

# Kinetic Modeling of Enzymatic Hydrolysis of Cellulosic Material

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## Research Article

**Keywords:** Cellulosic material, Cellobiose, Enzymatic hydrolysis, Glucose, Kinetic model

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**Kinetic modeling of enzymatic hydrolysis of cellulosic material**

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30 **Abstract**

31 Development of a kinetic model for analysis of processes for fermentable sugars production is a significant  
32 challenging issue due to the complexity of the enzymatic hydrolysis of cellulose. This paper presents a useful  
33 mathematical model for simulation and evaluation of enzymatic hydrolysis of cellulosic material. The  
34 simulation results were compared with a set of experimental results reported in the literature, in order to validate  
35 the proposed model. A comparison was also made between the proposed model and another model previously  
36 described in the literature. Numerical results indicate that the proposed model gives a more accurate prediction  
37 of time course production of glucose, cellobiose, and cellulose during enzymatic hydrolysis of cellulosic materials.

38

39 **Keywords:** Cellulosic material; Cellobiose, Enzymatic hydrolysis; Glucose; Kinetic model

40 **1. Introduction**

41 Cellulose as a major constituent of all plant materials is a linear biopolymer of glucose molecules,  $(C_6H_{10}O_5)_n$   
42 (Girometta, Zeffiro et al. 2017; Tian, Lu et al. 2017). Enzymatic hydrolysis of cellulose is performed by  
43 cellulases enzymes including Endo-Glucanase, Exo-Glucanase (cellobiohydrolases) and  $\beta$ -Glucosidase  
44 (cellobiase) under mild conditions (pH 4.5-5.0 and temperature 40–50°C) (Singhania, Saini et al. 2015; Niu,  
45 Shah et al. 2016). Enzymatic hydrolysis has various advantages compared to acid hydrolysis, including low  
46 utility consumption, less corrosion problems, higher glucose yield without sugar-degradation and producing  
47 inhibitory products (Tian, Lu et al. 2017). However, enzymatic hydrolysis of cellulose is still considered as the  
48 main bottleneck of the biological production of ethanol due to the low hydrolysis rates and the high cost of  
49 enzymes (Szi $\acute{a}$ rt $\acute{o}$ , Siika-aho et al. 2008; Soudham, Alriksson et al. 2011; Zheng, Zhang et al. 2013). Many  
50 parameters including cellulose structure (accessible area, degree of polymerization, crystallinity) and cellulase  
51 system (enzyme activities, adsorption, synergism, and inhibition) play an important role in the enzymatic  
52 hydrolysis of cellulose (Hu, Zhang et al. 2018). However, in view of the difficulty in analysis and measurement  
53 of parameters such as crystallinity and degree of polymerization of cellulose, a model based on observable  
54 properties would be useful in the analysis of processes for fermentable sugars or bioethanol production.

55 Several authors investigated the kinetics of enzymatic hydrolysis of cellulosic material applying Michaelis–  
56 Menten’s (M-M) equation due to its simplicity (Movagharnejad and Sohrabi 2003; Imai, Ikari et al. 2004; Peri,  
57 Karra et al. 2007; Zheng, Pan et al. 2009; Harun and Danquah 2011; Sakimoto, Kanna et al. 2017). Also, Ye and  
58 Berson (Ye and Berson 2011) proposed a mathematical model describing the kinetics of glucose formation from

59 cellulose with first order inactivation of adsorbed cellulase. In this model, substrate reactivity (transformation in  
60 the degree of polymerization, crystal structure, substrate availability, etc), cellulose to cellobiose and cellobiose  
61 to glucose conversion have not been included. Some authors believed that the substrate reactivity (SR) is one of  
62 the important parameter affecting the enzymatic hydrolysis of cellulosic material (Klyosov 1990; Weimer,  
63 French et al. 1991; Gama, Teixeira et al. 1994; Desai and Converse 1997; Lynd, Weimer et al. 2002; Kadam,  
64 Rydholm et al. 2004). They detected cellulose hydrolysis rate increased 3–30 times in amorphous cellulose  
65 compared to crystalline cellulose. However, it is difficult to evaluate substrate structural shapes and specify the  
66 different enzymes adsorbed onto crystalline or amorphous cellulose regions. Therefore, the variation of SR may  
67 be a more feasible way to model the biphasic demeanor compare to an assumed dichotomy between crystalline  
68 and amorphous regions.

69 The objective of this work was to modify the model proposed by Ye and Berson (Ye and Berson 2011) to  
70 incorporate substrate reactivity, enzyme inactivation and the kinetic of cellobiose and glucose formation from  
71 cellulose. Afterward, the proposed model and another model from the literature were compared together and  
72 with the experimental data reported in the literatures.

## 73 **2. Model description**

74 In this work, the proposed model by Zheng, et al (Zheng, Pan et al. 2009) is evaluated by the experimental  
75 data reported in four literatures. Thereafter, the modified kinetic model was used proposed to predict  
76 concentrations of cellulose, cellobiose, and glucose during enzymatic hydrolysis. Reaction scheme for the  
77 modeling of the cellulose hydrolysis is depicted in Fig. 1.

### 78 **Fig. 1**

79 The computer program, AQUASIM, is used as a tool to simulate enzymatic hydrolysis of cellulose (Reichert  
80 2014).

#### 81 **2.1. Kinetic model from the literature**

82 The model proposed in the literature by Zheng et al (Zheng, Pan et al. 2009) assumes that hydrolysis of  
83 cellulose occurs in three steps ". The cellulose is hydrolyzed to soluble cellobiose by the synergistic action of  
84 endo- $\beta$ -1,4-glucanase (EG) and exo- $\beta$ -1,4-cellobiohydrolase (CBH), the cellulose is hydrolyzed to glucose by  
85 the synergistic action of CBH and exo- $\beta$ -1,4-glucanase, and cellobiose is hydrolyzed to glucose by the action of  
86  $\beta$ -glucosidase. Reaction scheme (Fig. 1(a)) involves hydrolysis reactions of  $r_1$ ,  $r_2$ , and  $r_3$ . The reaction rates in  
87 the model are given by the following equations:

$$88 \quad r_1 = \frac{k_{1r} \cdot E \cdot SR \cdot C}{1 + \frac{G_2}{k_{1IG2}} + \frac{G}{k_{1IG}}} \quad (1)$$

$$89 \quad r_2 = \frac{k_{2r} \cdot E \cdot SR \cdot C}{1 + \frac{G_2}{k_{2IG2}} + \frac{G}{k_{2IG}}} \quad (2)$$

$$90 \quad r_3 = \frac{k_{3r} \cdot E \cdot G_2}{k_{3M} \cdot \left(1 + \frac{G}{k_{3IG}}\right) + G_2} \quad (3)$$

91 SR is substrate reactivity and expressed according to the following equation.

$$92 \quad SR = \frac{C}{C_0} \quad (4)$$

93 This model was developed based on end-product inhibition. In these rate equations,  $k_{ir}$  (1, 2) are the reaction  
 94 rate constants (ml/(mg h));  $k_{3r}$  is the reaction rate constant ( $h^{-1}$ );  $k_{1IG}$ ,  $k_{1IG2}$  (1, 2) are inhibition constants (mg/ml)  
 95 of glucose and cellobiose on enzymes, respectively;  $k_{3M}$  is cellobiose saturation constants (mg/ml); and C,  $G_2$ , G  
 96 are concentrations of cellulose, cellobiose, and glucose (mg/ml), respectively.  $C_0$ , and E are initial substrate  
 97 concentration (g/l) and initial enzyme concentration (mg/ml), respectively. The mass balance equations of  
 98 cellulose, cellobiose, and glucose can be written as:

$$99 \quad \frac{dC}{dt} = -r_1 - r_2 \quad (5)$$

$$100 \quad \frac{dG_2}{dt} = 1.056r_1 - r_3 \quad (6)$$

$$101 \quad \frac{dG}{dt} = 1.116r_2 + 1.053r_3 \quad (7)$$

## 102 2.2. Proposed kinetic model

103 The proposed model by Ye and Berson (Ye and Berson 2011) simulates straight conversion of cellulose to  
 104 glucose. Moreover, this model did not consider the substrate reactivity. Thus, the modification of this model was  
 105 conducted in an attempt to predict kinetic of cellobiose and glucose formation from cellulose and to consider  
 106 substrate reactivity and enzyme inactivation.

107 Inactivation of cellulases is described in the model proposed in this work with the use of an exponential  
 108 decay term. In the proposed model, it is assumed that hydrolysis of cellulose occurs in two steps: the cellulose is  
 109 hydrolyzed first to soluble cellobiose by the synergistic action of  $\beta$ -1,4-glucan cellobiohydrolase and endo- $\beta$ -  
 110 1,4-glucanase, and the cellobiose is hydrolyzed to glucose by the action of  $\beta$ -glucosidase (Fig. 1(b)). To simplify

111 the model  $r_2$  in the previous model is ignored. The reaction rates and mass balance equations in the new model  
 112 are demonstrated by the following equations:

$$113 \quad K_c = \frac{k_r}{k_r + k_f} + \frac{k_f}{k_r + k_f} \cdot \exp(-(k_r + k_f) \cdot t) \quad (8)$$

$$114 \quad r_1 = \frac{k_{1r} \cdot E \cdot SR \cdot K_c \cdot C}{C + K_m} \quad (9)$$

$$115 \quad r_2 = \frac{k_{2r} \cdot E \cdot K_c \cdot G_2}{G_2 + K_m} \quad (10)$$

$$116 \quad \frac{dC}{dt} = -r_1 \quad (12)$$

$$117 \quad \frac{dG_2}{dt} = 1.056r_1 - r_2 \quad (13)$$

$$118 \quad \frac{dG}{dt} = 1.053r_2 \quad (14)$$

119 In these rate equations,  $k_{1r}$  is the reaction rate constant (ml/ (mg h)); and  $k_{2r}$  is the reaction rate constant ( $h^{-1}$ );  
 120  $k_m$  is saturation constants (mg/ml); and C,  $G_2$ , G are concentrations of cellulose, cellobiose, and glucose (mg/ml),  
 121 respectively. In addition,  $k_f$  and  $k_r$  are inactivation and reactivation rate constants ( $h^{-1}$ ), respectively. SR is  
 122 substrate reactivity and expressed as described in Eq (4).

### 123 3. Evaluation of model predictions

124 The model parameters were estimated using the AQUASIM by minimizing the sum of the squares of the  
 125 weighted deviations ( $\chi^2(m)$ ) between the experimental data and the model results.  $\chi^2(m)$  is expressed as:

$$126 \quad \chi^2(m) = \sum_{i=1}^n \left( \frac{X_{\text{exp},i} - X_i(m)}{\sigma_{\text{exp},i}} \right)^2 \quad (15)$$

127 In this equation  $X_{\text{exp},i}$  is the i-th measurement,  $X_i(m)$  is the calculated value of the model variable  
 128 corresponding to the i-th measurement,  $\sigma_{\text{exp},i}$  is standard deviation, m is the model parameter and n is the  
 129 number of data points. AQUASIM performs a minimization of the sum of squares with the  
 130 restriction  $m_{\text{min},i} \leq m \leq m_{\text{max},i}$ , where  $m_{\text{min},i}$  and  $m_{\text{max},i}$  are the minimum and maximum of the constant  
 131 variable m. The numerical values of the parameters were obtained by fitting the model to experimental data  
 132 taken from the literature.

133 **4. Results and Discussion**

134 In this work, experimental results of enzymatic hydrolysis of cellulosic material under different conditions  
135 from four literatures are modeled. The conditions in the literatures are as follows:

136 1- Enzymatic hydrolysis of insoluble cellulose with initial substrate concentrations, 10, 20, 30, 40, 50, 75 g/l  
137 and cellulase concentration, 0.98 g/l; thereafter, initial substrate concentration, 50 g/l and cellulase  
138 concentrations, 0.49, 0.71, 0.98 g/l at 50°C, pH 4.5, in Fan and Lee study (Fan and Lee 1983).

139 2- Enzymatic hydrolysis of cellulose was prepared by a chemical treatment of cotton stalks, with initial  
140 substrate concentration, 40 g/l and cellulase concentration, 10 g/l; initial substrate concentration, 40 g/l and  
141 cellulase concentration, 25 g/l; initial substrate concentration, 80 g/l and cellulase concentration, 40 g/l; at 50°C,  
142 pH 4.5, in Gusakov et al study (Gusakov, Sinitsyn et al. 1985).

143 3- Enzymatic hydrolysis of noncrystalline cellulose (NCC) with initial substrate concentration, 11.5 g/l with  
144 1 and 3 FPU/g-glucan cellulase loading at 50°C, pH 4.5 in Peri et al study (Peri, Karra et al. 2007).

145 4- Enzymatic hydrolysis of pretreated creeping wild ryegrass with initial substrate concentration, 42.5 g/l  
146 with 5 and 150 FPU/g-glucan enzyme loading at 50°C, pH 5 in Zheng et al study. Cellulase activity of 90  
147 FPU/ml corresponding to 54 mg protein/ml (Zheng, Pan et al. 2009).

148 The main products of enzymatic hydrolysis of cellulosic material by cellulases are glucose and cellobiose.  
149 Therefore, the experimental results that were obtained from the literature containing glucose, cellobiose, and  
150 cellulose concentrations are evaluated by solving differential equations using AQUASIM software. The model  
151 parameters were estimated by minimizing the sum of the squares of the weighted deviations between  
152 measurements and calculated model results. The estimated parameters from the two models are shown in Table  
153 1-2.

154 **Table 1**

155 **Table 2**

156 The comparisons between the experimental and predicted results using two models for prediction of glucose,  
157 cellobiose and cellulose concentrations versus time have been depicted in Fig. (2-7).

158 Fig. 2(a) shows the comparison of measured and predicted concentration versus time with initial substrate  
159 concentration 50 g/l and cellulase concentration, 0.98 g/l. Also, Fig. 2 demonstrate comparison of measured and  
160 predicted reducing sugar concentration at different initial cellulose concentration (10, 20, 30, 40, 50, 75 g/l ) and

161 cellulase concentration, 0.98 g/l (Fig. 2(b)); and at different initial cellulase concentration (0.49, 0.71, 0.98 g/l)  
162 and initial substrate concentration 50 g/l (Fig. 2(c)), in Fan and Lee study(Fan and Lee 1983).

163 **Fig. 2**

164 As can be seen, reducing sugar concentration is dependent on hydrolysis time, initial cellulose concentration  
165 and cellulase concentration. Sum of the squares of the weighted deviations ( $\chi^2(m)$ ) have been calculated to be  
166 80.52 and 69.39 for model 1 and 2, respectively. As predicted and illustrated in Fig. 2(c), the experimental  
167 results, as well as the model predictions both, indicate an increase in the reducing sugar production rates with  
168 increasing enzyme concentration. However, the hydrolysis rate did not increase at high enzyme concentration  
169 level because of saturation of enzyme on the available surface of cellulose.

170 Comparison between the predicted and experimental result of enzymatic hydrolysis of cellulose with  
171 condition base on Gusakov, Sinitsyn et al study(Gusakov, Sinitsyn et al. 1985) is illustrated in Fig. 3.

172 **Fig. 3**

173 Sum of the squares of the weighted deviations have been computed to be 57.810 and 38.42 for model 1 and  
174 2, respectively. The simulation results represent that both models could successfully predict time course  
175 production of glucose, cellobiose, and cellulose during enzymatic hydrolysis.

176 Also, kinetic profiles of glucose, cellobiose, cellulose during enzymatic hydrolysis in Peri, Karra et al  
177 study(Peri, Karra et al. 2007) and Zheng, Y., et al. study (Zheng, Pan et al. 2009) are depicted in Fig. 4 and Fig.  
178 5, respectively.

179 **Fig. 4**

180 **Fig. 5**

181 As can be seen, hydrolysis rate of cellulose is dependent on hydrolysis time, and cellulase concentration.  
182 Hydrolysis rate increase because of increasing enzyme concentration, this behavior is demonstrated in Fig 4(a,  
183 b) and Fig 5(a, b). However, the hydrolysis rate did not increase at high enzyme concentration level due to  
184 saturation of enzyme on the available surface of cellulose.

185 Sum of the squares of the weighted deviations have been calculated to be 11.65 and 5.49 for model 1 and 2  
186 in Fig. 4, respectively. In addition,  $\chi^2(m)$  values were 84.11 and 79.16 for model 1 and 2 in Fig. 5,  
187 respectively.



188 It can be observed that both model prediction fits well with the experimental values. However, comparison of  
189  $\chi^2(m)$  values of two models shows that the model proposed in this work is more accurate than the previously  
190 reported model in all the cases studied.

## 191 **5. Conclusion**

192 In the present study, a comparative study was performed between two models used for prediction of enzymatic  
193 hydrolysis of cellulosic material. One model was taken from the literature and another one was developed in this  
194 work from the basis of the substrate reactivity and enzyme inactivation. The simulation results indicate that the  
195 model proposed in this work could successfully be used to predict the time course of glucose, cellobiose, and  
196 cellulose during enzymatic hydrolysis. Moreover, the predictions by the model proposed in this work was more  
197 accurate to the experimental data reported in the literature when it is compared with previously reported models.

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## 199 **Authors' contributions**

200 Hanieh Shokrkar: Conceptualization, Methodology, Data analysis and writing; Sirous Ebrahimi: Methodology

## 201 **Conflict of interest**

202 Conflict of Interest The authors declare no conflict of interest.

## 203 **Statement of informed consent, human/animal rights**

204 No conflicts, informed consent, or human or animal rights are applicable to this study.

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269 **Table Captions:**

270 **Table 1.** Estimate values of the parameters based on kinetic model from the literature

271 **Table 2.** Estimate values of the parameters based on proposed kinetic model

272 **Figures Captions:**

273 **Fig. 1.** Reaction scheme for modeling cellulose hydrolysis (a), literature model; (b), our model

274 **Fig. 2.** Comparison between predicted and experimental result of glucose, cellobiose and cellulose during  
275 enzymatic hydrolysis of cellulose with (a), initial substrate concentration, 50 g/l and cellulase concentration,  
276 0.98 g/l; (b), different initial cellulose concentration (10, 20, 30, 40, 50, 75 g/l ) and cellulase concentration, 0.98  
277 g/l; (c), different initial enzyme concentration (0.49, 0.71, 0.98 g/l) and initial substrate concentration, 50 g/l.  
278 Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.

279 **Fig. 3.** Comparison between predicted and experimental result of glucose, cellobiose, cellulose during  
280 enzymatic hydrolysis of cellulose (a), initial substrate concentration 40 g/l and concentration of cellulase, 10 g/l;  
281 (b), initial substrate concentration 40 g/l and concentration of cellulase, 25 g/l; (c), initial substrate concentration  
282 80 g/l and concentration of cellulase, 40 g/l; 50°C; pH 4.5. Solid and dashed lines show the simulation results of  
283 the previous model and our modified model, respectively.

284 **Fig. 4.** Comparison between predicted and experimental result of glucose; cellobiose, cellulose during  
285 enzymatic hydrolysis of cellulose with (a), 1 FPU/g-glucan enzyme loading; (b), 3FPU/g-glucan enzyme  
286 loading. Solid and dashed lines show the simulation results of the previous model and our modified model,  
287 respectively.

288 **Fig. 5.** Comparison between predicted and experimental result of glucose, cellobiose, cellulose during  
289 enzymatic hydrolysis of cellulose with (a), 5 FPU/g-glucan; (b), 150 FPU/g-glucan enzymes loading, Solid and  
290 dashed lines show the simulation results of the previous model and our modified model, respectively.

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**Table 1.** Estimate values of the parameters based on kinetic model from the literature

Parameters	Estimate values(Fan and Lee 1983)	Estimate values(Gusako v, Sinitsyn et al. 1985)	Estimate values (Peri, Karra et al. 2007)	Estimate values(Zheng, Pan et al. 2009)
$k_{1IG2}$ (mg/ml)	0.04	0.04	0.04	0.04
$k_{1IG}$ (mg/ml)	0.10	0.10	0.10	0.10
$k_{1r}$ (ml/(mg h))	7.62	0.294	160.2	16.5
$k_{2IG2}$ (mg/ml)	132.50	132.50	132.50	132.50
$k_{2IG}$ (mg/ml)	0.01	0.01	0.01	0.01
$k_{2r}$ (ml/(mg h))	1.248	0.168	99.60	7.08
$k_{3M}$ (mg/ml)	25.50	25.50	25.50	25.50
$k_{3IG}$ (mg/ml)	2.10	2.10	2.10	2.10
$k_{3r}$ (h <sup>-1</sup> )	1.626	0.204	312.60	267.60
$\chi^2(m)$	80.52	57.810	11.65	84.11

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**Table 2.** Estimate values of the parameters based on proposed kinetic model

Parameters	Estimate values(Fan and Lee 1983)	Estimate values(Gusakov, Sinitsyn et al. 1985)	Estimate values(Peri, Karra et al. 2007)	Estimate values(Zheng, Pan et al. 2009)
$k_r$ ( $h^{-1}$ )	0.033	0.033	0.033	0.033
$k_f$ ( $h^{-1}$ )	0.251	0.251	0.251	0.251
$k_{1r}$ (ml/(mg h))	10.86	0.36	612	42.30
$K_m$ (mg/ml)	38.638	38.638	38.638	38.638
$k_{2r}$ ( $h^{-1}$ )	7.02	0.36	1320.60	852.60
$\chi^2(m)$	69.39	38.42	5.49	79.16

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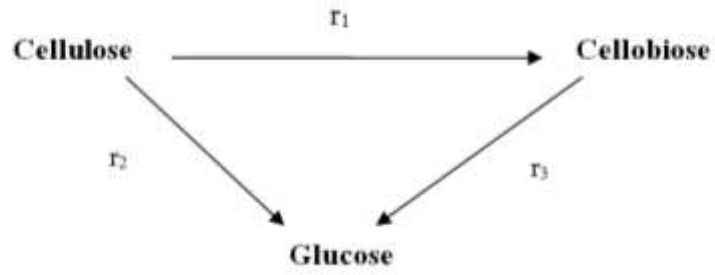
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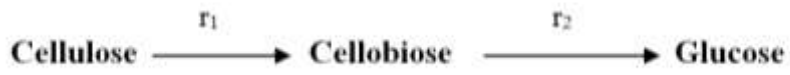
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(a)



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**Fig. 1.** Reaction scheme for modeling cellulose hydrolysis (a), literature model; (b), our model

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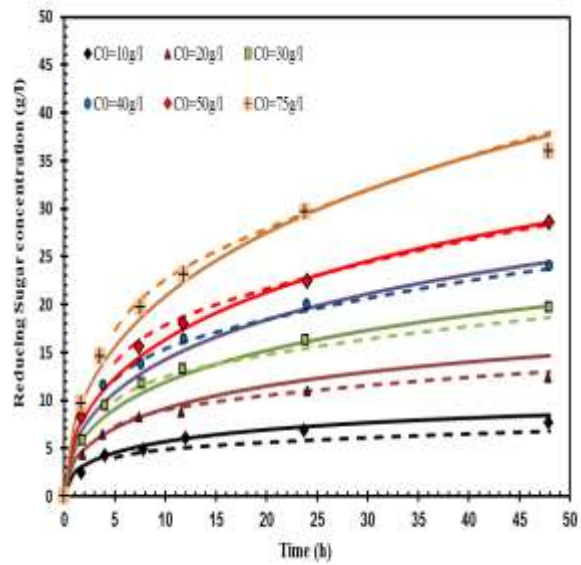
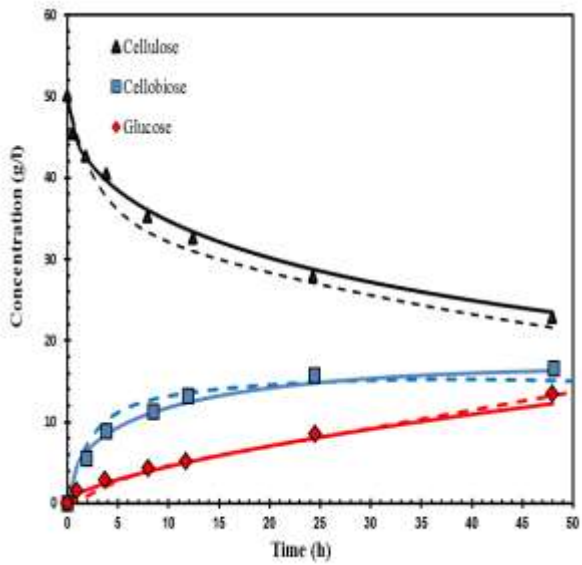
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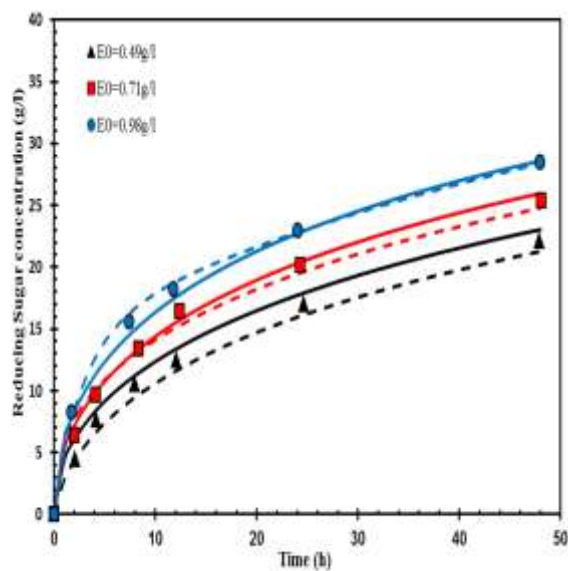


341

342

(a)

(b)



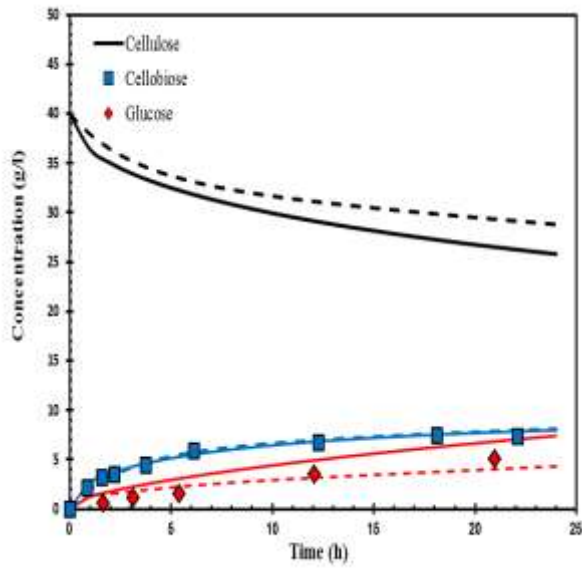
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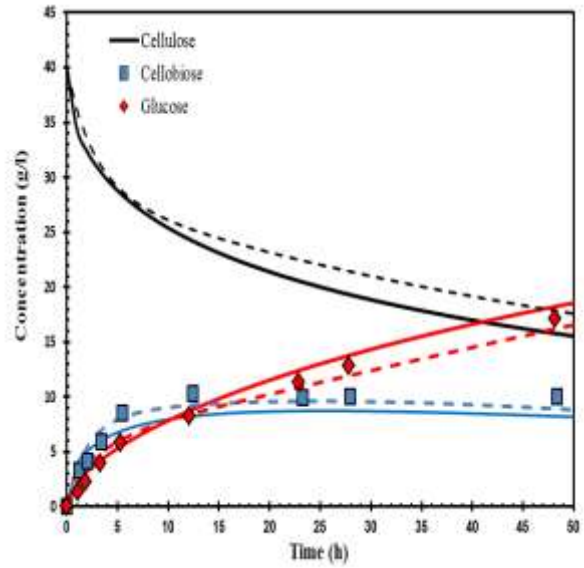
(c)

345 **Fig. 2.** Comparison between predicted and experimental result of glucose, cellobiose and cellulose during  
 346 enzymatic hydrolysis of cellulose with (a), initial substrate concentration, 50 g/l and cellulase concentration,  
 347 0.98 g/l; (b), different initial cellulose concentration (10, 20, 30, 40, 50, 75 g/l ) and cellulase concentration, 0.98  
 348 g/l; (c), different initial enzyme concentration (0.49, 0.71, 0.98 g/l) and initial substrate concentration, 50 g/l.  
 349 Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.

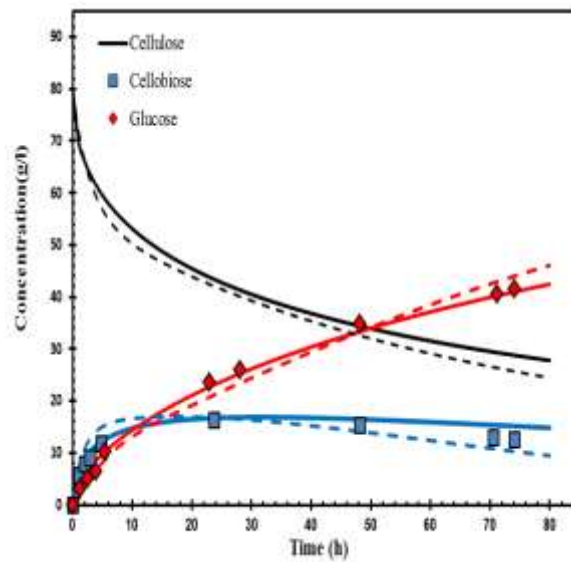




(a)

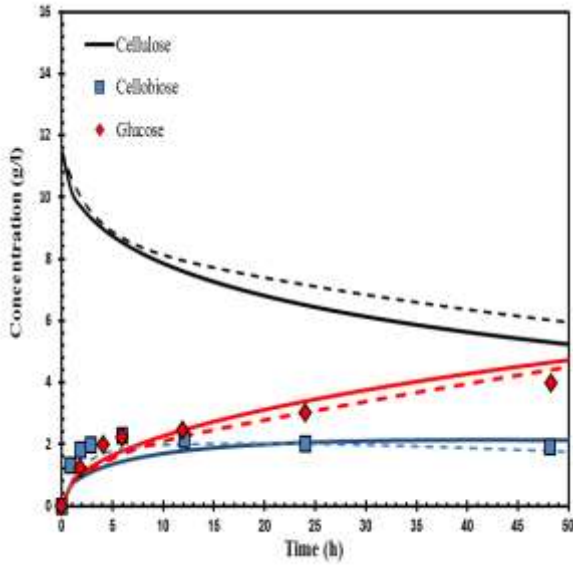


(b)

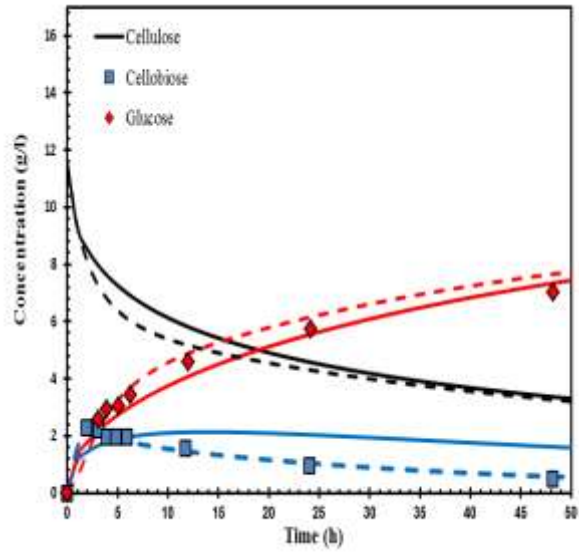


(c)

**Fig. 3.** Comparison between predicted and experimental result of glucose, cellobiose, cellulose during enzymatic hydrolysis of cellulose (a), initial substrate concentration 40 g/l and concentration of cellulase, 10 g/l; (b), initial substrate concentration 40 g/l and concentration of cellulase, 25 g/l; (c), initial substrate concentration 80 g/l and concentration of cellulase, 40 g/l; 50°C; pH 4.5. Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.



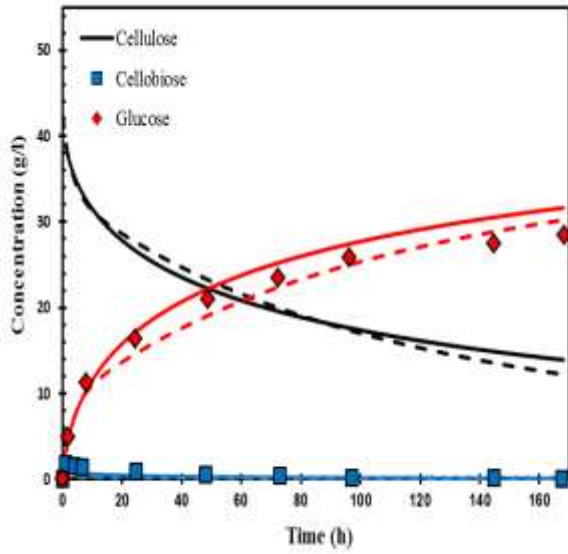
(a)



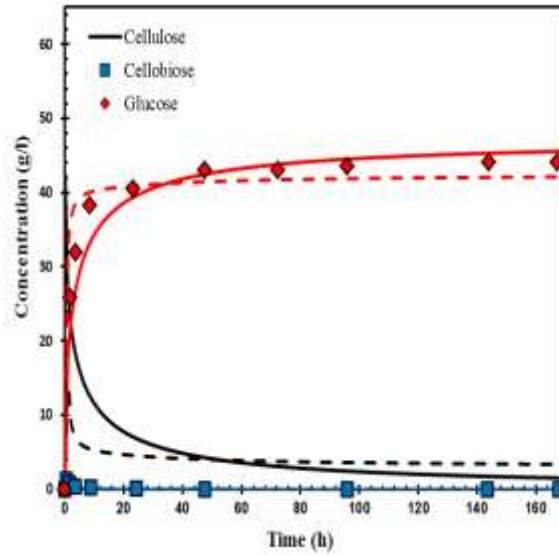
(b)

360  
 361  
 362  
 363 **Fig. 4.** Comparison between predicted and experimental result of glucose; cellobiose, cellulose during  
 364 enzymatic hydrolysis of cellulose with (a), 1 FPU/g-glucon enzyme loading; (b), 3FPU/g-glucon enzyme  
 365 loading. Solid and dashed lines show the simulation results of the previous model and our modified model,  
 366 respectively.

367  
 368  
 369  
 370



(a)



(b)

371  
 372  
 373 **Fig. 5.** Comparison between predicted and experimental result of glucose, cellobiose, cellulose during  
 374 enzymatic hydrolysis of cellulose with (a), 5 FPU/g-glucan; (b), 150 FPU/g-glucan enzymes loading, Solid and  
 375 dashed lines show the simulation results of the previous model and our modified model, respectively.

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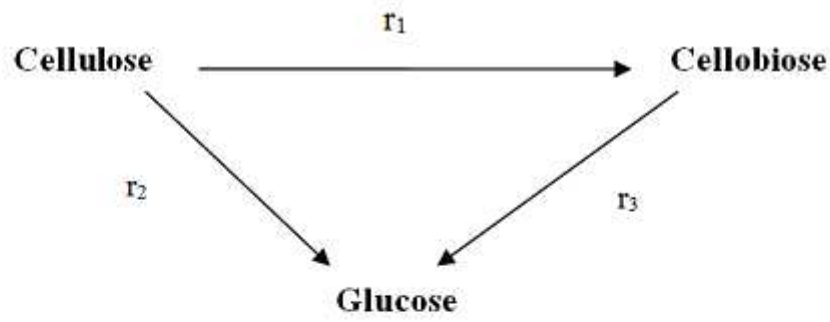
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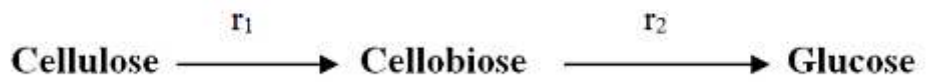
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# Figures



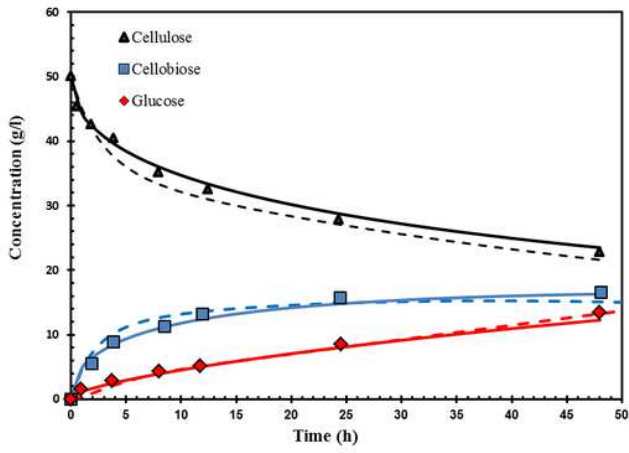
(a)



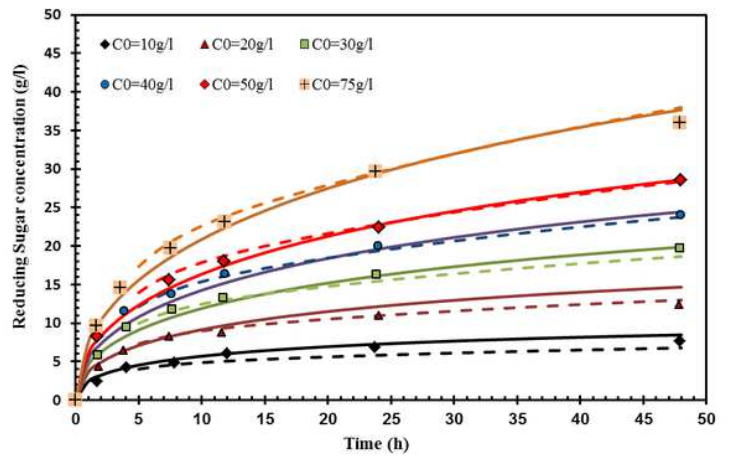
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Figure 1

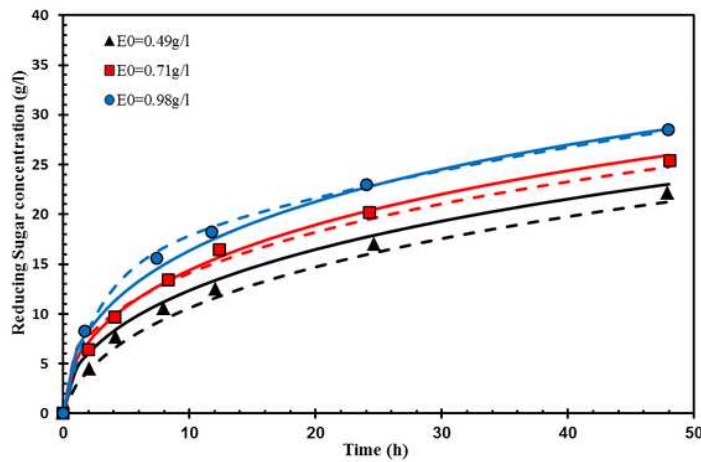
Reaction scheme for modeling cellulose hydrolysis (a), literature model; (b), our model



(a)



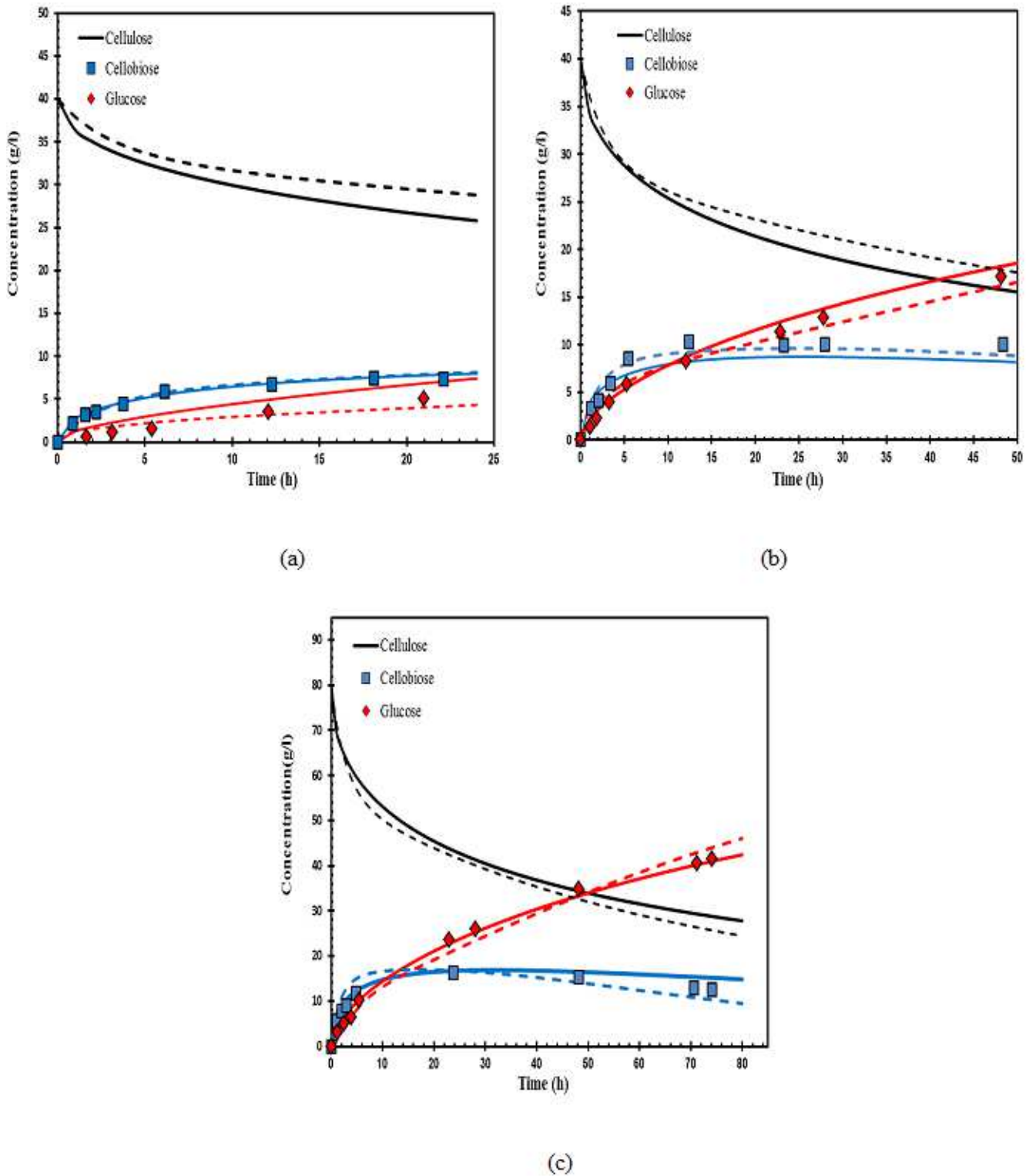
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(c)

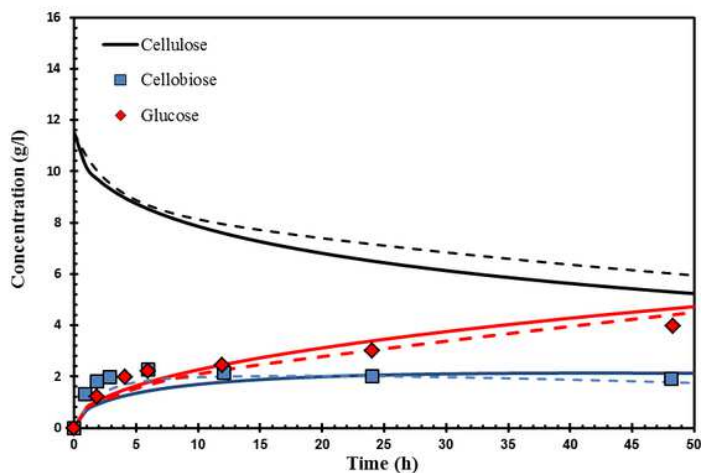
**Figure 2**

Comparison between predicted and experimental result of glucose, cellobiose and cellulose during enzymatic hydrolysis of cellulose with (a), initial substrate concentration, 50 g/l and cellulase concentration, 0.98 g/l; (b), different initial cellulose concentration (10, 20, 30, 40, 50, 75 g/l) and cellulase concentration, 0.98 g/l; (c), different initial enzyme concentration (0.49, 0.71, 0.98 g/l) and initial substrate concentration, 50 g/l. Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.

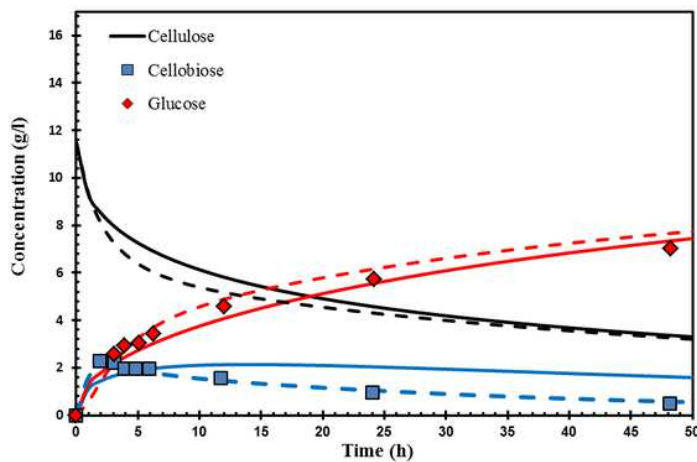


**Figure 3**

Comparison between predicted and experimental result of glucose, cellobiose, cellulose during enzymatic hydrolysis of cellulose (a), initial substrate concentration 40 g/l and concentration of cellulase, 10 g/l; (b), initial substrate concentration 40 g/l and concentration of cellulase, 25 g/l; (c), initial substrate concentration 80 g/l and concentration of cellulase, 40 g/l; 50°C; pH 4.5. Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.



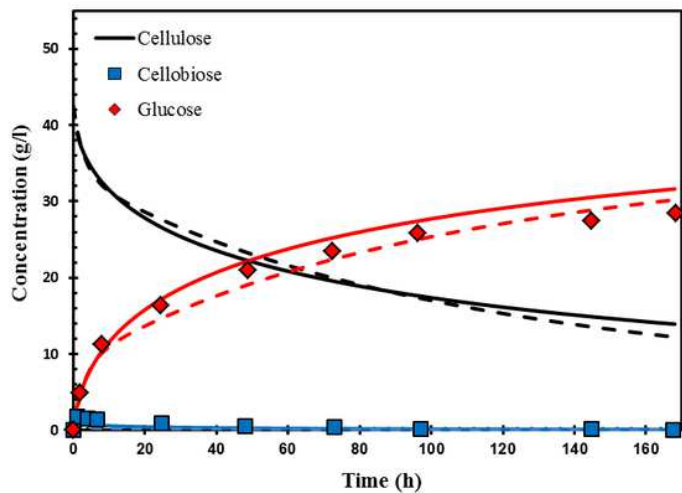
(a)



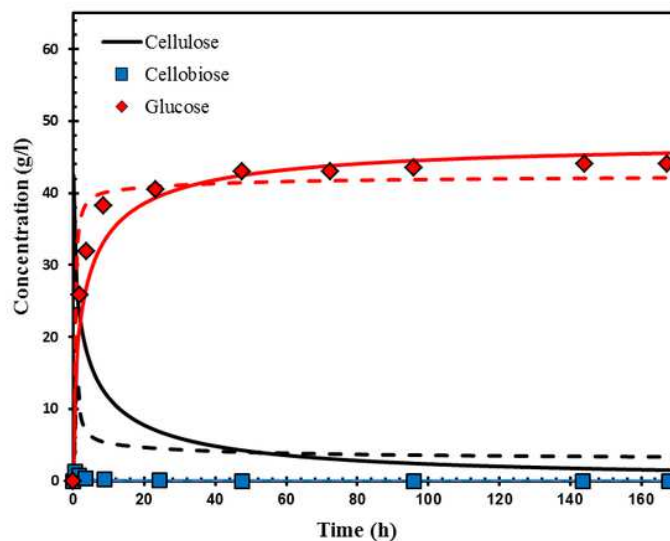
(b)

**Figure 4**

Comparison between predicted and experimental result of glucose; cellobiose, cellulose during enzymatic hydrolysis of cellulose with (a), 1 FPU/g-glucon enzyme loading; (b), 3FPU/g-glucon enzyme loading. Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.



(a)



(b)

**Figure 5**

Comparison between predicted and experimental result of glucose, cellobiose, cellulose during enzymatic hydrolysis of cellulose with (a), 5 FPU/g-glucon; (b), 150 FPU/g-glucon enzymes loading, Solid and dashed lines show the simulation results of the previous model and our modified model, respectively.