Event-Based Secure Consensus of Multiple AUVs Under DoS Attacks

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Abstract In this paper, the event-based triggering method is adopted to investigate the secure consensus issue of multiple autonomous underwater vehicles (AUVs) under denial-of-service (DoS) attacks. DoS attack is a form of time-sequence-based cyber attack, which can destroy the normal service of the control target or network. First, based on an event-triggered mechanism, a novel secure control protocol is proposed. Second, the upper bounds of attack duration and attack frequency are given to ensure that multiple AUVs under DoS attacks can reach consensus. Third, an event-triggered mechanism with exponential variables is developed to avoid the continuous update of the controller, thereby reducing the burdens of communication and calculation. Zeno behavior can be strictly ruled out for each AUV under this triggering mechanism. Finally, the simulation results illustrate the feasibility of the proposed scheme.

Keywords AUVs · Event-triggered mechanism · DoS attacks · Consensus

1 Introduction

Recently, collaborative control of multi-agent systems has become a hot research field because of its wide application in various fields, such as the formation of robots, unmanned vehicles, and surface ships [2–6]. As we all know, in the numerous studies of cooperative control, the consensus of multiple AUVs is one of the basic problems. The main goal of multiple AUV consensus is to design control protocols based on the local information of the neighbors of the AUV to ensure that the states of all AUVs finally reach consensus [7–9]. Due to the complex underwater environment, it is relatively difficult to share information among multiple AUVs, consensus control becomes a huge challenge.

Consensus of multiple AUVs is a hot topic in both practical and theoretical research. In [7, 8, 12], the consensus of multiple AUVs under fully actuated and underactuated was discussed. In [9, 14–17], the trajectory tracking issue of AUVs with disturbances under different conditions was studied by the sliding mode control method. An impulse network method was

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developed to study the formation problem of multiple AUVs in fixed or switched communication topologies in [10,11]. In [13], a coordinated controller was proposed based on the hybrid control theory, which enabled the controller to switch freely. To cope with the complex marine environment and achieve the purpose of long-term navigation, energy saving is a hot topic of research and the multiple AUVs control under the event-triggered mechanism is necessary.

It is worth noting that in reality, each AUV usually consists of some specific modules with limited energy, such as processor modules, communication modules, and drive modules. The event-triggered control strategy is an effective energy-saving control method, which can reduce the communication and calculation burden caused by continuous communication among multiple AUVs and frequent updates of controllers [18,19]. Therefore, this method can save energy while ensuring control performance. The mathematical model of the AUV can be regarded as a special second-order nonlinear system. Some efforts are made to second-order systems under the triggering mechanism [20–23], but there are few studies about AUVs under the triggering mechanism. In [24,25], the trajectory tracking problem of fully actuated and underactuated AUVs under the event-triggered mechanism was investigated.

The multi-agent systems can be regarded as fragile network systems, which are extremely vulnerable to cyber attacks. DoS attacks are a major form of cyber attacks. DoS attacks can affect or even destroy the normal services and communications of the target network. The design of the secure control protocol is an important issue in the network control systems. To eliminate the impact of DoS attacks on network systems, some secure control protocols were developed in [26,27,31]. Driven by [26,27], the leader-follower multi-agent systems under DoS attacks were discussed in [28]. The distributed consensus controller was designed in [29] under the framework of linear systems. In [30,32,33], the consensus of linear multi-agent systems under DoS attack based on event-triggered mechanism was studied. DoS attack frequency and attack duration are important indicators to determine the multiple AUVs can reach consensus. At present, the results of most DoS attacks are under the framework of linear systems. As we all know, AUV contains nonlinear dynamics. Therefore, the secure consensus problem of multiple AUVs under DoS attacks is worth studying.

Motivated by the above discussions, this paper is the first to investigate the consensus problem of multiple AUVs under DoS attacks based on the event-triggered mechanism. The main contributions of this paper are summarized as follows. First, the DoS attack scheme is applied to secure consensus controller design of multiple AUVs under the event-triggered mechanism. This attack scheme is originally proposed in [30] to solve the consensus problem of the linear systems. Second, the upper bounds of DoS attack frequency and attack duration can be obtained so that the positions of multiple AUVs can reach consensus. Third, an event-triggered mechanism including exponential variables is adopted, which is not involved in [31,32]. Under this triggering mechanism, a larger interevent interval can be obtained to avoid Zeno behavior. Finally, the simulation result shows the effectiveness of the algorithm.

The rest of this paper is summarized as follows. Section 2 provides some preliminaries and the problem statement. In section 3, event-based secure consensus of multiple AUVs under DoS attacks is investigated. In section 4, a simulation example is provided to demonstrate the merits and validness of the obtained results. In section 5, the conclusion and future work are presented.

Notation: R^n is the N dimensional column vectors. $\|\cdot\|$ is the Euclidean norm. $1_N(0_N)$ is a vector with entries being $1(0)$. I_M represents the identity matrix and the dimension is M.

$\lambda_{max}(H)(\lambda_{min}(H))$ is introduced to represent the maximum (minimum) eigenvalues of the matrix H. $diag\{\}$ denotes a diagonal matrix.

2 Preliminaries and problems definition

2.1 Communication topology

The topology among N agents can be described as a graph $G = \{\mathscr{V}, \mathscr{E}, \mathscr{A}\}$ consisting of a set of vertices $\mathscr{V} = \{v_1, v_2, ..., v_N\}$, a set of edges $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, and a weighted adjacency matrix $\mathscr{A} = (a_{ij})_{N\times N}$ with non-negative entries. The directed edge e_{ij} is denoted by a pair of vertices (v_i, v_j), which means v_j can obtain information from v_i, the neighbor set of the v_i is denoted by $\mathscr{N}_i = \{v_j \in \mathscr{V} | ((v_i, v_j)) \in \mathscr{E}\}$ and $e_{ij} \in \mathscr{E}$ if and only if $a_{ij} > 0$; $a_{ij} = 0$ otherwise. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$. For an undirected graph, it follows that $a_{ij} = a_{ji}, \forall i, j \in (1, 2, \ldots, N)$. If there is a reachable path between any two agents in G, then G is considered to be connected.

2.2 AUV model

Consider that a multi-agent system consists of N AUVs. The AUV obtains position information from its neighbors to determine position and orientation through the sensors. Suppose that the attitudes of AUVs are fixed. The kinematic and dynamic model of the i^{th} AUV can be described as [1]

$$\dot{\eta}_i = J_i(\Theta_i)v_i, \tag{1}$$

$$M_i\dot{v}_i = -C_i(v_i)v_i - D_i(v_i)v_i - g_i(\Theta_i) + \tau_i, \tag{2}$$

where $i \in \{1, 2, \ldots, N\}$ is the lable of the AUVs, $\eta_i = [x_i, y_i, z_i]^T \in R^3$ is the position vector of the i^{th} AUV in the inertial reference frame, $\Theta_i = [\theta_i, \phi_i, \psi_i]^T \in R^3$ is the attitude vector and θ_i, ϕ_i, ψ_i are the Euler angles (roll, pitch, yaw) of the i^{th} AUV in the inertial reference frame, $J_i(\Theta_i)$ represents the kinematic transformation matrix . $v_i = [u_i, q_i, w_i] \in R^3$ is the velocity vector and u_i, q_i, w_i are the linear velocities (surge, sway, heave) of the i^{th} AUV in the body-fixed reference frame. M_i denotes the inertia matrix. $C_i(v_i)$ is the Coriolis and centripetal matrix. $D_i(v_i)$ is the hydrodynamic drag matrix. $g_i(\Theta_i) \in R^3$ is the vector of restoring forces (gravity and buoyancy). $\tau_i \in R^3$ is the control input. For brevity, $J_i = J_i(\Theta_i)$, $C_i = C_i(v_i)$, $D_i = D_i(v_i)$, $g_i = g_i(\Theta_i)$. The kinematic transformation matrix is as follows:

$$J_i = \begin{pmatrix} c_{\psi_i}c_{\phi_i} & -s_{\psi_i}c_{\theta_i} + c_{\psi_i}s_{\phi_i}s_{\theta_i} & s_{\psi_i}s_{\theta_i} + c_{\psi_i}c_{\theta_i}s_{\phi_i} \\ s_{\psi_i}c_{\phi_i} & c_{\psi_i}c_{\theta_i} + s_{\psi_i}s_{\phi_i}s_{\theta_i} & -c_{\psi_i}s_{\theta_i} + s_{\psi_i}c_{\theta_i}s_{\phi_i} \\ -s_{\phi_i} & c_{\phi i}s_{\theta_i} & c_{\phi_i}c_{\theta_i} \end{pmatrix},$$

for an angle $\alpha \in R$, the symbol s_α and c_α denote $sin\alpha$ and $cos\alpha$.

$$C_i = \begin{pmatrix} 0 & 0 & -m_{i2}q_i \\ 0 & 0 & m_{i1}u_i \\ m_{i2}q_i & -m_{i1}u_i & 0 \end{pmatrix},$$

$M_i = diag\{m_{i1}, m_{i2}, m_{i3}\}$, $D_i = diag\{d_{i1}, d_{i2}, d_{i3}\}$, $m_{i1} = m - X_{i\dot{u}}$, $m_{i2} = m - Y_{i\dot{q}}$, $m_{i3} = m - Z_{i\dot{w}}$, $d_{i1} = -X_{iu} - X_{i|u|u}|u_i|$, $d_{i2} = -Y_{iq} - Y_{i|q|q}|q_i|$, $d_{i3} = -Z_{iw} - Z_{i|w|w}|w_i|$. $g_i = \left[(F_i - H_i)s_{\phi_i}, -(F_i - H_i)c_{\phi_i}s_{\theta_i}, -(F_i - H_i)c_{\phi_i}c_{\theta_i}\right]^T$, where F_i and H_i denote the gravitational and buoyancy forces. Note that $J_iJ_i^T = I_3$ and $x^TD_i(x)x > 0$, $\forall x \neq 0, x \in R^3$.

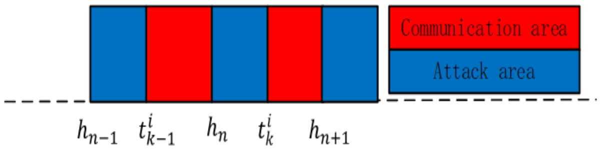

Figure 1 Sketch map of aperiodic sequences: t^i_k and h_n.

2.3 DoS attack model

DoS attacks, as a common form of cyber attacks, can disrupt the information transmission among agents. Under the DoS attacks, the target systems cannot provide normal services. Since the energy of DoS attacks is limited, the systems can enter the recovery phase after each attack to increase energy. Therefore, the entire time can be divided into attack areas and communication areas. As shown in Figure 1, the red areas represent the communication areas. Note that the event-triggered mechanism exists in this area. The blue areas represent the attack areas, in which the controller is not available. Suppose $\{h_n\}_{n\in N}$ is a DoS attack sequence, where h_n represents the instant of the nth attack. For Δ_n, the nth attack interval is $\bar{H}_n = [h_n, h_n + \Delta_n)$, in which $h_{n+1} > h_n + \Delta_n$. For $t \geq t_0$, the sets of time instants for the attack areas and the communication areas are depicted as follows:

$$\Lambda_a(t,t_0) = \bigcup_{n\in N} H_n \cap [t,t_0) \tag{3}$$

$$\Lambda_s(t,t_0) = [t,t_0] \setminus \Lambda_a(t,t_0). \tag{4}$$

Definition 1 (Attack Frequency): Define $\mathscr{N}_a(t,t_0)$ as the number of DoS attacks over $[t,t_0)$. The attack frequency can be depicted as follows:

$$\mathscr{F}_a(t,t_0) = \frac{\mathscr{N}_a(t,t_0)}{t-t_0}. \tag{5}$$

Definition 2 (Attack Duration Rate): For $t > t_0 > 0$, $\Lambda_a(t,t_0)$ denotes the total attack interval in $[t,t_0)$. The attack duration rate can be depicted as follows:

$$rate = \frac{\Lambda_a(t,t_0)}{t-t_0}. \tag{6}$$

Assumption 1: There is a Lipschitz constant $c > 0$, which can make $\|f(a) - f(b)\| \leq c\,\|a-b\|$, for $a,b \in R$.

The purpose of this paper is to design the event-triggered control protocol so that the positions of multiple AUVs under DoS attacks can reach consensus when the following conditions can be satisfied:

$$\lim_{t\to\infty} \|\eta_i - \eta_j\| \to 0. \tag{7}$$

3 Event-based secure consensus of multiple AUVs under DoS attacks

The control protocol of the ith AUV can be given as follows:

$$\tau_i = C_i(v_i)v_i + D_i(v_i)v_i - J_i^T(p_i + J_i v_i) + g_i, \tag{8}$$

where $p_i = \sum_{j=1}^{N} a_{ij}(\eta_i - \eta_j)$ is the relative position relationship between the i^{th} AUV and its neighbors.

According to the control protocol (7), a related vector containing four unknown variables is designed to be $\gamma_i = [\eta_i^T, \eta_j^T, v_i^T, v_j^T]^T$. The control protocol can be described as $\tau_i = \mu(\gamma_i)$.

From the angle of energy saving, the triggering mechanism is adopted to reduce the controller update times as much as possible. Under the triggering mechanism, time t can be divided into the triggering instants of $\{t_k^i\}_{k=0}^{\infty}$. The control protocol changes instantaneously at t_k^i and maintains through the zero-order hold until the next triggering moment t_{k+1}^i. Each agent is triggered within its own event-triggered time. During mutual execution, the controller keeps each segment unchanged. The event-triggered protocol can be depicted as

$$\tau_i(t) = \mu(\gamma_i(t_k^i)) = C_i(v_i(t_k^i))v_i(t_k^i) + D_i(v_i(t_k^i))v_i(t_k^i) - J_i^T(p_i(t_k^i) + J_i v_i(t_k^i)) + g_i. \tag{9}$$

Let $\beta_{ij} = \eta_i - \eta_j$ and $\xi_i = J_i v_i, \forall i, j \in \mathscr{V}$. Then, the kinematic and dynamic model of the ith AUV can be rewritten as

$$\begin{aligned} \dot{\beta}_{ij} &= \xi_i - \xi_j, \\ \dot{\xi}_i &= -J_i M^{-1} C_i J_i^T \xi_i - J_i M^{-1} D_i J_i^T \xi_i - J_i M^{-1} g_i + J_i M^{-1} \mu(\gamma_i(t_k^i)). \end{aligned} \tag{10}$$

According to the definition of DoS attacks, when the system suffers DoS attacks, the control protocol cannot be available, i.e., $\tau_i(t) = 0$. Therefore, the kinematic and dynamic model of the ith AUV can be given as

$$\begin{aligned} \dot{\beta}_{ij} &= \xi_i - \xi_j, \\ \dot{\xi}_i &= -J_i M^{-1} C_i J_i^T \xi_i - J_i M^{-1} D_i J_i^T \xi_i - J_i M^{-1} g_i. \end{aligned} \tag{11}$$

A measurement error applicable to the system (9) is defined as

$$e_i = \gamma_i(t_k^i) - \gamma_i = [\eta_i^T(t_k^i) - \eta_i^T, \eta_j^T(t_k^i) - \eta_j^T, v_i^T(t_k^i) - v_i^T, v_j^T(t_k^i) - v_j^T]^T. \tag{12}$$

Based on (9) and (11), one has

$$\begin{aligned} \dot{\beta}_{ij} &= \xi_i - \xi_j, \\ \dot{\xi}_i &= -J_i M^{-1} C_i J_i^T \xi_i - J_i M^{-1} D_i J_i^T \xi_i - J_i M^{-1} g_i + J_i M^{-1}(\mu_i(\gamma_i) + \mu(\gamma_i + e_i) - \mu_i(\gamma_i)). \end{aligned} \tag{13}$$

Next, a triggering function is designed to determine when the controller is updated, which can be described as follows:

$$f_i(e_i, \xi_i, \xi_j, t) = \|e_i\|^2 - \frac{1}{\rho_i}(\delta_i \|\xi_i\|^2 - \iota_i \sum_{j=1}^{N} \|\beta_{ij}\|^2 + k_i e^{-d_i t}), \tag{14}$$

where $\rho_i > 0$, $0 < \delta_i < 1$ and $k_i, d_i > 0$. The triggering instant t_k^i of the ith agent is determined by

$$t_{k+1}^i = inf\left\{t > t_k^i | f_i(e_i, \xi_i, \xi_j, t) \geq 0\right\}. \tag{15}$$

Remark 1. It is worth noting that the term $k_i e^{-d_i t}$ in (13) is exponential convergent function, which is the key factor for dynamically adjusting the interevent interval. The variable $k_i e^{-d_i t}$ reduces the number of triggers by extending the interevent interval, so that Zeno behavior can be excluded.

Theorem 1. Suppose Assumption 1 holds. Considering system (1) under DoS attacks with the event-triggered mechanism (14) can ensure that the positions of multiple AUVs reach consensus when the following conditions can be satisfied:

$$\mathscr{F}_a(t,t_0) = \frac{\mathscr{N}_a(t,t_0)}{t-t_0} \leq \frac{\varepsilon_1^*}{\ln\mu + (\varepsilon_1+\varepsilon_2)\Delta_*} \tag{16}$$

$$rate < \frac{\varepsilon_1 - \varepsilon_1^*}{\varepsilon_1+\varepsilon_2} \tag{17}$$

where $\varepsilon_1^* \in (0,\varepsilon_1)$, $\varsigma_{max}(\varsigma_{min}) > 0$, $\varepsilon_1 = min\{2(1-\delta_{max})/\lambda_{max}(H), 2\iota_{min}/\lambda_{max}(L)\}$ and $\varepsilon_2 = max\left\{2(\varsigma_{max} + \lambda_{max}(J_i(C_i+D_i)J_i^T))/\lambda_{min}(H), 2\lambda_{max}(L)/\varsigma_{min}\lambda_{min}(L)\right\}$.

Proof: When DoS attacks exist, there are two time sequences in the systems: triggering sequence $\left\{t_k^i\right\}_{k=0}^{\infty}$ and attack sequence $\{h_n\}_{n\in N}$. The update set of agents in the attack areas: $\aleph = \left\{(i,k) \in \mathscr{V}\times N | t_k^i \in \bar{H}_n\right\}$. After each DoS attack, agent need time Δ_n to recover communication. Meanwhile, there is a time delay Δ_* between two attacks and the interval between two attacks is greater than Δ_*. $\Pi_n = [h_n, h_n+\Delta_n+\Delta_*)$ denotes the nth interval where the triggering condition (14) cannot work. For any time interval $[t,t_0)$, we have $[t,t_0) = \bar{\Lambda}_s(t,t_0) \cup \bar{\Lambda}_a(t,t_0)$, where $\bar{\Lambda}_a(t,t_0) = \cup\Pi_n \cap [t,t_0]$ and $\bar{\Lambda}_s(t,t_0) = [t,t_0] \setminus \bar{\Lambda}_a(t,t_0)$.

First, the system without DoS attacks is considered, i.e., the system is in the communication areas $\bar{\Lambda}_s(t,t_0)$. Choose the Lyapunov function candidate as

$$V = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\beta_{ij}^T\beta_{ij} + \frac{1}{2}\sum_{i=1}^{N}\xi_i^T H \xi_i, \tag{18}$$

where $H = (J_i M_i^{-1} J_i^T)^{-1}$.
Then, one can obtain that

$$\begin{aligned}
\dot{V} = &\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\beta_{ij}^T(\xi_i - \xi_j) + \sum_{i=1}^{N}\xi_i^T H(-J_i M^{-1} C_i J_i^T \xi_i - J_i M^{-1} D_i J_i^T \xi_i \\
&- J_i M^{-1} g_i + J_i M^{-1}(\mu_i(\gamma_i) + \mu(\gamma_i + e_i) - \mu_i(\gamma_i))) \\
= &\sum_{i=1}^{N}\xi_i^T \sum_{j=1}^{N} a_{ij}\beta_{ij} - \sum_{i=1}^{N}\xi_i^T J_i C_i J_i^T \xi_i - \sum_{i=1}^{N}\xi_i^T J_i D_i J_i^T \xi_i - \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} g_i \\
&+ \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} C_i J_i^T \xi_i + \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} D_i J_i^T \xi_i - \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} J_i^T p_i \\
&- \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} J_i^T \xi_i + \sum_{i=1}^{N}\xi_i^T H J_i M^{-1} g_i + \sum_{i=1}^{N}\xi_i^T H J_i M^{-1}(\mu(\gamma_i + e_i) - \mu_i(\gamma_i)).
\end{aligned} \tag{19}$$

According to Assumption 1, suppose the Lipschitz constant is ℓ. Then, we have $\|\mu(\gamma_i + e_i) - \mu_i(\gamma_i)\| \leq \ell\|e_i\|$.

Thus

$$
\begin{aligned}
\dot{V} \leq & \sum_{i=1}^{N} \xi_i^T \sum_{j=1}^{N} a_{ij}\beta_{ij} - \sum_{i=1}^{N} \xi_i^T J_i C_i J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T J_i D_i J_i^T \xi_i + \sum_{i=1}^{N} \xi_i^T J_i C_i J_i^T \xi_i \\
& + \sum_{i=1}^{N} \xi_i^T J_i D_i J_i^T \xi_i - \sum_{i=1}^{N} \xi_i^T \sum_{j=1}^{N} a_{ij}\beta_{ij} - \sum_{i=1}^{N} \xi_i^T \xi_i + \sum_{i=1}^{N} \xi_i^T J_i \ell \, \|e_i\| \\
\leq & -\sum_{i=1}^{N} \xi_i^T \xi_i + \sum_{i=1}^{N} \rho_i \|e_i\|^2 \\
= & -\sum_{i=1}^{N} \|\xi\|^2 + \sum_{i=1}^{N} \rho_i \|e_i\|^2
\end{aligned}
\tag{20}
$$

where $\rho_i = \frac{\|J_i\|\,\|\ell\|}{2}$. Based on the event-triggered mechanism (14), it can be obtained that

$$
\|e_i\|^2 \leq \frac{1}{\rho_i}(\delta_i \|\xi_i\|^2 - \iota_i \sum_{j=1}^{N} \left\|\beta_{ij}\right\|^2 + k_i e^{-d_i t}). \tag{21}
$$

From the inequations (17) and (18), one has

$$
\begin{aligned}
\dot{V} & \leq \sum_{i=1}^{N} [-(1-\delta_i)\|\xi_i\|^2 - \iota_i \sum_{j=1}^{N} \left\|\beta_{ij}\right\|^2 + k_i e^{-d_i t}] \\
& \leq -\varepsilon_1 V(t)
\end{aligned}
\tag{22}
$$

where $\varepsilon_1 = min\{2(1-\delta_{max})/\lambda_{max}(H), 2\iota_{min}/\lambda_{max}(L)\}$.

Next, the system with DoS attacks is considered, i.e., the system is in the attack areas $\bar{\Lambda}_a(t,t_0)$. Choose the same Lyapunov function as (15). Then, one has

$$
\begin{aligned}
\dot{V} & \leq \sum_{i=1}^{N} \xi_i^T \sum_{j=1}^{N} a_{ij}\beta_{ij} + \sum_{i=1}^{N} \xi_i^T J_i (C_i + D_i) J_i^T \xi_i \\
& \leq \varsigma_i \sum_{i=1}^{N} \xi_i^T \xi_i + \frac{1}{\varsigma_i} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\beta_{ij}^T \beta_{ij} + \sum_{i=1}^{N} \xi_i^T J_i (C_i + D_i) J_i^T \xi_i \\
& \leq \varepsilon_2 V(t)
\end{aligned}
\tag{23}
$$

where $\varepsilon_2 = max\left\{2(\varsigma_{max} + \lambda_{max}(J_i(C_i+D_i)J_i^T))/\lambda_{min}(H), 2\lambda_{max}(L)/\varsigma_{min}\lambda_{min}(L)\right\}$. For $t \in [h_{n-1}+\Delta_{n-1}, h_n)$ and $t \in [h_n, h_n+\Delta_n+\Delta_*)$, we define $V(t) = V_a(t)$ and $V(t) = V_b(t)$, respectively. Inspired by [24], we have

$$
V(t) \leq \begin{cases} e^{-\varepsilon_1 (t-h_{n-1}-\Delta_{n-1})} V_a(t)(h_{n-1}+\Delta_{n-1}) \\ e^{\varepsilon_2 (t-h_n)} V_b(t)(h_n). \end{cases} \tag{24}
$$

When $t \in [h_{n-1}+\Delta_{n-1}, h_n)$, one has

$$
\begin{aligned}
V(t) & \leq e^{-\varepsilon_1 (t-h_{n-1}-\Delta_{n-1})} V_a(t)(h_{n-1}+\Delta_{n-1}) \\
& \leq \mu e^{-\varepsilon_1 (t-h_{n-1}-\Delta_{n-1})} V_b(t)(h_{n-1}^- + \Delta_{n-1}^-) \\
& \leq \cdots \\
& \leq \mu^n e^{-\varepsilon_1 \left|\bar{\Lambda}_s(t,t_0)\right|} e^{\varepsilon_2 \left|\bar{\Lambda}_a(t,t_0)\right|} V_a(t_0).
\end{aligned}
\tag{25}
$$

When $t \in [h_n, h_n + \Delta_n + \Delta_*)$, one has

$$\begin{aligned} V(t) &\leq e^{\varepsilon_2(t-h_n)} V_b(t)(h_n) \\ &\leq \mu e^{\varepsilon_2(t-h_n)} V_a(t)(h_n^-) \\ &\leq \cdots \\ &\leq \mu^{n+1} e^{-\varepsilon_1 \left|\bar{\Lambda}_s(t,t_0)\right|} e^{\varepsilon_2 \left|\bar{\Lambda}_a(t,t_0)\right|} V_a(t_0). \end{aligned} \tag{26}$$

where $\mu = max\{\lambda_{max}(L), \lambda_{max}(H)\}$

Based on the Definition 1 of attack frequency , we can get the number of the communication areas and the attack areas as $\mathcal{N}_a(t,t_0) = n$ and $\mathcal{N}_a(t,t_0) = n+1$, respectively. For $[t_0,t]$, note that $\left|\bar{\Lambda}_s(t,t_0)\right| = t - t_0 - \left|\bar{\Lambda}_a(t,t_0)\right|$ and $\left|\bar{\Lambda}_a(t,t_0)\right| \leq |\Lambda_a(t,t_0)| + (1+\mathcal{N}_a(t,t_0))\Delta_*$. From (23), one has

$$\begin{aligned} V(t) &\leq \mu^{\mathcal{N}_a(t,t_0)} e^{-\varepsilon_1 \left|\bar{\Lambda}_s(t,t_0)\right|} e^{\varepsilon_2 \left|\bar{\Lambda}_a(t,t_0)\right|} V(t_0) \\ &= \mu^{\mathcal{N}_a(t,t_0)} e^{-\varepsilon_1 (t-t_0-\left|\bar{\Lambda}_a(t,t_0)\right|)} e^{\varepsilon_2 \left|\bar{\Lambda}_a(t,t_0)\right|} V(t_0) \\ &\leq e^{\mathcal{N}_a(t,t_0)\ln\mu} e^{-\varepsilon_1(t-t_0)+(\varepsilon_1+\varepsilon_2)[|\Lambda_a(t,t_0)|+(1+\mathcal{N}_a(t,t_0))\Delta_*]} V(t_0) \\ &= e^{\mathcal{N}_a(t,t_0)\ln\mu} e^{-\varepsilon_1(t-t_0)+(\varepsilon_1+\varepsilon_2)[(t-t_0)rate+(1+\mathcal{N}_a(t,t_0))\Delta_*]} V(t_0) \\ &= e^{(\varepsilon_1+\varepsilon_2)\Delta_*} e^{[-\varepsilon_1+(\varepsilon_1+\varepsilon_2)rate](t-t_0)} e^{\mathcal{N}_a(t,t_0)[(\varepsilon_1+\varepsilon_2)\Delta_*+\ln\mu]} V(t_0). \end{aligned} \tag{27}$$

Since $\mathcal{F}_a(t,t_0) = \dfrac{\mathcal{N}_a(t,t_0)}{t-t_0} \leq \dfrac{\varepsilon_1^*}{\ln\mu + (\varepsilon_1+\varepsilon_2)\Delta_*}$ and $rate < \dfrac{\varepsilon_1 - \varepsilon_1^*}{\varepsilon_1+\varepsilon_2}$, we have

$$\begin{aligned} V(t) &\leq e^{(\varepsilon_1+\varepsilon_2)\Delta_*} e^{[-\varepsilon_1+(\varepsilon_1+\varepsilon_2)rate](t-t_0)} e^{\varepsilon_1^*(t-t_0)} V(t_0) \\ &= e^{(\varepsilon_1+\varepsilon_2)\Delta_*} e^{[-\varepsilon_1+(\varepsilon_1+\varepsilon_2)rate+\varepsilon_1^*](t-t_0)} V(t_0) \\ &\leq e^{(\varepsilon_1+\varepsilon_2)\Delta_*} e^{-\bar{\varepsilon}_1(t-t_0)} V(t_0) \end{aligned} \tag{28}$$

where $\bar{\varepsilon}_1 = \varepsilon_1 - (\varepsilon_1+\varepsilon_2)rate - \varepsilon_1^* > 0$. According to (25), we can obtain that

$$\lim_{t\to\infty} V(t) \leq \lim_{t\to\infty} (e^{(\varepsilon_1+\varepsilon_2)\Delta_*} e^{-\bar{\varepsilon}_1(t-t_0)} V(t_0)) \leq 0. \tag{29}$$

From (26), it can be seen from the above formula that multiple AUVs can achieve position consensus i.e, $\eta_i - \eta_j \to 0, \forall i,j \in \mathcal{V}$, $v_i \to 0, \forall i \in \mathcal{V}$.

Theorem 2. Zeno behavior can be ruled out for each AUV in the communication area $\bar{\Lambda}_s(t,t_0)$ under the proposed event-triggered protocol (8) if he following conditions are satisfied:

$$t_{k+1}^i - t_k^i \geq \tau_i \geq \frac{1}{\ell} \ln\left[\frac{\ell}{\sigma_i}\sqrt{\frac{1}{\bar{\rho}_i} k_i e^{-d_i t}} + 1\right] \tag{30}$$

where $\bar{\rho}_i = max\{\rho_i\}$, $i = (1,2,\ldots,N)$.

Proof: For $t \in \bar{\Lambda}_s(t,t_0)$, Zeno phenomenon can be excluded for any AUV under the triggering mechanism (14). From the Lyapunov function V, it can be obtained that $\left\|[\eta_i^T, \eta_j^T, v_i^T, v_j^T]^T\right\| \leq$

$\omega_i = \sqrt{max\lambda(L), \lambda(H)/min\lambda(L), \lambda(H)\bar{\omega}_i^2}$, and $\left\|[\eta_i^T(t_0^i), \eta_j^T(t_0^j), v_i^T(t_0^i), v_j^T(t_0^i)]^T\right\| \leq \bar{\omega}_i$, $\bar{\omega}_i > 0$. According to the property of the triangle inequality. Then, one has

$$\begin{aligned} \frac{d\left\|e_i\right\|}{dt} &= \left\|[\dot{\eta}_i^T, \dot{\eta}_j^T, \dot{v}_i^T, \dot{v}_j^T]^T\right\| \\ &\leq ||[J_i^T v_i^T, J_j^T v_j^T, [-M^{-1}C_i J_i^T \xi_i - M^{-1} D_i J_i^T \xi_i - M^{-1} g_i \\ &\quad + M^{-1}(\mu_i(\gamma_i))]^T]^T|| + ||M^{-1}(\mu(\gamma_i + e_i) - \mu_i(\gamma_i))|| \end{aligned} \tag{31}$$

Based on Assumption 1, for the right side formula of the i^{th} AUV, we define a Lipschitz constant ℓ_1. then we have

$$\frac{d\left\|e_i\right\|}{dt} \leq \ell_1 \left\|[\eta_i^T, \eta_i^T, v_i^T, v_j^T]^T\right\| + \ell\left\|e_i\right\| \leq \ell\left\|e_i\right\| + \sigma_i \tag{32}$$

where $\sigma_i = \ell_1 \omega_i > 0$.

By contradiction, we prove that each AUV can exclude Zeno behavior. Suppose Zeno behavior for the i^{th} AUV exists, we define a finite value $\Xi_i > 0$, which has the following relationship with the triggering time series t_k^i, where $k = (0, 1, \ldots, \infty)$ and $t_k^i \leq \Xi_i$, then $\lim_{k\to\infty} t_k^i = \Xi_i$. Based on the definition of the finite sequence, there is $\kappa_i > 0$, so $\Delta_i - \kappa_i < t_k^i \leq \Xi_i$ can be established, when $k \geq l_i$ and $l_i > 0$.

For $t \in [t_k^i, t_{k+1}^i)$, based on (18), we have

$$\left\|e_i\right\| \leq \frac{\sigma_i}{\ell}[e^{\ell(t-t_k^i)} - 1] \tag{33}$$

From the event-triggered function (14), when the measurement error $\left\|e_i\right\|^2$ goes from 0 to $\frac{1}{\rho_i}(\delta_i \left\|\xi_i\right\|^2 - \iota_i \sum_{j=1}^N \left\|\beta_{ij}\right\|^2 + k_i e^{-z_i t})$, the interevent interval can be calculated. Then, based on (19), a lower interevent interval τ_i can be given from the following equation

$$\frac{\sigma_i}{\ell}[e^{\ell\tau_i} - 1] = \sqrt{\frac{1}{\bar{\rho}_i} k_i e^{-d_i t}} \tag{34}$$

where $\bar{\rho}_i$ is the $\bar{\rho}_i = max\{\rho_i\}$, $i = (1, 2, \ldots, N)$. According to equation (32), we have

$$\kappa_i = \frac{1}{2\ell} \ln[\frac{\ell}{\sigma_i}\sqrt{\frac{1}{\bar{\rho}_i} k_i e^{-d_i t}} + 1] \tag{35}$$

where $\kappa_i > 0$ for $k \geq l_i$. By solution τ_i of the equation (32), it can be obtained that $\tau_i \geq 2\kappa_i$, which means that $t_{k+1}^i \geq t_k^i + \tau_i > \Xi_i$. The above result contradicts hypothesis $\Xi_i - \kappa_i < t_{k+1}^i \leq \Xi_i$. The above analysis shows that the Zeno behavior does not exist for each AUV.

4 Simulation example

In this section, a simulation example is given to illustrate the effectiveness of the proposed algorithms. The communication topology is described as Figure 1. It can be seen from the figure that the topology is connected and contains four AUVs. The parameters of the i^{th} AUV dynamics model are given in Table 1.

The DoS attack sequence is shown in Figure 3 under the event-triggered control protocol. The five DoS attacks in the figure are random and unknown. According to Theorem

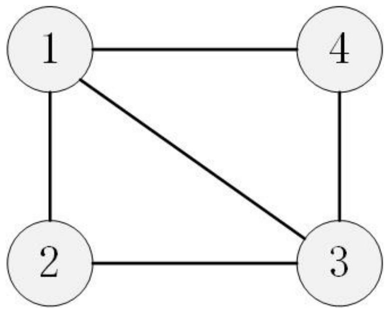

Figure 2 Topological structure of the communication graph.

Table 1 Parameters of i^{th} AUV model

m=100kg	F_i=1148N	H_i=1108N
X_{iu}=-120kg/s	Y_{iq}=-90kg/s	Z_{iw}=-150kg/s
$X_{i\dot{u}}$=-75.4kg	$Y_{i\dot{q}}$=-40.8kg	$Z_{i\dot{w}}$=-40.8kg
$X_{i\|u\|\|u\|}$ $\|u_i\|$=-90kg/m	$Y_{i\|q\|\|q\|}$ $\|q_i\|$-90kg/m	$Z_{i\|w\|\|w\|}$ $\|w_i\|$=-120kg/m

1, the attack duration rate is $rate = 0.36 < \dfrac{\varepsilon_1 - \varepsilon_1^*}{\varepsilon_1 + \varepsilon_2} = 0.378$ and the attack frequency is $\mathscr{F}_a(t,t_0) \leq \dfrac{\varepsilon_1^*}{\ln\mu + (\varepsilon_1 + \varepsilon_2)\Delta_*} = 0.0016$.

The simulation parameters are selected as follows: $\rho_i = 1$, $\delta_i = 0.5$, $k_i = 3$, $d_i = 1$ and $\Theta_i = [\frac{\pi}{5}, -\frac{\pi}{10}, \frac{\pi}{12}]^T$. The initial states are chosen as $\eta_i(0) = [i,i,i]^T$ and $v_i(0) = [i,i,i]^T$, where $i = 1,2,3,4$. Figures 4-6 show that the states of four AUVs are asymptotical consensus. The total number of triggering instants for AUVs can be seen in Figure 7.

In order to verify the superiority of the triggering mechanism (14) (Case: A) proposed in this paper. The same DoS attack is adopted, compared with the triggering mechanism in [30] (Case: B)that does not contain exponential variables. From Figure 7 and Figure 8, it can be seen that the event-triggered mechanism mentioned in this paper can effectively reduce the number of triggers.

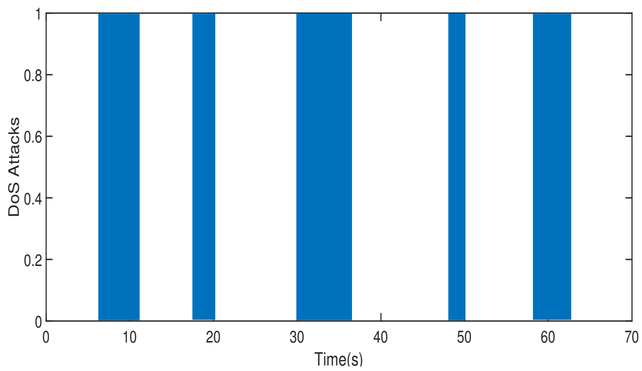

Figure 3 Sequence of DoS attacks.

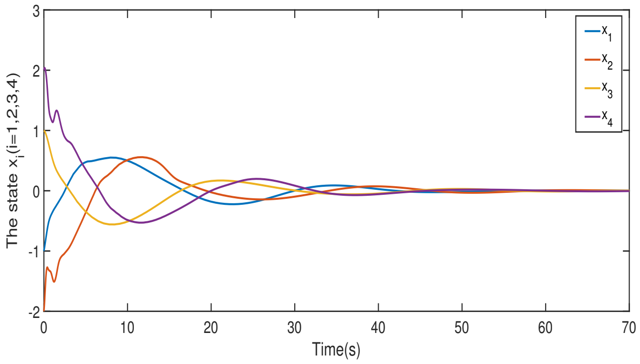

Figure 4 State trajectories of the AUVs in x direction.

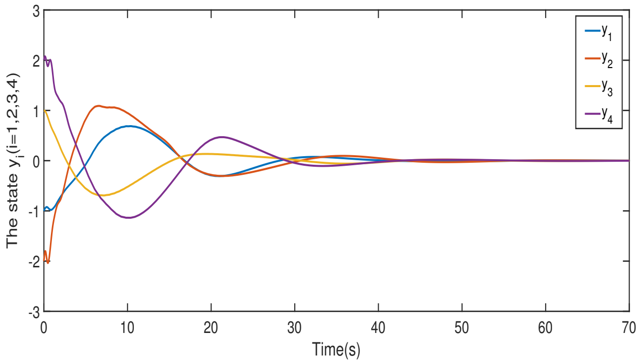

Figure 5 State trajectories of the AUVs in y direction.

5 Conclution

In this paper, we study the secure consensus of multiple AUVs with DoS attacks by adopting event-triggered mechanism method. Based on the DoS attack characteristics adopted in this paper, we know that the attacks are unknown and occur irregularly. Compared with the existing results on the consensus problem of DoS attacks, we propose a novel triggering condition. Under the proposed triggering conditions, the attack frequency and attack duration of DoS attacks are discussed. As long as these conditions are satisfied, the consensus problem can be solved. In future work, we will try to study the consensus of multiple AUVs in the case of directed topology.

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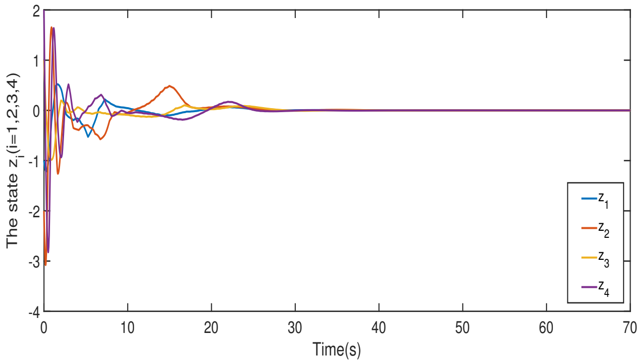

Figure 6 State trajectories of the AUVs in z direction.

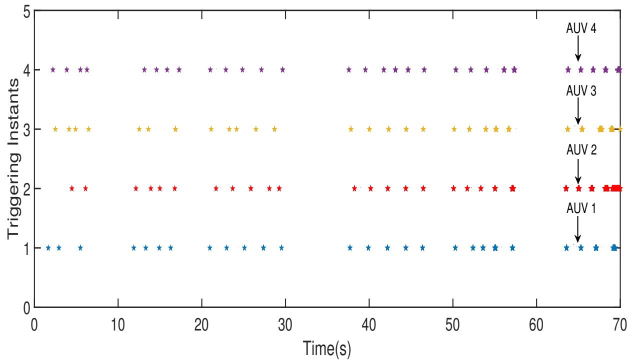

Figure 7 Triggering instants for AUVs in x direction (Case: A).

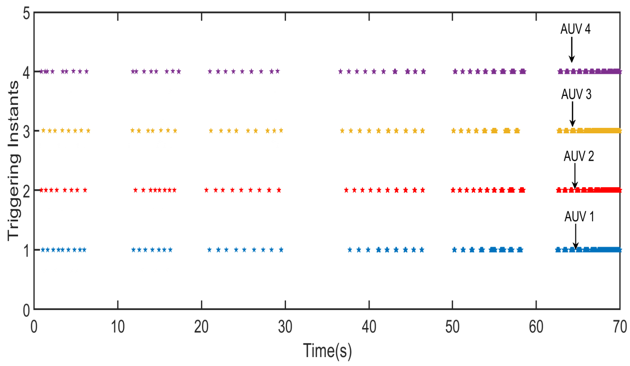

Figure 8 Triggering instants for AUVs in x direction (Case: B).

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Dear editors:

We declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figures

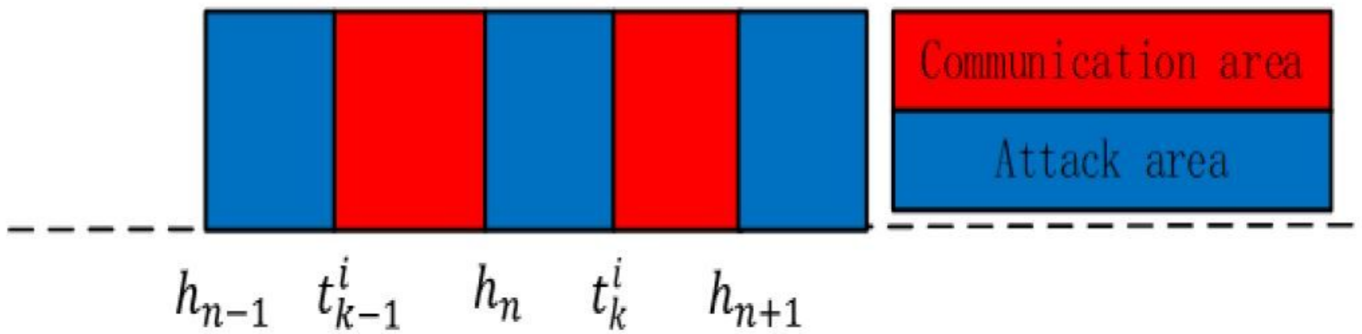

Figure 1

Sketch map of aperiodic sequences: tik and hn.

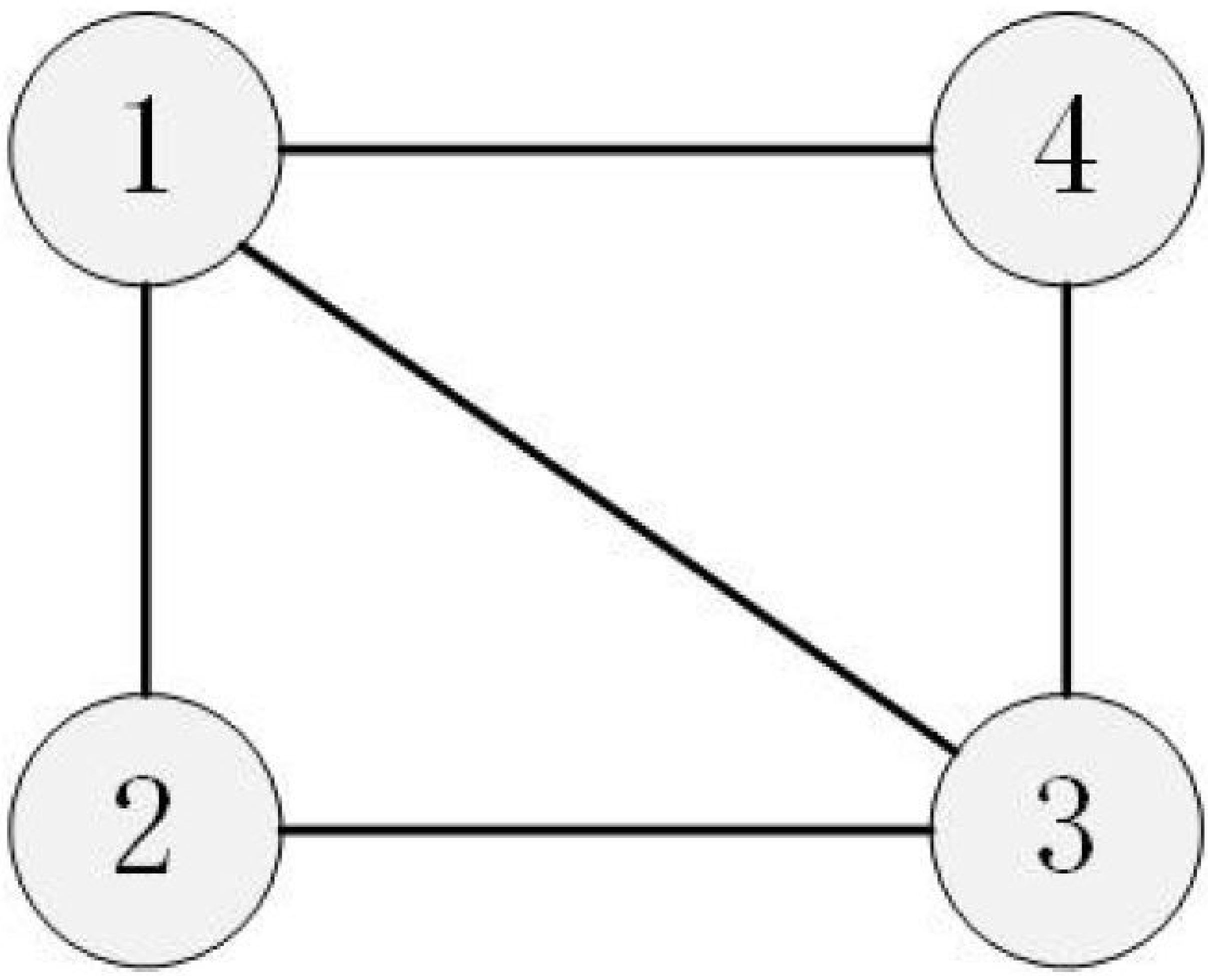

Figure 2

Topological structure of the communication graph.

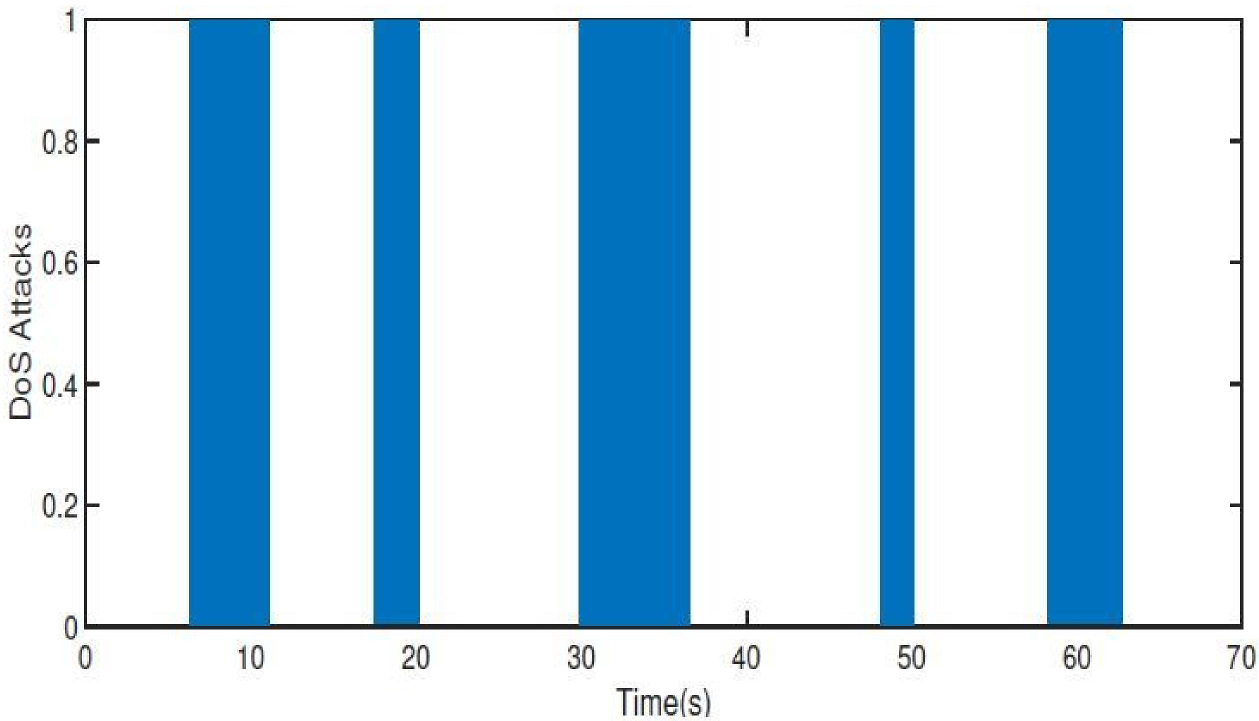

Figure 3

Sequence of DoS attacks.

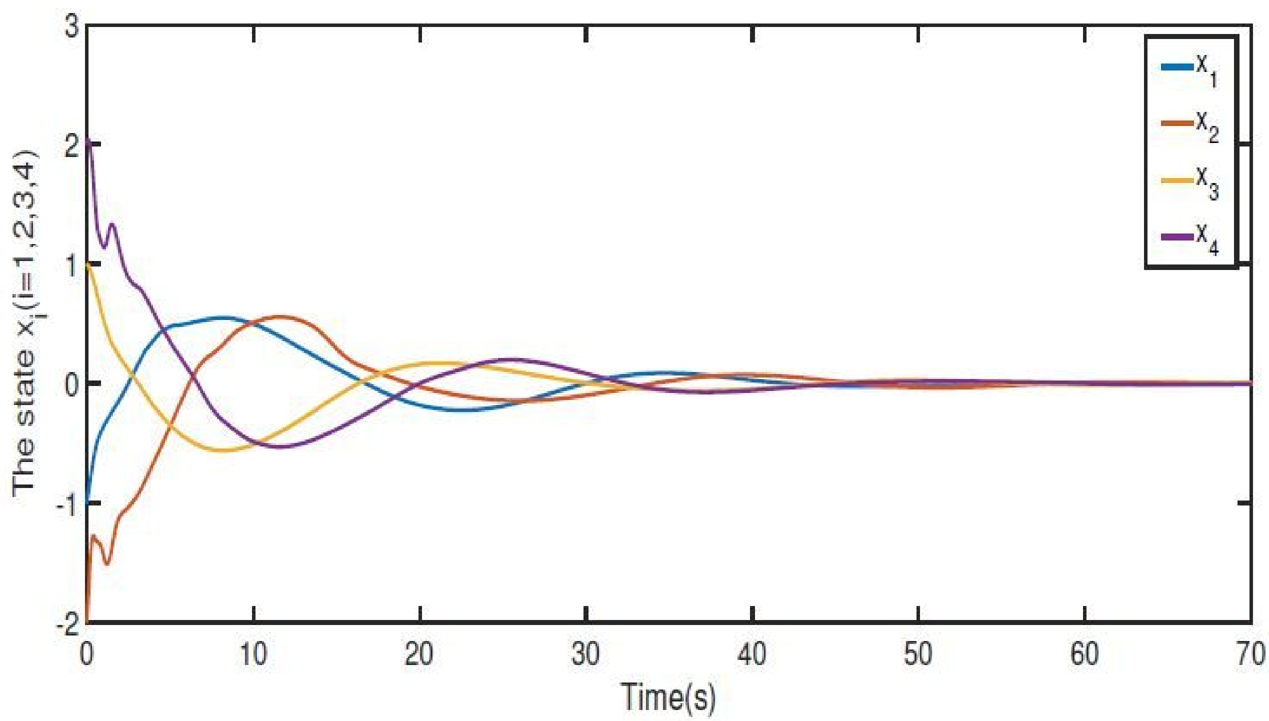

Figure 4

State trajectories of the AUVs in x direction.

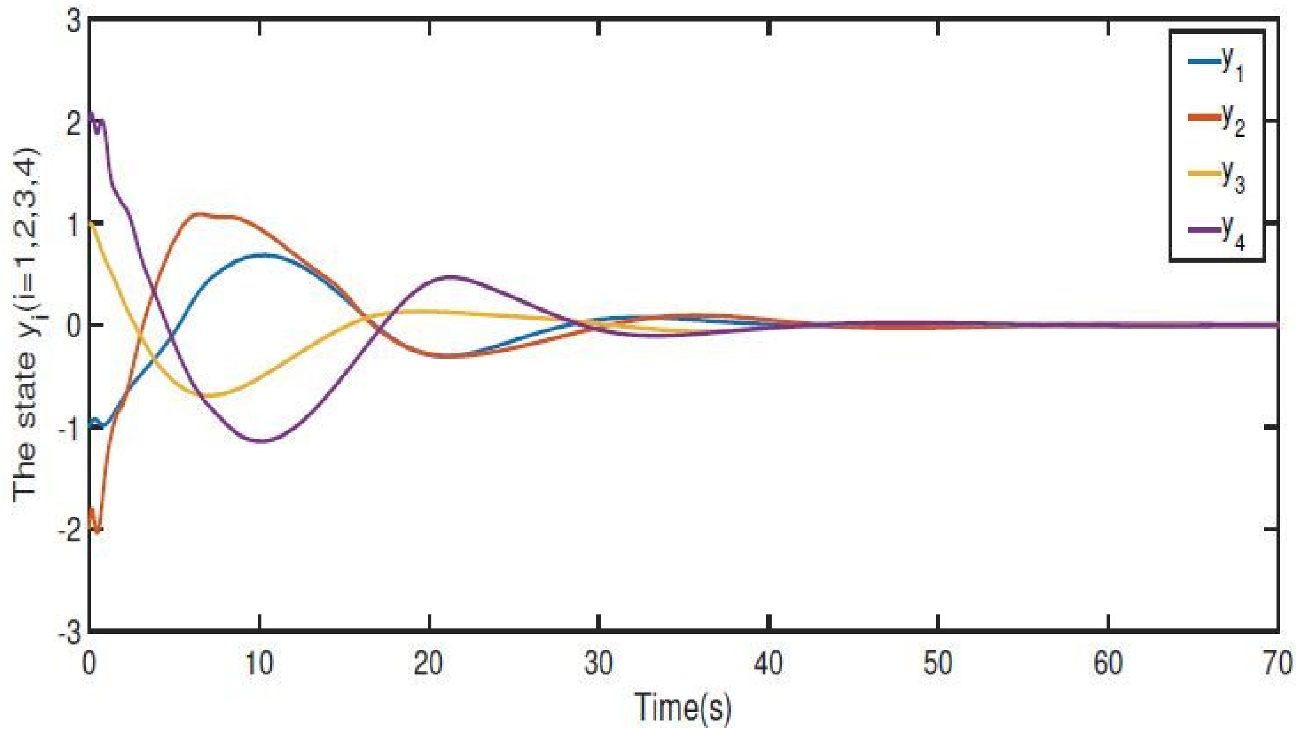

Figure 5

State trajectories of the AUVs in y direction.

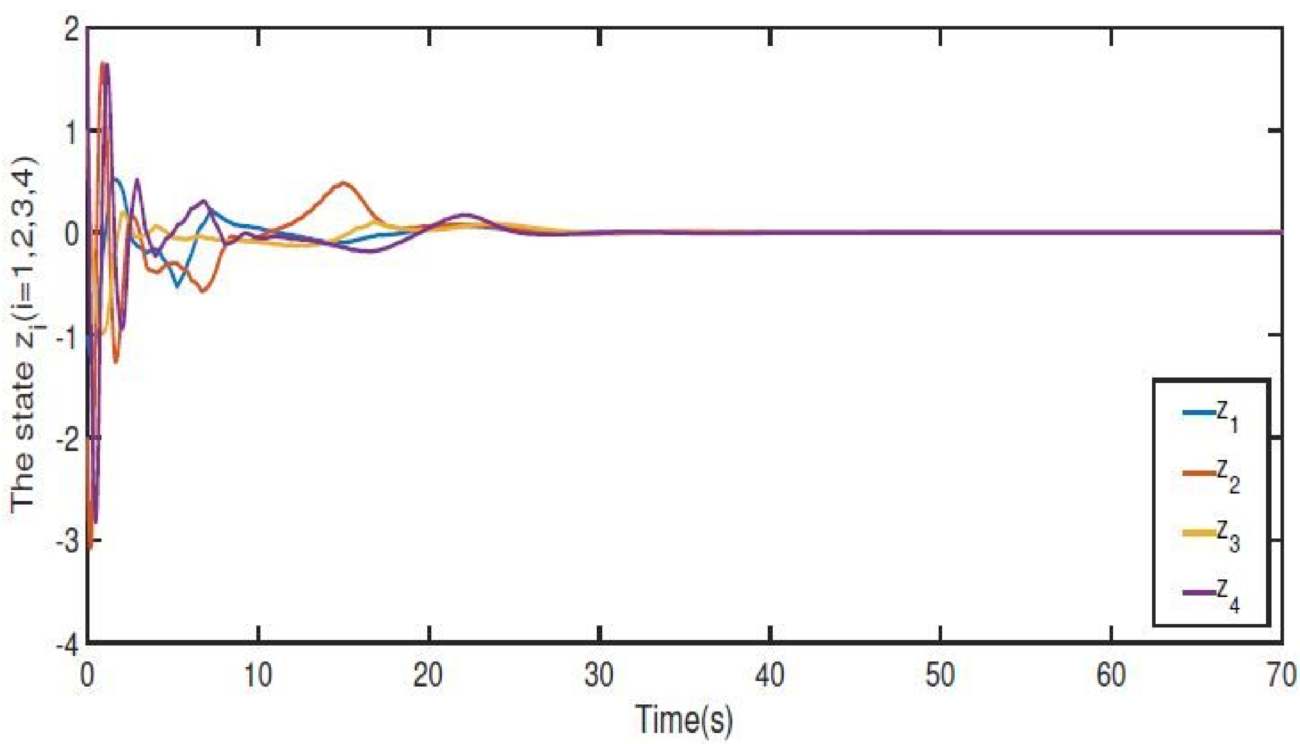

Figure 6

State trajectories of the AUVs in z direction.

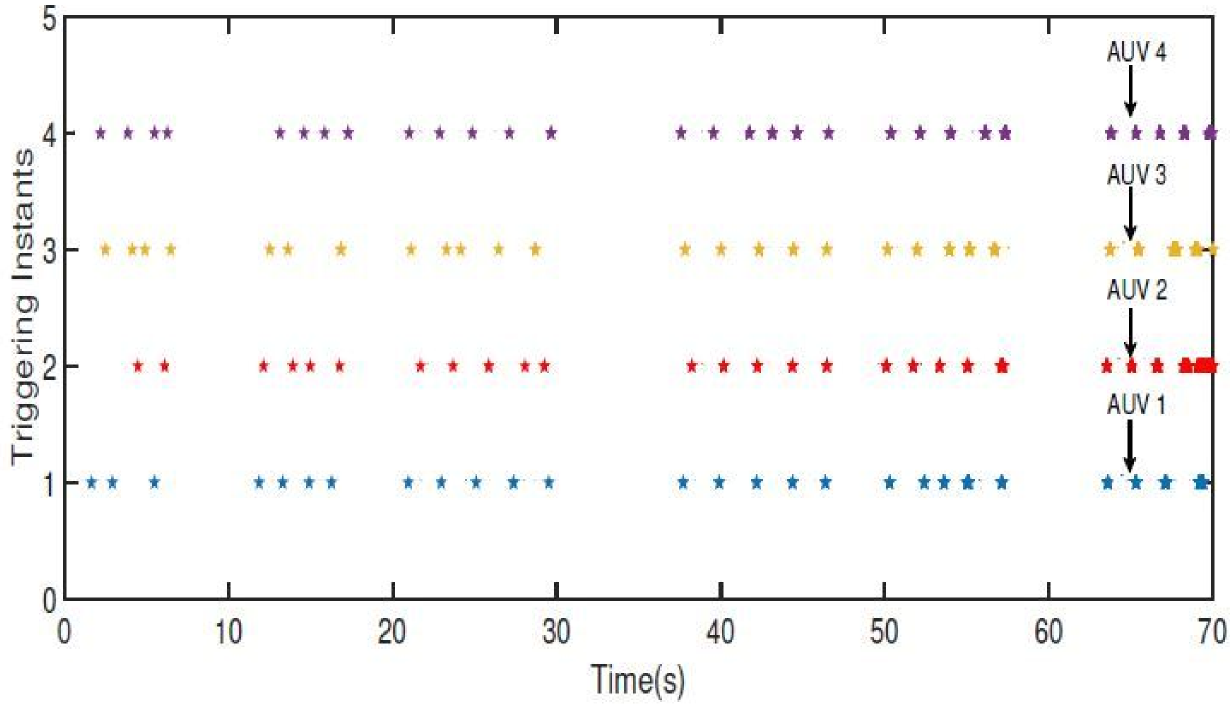

Figure 7

Triggering instants for AUVs in x direction (Case: A).

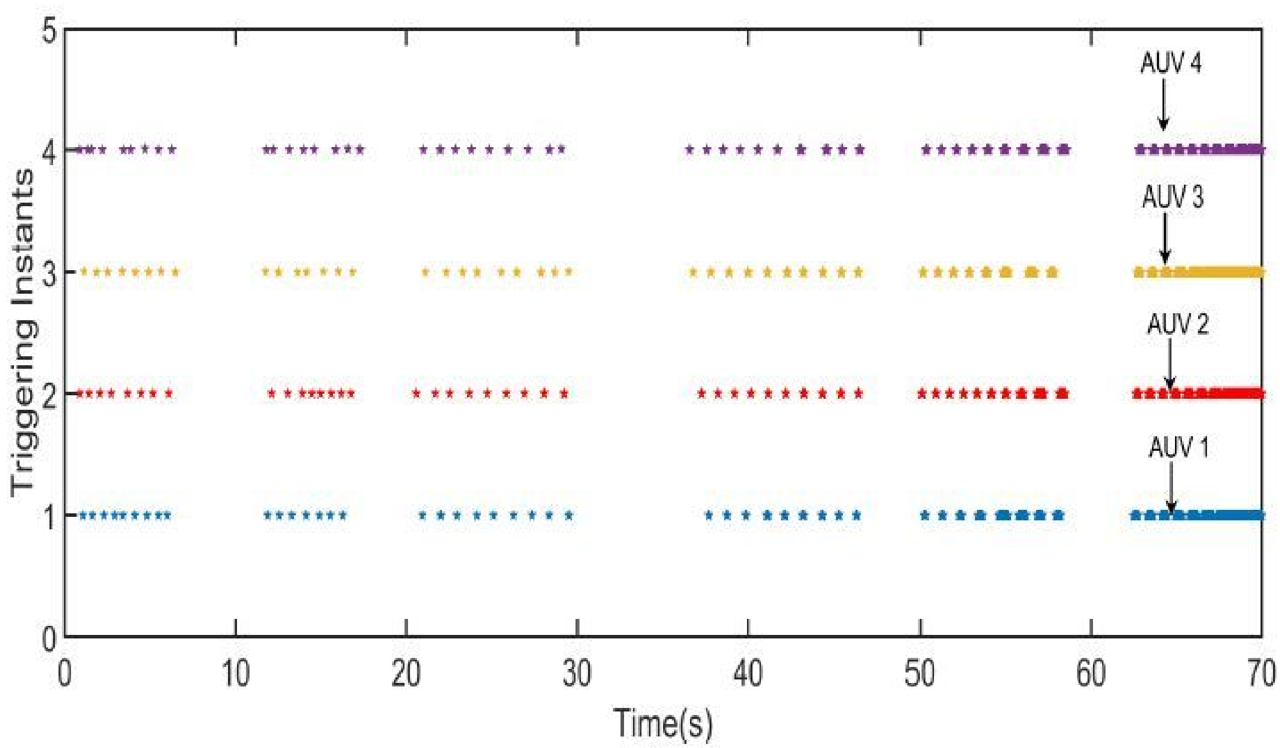

Figure 8

Triggering instants for AUVs in x direction (Case: B).

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