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Impulsive State Estimation for Nonlinear Systems with Redundant Communication Channels and Markov Delay

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Abstract The state estimation issue for the discrete-time nonlinear systems with Markov delay is investigated in this paper, where the redundant communication channel is considered to ensure the reliability of transmission. Because the channel capacity is limited, the packet dropout conditions of the main channel and the redundant channel are described by the Bernoulli stochastic variables. In addition, a mode-dependent estimator is proposed based on the current state and the delayed state, simultaneously. Combining with the impulsive control strategy, the efficiency of estimator is improved. An augmented estimation error system is proposed to deal with the Markov delay in the nonlinear system, subsequently, a sufficient condition that ensures the asymptotic stability of the augmented error system is obtained by a constructed Lyapunov functional candidate and the gains of the impulsive estimator are derived. Finally, a numerical example of moving vehicle is utilized to illustrate the developed results.

Keywords State estimation · redundant channel · Markov delay · impulsive approach.

1 Introduction

Time delay phenomenon widely existing in natural world and industrial process, such as perceptual process, network transmission, machine manufacturing, and so on [1–4]. Delay systems are proposed for this series of issues, and many effective methods for analysis of de-

lay systems are developed, such as the new type of augmented Lyapunov functional stability analysis with less conservative for linear systems, and a multi-bound dependent stability analysis with less computation for nonlinear systems, etc. [5–8]. Furthermore, the models of delay attracted much attention from researchers, which have different characteristics. For example, the earlier constant model and the Bernoulli delay model took limited possible values, which are simple but insufficient in describing the random property of the delay [9]. On the other hand, the random time-varying delay had less conservatism but possesses complex model. Therefore, Markov chain as an appropriate method for modeling delay was applied in systems [10–12]. In [13], the random delays in sensors were modeled by homogeneous Markov chains. In [14], the Markovian delays with uncertain transition probabilities were designed for networked control systems. All in all, the research of the Markov chains that applied to the time delay not be fully investigated, which deserves further study.

Networked systems (NSs) consist of several interconnected subsystems, which are connected via the shared communication channel (CC) [15–17]. On the one hand, NSs have some excellent advantages, for instances, low installation and maintain costs, simplification of system wiring, high system flexibility, and so on [18–20]. On the other hand, there existing phenomena of packet dropout and transmission delay, which is caused by the network bandwidth and the packet size constraints [21–24]. Aiming at the issue of channel capacity constraint, the redundant channel method is developed by researchers to improve the transmission efficiency of NCs. In [25–27], the redundant channels prepared for packet dropout conditions were designed, which playing their role when the main channel drop packets. That is, the redundant channels have low utilization rate and

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most of the time is idle. In order to further improve the channel efficiency and reduce the packet dropout rate, the usage of redundant channels and transmission strategy requires more attention.

As is known to all, few systems in industrial process can realize synchronization or estimation by their system parameter without outer interference [28–31]. Impulsive control, as an outer force, provides instantaneous energy to a system, which is always used in synchronization control and state estimation. For some cases, the impulsive method becomes a good choice for feedback, updating system state, controller and estimator design, and so on [32–34]. In this regard, impulsive control motivates some researchers to focus on it. In [35], an observer operating in a fixed period of time was proposed to estimate state of linear time delay system, which is updated by adopting the impulsive approach. In [36], the existence of an observer for the impulsive switched nonlinear systems was proved and a sufficient condition of exponential stability for above system based on Lyapunov functional candidate (LFC) was studied. In [37], impulsive control was used in non-fragile controller to cut down the communication times between leader neural network and coupled networks. Not only in these conditions impulse can be applied, but also in estimation problems. Combined with the redundant CC and different transmission strategy, how to realize the impulsive state estimation is a challenging issue.

According to the above descriptions, this work investigates the state estimation of the Markov delay systems with impulsive approach. A new transmission strategy for the redundant channel is proposed and a sufficient condition is derived to design the mode-dependent impulsive estimator. A numerical simulation is given to verify the correctness of the conclusion. The contributions are stated as follows.

1. The redundant CC is used to increase the transmission channels capacity, and a novel transmission strategy is proposed to schedule the information of the nonlinear systems with Markov delay. Two different Bernoulli stochastic variables are employed to describe the packet dropout conditions for the main CC and the redundant CC. Compared to the existing methods [26,27], the redundant CC can transmit not only the current state, but also the delayed state, which greatly improves the channel utilization rate.
2. A mode-dependent estimator is proposed combined with the new transmission strategy, which is based on current error and time delay error of the Markov delay systems, simultaneously. In order to cut down the communication loads of system and increase the efficiency of estimator, the impulsive control strategy is introduced, where the impulse satisfies time-triggered condition.
3. A sufficient condition that guarantees the asymptotic stability of the augmented estimation error system (AEES) is established, and the convergence rate of the AEES is depicted by the scalars μ_1 and μ_2 for the cases without and with impulsive signals. Then the allowed maximal interval h of the impulsive signals and the gains of mode-dependent estimator are obtained.

In this paper, section 2 consists of the nonlinear systems with Markov delay, the redundant CC, the state estimator and the AEES. The sufficient condition about the asymptotic stability of the AEES is proved in Section 3. In Section 4, the mode-dependent impulsive estimator is designed and its gains are obtained. Then, a numerical simulation for moving vehicle is given in Section 5 and the conclusion regarding the main results is summarized in Section 6.

Notations: The symbols \mathbb{Z}^+ , \mathbb{R}^n and $\mathbb{R}^{m \times n}$ denote positive integer numbers, the n dimensional vector, and the $m \times n$ matrices, respectively. $\|\cdot\|_2$ stands for the Euclidean norm for a vector, and $\mathbf{P}\{\cdot\}$ denotes the probability of a stochastic variable. The transpose of a matrix X is denoted by X^T . $\lambda_{max}(X)$ and $\lambda_{min}(X)$ are represented as the largest eigenvalue and smallest eigenvalue for the matrix X , respectively. The positive definite matrix is expressed as $X > 0$.

2 Problem Formulation

2.1 Nonlinear Systems with Markov Delay

This work considers a class of nonlinear systems with Markov delay, which are presented as

$$\begin{aligned} \mathbf{x}(k+1) &= A\mathbf{x}(k) + B\mathbf{g}(\mathbf{x}(k - \tau(k))) \\ \mathbf{y}(k) &= C\mathbf{x}(k) \\ \mathbf{x}(k) &= \varphi(k), -\tau \leq k \leq 0 \end{aligned} \quad (1)$$

where the vectors $\mathbf{x}(k) \in \mathbb{R}^n$, $\mathbf{y}(k) \in \mathbb{R}^m$ and $\varphi(k) \in \mathbb{R}^n$ represent the system state, measurement output and the given initial state, respectively. The parameter matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n}$ and $C \in \mathbb{R}^{m \times n}$ are known. $\mathbf{g}(\cdot)$ is nonlinear activation function, which meets the following assumption:

Assumption 1 *The nonlinear activation function $\mathbf{g}(\cdot)$ is subject to the inequality (2) for the vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ [38]:*

$$\begin{aligned} &(\mathbf{g}(\mathbf{x}) - \mathbf{g}(\mathbf{y}) - \kappa_1(\mathbf{x} - \mathbf{y}))^T \\ &\times (\mathbf{g}(\mathbf{x}) - \mathbf{g}(\mathbf{y}) - \kappa_2(\mathbf{x} - \mathbf{y})) \leq 0 \end{aligned} \quad (2)$$

where the matrices $\kappa_1, \kappa_2 \in \mathbb{R}^{n \times n}$ are known and $\kappa_1 > \kappa_2$.

We consider that the time-varying delay $\tau(k) \in \phi_\tau \triangleq \{1, 2, \dots, \tau\}$ in the nonlinear term is governed by a Markov chain, whose transition probability matrix is denoted as $\pi = [\pi_{\omega\nu}] \in \mathbb{R}^{\tau \times \tau}$. The transition probability $\pi_{\omega\nu}$ depicts that the probability from the state ω at time instant k jump to the state ν at time instant $k+1$ during a transfer, which satisfies

$$\mathbf{P}\{\tau(k+1) = \nu | \tau(k) = \omega\} = \pi_{\omega\nu}$$

where $0 \leq \pi_{\omega\nu} \leq 1$, and $\sum_{\nu=1}^{\tau} \pi_{\omega\nu} = 1, \forall \omega \in \phi_\tau$.

2.2 Redundant Communication Channel

For NSs, the limited CC capacity always leads to packet dropouts. To cut down the packet dropout rate, the redundant CC has been considered by researchers. We use the two different Bernoulli stochastic variables $\mathbf{a}(k)$ and $\mathbf{b}(k)$ to depict the packet dropout conditions for the main CC and the backup CC, respectively. That is, the successful probabilities for the information transmission of the channels satisfy

$$\mathbf{P}\{\mathbf{a}(k) = 1\} = \hat{\mathbf{a}}, \quad \mathbf{P}\{\mathbf{b}(k) = 1\} = \hat{\mathbf{b}} \quad (3)$$

where the probabilities $\hat{\mathbf{a}}, \hat{\mathbf{b}} \in [0, 1]$.

Some systems sharing one CC is a characteristic of NSs. As shown in Figure. 1, we assume that the system (1) has one time permission to access the main channel and the redundant channel at each instant time, respectively. This work considers a new transmission protocol for the redundant CC, that is, the main channel transmits the measurement output firstly, then it determines the information which is transmitted via the redundant channel. The detail is illustrated as follows,

1. If the measurement output $\mathbf{y}(k)$ of the system (1) is transmitted successfully over the main channel, that is, $\mathbf{a}(k) = 1$. Then the redundant channel transmits the delayed output state $\mathbf{y}(k - \tau(k))$.
2. Once the measurement output $\mathbf{y}(k)$ of the system (1) is dropped by the main channel, that is, $\mathbf{a}(k) = 0$. Then the redundant channel transmits the measurement output $\mathbf{y}(k)$ again.

Remark 1 The existed literatures dealt with the packet dropout utilizing the redundant channels have some limitations. For instance, in [25, 26], the main CC and the redundant CC transmit the same information simultaneously. Therefore, when packet dropout occurs on neither channel, the information transmitted by the main CC and the redundant CC don't transmits any

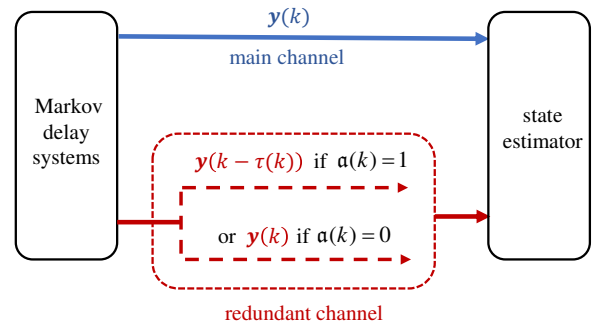


Fig. 1 The diagram of the state estimation with a redundant channel.

value. This transmission mechanism is precisely the weakness of this approach, that is, although redundant channels increase the probability of successful transmission, the usage rate of redundant channels is low, which makes the ratio of using value to cost smaller. In a different point of view, the existed of the redundant CC not only transmits the current state, but also transmits the time-delayed state when main channel successfully transmits in this paper, which makes the redundant channel not be left unused, and increases its utilization to a large extent.

2.3 Estimator Design

An impulsive estimator (4) is proposed for the discrete-time Markov delay systems (1),

$$\begin{aligned} \hat{\mathbf{x}}(k+1) = & A\hat{\mathbf{x}}(k) + B\mathbf{g}(\hat{\mathbf{x}}(k - \tau(k))) \\ & + \left((\mathbf{a}(k) + (1 - \mathbf{a}(k))\mathbf{b}(k))L_{1,\tau(k)} \right. \\ & \times (\mathbf{y}(k) - C\hat{\mathbf{x}}(k)) + \mathbf{a}(k)\mathbf{b}(k)L_{2,\tau(k)} \\ & \times (\mathbf{y}(k - \tau(k)) - C\hat{\mathbf{x}}(k - \tau(k))) \left. \right) \\ & \times \delta(k - k_m) \end{aligned} \quad (4)$$

$$\hat{\mathbf{y}}(k) = C\hat{\mathbf{x}}(k)$$

$$\hat{\mathbf{x}}(k) = \psi(k), \quad -\tau \leq k \leq 0$$

where $\hat{\mathbf{x}}(k) \in \mathbb{R}^n$, $\hat{\mathbf{y}}(k) \in \mathbb{R}^m$ and $\psi(k) \in \mathbb{R}^n$ stand for the estimation of state $\mathbf{x}(k)$, the estimator measurement output and the initial state, respectively. Matrices $L_{1,\tau(k)}$ and $L_{2,\tau(k)}$ are the mode-dependent estimator gains. $\delta(k - k_m)$ is the dirac impulse, where $\{k_m\}$ is a monotone increasing set denoting driven instants, which satisfies $-1 = k_0 < k_1 < \dots < k_m < \dots$, $m \in \mathbb{Z}^+$. Assume that the interval h_m between instants k_{m-1} and k_m subjects to the following condition:

$$0 < h_m \leq h \quad (5)$$

where $h \in \mathbb{Z}^+$ is the maximum value of the impulsive interval.

2.4 Augmented Estimation Error System

Define the extended state variables $\xi(\mathbf{x}(k))$ and $\hat{\xi}(\hat{\mathbf{x}}(k))$ as

$$\xi(\mathbf{x}(k)) \triangleq \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \\ \mathbf{x}(k-2) \\ \vdots \\ \mathbf{x}(k-\tau) \end{bmatrix}, \quad \hat{\xi}(\hat{\mathbf{x}}(k)) \triangleq \begin{bmatrix} \hat{\mathbf{x}}(k) \\ \hat{\mathbf{x}}(k-1) \\ \hat{\mathbf{x}}(k-2) \\ \vdots \\ \hat{\mathbf{x}}(k-\tau) \end{bmatrix}. \quad (6)$$

Based on the time delay system (1) and taking account of the state estimator (4) with impulsive signals, an AEES can be expressed as

$$\xi(\mathbf{x}(k+1)) = \bar{A}\xi(\mathbf{x}(k)) + \bar{B}I_{\tau(k)}\tilde{\mathbf{g}}(\mathbf{x}(k)) \quad (7)$$

and

$$\begin{aligned} \hat{\xi}(\hat{\mathbf{x}}(k+1)) &= \bar{A}\hat{\xi}(\hat{\mathbf{x}}(k)) + \bar{B}I_{\tau(k)}\hat{\mathbf{g}}(\mathbf{x}(k)) \\ &+ \left((\mathbf{a}(k) + (1 - \mathbf{a}(k))\mathbf{b}(k))\bar{L}_{1,\tau(k)}^0 \right. \\ &\times (\xi(\mathbf{x}(k)) - \hat{\xi}(\hat{\mathbf{x}}(k))) + \mathbf{a}(k)\mathbf{b}(k)\bar{L}_{2,\tau(k)}^{\tau(k)} \\ &\times (\xi(\mathbf{x}(k)) - \hat{\xi}(\hat{\mathbf{x}}(k))) \left. \right) \delta(k - k_m) \end{aligned} \quad (8)$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & \cdots & 0 & 0 \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\tilde{\mathbf{g}}(\mathbf{x}(k)) \triangleq \begin{bmatrix} \mathbf{g}(\mathbf{x}(k)) \\ \mathbf{g}(\mathbf{x}(k-1)) \\ \mathbf{g}(\mathbf{x}(k-2)) \\ \vdots \\ \mathbf{g}(\mathbf{x}(k-\tau)) \end{bmatrix}$$

$$\hat{\mathbf{g}}(\hat{\mathbf{x}}(k)) \triangleq \begin{bmatrix} \mathbf{g}(\hat{\mathbf{x}}(k)) \\ \mathbf{g}(\hat{\mathbf{x}}(k-1)) \\ \mathbf{g}(\hat{\mathbf{x}}(k-2)) \\ \vdots \\ \mathbf{g}(\hat{\mathbf{x}}(k-\tau)) \end{bmatrix}$$

$$\bar{L}_{1,\tau(k)}^0 = \begin{bmatrix} L_{1,\tau(k)}C & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\bar{L}_{2,\tau(k)}^{\tau(k)} = \begin{bmatrix} L_{2,\tau(k)}C \\ 0 \\ \vdots \\ 0 \end{bmatrix} I_{\tau(k)}$$

$$I_{\tau(k)} = \underbrace{[0 \ \cdots \ 0]}_{\tau(k)} I \underbrace{[0 \ \cdots \ 0]}_{\tau-\tau(k)}.$$

Define the state error as $\mathbf{e}(k) \triangleq \mathbf{x}(k) - \hat{\mathbf{x}}(k)$ and the extended state error as $\zeta(\mathbf{e}(k)) \triangleq \xi(\mathbf{x}(k)) - \hat{\xi}(\hat{\mathbf{x}}(k))$. The nonlinear function is defined as $\mathbf{f}(\mathbf{e}(k)) \triangleq \tilde{\mathbf{g}}(\mathbf{x}(k)) - \hat{\mathbf{g}}(\hat{\mathbf{x}}(k))$, according to (7) and (8), it follows that

$$\begin{aligned} \zeta(\mathbf{e}(k+1)) &= \xi(\mathbf{x}(k+1)) - \hat{\xi}(\hat{\mathbf{x}}(k+1)) \\ &= \bar{A}\zeta(\mathbf{e}(k)) + \bar{B}I_{\tau(k)}\mathbf{f}(\mathbf{e}(k)) \\ &\quad - \left((\mathbf{a}(k) + (1 - \mathbf{a}(k))\mathbf{b}(k))\bar{L}_{1,\tau(k)}^0 \zeta(\mathbf{e}(k)) \right. \\ &\quad \left. + \mathbf{a}(k)\mathbf{b}(k)\bar{L}_{2,\tau(k)}^{\tau(k)} \zeta(\mathbf{e}(k)) \right) \delta(k - k_m). \end{aligned} \quad (9)$$

For the sake of simplifying the computations, the terms containing the stochastic variables $\mathbf{a}(k)$ and $\mathbf{b}(k)$ in (7) are processed as follows based on the equality (3),

$$\begin{aligned} \mathbf{a}(k) + (1 - \mathbf{a}(k))\mathbf{b}(k) &= (\mathbf{a}(k) - \hat{\mathbf{a}} + \hat{\mathbf{a}}) + (1 - \hat{\mathbf{a}} - \mathbf{a}(k) + \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}} + \hat{\mathbf{b}}) \\ &= (\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} \\ &\quad - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) \end{aligned} \quad (10)$$

and

$$\begin{aligned} \mathbf{a}(k)\mathbf{b}(k) &= (\mathbf{a}(k) - \hat{\mathbf{a}} + \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}} + \hat{\mathbf{b}}) \\ &= (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) \\ &\quad + \hat{\mathbf{a}}\hat{\mathbf{b}} + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}). \end{aligned} \quad (11)$$

Based on the statement in the above, the AEES (9) can be rewritten as

$$\begin{aligned} \zeta(\mathbf{e}(k+1)) &= \bar{A}\zeta(\mathbf{e}(k)) + \bar{B}I_{\tau(k)}\mathbf{f}(\mathbf{e}(k)) \\ &\quad - \left((\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \right. \\ &\quad \left. + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) \right) \\ &\quad \times \bar{L}_{1,\tau(k)}^0 \zeta(\mathbf{e}(k)) \delta(k - k_m) \\ &\quad - \left((\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}})\hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}) \right. \\ &\quad \left. + \hat{\mathbf{a}}\hat{\mathbf{b}} \right) \bar{L}_{2,\tau(k)}^{\tau(k)} \zeta(\mathbf{e}(k)) \delta(k - k_m). \end{aligned} \quad (12)$$

Definition 1 The AEES (12) reaches asymptotic stability based on the estimator (4), if the following equation holds [39]:

$$\lim_{k \rightarrow \infty} \mathbf{E}\{\|\mathbf{e}(k)\|_2\} = 0. \quad (13)$$

3 Main Result

In this section, a sufficient condition for asymptotic stability of the AEES is proposed, and the main result is stated as follows.

Theorem 1 Given scalars $\mu_1 > 1$ and $0 < \mu_2 < 1$, the AEES reaches asymptotic stability with impulsive method, if there exist matrices $P_\omega > 0$, $Q > 0$, $\bar{L}_{1,\omega}^0$ and $\bar{L}_{2,\omega}^\omega$ such that the following conditions (14) to (16) hold with $\omega \in \phi_\tau$:

$$\begin{bmatrix} -\epsilon\bar{\kappa}_1^T\bar{\kappa}_2 - \mu_1 P_\omega & 1/2\epsilon(\bar{\kappa}_1^T + \bar{\kappa}_2^T) & \bar{A}^T \\ * & -\epsilon I & I_\omega^T \bar{B}^T \\ * & * & -\mathcal{P}_\nu^{-1} \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \bar{A}^T & \Psi_{14} & \Psi_{15} \\ * & -\epsilon I & I_\omega^T \bar{B}^T & 0 & 0 \\ * & * & -\mathcal{P}_\nu^{-1} & 0 & 0 \\ * & * & * & -\mathcal{P}_\nu^{-1} & 0 \\ * & * & * & 0 & -\mathcal{P}_\nu^{-1} \end{bmatrix} < 0 \quad (15)$$

and

$$0 < \mu_1^{h-1} \mu_2 < 1 \quad (16)$$

where

$$\mathcal{P}_\nu = \sum_{\nu=1}^{\tau} \pi_{\omega\nu} P_\nu$$

$$\begin{aligned} \Psi_{11} = & -(\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})(\bar{A}^T \mathcal{P}_\nu \bar{L}_{1,\omega}^0 + (\bar{L}_{1,\omega}^0)^T \mathcal{P}_\nu \bar{A}) \\ & - \hat{\mathbf{a}}\hat{\mathbf{b}}(\bar{A}^T \mathcal{P}_\nu \bar{L}_{2,\omega}^\omega + (\bar{L}_{2,\omega}^\omega)^T \mathcal{P}_\nu \bar{A}) - \mu_2 P_\omega \\ & - \epsilon \bar{\kappa}_1^T \bar{\kappa}_2 \end{aligned}$$

$$\begin{aligned} \Psi_{12} = & -(\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})(\bar{L}_{1,\omega}^0)^T \mathcal{P}_\nu \bar{B} I_\omega \\ & - \hat{\mathbf{a}}\hat{\mathbf{b}}(\bar{L}_{2,\omega}^\omega)^T \mathcal{P}_\nu \bar{B} I_\omega + 1/2\epsilon(\bar{\kappa}_1^T + \bar{\kappa}_2^T) \end{aligned}$$

$$\Psi_{14} = \sqrt{\hat{\mathbf{a}} + \hat{\mathbf{b}} - 2\hat{\mathbf{a}}\hat{\mathbf{b}}} (\bar{L}_{1,\omega}^0)^T$$

$$\Psi_{15} = \sqrt{\hat{\mathbf{a}}\hat{\mathbf{b}}} ((\bar{L}_{1,\omega}^0)^T + (\bar{L}_{2,\omega}^\omega)^T).$$

Proof A LFC is constructed to prove the asymptotic stability of the AEES (12) as follows,

$$\mathcal{V}(\mathbf{e}(k)) = \zeta(\mathbf{e}(k))^T P_{\tau(k)} \zeta(\mathbf{e}(k)) \quad (17)$$

where the matrix $P_{\tau(k)}$ is positive definite. To facilitate the expression, define $\tau(k) \triangleq \omega$ and $\tau(k+1) \triangleq \nu$, and define the difference of the LFC in the mean sense as

$$\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} = \mathbf{E}\{\mathcal{V}(\mathbf{e}(k+1)) - \mu_i \mathcal{V}(\mathbf{e}(k))\} \quad (18)$$

where the scalar μ_i belongs to the set $\{\mu_1, \mu_2\}$. In the rest part, the impulsive approach is employed to prove Theorem 1 in three steps.

Step 1: For the instant $k \neq k_m$ which means $\delta(k - k_m) = 0$, the difference of the LFC $\Delta\mathcal{V}(\mathbf{e}(k))$ in (18) is

as follows,

$$\begin{aligned} \mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} &= \mathbf{E}\{\mathcal{V}(\mathbf{e}(k+1)) - \mu_1 \mathcal{V}(\mathbf{e}(k))\} \\ &= \mathbf{E}\{\zeta^T(\mathbf{e}(k+1)) P_\nu \zeta(\mathbf{e}(k+1)) \\ &\quad - \mu_1 \zeta^T(\mathbf{e}(k)) P_\omega \zeta(\mathbf{e}(k))\} \\ &= \mathbf{E}\{\zeta^T(\mathbf{e}(k)) \bar{A}^T P_\nu \bar{A} \zeta(\mathbf{e}(k)) \\ &\quad - \mathbf{f}^T(\mathbf{e}(k)) I_\omega^T \bar{B}^T P_\nu \bar{B} I_\omega \mathbf{f}(\mathbf{e}(k)) \\ &\quad + 2\zeta^T(\mathbf{e}(k)) \bar{A}^T P_\nu \bar{B} I_\omega \mathbf{f}(\mathbf{e}(k)) \\ &\quad - \mu_1 \zeta^T(\mathbf{e}(k)) P_\omega \zeta(\mathbf{e}(k))\} \\ &= \chi^T(\mathbf{e}(k)) \begin{bmatrix} \bar{A}^T P_\nu \bar{A} - \mu_1 P_\omega & \bar{A}^T P_\nu \bar{B} I_\omega \\ * & I_\omega^T \bar{B}^T P_\nu \bar{B} I_\omega \end{bmatrix} \chi(\mathbf{e}(k)) \end{aligned} \quad (19)$$

where $\chi(\mathbf{e}(k)) \triangleq [\zeta^T(\mathbf{e}(k)) \mathbf{f}^T(\mathbf{e}(k))]^T$.

The nonlinear inequality (20) holds for any $\epsilon > 0$ based on Assumption 1,

$$\epsilon(\mathbf{f}(\mathbf{e}(k)) - \bar{\kappa}_1 \zeta(\mathbf{e}(k)))^T (\mathbf{f}(\mathbf{e}(k)) - \bar{\kappa}_2 \zeta(\mathbf{e}(k))) \leq 0 \quad (20)$$

which implies

$$\chi^T(\mathbf{e}(k)) \begin{bmatrix} \epsilon \bar{\kappa}_1^T \bar{\kappa}_2 - 1/2\epsilon(\bar{\kappa}_1^T + \bar{\kappa}_2^T) \\ * & \epsilon I \end{bmatrix} \chi(\mathbf{e}(k)) \leq 0. \quad (21)$$

Then, combined formula (19) with (21), we derive

$$\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} \leq \chi^T(\mathbf{e}(k)) \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ * & \Upsilon_{13} \end{bmatrix} \chi(\mathbf{e}(k)) \quad (22)$$

where

$$\begin{aligned} \Upsilon_{11} &= \bar{A}^T P_\nu \bar{A} - \epsilon \bar{\kappa}_1^T \bar{\kappa}_2 - \mu_1 P_\omega \\ \Upsilon_{12} &= \bar{A}^T P_\nu \bar{B} I_\omega + 1/2\epsilon(\bar{\kappa}_1^T + \bar{\kappa}_2^T) \\ \Upsilon_{13} &= I_\omega^T \bar{B}^T P_\nu \bar{B} I_\omega - \epsilon I. \end{aligned}$$

In view of the condition (14), we have $\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} < 0$, which implies that

$$\mathbf{E}\{\mathcal{V}(\mathbf{e}(k+1))\} < \mu_1 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k))\}. \quad (23)$$

Step 2: For the case $k = k_m$ where the impulsive signals take effect, substituting the AEES (12) into

(18), gives

$$\begin{aligned}
\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} &= \mathbf{E}\{\zeta^T(\mathbf{e}(k+1))P_\nu\zeta(\mathbf{e}(k+1)) \\
&\quad - \mu_2\zeta^T(\mathbf{e}(k))P_\omega\zeta(\mathbf{e}(k))\} \\
&= \mathbf{E}\{\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{A}\zeta(\mathbf{e}(k)) \\
&\quad + \mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{B}I_\omega \mathbf{f}(\mathbf{e}(k)) \\
&\quad + ((\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}))^2 \zeta^T(\mathbf{e}(k))(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + ((\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}}\hat{\mathbf{b}} \\
&\quad + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}))^2 \zeta^T(\mathbf{e}(k))(\bar{L}_{2,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + 2\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{B}I_\omega \mathbf{f}(\mathbf{e}(k)) \\
&\quad - 2((\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}))\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - 2((\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}}\hat{\mathbf{b}} \\
&\quad + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}))\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - 2((\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}})) \\
&\quad \times \mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - 2((\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}}\hat{\mathbf{b}} \\
&\quad + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}))\mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + 2((\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}} + (1 - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) \\
&\quad + (1 - \hat{\mathbf{a}})\hat{\mathbf{b}} - (\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) - \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}})) \\
&\quad \times ((\mathbf{a}(k) - \hat{\mathbf{a}})(\mathbf{b}(k) - \hat{\mathbf{b}}) + \hat{\mathbf{b}}(\mathbf{a}(k) - \hat{\mathbf{a}}) + \hat{\mathbf{a}}\hat{\mathbf{b}} \\
&\quad + \hat{\mathbf{a}}(\mathbf{b}(k) - \hat{\mathbf{b}}))\zeta(\mathbf{e}(k))^T(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - \mu_2\zeta^T(\mathbf{e}(k))P_\omega\zeta(\mathbf{e}(k))\}.
\end{aligned} \tag{24}$$

With reference to the properties (3) of stochastic variables $\mathbf{a}(k)$ and $\mathbf{b}(k)$, the equation (24) is equal to

$$\begin{aligned}
\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} &= \mathbf{E}\{\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{A}\zeta(\mathbf{e}(k)) \\
&\quad + \mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{B}I_\omega \mathbf{f}(\mathbf{e}(k)) \\
&\quad - \mu_2\zeta^T(\mathbf{e}(k))P_\omega\zeta(\mathbf{e}(k)) \\
&\quad + (\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})\zeta^T(\mathbf{e}(k))(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + \hat{\mathbf{a}}\hat{\mathbf{b}}\zeta^T(\mathbf{e}(k))(\bar{L}_{2,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + 2\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{B}I_\omega \mathbf{f}(\mathbf{e}(k)) \\
&\quad - 2(\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - 2\hat{\mathbf{a}}\hat{\mathbf{b}}\zeta^T(\mathbf{e}(k))\bar{A}^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad - 2(\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})\mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{L}_{1,\omega}^0 \zeta(\mathbf{e}(k))
\end{aligned} \tag{25}$$

$$\begin{aligned}
&\quad - 2\hat{\mathbf{a}}\hat{\mathbf{b}}\mathbf{f}^T(\mathbf{e}(k))I_\omega^T \bar{B}^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k)) \\
&\quad + 2\hat{\mathbf{a}}\hat{\mathbf{b}}\zeta^T(\mathbf{e}(k))(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 \zeta(\mathbf{e}(k))\} \\
&= \chi^T(\mathbf{e}(k)) \begin{bmatrix} \Upsilon_{21} & \Upsilon_{22} \\ * & \Upsilon_{23} \end{bmatrix} \chi(\mathbf{e}(k))
\end{aligned} \tag{26}$$

where

$$\begin{aligned}
\Upsilon_{21} &= \bar{A}^T P_\nu \bar{A} + (\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{1,\omega}^0 \\
&\quad + \hat{\mathbf{a}}\hat{\mathbf{b}}(\bar{L}_{2,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 - 2(\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})\bar{A}^T P_\nu \bar{L}_{1,\omega}^0 \\
&\quad - 2\hat{\mathbf{a}}\hat{\mathbf{b}}\bar{A}^T P_\nu \bar{L}_{2,\omega}^0 + 2\hat{\mathbf{a}}\hat{\mathbf{b}}(\bar{L}_{1,\omega}^0)^T P_\nu \bar{L}_{2,\omega}^0 - \mu_2 P_\omega \\
\Upsilon_{22} &= \bar{A}^T P_\nu \bar{B}I_\omega - (\hat{\mathbf{a}} + \hat{\mathbf{b}} - \hat{\mathbf{a}}\hat{\mathbf{b}})(\bar{L}_{1,\omega}^0)^T P_\nu \bar{B}I_\omega \\
&\quad - \hat{\mathbf{a}}\hat{\mathbf{b}}(\bar{L}_{2,\omega}^0)^T P_\nu \bar{B}I_\omega \\
\Upsilon_{23} &= I_\omega^T \bar{B}^T P_\nu \bar{B}I_\omega.
\end{aligned}$$

According to the nonlinear inequality (21), the following formula holds based on the equation (25):

$$\begin{aligned}
\mathbf{E}\{\Delta\mathcal{V}(\mathbf{e}(k))\} &\leq \\
\chi^T(\mathbf{e}(k)) &\begin{bmatrix} \Upsilon_{21} - \epsilon\kappa_1^T \kappa_2 & \Upsilon_{22} + 1/2(\bar{\kappa}_1^T + \bar{\kappa}_2^T) \\ * & \Upsilon_{23} - \epsilon I \end{bmatrix} \chi(\mathbf{e}(k)).
\end{aligned} \tag{27}$$

Taking account into the inequality (15) in Theorem 1, we derive

$$\mathbf{E}\{\mathcal{V}(\mathbf{e}(k+1))\} < \mu_2 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k))\}. \tag{28}$$

Step 3: On the basis of the results obtained in (23) and (28), we begin to study the characteristic of augmented error $\zeta(\mathbf{e}(k))$ under the impulsive strategy. The LFC $\mathcal{V}(\mathbf{e}(k))$ are separated two parts to analyze, in the absence of the impulsive signal, we get

$$\begin{aligned}
&\mathbf{E}\{\mathcal{V}(\mathbf{e}(k))\} \\
&< \begin{cases} \mu_1 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k-1))\} < \mu_1^2 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k-2))\} \\ < \dots < \mu_1^k \mathbf{E}\{\mathcal{V}(0)\}, & k \in (0, k_1] \\ \mu_1 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k-1))\} < \dots \\ < \mu_1^{k-k_1-1} \mathbf{E}\{\mathcal{V}(\mathbf{e}(k_1+1))\}, & k \in (k_1+1, k_2] \\ \mu_1 \mathbf{E}\{\mathcal{V}(\mathbf{e}(k-1))\} < \dots \\ < \mu_1^{k-k_2-1} \mathbf{E}\{\mathcal{V}(\mathbf{e}(k_2+1))\}, & k \in (k_2+1, k_3] \\ \dots \end{cases}
\end{aligned} \tag{29}$$

At the moment when the impulsive signals take effect, the following inequalities hold:

$$\mathbf{E}\{\mathcal{V}(e(k))\} < \begin{cases} \mu_2 \mathbf{E}\{\mathcal{V}(e(k_1))\} < \mu_1^{k_1} \mu_2 \mathbf{E}\{\mathcal{V}(0)\}, & k = k_1 + 1 \\ \mu_2 \mathbf{E}\{\mathcal{V}(e(k_2))\} < \mu_1^{k_2 - k_1 - 1} \mu_2 \mathbf{E}\{\mathcal{V}(e(k_1 + 1))\}, & k = k_2 + 1 \\ \mu_2 \mathbf{E}\{\mathcal{V}(e(k_3))\} < \mu_1^{k_3 - k_2 - 1} \mu_2 \mathbf{E}\{\mathcal{V}(e(k_2 + 1))\}, & k = k_3 + 1 \\ \dots & \end{cases} \quad (30)$$

The above formulas (29) and (30) can be generalized as the following inequalities (31) by iterative method:

$$\mathbf{E}\{\mathcal{V}(e(k))\} < \begin{cases} \mu_1^{m(h-1)} \mu_2^{m-1} \mathbf{E}\{\mathcal{V}(0)\}, & \text{if } k \in (k_{m-1} + 1, k_m] \\ \mu_1^{m(h-1)} \mu_2^m \mathbf{E}\{\mathcal{V}(0)\}, & \text{if } k = k_m + 1 \end{cases} \quad (31)$$

which implies that

$$\mathbf{E}\{\|\zeta(e(k))\|_2\} < \begin{cases} \frac{\max\{\lambda_{\max}(P_\omega)\}}{\min\{\lambda_{\min}(P_\omega)\}} (\mu_1^{m(h-1)} \mu_2^{m-1}) \|\zeta(0)\|_2, & \text{if } k \in (k_{m-1} + 1, k_m] \\ \frac{\max\{\lambda_{\max}(P_\omega)\}}{\min\{\lambda_{\min}(P_\omega)\}} (\mu_1^{h-1} \mu_2)^m \|\zeta(0)\|_2, & \text{if } k = k_m + 1. \end{cases} \quad (32)$$

According to the inequality (16), when k tends to infinity, we have

$$\lim_{k \rightarrow \infty} \mathbf{E}\{\|\zeta(e(k))\|_2\} = 0. \quad (33)$$

Therefore, the AEES (12) is asymptotically stable.

4 Estimator gains design

According to the results in the above, the estimator gains are designed in this section, which guarantee the stability of the AEES (12).

Theorem 2 *Given scalars $\mu_1 > 1$ and $0 < \mu_2 < 1$, the AEES (12) achieves asymptotic stability with impulsive*

approach, if there exist matrices $P_\omega > 0$, $\mathcal{L}_{1,\omega}$ and $\mathcal{L}_{2,\omega}$ such that the inequalities (16), (34) and (35) hold,

$$\begin{bmatrix} -\epsilon \bar{\kappa}_1^T \bar{\kappa}_2 - \mu_1 P_\omega & 1/2\epsilon(\bar{\kappa}_1^T + \bar{\kappa}_2^T) & \bar{A}^T \mathcal{P}_\nu \\ * & -\epsilon I & I_\omega^T \bar{B}^T \mathcal{P}_\nu \\ * & * & -\mathcal{P}_\nu \end{bmatrix} < 0 \quad (34)$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \bar{A}^T \mathcal{P}_\nu & \Psi_{24} & \Psi_{25} \\ * & -\epsilon I & I_\omega^T \bar{B}^T \mathcal{P}_\nu & 0 & 0 \\ * & * & -\mathcal{P}_\nu & 0 & 0 \\ * & * & * & -\mathcal{P}_\nu & 0 \\ * & * & * & 0 & -\mathcal{P}_\nu \end{bmatrix} < 0 \quad (35)$$

where

$$\Psi_{24} = \sqrt{\hat{\mathbf{a}} + \hat{\mathbf{b}} - 2\hat{\mathbf{a}}\hat{\mathbf{b}}\mathcal{L}_{1,\omega}^T}$$

$$\Psi_{25} = \sqrt{\hat{\mathbf{a}}\hat{\mathbf{b}}(\mathcal{L}_{1,\omega}^T + \mathcal{L}_{2,\omega}^T)}.$$

Then the estimator gains are obtained as

$$\bar{L}_{1,\omega}^0 = \mathcal{P}_\nu^{-1} \mathcal{L}_{1,\omega}, \quad \bar{L}_{2,\omega}^\omega = \mathcal{P}_\nu^{-1} \mathcal{L}_{2,\omega}, \quad \omega \in \phi_\tau. \quad (36)$$

Proof Premultiplying and postmultiplying the inequality (34) with diagonal matrices $\text{diag}\{I, I, \mathcal{P}_\nu^{-1}\}$ and $\text{diag}_3\{I, I, (\mathcal{P}_\nu^{-1})^T\}$, the formula (14) is achieved. Then, premultiplying and postmultiplying the inequality (35) with $\text{diag}_5\{I, I, \mathcal{P}_\nu^{-1}, \mathcal{P}_\nu^{-1}, \mathcal{P}_\nu^{-1}\}$ and its transpose, respectively, the inequality (37) holds

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \bar{A}^T & \Phi_{14} & \Phi_{15} \\ * & -\epsilon I & I_\omega^T \bar{B}^T & 0 & 0 \\ * & * & -\mathcal{P}_\nu^{-1} & 0 & 0 \\ * & * & * & -\mathcal{P}_\nu^{-1} & 0 \\ * & * & * & * & -\mathcal{P}_\nu^{-1} \end{bmatrix} < 0 \quad (37)$$

where

$$\Phi_{14} = \sqrt{\hat{\mathbf{a}} + \hat{\mathbf{b}} - 2\hat{\mathbf{a}}\hat{\mathbf{b}}\mathcal{L}_{1,\omega}^T (\mathcal{P}_\nu^{-1})^T}$$

$$\Phi_{15} = \sqrt{\hat{\mathbf{a}}\hat{\mathbf{b}}(\mathcal{L}_{1,\omega}^T + \mathcal{L}_{2,\omega}^T) (\mathcal{P}_\nu^{-1})^T}.$$

Define $\mathcal{L}_{1,\omega} \triangleq \mathcal{P}_\nu \bar{L}_{1,\omega}^0$ and $\mathcal{L}_{2,\omega} \triangleq \mathcal{P}_\nu \bar{L}_{2,\omega}^\omega$, and substitute them into (37), the inequality (15) holds.

From the aforementioned analysis, we have the conclusions that formulas (34), (35) and condition (16) ensure the asymptotic stability of the AEES.

5 Simulation

In this section, a real vehicle system is considered to prove the correctness of derived results. The real model is as follows [40],

$$\begin{cases} x_1(k+1) = x_1(k) + x_2(k)t + 0.5t^2 x_3(k) \\ x_2(k+1) = x_2(k) + x_3(k)t \\ x_3(k+1) = \rho x_3(k) + bu_1(k) \end{cases} \quad (38)$$

where t is the sampling period and ρ is the acceleration parameter. The states $x_1(k)$, $x_2(k)$ and $x_3(k)$ are the position, velocity and acceleration of the vehicle, respectively. $u_1(k)$ is the control term acting at acceleration and b is a constant coefficient. Define $\mathbf{x}(k) \triangleq [x_1(k) \ x_2(k) \ x_3(k)]^T$, and the state space model of the vehicle system is

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + B\mathbf{u}(k) \quad (39)$$

where

$$A = \begin{bmatrix} 1 & t & 0.5t^2 \\ 0 & 1 & t \\ 0 & 0 & \rho \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b \end{bmatrix}.$$

Given the sampling period $t = 0.1$, the acceleration parameter $\rho = 0.86$ and $b = 0.02717$. Define the controller of vehicle as

$$\begin{aligned} \mathbf{u}(k) &\triangleq \mathbf{g}(\mathbf{x}(k - \tau(k))) \\ &= 0.028(|\mathbf{x}(k - \tau(k)) + 1| - |\mathbf{x}(k - \tau(k))|) \end{aligned} \quad (40)$$

where the nonlinear function $\mathbf{g}(\cdot)$ is designed to be the saturated form and regarded as the accelerator of vehicle with time delay, which promotes maximum speed in a fixed period of time. The term $\tau(k) \in \{1, 2, 3\}$ is the Markov delay, and the probability transition matrix π of the Markov chain describing the stochastic time-varying delay is

$$\pi = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.4 & 0.1 & 0.5 \\ 0.6 & 0.2 & 0.2 \end{bmatrix}.$$

Assume that the position $x_1(k)$, velocity $x_2(k)$ and acceleration $x_3(k)$ are measurable where the measurement output is

$$\mathbf{y}(k) = C\mathbf{x}(k)$$

where $C = [0.3 \ 0.4 \ 0.5]$.

For convergence rates $\mu_i, i \in \{1, 2\}$ in (18), we find the scalar $\mu_1 = 1.023$ satisfying the inequality (34), and consider $\mu_2 = 0.95$, which makes (34) and (35) have solution, simultaneously. According to the inequality (16), the maximal impulsive interval $h = 3$ is obtained. Based on the above conditions, the estimator gains are

$$\begin{aligned} L_{11} &= \begin{bmatrix} 0.0371 \\ 0.4050 \\ 0.6845 \end{bmatrix}, L_{12} = \begin{bmatrix} 0.0383 \\ 0.4249 \\ 0.7229 \end{bmatrix}, L_{13} = \begin{bmatrix} 0.0504 \\ 0.5535 \\ 0.9495 \end{bmatrix} \\ L_{21} &= \begin{bmatrix} 0.0113 \\ 0.1917 \\ 0.3412 \end{bmatrix}, L_{22} = \begin{bmatrix} 0.0081 \\ 0.1483 \\ 0.2405 \end{bmatrix}, L_{23} = \begin{bmatrix} 0.0123 \\ 0.2373 \\ 0.3664 \end{bmatrix}. \end{aligned}$$

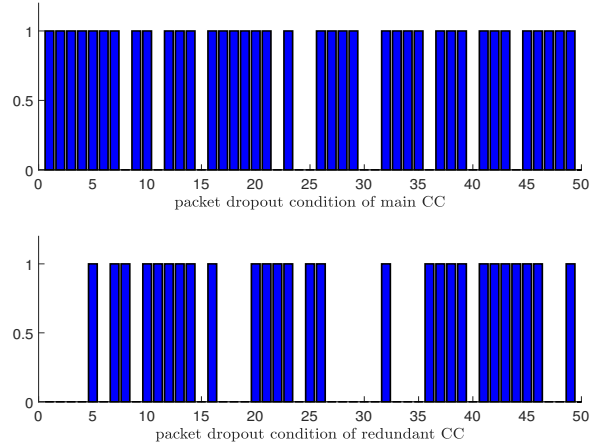


Fig. 2 The packet dropout condition of the main and redundant CC.

Given the initial state of $\hat{\mathbf{x}}(k)$ is zero, and the initial state of systems (1) is

$$\varphi(k) = [1.5 \ 4.1 \ 3.1]^T, -3 \leq k \leq 0.$$

Suppose that the probabilities of successful transmission for the main CC and the redundant CC are $\hat{a} = 0.8$ and $\hat{b} = 0.6$, respectively. We can intuitively see that the packet dropout rates for the main CC and the redundant CC are 20% and 40% in Figure 2. It is clear that the existence of redundant CC compensates the packet dropout rates of main CC and transmits a lot of effective information of the delayed states simultaneously.

The impulsive signals are shown in Figure 3 with $h = 3$. Referring to Table 1, with the decreasing of maximum interval h , the denser the impulse signal is, the better the control performance is. As a result, the AEES converges faster.

h	2	3	4	5	6
$\mu_1^{h-1} \mu_2$	0.9719	0.9942	1.0171	1.0405	1.0644

Table 1 Impulse interval and converge rate.

In figures 4-6, the trajectories of state $\mathbf{x}(k)$ and its estimation $\hat{\mathbf{x}}(k)$ are represented in the case of redundant CC existence. It can be seen from figures that the states of the moving vehicle system are estimated without error for a short period of time in the case of possessing impulsive estimator and the new transmission strategy. Furthermore, to reveal the effectiveness and availability of the proposed redundant CC, we add in the comparison condition. Considering the system without redundant CC, that is, $\hat{b} = 0$, we redefine the estimate state as $\tilde{\mathbf{x}}(k)$ based on the above condition,

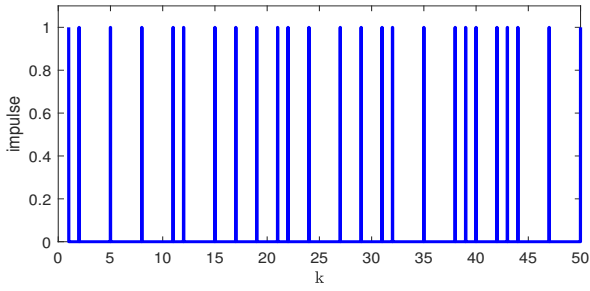


Fig. 3 The condition of the impulsive signals.

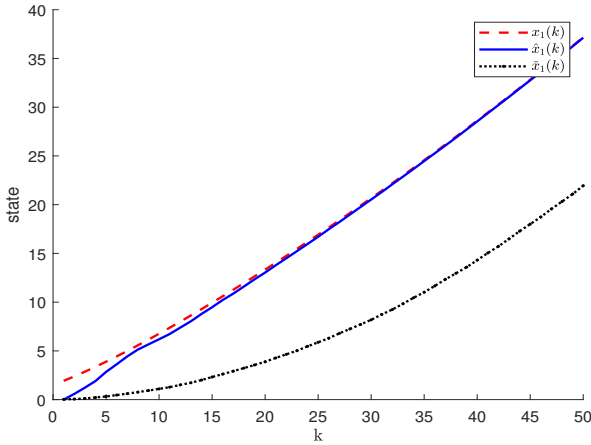


Fig. 4 The trajectories of state $x_1(k)$ and its estimation with and without redundant CC.

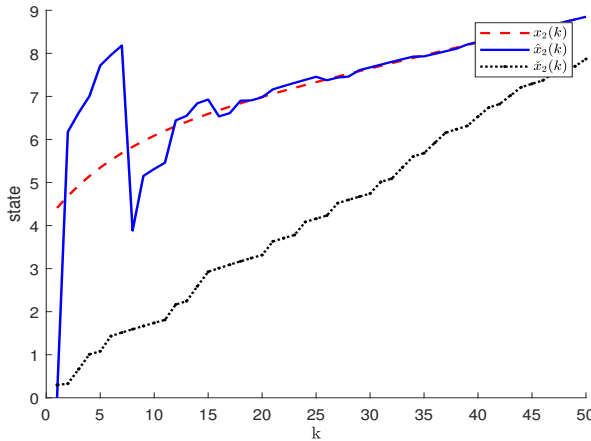


Fig. 5 The trajectories of state $x_2(k)$ and its estimation with and without redundant CC.

and obtain the trajectories in figures 4-6 with all other things being same. Compared the trajectory of estimation state $\hat{x}(k)$ with $\tilde{x}(k)$, we can conclude that since the redundant CC transmits extra information to compensate the packet dropout condition of the main CC, the estimate performance possessing redundant CC is better than the case without it.

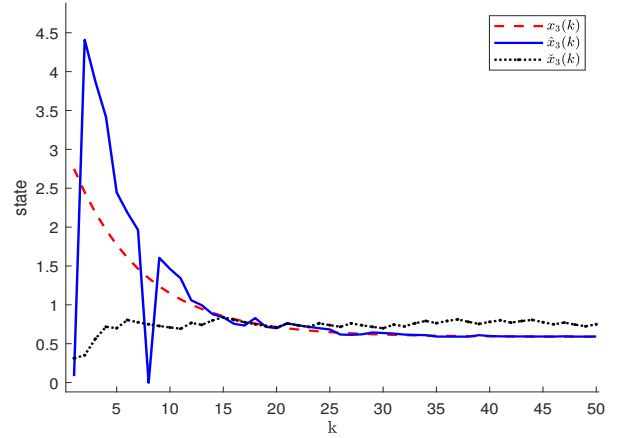


Fig. 6 The trajectories of state $x_3(k)$ and its estimation with and without redundant CC.

6 Conclusion

In this paper, the state estimation for the nonlinear systems with Markov delay and redundant CC has been addressed based on the impulsive method. The redundant CC has been proposed to compensate the packet dropout rate of the main CC aiming at the problem of limited channel capacity. The impulse strategy has been applied to the novel mode-dependent estimator to improve the efficiency of estimator. An AEES has been constructed to deal with the Markov delay. Furthermore, the sufficient condition for asymptotic stability of the AEES and its convergence rate have been obtained. A numerical example of the moving vehicle system has been employed to verify the correctness of the results.

7 Compliance with Ethical Standards

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Figures

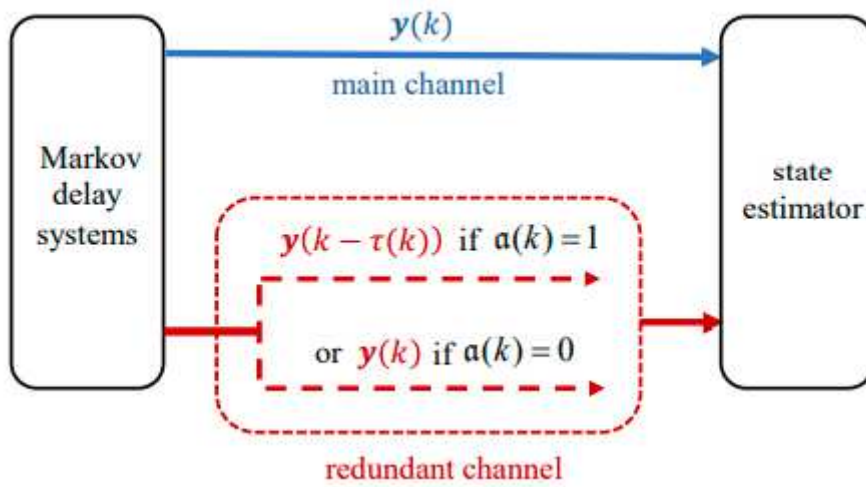


Figure 1

The diagram of the state estimation with a redundant channel.

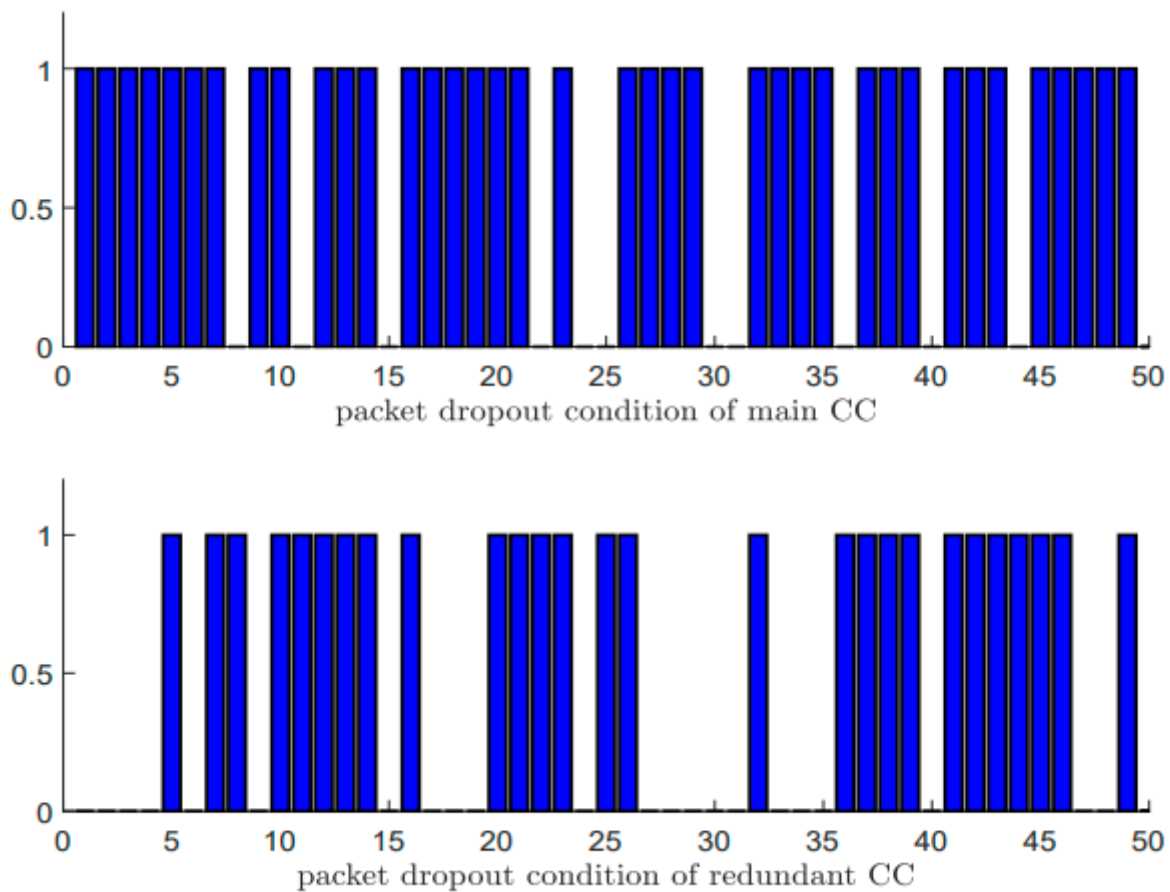


Figure 2

The packet dropout condition of the main and redundant CC.

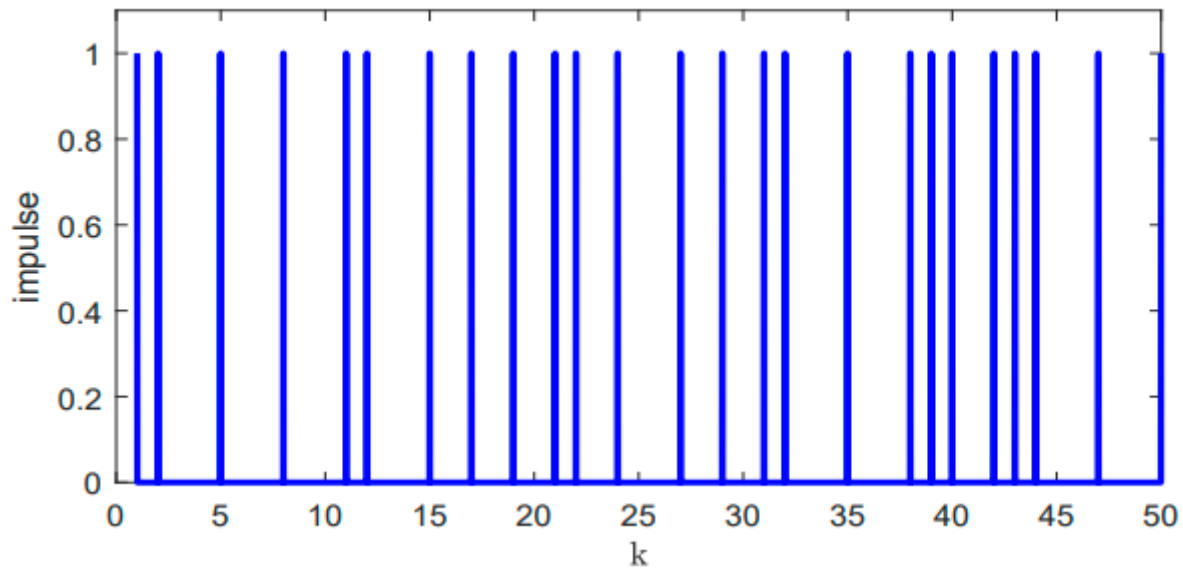


Figure 3

The condition of the impulsive signals.

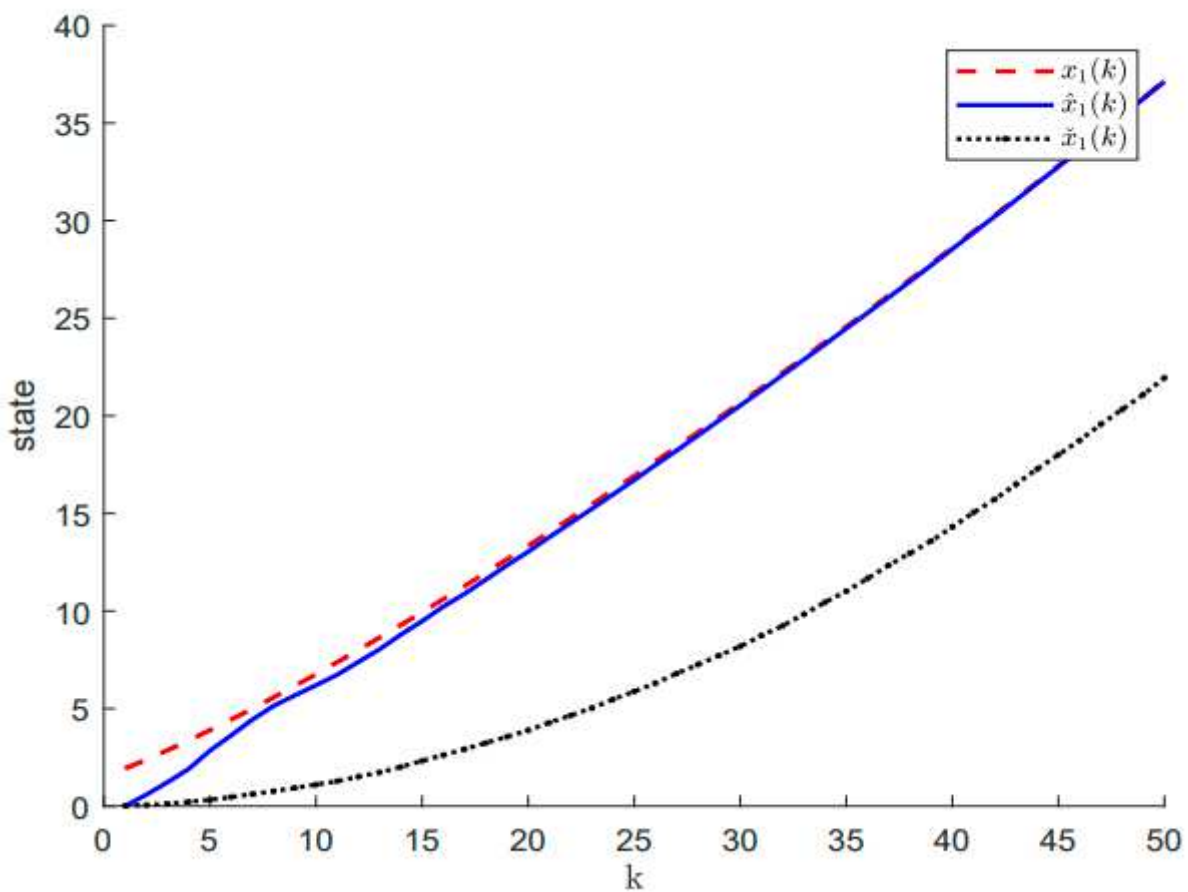


Figure 4

The trajectories of state $x_1(k)$ and its estimation with and without redundant CC.

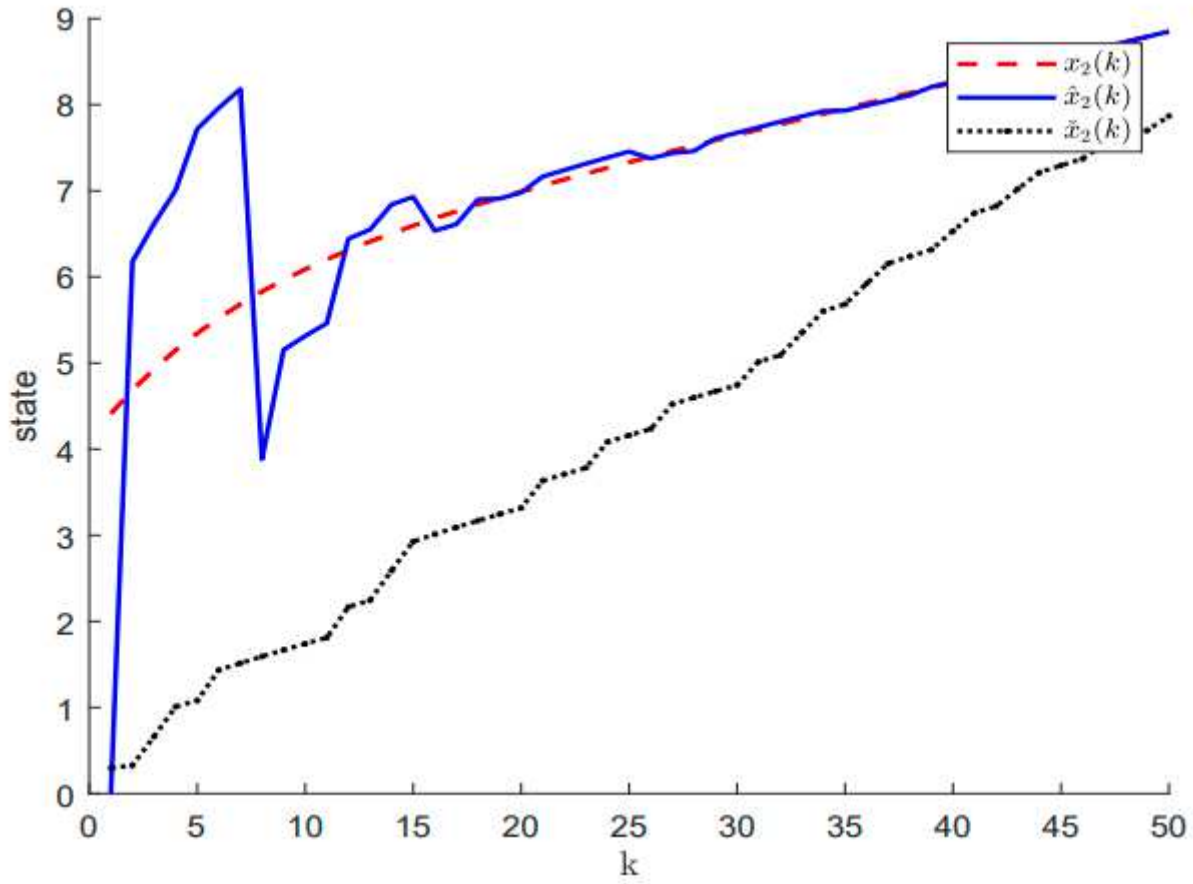


Figure 5

The trajectories of state $x_2(k)$ and its estimation with and without redundant CC.

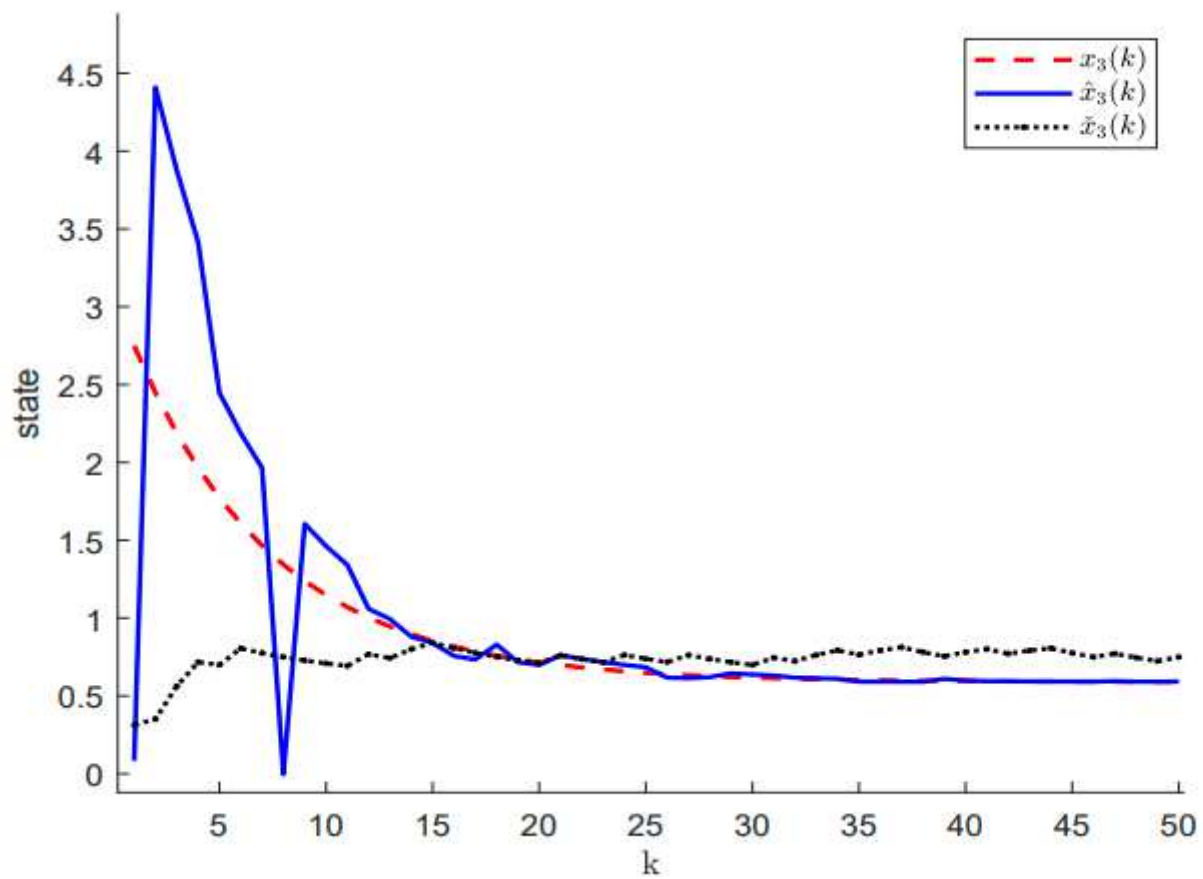


Figure 6

The trajectories of state $x_3(k)$ and its estimation with and without redundant CC.