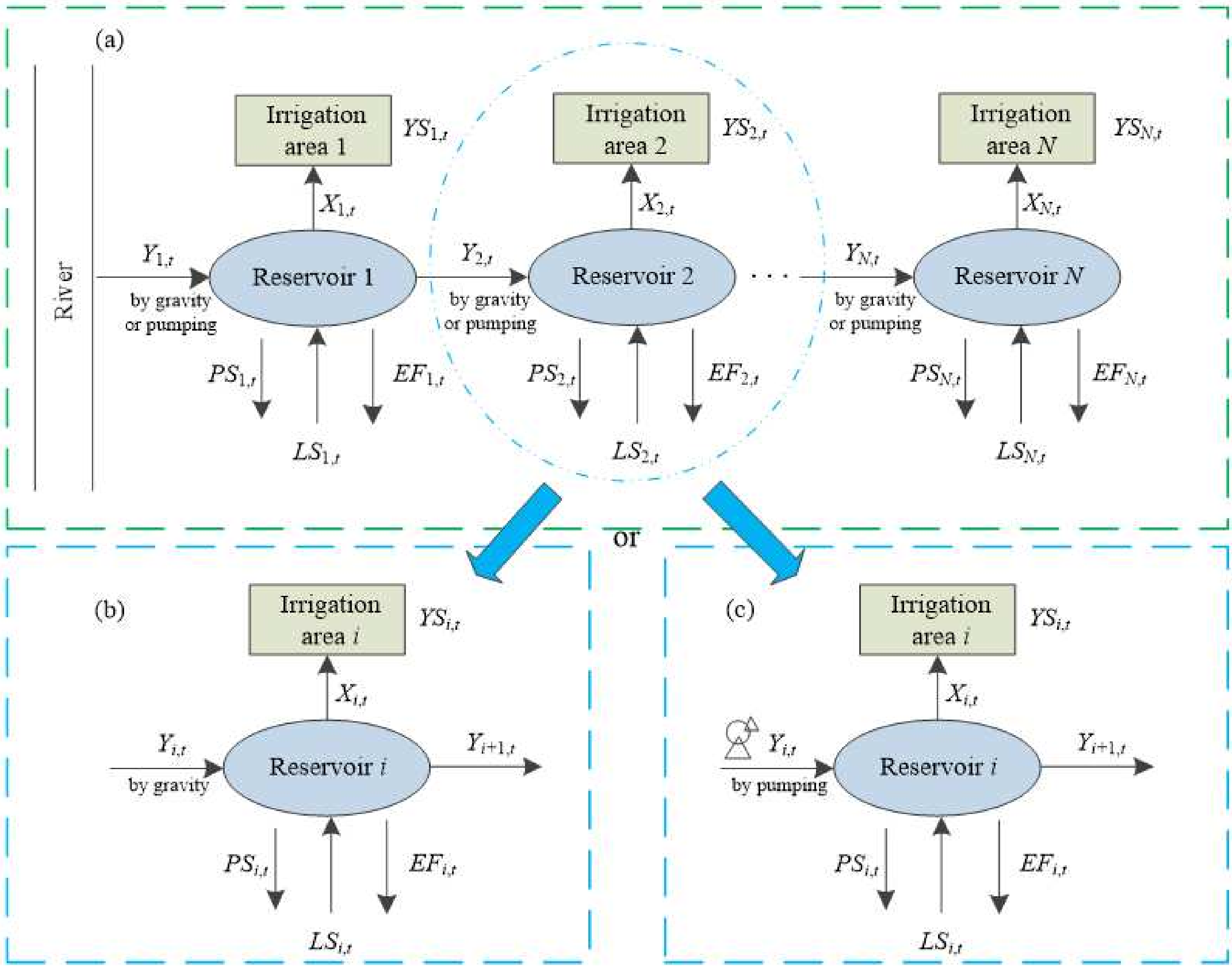

Figure 1

A system of reservoirs in series Note: (a) a system of reservoirs in series; (b) a subsystem of a reservoir; (c) a subsystem of a reservoir and a pumping station

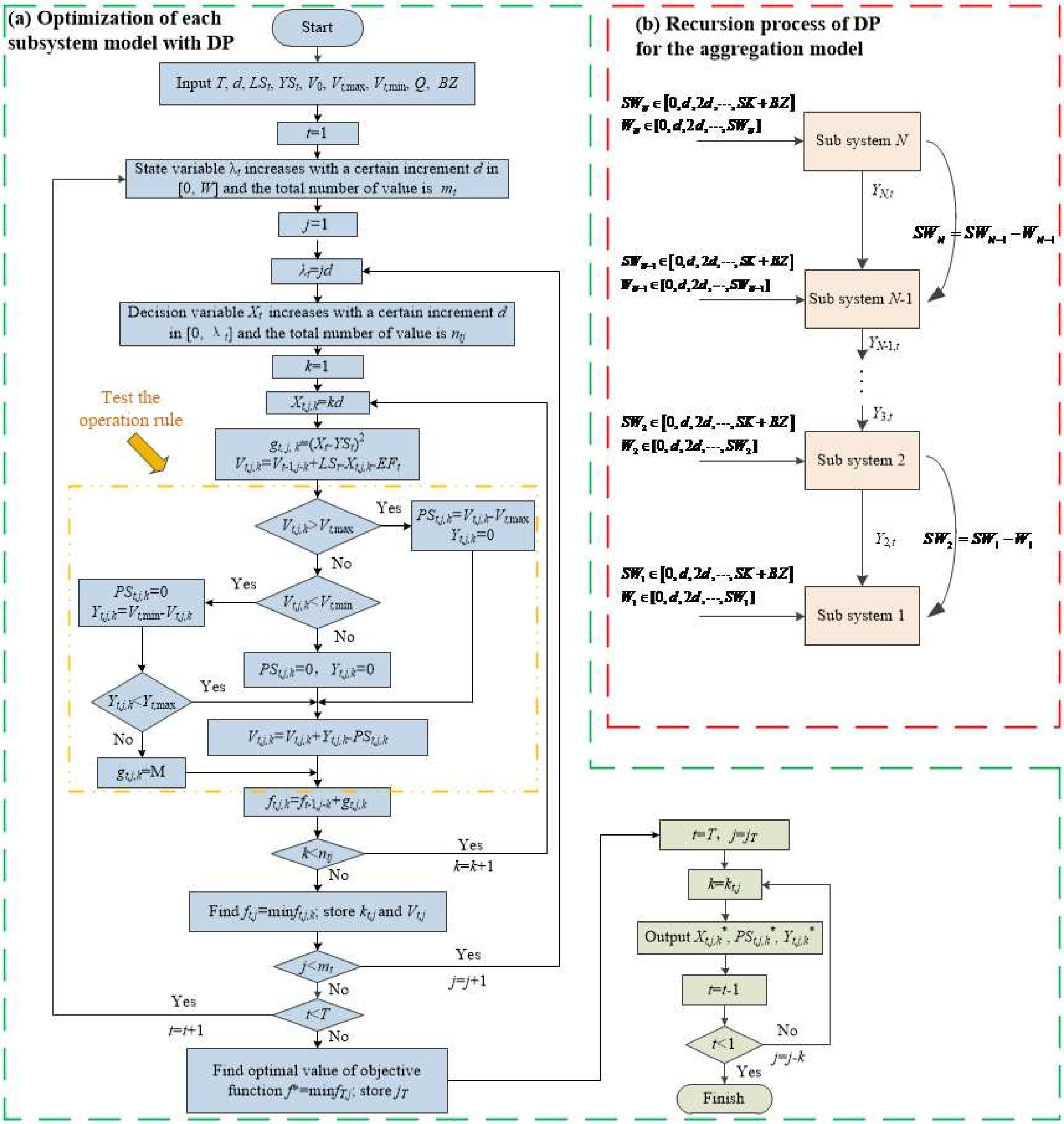

Figure 2

Flow chart of DDPA Note: d is a certain increment; the aggregation model is optimized in reverse order so that Yi+1 is a known quantity when Vi is calculated according to Equation (6); M is a positive number, which is large enough.

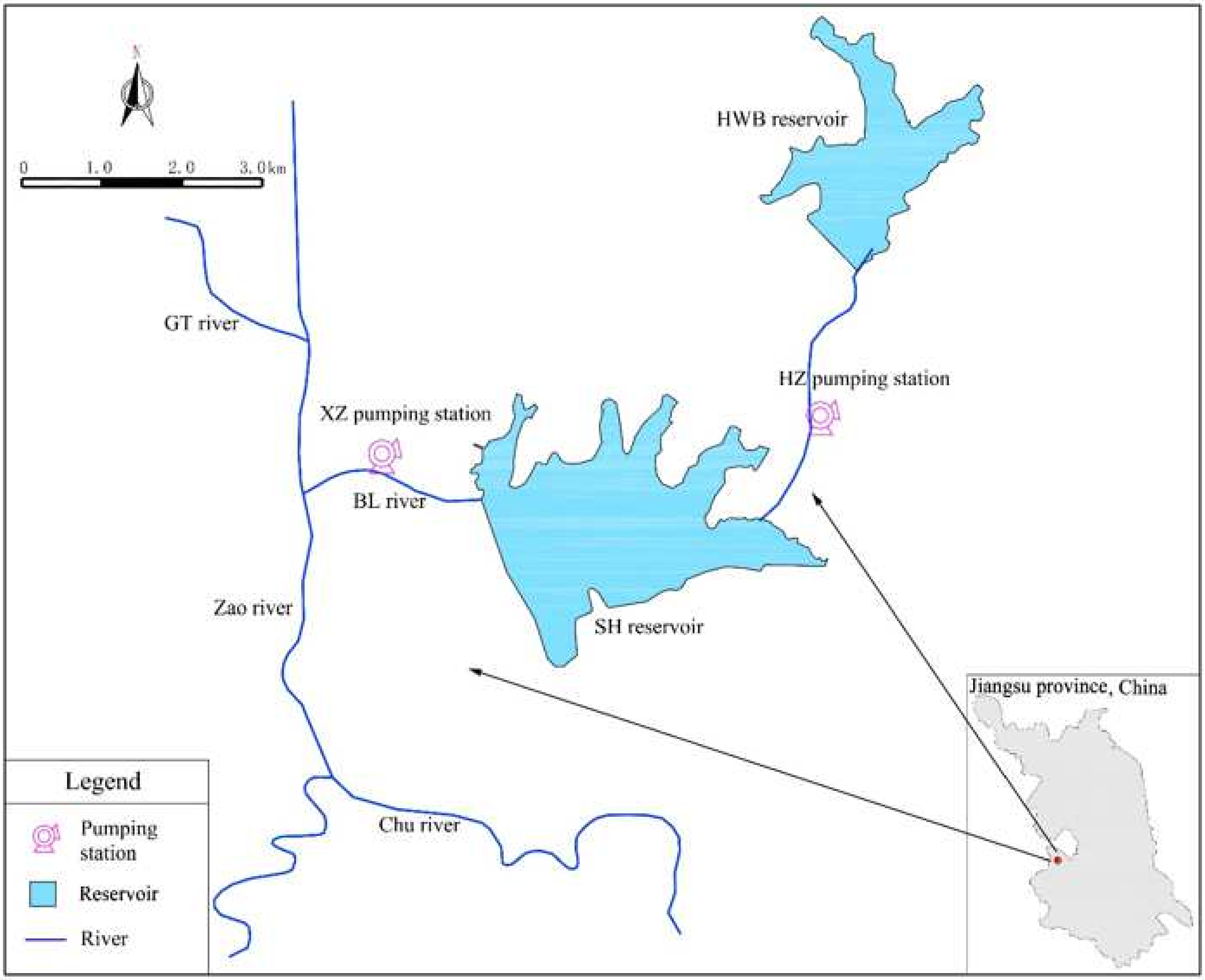

Figure 3

Location of the system Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

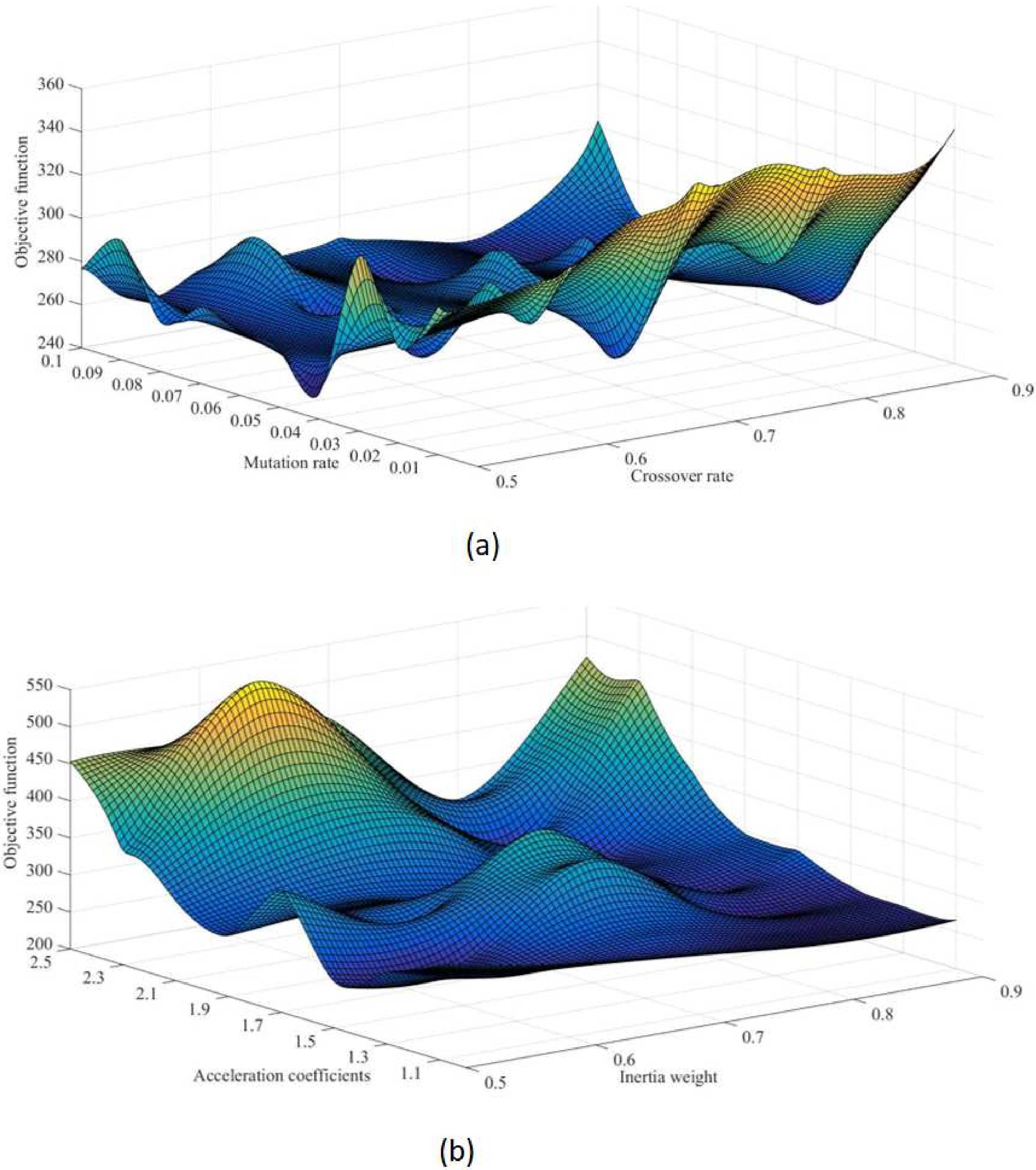

Figure 4

The objective function values obtained with GA and PSO (a) Genetic algorithm (b) Particle swarm optimization algorithm Notes: Each objective function value under different combinations of parameters is the minimum value among five runs.