A closer look from diversification perspectives with Bitcoin, Gold, Oil and Stock Markets during COVID-19 Pandemic

Sana Braiek
sanahecc@yahoo.fr

University of Sousse

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Abstract

The emergence of new asset classes such as Islamic assets and digital assets offers avenues to international investment community however understanding relationship between any assets in a single portfolio is important. We investigate the risk dependence between daily Bitcoin DJWI, DJIM, Gold and Oil prices spanning over from the 2nd of December 2019 to the 31st of May 2021. We start by examining long memory properties and report significant results. Our methodology consists to construct two types of portfolio, one with the Islamic Dow Jones Islamic index and one with the Conventional Dow Jones index. We refer to the model proposed by Braïek et al (2020) the Mean-CoVaR to obtain the optimal weights. We find that the frontier of the portfolio with (DJWI) out performs the frontier of the portfolio with DJIM. Our results suggest that the inclusion of DJIW into diversified portfolio may be profitable during the pandemic, serving therefore as risk diversifier. We show that our results are robust to the period of the recent market fluctuations caused by COVID-19.

**JEL classification**: C51; C52; C58; G11; G17

**Keywords**: Bitcoin, Gold, Brent, Mean-CoVaR, stock market.
1 Introduction

Increasing integration of financial markets at one end provides stability and efficient functioning, yet at the other end becomes challenging for investors in achieving high diversification benefits. In a real world, financial assets are in the same policy and economic environment, and thus their connection exists objectively. Moreover, with the development of economic globalization, the dependence structure among financial assets has been strengthened unprecedentedly, which is of vital importance for portfolio selection, see Yang et al. (2019).

Assets, like gold, crypto currencies and Islamic equities, with superior hedging characteristics against traditional asset markets have gained attention for empirical contribution on diversification of risk and return trade-off (Lim Masih, 2017; Kenourgios et al., 2016; Evans, 2015). Islamic equity market sustained the financial meltdown of 2008-09 and out beat the performance of conventional equity markets (Milly Sultan, 2012; Hayat Kraussl, 2011) which sparked investor’s interest in Sharia compliance products that prohibit interest and operates on the principles of profit and loss sharing (Hkiri et al., 2017; Nazlioglu et al., 2015). Islamic finance is based on the principles of Islamic law, which also prohibit gambling, derivatives, short selling and promotes risk sharing over risk shifting (Sadaoui, 2017; Maghyereh Awartani, 2016; Abbes, 2015).

According to Dewandaru et al. (2015), though Islamic equity markets exhibit low correlation with their conventional counterparts, such correlation are more evident under shorter investment horizons. The selection of Islamic equity indices for better portfolio diversification has been suggested by many studies mainly due to their asset based financing and Shariah complaint activities (see Hayat Kraussl, 2011; Beck et al. 2013).

Nowadays, it has one word that many people especially investors interested in "Cryptocurrency" In the aftermath of the Financial Crisis in 2008. Many investors lose their faith in the traditional currency as the central banks had the authority to produce money without any backup asset. The evolution of Bitcoin and other flowering cryptocurrencies in the market has been analyzed in detail during this decade. In particular, from 2013 when the value of Bitcoin increased rapidly from around 150 in mid-2013 to over 1000 in late 2013, which is known as the 2013 bubble.

Cryptocurrencies are more of safe havens than substitutes to commodity markets. Wang, et al. (2019) and Shahzad et al. (2019) showed empirical evidence to the claim of Bitcoin being a safe haven. As such, cryptocurrencies interrelate with other investment options. On this note, active trading and mining of cryptocurren-
cies relate significantly with the electricity market (Hayes, 2017) cryptocurrencies relate with commodity markets (Bouri, et al., 2018a), cryptocurrencies relate with energy commodities (Ji, et al., 2019c), cryptocurrencies can hedge crude oil prices (Selmi, et al., 2018), hedge stocks (Okorie, 2019; Chan, et al., 2019), hedge Financial Times stocks (Dyhrberg, 2015), hedge gold (Pal Mitra, 2019), etc. Bouri, et al. (2018) investigated the relationships between Bitcoin, aggregate commodity and gold prices and his results show that gold price and aggregate commodity price information is capable of predicting the Bitcoin prices even though their relationship is asymmetric, nonlinear and quantiles dependent.

Bitcoin and gold are capable of hedging or providing diversification for SP GSCI and crude oil (Al-Yahyae, et al., 2019) while Okorie (2019) proved that Bitcoin and SP500 are vital for portfolio hedging and diversification. Bitcoin is a hedge for global EPU (global bonds and global equities) under specific economic uncertainty conditions (Fang, et al., 2019). Bitcoin is capable of hedging against stocks, bonds and Shanghai Interbank Offered Rate (Shahzad, et al., 2019; Wang, et al., 2019). Brian (2015) and Corbet, et al., (2019) added that Bitcoin qualifies as an asset and it has no fundamental value to qualify as a conventional currency. Philippas et al. (2019) showed that Bitcoin prices are significantly driven by media attention in social networks.

Pal and Mitra (2019) stipulate that gold, amongst other assets, is capable of hedging against Bitcoin risks. The gold market dominates crude oil and Bitcoin markets in absorbing new information while the Bitcoin market is susceptible to price fluctuations from gold and oil markets (Huang, et al., 2018).

The Markowitz Portfolio Framework remained a cornerstone in the modern portfolio optimization theory by its initial mathematical application in constructing an optimal portfolio by quantitative methods. However, the variance is only efficient in measuring the symmetric risks of financial assets. However, the distribution of securities in the market is not symmetric.

Value-at-Risk (VaR) measurement is one of the most popular risk measurement factors to estimate the downside risk of financial instruments, and this factor is widely used by financial institutions, banking, investors, and practitioners. However, many studies proved the non-efficiency of VaR in optimizing investment portfolio because of mathematical problems such as non-subadditivity or non-convexity (Rockafellar et a., 2000; Artzner, 1999; Kolm et al., 2014). On the other hand, Conditional Value at risk has been recently submitted as an effective risk measurement factor to measure downside risks of the assets with asymmetric return distribution. The efficient application and the trustworthiness of CVaR outperform the VaR that can be easily seen by the replacement of risk measurement
of Basel III from VaR to CVaR.

Zhang (2016) conducted the first research about optimization with the Mean-CVaR framework for multiple portfolios instead of a single portfolio. The numerical results of the study have shown that the performance of Mean-CVaR to multiple portfolios is relatively better than the individual portfolio in terms of improvement rate from independent optimization. Additionally, the research has also shown that the Mean-CVaR framework supports the multi portfolios to reduce the market impact cost than the independent decision. The contribution of this research is significantly important, since it has extended the application and the development of the Mean-CVaR framework.

Nguyen et al. (2018) conducted a study to give a precise answer of Does Mean-CVaR outperform Mean-Variance in both theoretical and practical perspectives. The study examined the performance of these two frameworks for the whole stocks in the US market in almost a century, from 1926 to 2006. The research analysis shows the better performance of the Mean-CVaR framework compared to the Mean-Variance framework by the influence of different features of stock returns such as means, volatilities, skewness, and kurtosis. Furthermore, by comparing these two frameworks under different market conditions, the Mean-CVaR framework also shown a positive effect in distress markets.

More recently, Braiek et al (2020) developed a new methodology based on Mean-CoVaR to optimize Islamic portfolio. Using the vine-copula, they computed downside and upside risk with CoVaR measure. They found that the mean-CoVaR model outperforms the mean-Variance model and that The minimum risk portfolio allocation is influenced by the existence of systemic risk, the interdependence structure between sectors and the optimization model.

This paper aims to answer the question whether adding DJIM into an investment portfolio makes the portfolio more efficient. The topic is relevant currently due to the fast development of the crypto market and the numerous contradictions among researchers. This study employs the Mean-CoVaR model of Braiek et al (2020).

In the next section, the concept and model used in the study are explained, followed by information used in the study and empirical result. Conclusion provides in Section 5.
2 Methodology

2.1 Modeling Marginal distributions

Volatility often displays the long-memory property in which the autocorrelations of the absolute and squared returns are characterized by a very slow decay over long periods. To accommodate this phenomenon, Baillie et al (1996) extended the standard GARCH model by introducing a fractionally integrated process that resulted in the FIGARCH model. Unlike the knife-edge distinction between I(0) and I(1) processes, the fractionally integrated process, I(d), can distinguish between short memory and long-memory in conditional variances.

The FIGARCH (p, d, q) model is given by:

\[ \varphi(L)(1 - L)^d \epsilon_t^2 = \alpha + [1 - \beta(L)] \omega_t \]  

(1)

Conditional variance of \( \epsilon_t \) is:

\[ \sigma_t^2 = \frac{\alpha}{1 - \beta(1)} + \gamma(L) \epsilon_t^2 \]  

(2)

where \( \gamma(L) = \gamma_1 L1 + \gamma_2 L2 ... \gamma_k L_k \).

When \( 0 < d < 1 \), the coefficients capture the short term dynamics of volatility while fractional difference parameter \( d \) models the long-term characteristics of volatility.

Tse (1998) extended the FIGARCH model by adding the function \((|\epsilon_t| - \gamma \epsilon_t)^\delta\) of the Asymmetric Power Autoregressive Conditional Heteroscedasticity (APARCH) model to evaluate asymmetry and long memory property in the conditional variance. The FIAPARCH (p, d, q) model can be expressed as follows:

\[ \sigma_t^2 = \lambda(L) + (|\epsilon_t| - \gamma \epsilon_t)^\delta \]  

(3)

where \( \lambda, \gamma \) are the parameters of model and \( \delta \) is power parameter.

Furthermore, \( \lambda(L) = \sum_{i=1}^{\infty} \lambda_i L^i \), with \( \lambda(L) \) an infinite summation, \( d \) parameter is the long memory parameter and when \( 0 < d < 1 \), conditional variance has long memory property.

2.2 Vine-copula CoVaR for systemic risk

Adrian and Brunnermeier (2011) and Girardi and Ergün (2013) quantify the systemic impact of assets on the financial market with the CoVaR risk measure. CoVaR
allows the impact of financial distress in the financial market on the assets to be measured by providing information on the VaR of the financial market conditional on the fact that the asset is in financial distress.

Let \( x^f_t \) be the returns for the financial market at time \( t \) and let \( x^d_t \) be the returns for the asset at time \( t \). CoVaR for a confidence level of \( (1 - \alpha') \) can be formally characterized as the \( \alpha' \)-quantile of the conditional \( x^f_t \), as follows:

\[
Pr(x^f_t \leq \text{CoVaR}^f|d_{\alpha'}, t | x^d_t \leq \text{VaR}^d t) = \alpha' 
\]  \( (4) \)

where \( \text{VaR}^d t \) denotes the VaR of the market that measures the maximum loss that may be experienced by the market for a confidence level \( (1 - \alpha) \) at time \( t \). Formally, it is the \( \alpha \)-quantile of the return distribution for the market: \( Pr(x^d_t \leq \text{VaR}_n^d) = \alpha \).

We can compute CoVaR by determining the quantile of a conditional distribution or by using the quantile of an unconditional bivariate distribution, given that Eq (4) can be written as:

\[
Pr(x^f_t \leq \text{CoVaR}^f|d_{\alpha'}, t | x^d_t \leq \text{VaR}^d t) = \frac{\alpha'}{Pr(x^d_t \leq \text{VaR}^d t)} = \alpha' 
\]  \( (5) \)

or alternatively as:

\[
Pr(x^f_t \leq \text{CoVaR}^f|d_{\alpha'}, x^d_t \leq \text{VaR}^d t) = \alpha \alpha' 
\]  \( (6) \)

To obtain CoVaR from Eq.(6), we used copulas to characterize the joint distribution function. Eq.(6) can be expressed in terms of the joint distribution function of \( x^f_t \) and \( x^d_t \), \( F^f(x^f_t), F^d(x^d_t) \), as \( F^d(x^d_t | \text{CoVaR}^f|d_{\alpha'}, t) = \alpha \alpha' \), furthermore, Sklar’s (1959) theorem relates the joint distribution function and the copula as follows:

\[
F^d(x^f_t, x^d_t) = C(u^f, u^d) 
\]  \( (7) \)

where \( C(., .) \) is a copula function, \( u^f = F^f(x^f_t) \) and \( u^d = F^d(x^d_t) \) and \( F^f \) and \( F^d \) are the marginal distribution functions of \( x \) respectively. Consequently, we can express Eq. (6) in terms of copulas as:

\[
C(F^f(\text{CoVaR}^f|d_{\alpha'}), F^d(\text{VaR}^d t)) = \alpha \alpha' 
\]  \( (8) \)

CoVaR can be computed from Eq.(8) using a simple two-step procedure (Reboredo et al(2015)).
1. We obtain the value of \( F_j(\text{CoVaR}^{\text{fd}}_{\alpha,t}) \) from Eq. (8). Given that \( C(u_f, u_d) = \alpha \alpha \tau \), where \( \alpha, \alpha \tau \) and \( u_d \) are given (note that \( u_d = \alpha \)), from the copula function specification we can solve to determine the value of \( u_f = F_j(\text{CoVaR}^{\text{fd}}_{\alpha,t}) \).

2. From \( u \) we can obtain CoVaR as the quantile of the distribution of \( x'_j \) with a cumulative probability equal to \( u \), by inverting the marginal distribution function of \( x'_j: (\text{CoVaR}^{\text{fd}}_{\alpha,t}) = F^{-1}_j(u_f) \)

The focus of CoVaR is to examine the spillover effect from one entity’s failure to the safety of another entity or the whole financial system. Using copulas to obtain CoVaR is enticing because of their flexibility, compared to parametric bivariate functions, in allowing separate modeling of the marginals and the dependence structure. This is crucial because marginals and dependence functions may have different tail dependence characteristics affecting CoVaR. Furthermore, computing CoVaR using the above two-step procedure is simple and only requires information on confidence levels (note that only the exogenous confidence level for VaR is required and not its value).

Since we wished to analyze the systemic impact of a distressed asset on other asset, we considered a vine copula with four variables. Let \( x^d_t \) be the returns for the sector at time \( t \) with distribution function \( F_d(x^d_t) = u_d* \). According to Bedford and Cooke (2002), and taking the example of three variables the joint density can be expressed as the product of the marginal densities and a set of conditional bivariate copulas as:

\[
\begin{align*}
f(x', x^d, x^d) &= C_{f,d|d}(F_{f|d}(x'), F_{d|d}(x^d))C_{f,d}(F_f(x')) \times \times (x^d)), C_{d,d}(F_d(x^d), F_d(x^d))f_f(x')f_d(x^d)f_d(x^d)^2 \end{align*}
\]

where \( C(.,.) \) and \( f() \) denote the copula density and the marginal densities, respectively, and where \( C_{f,d|d} \) is referred to as the pair-copula. The conditional distribution functions in Eq. (9) for any two random variables \( x \) and \( y \) can be obtained as (Joe, 1997):

\[
F(x|y) = \frac{\partial C_{x,y}(F(x), F(y))}{\partial F(y)}
\]

In this multivariate conditional setup, CoVaR is given by:

\[
\begin{align*}
Pr(x'_j \leq \text{CoVaR}^{\text{fd}}_{\alpha,t} | x^d_t \leq VaR^{\text{fd}}_{\alpha,t}, x^d_t) = \alpha \tau
\end{align*}
\]

which can be expressed in terms of the conditional joint distribution function as: \( F_{f,d|d}(\text{CoVaR}^{\text{fd}}_{\alpha,t}) = \alpha \alpha \tau \). Hence, to obtain CoVaR from the vine-copula specification, we have to take into account information provided by the conditional joint
distribution function of $x^f_t$ and $x^d_t$ given, in terms of the copula, as:

$$F_{f,d|d^*}((x^f_t|x^{d*}), (x^d_t|x^{d*}) = C_{f,d|d^*}((u_f|u_{d^*}), (u_d|u_{d^*}))$$  \hspace{1cm} (12)

Thus, CoVaR can be obtained from the vine copula using a three-steep procedure:

1. For given values of $\alpha^*, u_d = \alpha$ and $u_{d^*}$ and for the copula specification in Eq. (12) we can solve to determine the value of $(u^*_f|u_{d^*})$.

2. From the value of $(u^*_f|u_{d^*})$ we obtain the value of $u_f$ by solving from the conditional distribution of $x^f_t$ given by Eq. (10).

3. From $u_f$ we obtain CoVaR as the quantile of the distribution of $x^f_t$ with a cumulative probability equal to $u_f$ by inverting the marginal distribution function of $x^f_t$: $\text{CoVaR}_{f|d^*, d^*}^{1-q, t} = F^{-1}_f(u_f)$

### 2.3 Mean-CoVaR portfolio optimization

In line with the concept of mean-risk optimization, we use the Mean-CoVaR portfolio optimization developed by Braiek et al (2020) to minimize a portfolio’s potential loss caused by systemic risk. We select CoVaR as the objective function rather than VaR because of the drawbacks of VaR optimization. Let $R^i_t$ be the return of asset $i$. The subscript $t$ stands for a time period. We assume that any return distribution is continuous. Let $C(R^i_t)$ be a certain condition of $R^i_t$. Then, the CoVaR of asset $j$ on the condition $C(R^i_t)$ at the confidence level $1 - q$ is defined as:

$$\text{CoVaR}_{q,t}^{j|C(R^i_t)} = \frac{1}{q} \int_{0}^{q} \text{CoVaR}_{p,t}^{j|C(R^i_t)} dp = -E \left( R^j_t \left| R^i_t < -\text{CoVaR}_{q,t}^{j|C(R^i_t)} \right. \right) \cap C(R^i_t)$$  \hspace{1cm} (13)

Where $\text{CoVaR}_{q,t}^{j|C(R^i_t)}$ is the VaR of asset $j$ on the condition $C(R^i_t)$ at the confidence level $1 - q$.

Let $w_t = (w^1_t, w^2_t, w^3_t, ..., w^J_t)$ be a set of weights of asset $1, 2, ..., J$ in the portfolio. We now formulate the mean-CoVaR portfolio optimization as follows:

$$\min \text{CoVaR}_{q,t}^{\text{pot}(w_t)|C^d(R^\text{index})} \quad \text{s.t.} \quad \sum_{j=1}^{J} w^j_t \mu^j_t = \bar{\mu}_t$$

$$\sum_{j=1}^{J} w^j_t = 1$$

$$w^j_t \geq 0, \forall j$$  \hspace{1cm} (14)
where $\mu^t_j$ is a conditional mean of $R^t_j$ on the information up to $t-1$ and $\mu_t$ is an expected return of the portfolio. Note that CoVaR is defined for the portfolio return $R^\text{port}(w_t) = \sum_{j=1}^J w_j R^t_j$ against the market index return $R^\text{index}$. The distress condition $C^d(R^\text{index})$ is defined as the loss of the index being above its VaR:

$$C^d(R^\text{index}) = \left\{ R^\text{index} \leq -VaR^{\text{index}}_{q,t} \right\}$$ (15)

For simplicity, we do not take transaction costs into account.

3 Data

The dataset comprises of daily observations of the Dow Jones world stock market index (DJIW), the Dow Jones Islamic stock market index (DJIM), the daily oil price data of Europe Brent crude, Bitcoin and Gold prices ranging from the 2nd of December 2019 to the 31st of May 2021. The database was collected from the DataStream. Table 1 provides descriptive statistics for the data. All series present clear signs of non-normality. The J-B test statistics also reveal that the residuals appear to be leptokurtic. The kurtosis of the returns are significantly higher than 3 indicating that the empirical distributions of returns display heavy tails characteristics comparing to Gaussian distribution. The skewness is negative, the data are negatively skewed, meaning that the left tail is longer. The Augmented Dickey-Fuller (ADF) implying that the return series are stationary. Daily returns are defined by $r_t = (p_t / p_{t-1})$, with $p_t$ standing for the data closing price on day $t$. Figures of returns volatility (Figure 1) yield some remarks. (1) The volatility of all assets, in general, is more generous in March in comparison with other months, which indicates that COVID-19 has substantially impacted the stock markets. (2) The Gold keep his high volatility after March especially in September 2020, the gold fluctuates ups and downs relatively more than bitcoin does.

In order to better detect various characteristics of asset returns, we employ long memory tests before modeling the volatility of these series. According to the results of these long memory tests, we later estimate the appropriate GARCH class model, which accurately takes into account the asymmetry in the relevant series. Long memory process could be described as a slowly decaying autocovariance function. We employ two different statistics: The Hurst-Mandelbrot Rescaled Range (R/S) statistics and Lo (1991) Rescaled Range R/S. When testing for long-term memory for all assets, we use the squared returns as proxies of daily volatility following Arouri et al. (2012). As shown in Table 2, we should not reject the null
Table 1: Descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>DJIM</th>
<th>DJWI</th>
<th>Bitcoin</th>
<th>Gold</th>
<th>Brent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std.Dev</td>
<td>0.014</td>
<td>0.049</td>
<td>0.011</td>
<td>0.011</td>
<td>0.059</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0008</td>
<td>0.004</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0002</td>
</tr>
<tr>
<td>Skewness</td>
<td>-1.19</td>
<td>-1.58</td>
<td>-2.57</td>
<td>-0.83</td>
<td>-2.58</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>12.14</td>
<td>14.53</td>
<td>26.41</td>
<td>4.22</td>
<td>44.19</td>
</tr>
<tr>
<td>JB</td>
<td>2490.7***</td>
<td>3598.5***</td>
<td>11766***</td>
<td>335.47***</td>
<td>32169***</td>
</tr>
<tr>
<td>ADF</td>
<td>-9.36***</td>
<td>-9***</td>
<td>-9.93***</td>
<td>-11.35***</td>
<td>-12.35***</td>
</tr>
</tbody>
</table>

Note: LM statistic is used with respect to the ARCH test, ***,** and * Indicates statistical significance at 1%, 5% and 10% level respectively.

hypothesis of no long term dependence (short memory), however the squared returns test statistic indicates an evidence of long memory property for all assets. These results are in accordance with the findings of Bouri et al. (2016) show evidence of a long memory in two measures of volatility of Bitcoin. This is suited for risk-seeking investors looking for a way to invest or enter into technology markets.

Table 2: Long memory test.

<table>
<thead>
<tr>
<th></th>
<th>Lo-Statistic</th>
<th>Hurst-Mandelbrot R/S Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJIM</td>
<td>R</td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>2.56</td>
</tr>
<tr>
<td>DJWI</td>
<td>R</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>2.54</td>
</tr>
<tr>
<td>Bitcoin</td>
<td>R</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>1.12</td>
</tr>
<tr>
<td>Gold</td>
<td>R</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>1.77</td>
</tr>
<tr>
<td>Brent</td>
<td>R</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td>R²</td>
<td>2.54</td>
</tr>
</tbody>
</table>
4 Results and discussion

We firstly consider the two ordinary long-memory GARCH(1,d,1) models: FI-GARCH for DJIM, DJWI and Gold and IGARCH(1,1) for Bitcoin and Brent. The estimation results are reported in Table 3. The IGARCH model falls within the standard GARCH framework and contains a conditional volatility process, which is highly persistent (with infinite memory), and this has been shown in the literature (Caporale et al. (2003)). Our analysis assumes that we are looking at Bitcoin in terms of financial assets, where most users are trading them for investment purposes: either as a long-term investment in new technology or looking to make a short-term profit. Investigating the volatility of assets is important in terms of
financial investment like hedging or pricing instruments. Therefore, these results would be particularly useful in terms of portfolio and risk management and could help others make better informed decisions with regard to financial investments and the potential benefits and pitfalls of utilizing Bitcoin, Gold and Oil. In general, for the variance equation, all coefficients were found to be highly significant for all specifications. A more in-depth analysis reveals that the value of the fractionally integrated parameter $d – FIGARCH$ are statistically significant at 5% level, which implies the existence of long memory in the selected assets volatility. Besides, the estimates of the tail parameter is statistically significant at the 1% level suggesting the usefulness of Skewed-Student-t distribution.

Table 3: AR-FIGARCH(1,d,1) and AR(1)-IGARCH(p,q) estimation parameter.

<table>
<thead>
<tr>
<th></th>
<th>DJIM</th>
<th>DJWI</th>
<th>Bitcoin</th>
<th>Gold</th>
<th>Brent</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0014***</td>
<td>0.0012***</td>
<td>0.0047**</td>
<td>0.0009</td>
<td>0.0010</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.0465</td>
<td>0.0324</td>
<td>-0.0973**</td>
<td>0.0019</td>
<td>-0.045</td>
</tr>
<tr>
<td>$\omega \times 10^4$</td>
<td>0.0826**</td>
<td>0.0435</td>
<td>1.1820</td>
<td>0.9032**</td>
<td>0.669***</td>
</tr>
<tr>
<td>$d – FIGARCH$</td>
<td>0.8904***</td>
<td>0.7409***</td>
<td>/</td>
<td>0.2691**</td>
<td>/</td>
</tr>
<tr>
<td>$ARCH(\phi_1)$</td>
<td>-0.3857***</td>
<td>-0.3254**</td>
<td>0.0951**</td>
<td>-0.3633</td>
<td>0.2472***</td>
</tr>
<tr>
<td>$GARCH(\beta_1)$</td>
<td>0.4728**</td>
<td>-0.3495</td>
<td>0.9048**</td>
<td>-0.107</td>
<td>0.7527</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>-0.2073***</td>
<td>-0.2222***</td>
<td>0.0442</td>
<td>-0.1545**</td>
<td>-0.159**</td>
</tr>
<tr>
<td>Tail</td>
<td>5.7311***</td>
<td>5.4380***</td>
<td>2.4997***</td>
<td>4.5249***</td>
<td>3.2679***</td>
</tr>
</tbody>
</table>

Note:***,** and * Indicates statistical significance at 1% 5% and 10% level respectively.

Precious metals such as gold can serve as portfolio diversification assets and hedge to equity markets against extreme market conditions, especially during the health crisis, causing uncertainty and hard economic consequences such as the COVID-19 pandemic. Results of Vine copula (Table 4) show lower dependence between DJIM and Gold, which means gold is a good tool for diversification for Islamic securities. These finding are in accordance with the findings of Maghyereh et al (2019). Investors in advanced and emerging markets often switch between oil and gold or combine them to diversify their portfolios (Soytas et al., 2009). Gold carries important psychological commonly known as a "safe haven" from the increasing risks in financial markets.

The role of oil is vital in driving the economy irrespective of the country’s growth rate. Oil price change affects not only the real economy but the financial sector as well. International Brent oil prices declined substantially from 66.5$ barrel on
January 1, 2020, to around 18$barrel on April 22, 2020, one of the largest drops in 20 years. Thus, the oil price is the important determinant of stock returns (Huang et al., (1996)). Based on this theoretical premise, oil would have a significant impact on both conventional and Islamic assets (Tables 4 and 5). These findings are in accordance with Mohanty et al., (2011) who have shown that the effect of oil price on stock returns of conventional and Islamic industry (in the same market) are different. Even the response of Islamic securities is not homogeneous on oil price change. The dependence between oil and Bitcoin markets can be explained through the extensive usage of energy in Bitcoin mining. The annual electrical consumption of Bitcoin mining has increased from 37 TWh on January 1, 2019, to 143 TWh on April 11, 2021, making it larger than the electrical energy usage of Argentina. In fact, the rise/decline in crude oil prices is expected to affect the cost of production and value of Bitcoin (Bouri, Jalkh, et al., 2017; Das & Dutta, 2020; Das et al., 2020), and evidence on the Bitcoin-energy markets’ interrelationships is provided by Corbet et al. (2021).

Bearing in the mind the enormous swings in commodity prices, there is a growing interest in unveiling how the price uncertainty in commodity markets is hedged. Fittingly, cryptocurrencies emerged when investors were seeking new asset classes possessing hedging and safe-haven properties. Cryptocurrencies offer distant features such as low transaction costs, little regulation, decentralization, and anonymity. Despite the high volatility dynamics of cryptocurrencies, the trading volume is increasing, which highlights the potential growth and development opportunities (Shahzad et al. 2019a). Despite the high return variances for Bitcoin, the investment community has demonstrated its attraction towards investments in Bitcoin (Rehman Apergis, 2019; Al-Yahyaee et al., 2019). Also, Islamic assets are documented to possess such hedging abilities that may reduce the overall variances in a portfolio, mainly because of their asset based financing (see Shahzad et al., 2017; Kenourgios et al., 2016). Therefore, as our aim is to include these two assets as our sample so that investors may invest in speculative assets like Bitcoin, however at the same time hedging the variances by including assets with hedging abilities. In tree 3 of vine structure (Table 4 and 5) the inclusion of Bitcoin and DJIM (the same for DJWI) to (Brent,Gold) portfolio possess good safe haven abilities (negative and lower dependence). Turning our analysis to the systemic risk or contagion between assets. First, we find that oil is the most systemically risky asset comparing with Gold and Bitcoin for the two case :Islamic and conventional markets. The systemic risk of all assets, behaves very similarly for the COVID-19

1Source: https://www.reuters.com/finance/commodities/energy
2Source: https://cbeci.org/
Table 4: C-Vine copula parameter estimation between DJIW, Bitcoin, Gold and Brent.

<table>
<thead>
<tr>
<th>Vine structure</th>
<th>Cop</th>
<th>$\hat{\theta}_1$</th>
<th>$\hat{\theta}_2$</th>
<th>$\tau$</th>
<th>$\hat{\lambda}_U$</th>
<th>$\hat{\lambda}_L$</th>
<th>CoVaR(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tree 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DJIM,Gold</td>
<td>t</td>
<td>0.19 (0.06)</td>
<td>4.34 (1.30)</td>
<td>0.12</td>
<td>0.11</td>
<td>0.11</td>
<td>-3.60%</td>
</tr>
<tr>
<td>DJIM,Bitcoin</td>
<td>t</td>
<td>0.25 (0.05)</td>
<td>8.00 (4.46)</td>
<td>0.16</td>
<td>0.05</td>
<td>0.05</td>
<td>-3.33%</td>
</tr>
<tr>
<td>Brent,DJIM</td>
<td>t</td>
<td>0.30 (0.05)</td>
<td>5.34 (1.93)</td>
<td>0.20</td>
<td>0.11</td>
<td>0.11</td>
<td>-3.75%</td>
</tr>
<tr>
<td><strong>Tree 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bitcoin,Gold</td>
<td>DJIM</td>
<td>G</td>
<td>1.11 (0.04)</td>
<td>-</td>
<td>-</td>
<td>0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>Brent,Bitcoin</td>
<td>DJIM</td>
<td>N</td>
<td>0.06 (0.05)</td>
<td>-</td>
<td>-</td>
<td>0.04</td>
<td>-</td>
</tr>
<tr>
<td><strong>Tree 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brent,Gold</td>
<td>Bitcoin</td>
<td>DJIM</td>
<td>N</td>
<td>-0.02 (0.05)</td>
<td>-</td>
<td>-</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

period. The systemic risk reaches its highest value in Tree 2.
Table 5: C-Vine copula parameter estimation between DJIM, Bitcoin, Gold and Brent.

<table>
<thead>
<tr>
<th>Vine structure</th>
<th>Cop  $\theta_1$</th>
<th>$\hat{\theta}_2$</th>
<th>$\tau$</th>
<th>$\hat{\lambda}_U$</th>
<th>$\hat{\lambda}_L$</th>
<th>CoVaR(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tree 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DJWI, Gold</td>
<td>t 0.15 (0.06)</td>
<td>3.90 (1.06)</td>
<td>0.10</td>
<td>0.12</td>
<td>0.12</td>
<td>-3.62%</td>
</tr>
<tr>
<td>DJWI, Bitcoin</td>
<td>N 0.28 (0.05)</td>
<td>-</td>
<td>-</td>
<td>0.18</td>
<td>-</td>
<td>-2.85%</td>
</tr>
<tr>
<td>Brent, DJWI</td>
<td>t 0.35 (0.05)</td>
<td>6.21 (2.62)</td>
<td>0.23</td>
<td>0.10</td>
<td>0.10</td>
<td>-3.71%</td>
</tr>
<tr>
<td><strong>Tree 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bitcoin, Gold</td>
<td>G 1.12 (0.04)</td>
<td>-</td>
<td>-</td>
<td>0.11</td>
<td>0.15</td>
<td>-7.44%</td>
</tr>
<tr>
<td>Brent, Bitcoin</td>
<td>N 0.04 (0.05)</td>
<td>-</td>
<td>-</td>
<td>0.03</td>
<td>-</td>
<td>-6.44%</td>
</tr>
<tr>
<td><strong>Tree 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brent, Gold</td>
<td>N -0.01 (0.05)</td>
<td>-</td>
<td>-</td>
<td>-0.01</td>
<td>-</td>
<td>-6.26%</td>
</tr>
</tbody>
</table>

Table 6: Optimal Weights of 10 portfolios with DJIW.

<table>
<thead>
<tr>
<th>P</th>
<th>DJWI</th>
<th>Bitcoin</th>
<th>Gold</th>
<th>Brent</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2378</td>
<td>0.2539</td>
<td>0.2552</td>
<td>0.2530</td>
<td>0.13%</td>
<td>46%</td>
</tr>
<tr>
<td>2</td>
<td>0.3216</td>
<td>0.2295</td>
<td>0.2314</td>
<td>0.2175</td>
<td>0.16</td>
<td>47%</td>
</tr>
<tr>
<td>3</td>
<td>0.4054</td>
<td>0.2050</td>
<td>0.2077</td>
<td>0.1819</td>
<td>0.19%</td>
<td>50%</td>
</tr>
<tr>
<td>4</td>
<td>0.4893</td>
<td>0.1805</td>
<td>0.1839</td>
<td>0.1463</td>
<td>0.22%</td>
<td>54%</td>
</tr>
<tr>
<td>5</td>
<td>0.5731</td>
<td>0.1561</td>
<td>0.1601</td>
<td>0.1107</td>
<td>0.25%</td>
<td>60%</td>
</tr>
<tr>
<td>6</td>
<td>0.6569</td>
<td>0.1316</td>
<td>0.1364</td>
<td>0.0751</td>
<td>0.28%</td>
<td>67%</td>
</tr>
<tr>
<td>7</td>
<td>0.7407</td>
<td>0.1072</td>
<td>0.1126</td>
<td>0.0396</td>
<td>0.31%</td>
<td>74%</td>
</tr>
<tr>
<td>8</td>
<td>0.8245</td>
<td>0.0827</td>
<td>0.0888</td>
<td>0.0040</td>
<td>0.34%</td>
<td>82%</td>
</tr>
<tr>
<td>9</td>
<td>0.9120</td>
<td>0.0406</td>
<td>0.0474</td>
<td>0</td>
<td>0.37%</td>
<td>91%</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.40%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Table 7: Optimal Weights of 10 portfolios with DJIM.

<table>
<thead>
<tr>
<th>P</th>
<th>DJIM</th>
<th>Bitcoin</th>
<th>Gold</th>
<th>Brent</th>
<th>Return</th>
<th>Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2383</td>
<td>0.2548</td>
<td>0.2540</td>
<td>0.2528</td>
<td>0.054%</td>
<td>45%</td>
</tr>
<tr>
<td>2</td>
<td>0.2762</td>
<td>0.2614</td>
<td>0.2610</td>
<td>0.2014</td>
<td>0.057%</td>
<td>46%</td>
</tr>
<tr>
<td>3</td>
<td>0.3141</td>
<td>0.2680</td>
<td>0.2680</td>
<td>0.1499</td>
<td>0.060%</td>
<td>47%</td>
</tr>
<tr>
<td>4</td>
<td>0.3520</td>
<td>0.2746</td>
<td>0.2750</td>
<td>0.0984</td>
<td>0.063%</td>
<td>50%</td>
</tr>
<tr>
<td>5</td>
<td>0.3898</td>
<td>0.2812</td>
<td>0.2820</td>
<td>0.0469</td>
<td>0.065%</td>
<td>53%</td>
</tr>
<tr>
<td>6</td>
<td>0.4368</td>
<td>0.2810</td>
<td>0.2822</td>
<td>0.</td>
<td></td>
<td>56%</td>
</tr>
<tr>
<td>7</td>
<td>0.5776</td>
<td>0.2104</td>
<td>0.2120</td>
<td>0</td>
<td>0.071%</td>
<td>56%</td>
</tr>
<tr>
<td>8</td>
<td>0.7184</td>
<td>0.1398</td>
<td>0.1418</td>
<td>0</td>
<td>0.074%</td>
<td>73%</td>
</tr>
<tr>
<td>9</td>
<td>0.8592</td>
<td>0.0692</td>
<td>0.0716</td>
<td>0</td>
<td>0.077%</td>
<td>85%</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.08%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Figure 2: Efficient frontiers comparison.

With the data input of full 4 assets, the study estimates a 95% CoVaR of six different portfolios based on the given return of each portfolio and the constraints given above. Then, we construct ten efficient portfolios using a Mean-CoVaR method. Figure 2 presents the results given in Tables 6 and 7 to plot the 95% efficient frontiers. It can be observed that the frontier of DJWI,Bitcoin,Gold,Brent out performs and is always higher than the frontier of DJIM,Bitcoin,Gold,Brent.

We can observe the loss between the Portfolio with DJIM and the Portfolio
with DJWI is different. We find that DJIM could increase the loss of the Portfolio. Table 6 shows the proportions of each asset in the portfolios with DJWI. We can see that portfolio 1 has the lowest expected risk 46% while the expected return is 0.13%. On the other hand, portfolio 10 has the highest expected risk which is 100% while the expected return is 0.40%. In portfolio 1, it can see that the proportion of Gold is highest which is 25.52%. Followed by Bitcoin (25.39%), Brent (25.30%) and DJWI (23.78%). However, if we choose to invest the only DJWI to achieve the highest return at 0.40%, investors have to be aware of the risks of portfolios.

Table 7 shows the proportions of each asset in portfolios with DJIM. It is found that the lowest expected risk in portfolio 1 is 45% while the expected return is 0.054%. On the other hand, the highest expected risk in portfolio 10 is 100% while the expected return is 0.08%. In portfolio 1, it can be seen that Bitcoin shows the highest proportion at 25.48% of portfolios. The proportion of other assets are the followings, Gold (25.40%), Brent (25.28%), and DJIM (23.83%).

Furthermore, our results suggest that the inclusion of DJIW into diversified portfolio may be profitable, serving therefore as risk diversifiers. Nevertheless, the investors who hold portfolios containing stocks, bonds, and other asset may face great losses during bear states. We should not forget to mention the major challenges facing investors in digital monies. Given the short track record of these assets, there is not a standard valuation tool that is mostly accepted to predict the trading prices of Bitcoin, and there is no consensus on the best method able to estimate the price trend. Moreover, the cryptocurrency market exposes to severe speculations, and new players enter the market every day, making the application of any valuation method problematic.

5 Conclusion

We study the portfolio allocation using the Mean-CoVaR methodology proposed by Braiek et al (2020). We investigate the risk dependence between daily Bitcoin DJWI,DJIM, Gold and Oil prices spanning over from the 2nd of December 2019 to the 31st of May 2021. Results of dependence show lower dependence between DJIM and Gold, which means gold is a good tool for diversification for Islamic securities. These finding are in accordance with the findings of Maghyereh et al (2019). Investors in advanced and emerging markets often switch between oil and gold or combine them to diversify their portfolios (Soytas et al., 2009). Gold carries important psychological commonly known as a "safe haven" from the increasing risks in financial markets. Our findings suggest that the inclusion
of DJIW into diversified portfolio may be profitable, serving therefore as risk diversifiers. Nevertheless, the investors who hold portfolios containing stocks, bonds, and others asset may face great losses during bear states. This result corresponds to the result of the efficiency of investment frontier which indicates that investing in DJWI can efficiently increase returns. In conclusion, this study suggests that one can invest in DJIM should one be able to bear the risks as the theory goes "High risk, high return."

On behalf of all authors, the corresponding author states that there is no conflict of interest.
References


The data that support the findings of this study are available from the corresponding author upon reasonable request.