Reliable H-Infinity Fusion Estimation of Time-Delayed Nonlinear Systems With Energy Constraints: The Finite-Horizon Case

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Abstract The fusion estimation issue of sensor networks is investigated for nonlinear time-varying systems with energy constraints, time-delays as well as packet loss. For the addressed problem, some local estimations are first obtained by using the designed Luenberger-type local estimator and then transmitted to a fusion center (FC) to generate a desired fusion value, where two classes of channels, whose schedules are governed by a diagonal matrix, are utilized to perform the information transmission. With the help of the Lyapunov stability theory, sufficient conditions are established to ensure the predetermined local and fused H_∞ performances over a finite horizon. Furthermore, by virtue of the well-known Schur complement lemma, the desired gains of local estimators and the suboptimal fusion weight matrices are obtained in light of the solution of linear matrix inequalities. It should be pointed out that the developed scheme is a two-step process under which the design of fusion weight matrices is based on the obtained estimator gains. Finally, a simulation example for sensor networks is performed to check the effectiveness of the proposed fusion scheme.

Keywords Fusion estimation · energy constraints · nonlinear systems · time-delays · sensor networks

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1 Introduction

With the rapid development and increasing maturity of communication technologies, sensor networks composed of many micro-sensors with limited computing power and communication capacity have been widely used in various engineering fields such as target tracking, traffic control, environmental monitoring, industrial automation, counter-terrorism and disaster prevention [1–6]. In contrast to a single sensor, sensor networks, whose main function is to perform information collection, have more advantages, such as information redundancy, complementarity and so on. Furthermore, the collected information is usually fused on a fusion center (FC) in a centralized way or a distributed one. It should be pointed out that the basic goal of information fusion is to derive more effective information by optimizing and combining the obtained data. In other words, the effectiveness of multiple sensor systems can be dramatically improved by taking advantage of the common or joint operation of multiple sensors. In practical engineering, a considerable challenge is how to design a feasible algorithm to obtain the optimal or sub-optimal fusion weights. To this end, many interesting results on information fusion under networked scenarios have been reported in the literature, see [1, 7–11] and the references therein. It should be pointed out that typical algorithms based on minimum variance technique or linear matrix inequality techniques can be roughly divided into centralized fusion algorithms, decentralized fusion algorithm [12] as well as distributed ones [11, 13]. Furthermore, the communication burden of distributed fusion estimation is great reduced although the accuracy is lower than that of the centralized one.

Although bringing a series of advantages, the resultant exploited communication network inevitably leads to various network-induced phenomena and plenty of energy consumption, especially for wireless sensor nodes, which reduces the service time of multiple sensor systems. On one hand, network congestion and packet loss will inevitably occur due to the limitation of communication resources or transmission power. Such phenomena could degrade the fusion performance if they are not handled rightly [11, 14–17, 38]. On the other hand, sensor nodes, usually powered by small batteries, are endowed with the low power constraint and limited communication and computation capabilities. Because of the difficulty of obtaining energy from the outside in a relatively harsh environment, reducing data transmission is undoubtedly an effective approach to achieve the energy constraint requirement. There is no doubt that the power constraint and the fusion performance usually need to achieve a balance. Up to date, some theory research on energy-constrained communication networks has focused on the design and optimization of transmission schemes [18–24]. For instance, optimal strategies of tradeoffs between bandwidth and power have been discussed in [20] to minimize the bit energy required for reliable transmission in broadband transmission methods. The best modulation strategy has been established in [18] to minimize the total energy consumption required to send a given number of bits. Furthermore, in [25], the total sensor transmission energy has been minimized by determining the optimal transmit power levels while ensuring the target mean squared error requirements. However, the precondition of developing these results is that the considered system must be a simple linear one. That is to say, the adopted methods in the above literature are unapplicable for complex dynamical systems including nonlinear terms, time-delays, or time-varying parameters. It is worth noting that, besides communication scheduling [9, 26], another typical strategy of energy management is the utilization of low-energy channels for information transmission while adopting suitable information compensation techniques to alleviate the negative effect from missing data [27], which constitutes one of the motivations for this investigation.

Time-delayed phenomena are ubiquitous in various actual engineering systems and usually aroused by material transmission and signal transmission. For example, communication delays may occur during data transmission over communication channels due to the limitation of communication bandwidth of sensor networks. There is no doubt that the existence of time delays usually degrades the fusion performance of multi-sensor systems if they cannot be adequately taken into consideration in the design process of the fusion algorithm. For the fusion based on Kalman filtering, time-delays can be effectively handled by the reorganized innovation [13,28–30], which, unfortunately, results in the increase of variable dimensions. Furthermore, the induced challenge for performance analysis and parameter design can also be handled by the analytical convenience of Lyapunov stability theory combined with linear matrix inequalities, see [31] for examples. Specially, the delay-dependent result can be established to reduce the conservatism of performance analysis and gain design for time-varying systems. On the other hand, the nonlinear characteristic of engineering systems is always regarded as an important source of complex dynamical behavior. As such, the fusion estimation with nonlinear or time-delayed influences has received ever-increasing research attention, see [26, 31–37] and the references therein. For instance, general nonlinear systems are first linearized as time-varying systems, and then an ingenious approach has been provided to find an upper bound of fusion estimation error while achieving the proposed weighting fusion criterion in [32]. A novel fusion framework has been developed in [31] to deal with the state estimation issue subject to partial-nodes-based measurements with the help of constructed Lyapunov-Krasovskii functionals. Note that the fusion estimation of nonlinear time-delayed systems with time-varying parameters still lies in an infant stage, not to mention the case that information transmission is enslaved to the limitation of energy. As such, the purpose of this paper is to attempt to fill in such a gap.

Inspired by the previous discussion, we begin to cope with the fusion estimation issue of the nonlinear time-delayed systems with energy constraints. Such an issue is nontrivial and appears some unavoidable challenges identified as follows: 1) how to evaluate the local estimation performance and the fusion estimation performance for time-varying systems with time delays in a finite horizon? and 2) how to obtain the desired fusion weights for complex dynamical systems where two kinds of channels named as reliable channels and general channels are employed to transmit information. As such, we devote to handle these two challenges and the main contributions of this paper are highlighted as follows: *1) a synthesis contributing to the complexity of networked systems is investigated within a unified framework that evaluates the influence from energy constraints based on channel scheduling, nonlinear terms, time-delays as well as time-varying parameters; 2) by means of Lyapunov stability analysis, sufficient conditions are derived to guarantee the predetermined fusion performance with a form of H_∞ index in a finite horizon; and 3) in light of established sufficient conditions, the desired local estimator gains and the suboptimal fusion weight matrices are obtained by using the solution of linear matrix inequalities.*

Notation The notations used here are fairly standard except where otherwise stated. $\mathbb{R}^n$ denote the n dimensional Euclidean space. The superscripts "T" and "-1" denote, respectively, the transpose operation of matrices and the inverse of matrices. The symbols I and 0 denote an identity matrix and a zero matrix with appropriate dimensions, respectively. The notations $\text{diag}_L\{\cdot\}$ and $\text{col}_L\{\cdot\}$ represent a diagonal matrix and a column vector formed by L same elements. In symmetric block matrices, an asterisk (i.e. $*$) stands for a term induced by symmetry. The probability of the event A is represented by $\mathbb{P}\{A\}$.

2 Problem Formulation and Preliminaries

In this paper, the considered sensor network, consisting of L nodes, monitors a class of discrete time-varying nonlinear targets over a finite time-horizon [0, N]

$$\begin{cases} x(t+1) = A(t)x(t) + F(t)x(t-\tau) \\ \qquad\qquad + B(t)g(x(t)) + \Gamma(t)w(t), \\ \quad y_i(t) = C_i(t)x(t) + D_i(t)v_i(t), \ \ i = 1,2,\cdots,L \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state of interesting targets that cannot be observed directly, $y_i(t) \in \mathbb{R}^{n_y}$ is the measured output from the sink node i, and $w(t)$ and $v_i(t)$ are, respectively, the process noise and the measurement noise satisfying $w(t) \in ([0,N];\mathbb{R}^q)$ and $v_i(t) \in ([0,N];\mathbb{R}^p)$. The positive integer τ describes the known time delay. $A(t)$, $B(t)$, $G(t)$, $F(t)$, $\Gamma(t)$, $C_i(t)$ and $D_i(t)$ are time-varying matrices with compatible dimensions.

In the above system, the nonlinear function $g(x(t))$ is supposed to be continuous and satisfies both $g(0) = 0$ and the bounded condition

$$\begin{aligned} &(g(x) - g(y) - \phi_1^g(x-y))^T \\ \times\ &(g(x) - g(y) - \phi_2^g(x-y)) \le 0, \forall x,y \in \mathbb{R}^{n_y} \end{aligned} \tag{2}$$

where ϕ_1^g and ϕ_2^g are two known matrices.

By resorting to the measurement $y_i(t)$, the local state estimation (LSE) $\hat{x}_i(t)$ is obtained via the following estimator on the ith node

$$\begin{aligned} \hat{x}_i(t+1) = &A(t)\hat{x}_i(t) + F(t)\hat{x}_i(t-\tau) + B(t)g(\hat{x}_i(t)) \\ &+ K_i(t)\big(y_i(t) - C_i(t)\hat{x}_i(t)\big), \end{aligned} \tag{3}$$

where $K_i(t)$ is the ith estimator gain to be designed.

As we all know, the information fusion has the capability of compensating the shortage of the local estimation performance. In order to execute the information fusion, LSE needs to be transmitted to FC, which leads to high data rates in communication. Furthermore, the lifetime of sensor nodes powered by an embedded battery with limited energy is also a considerable concern from the viewpoint of practical applications. As such, to reduce the energy consumption and improve the communication burden, two kinds of channels are exploited to properly facilitate data transmission. These two kinds of channels are, respectively, named as reliable channels owning the merit of high reliability and general channels subject to packet loss, see [27] for more details. For the convenience of mathematical analysis, a set of Bernoulli-distributed white sequences $\beta_i(t)$ are employed to describe the phenomenon of packet loss and their statistical characteristics are assumed to be satisfied as follows

$$\mathbb{P}(\beta_i(t) = 1) = \beta, \ \ \mathbb{P}(\beta_i(t) = 0) = 1 - \beta,$$

where β is a known positive scalar. Furthermore, let us introduce a binary variable $\chi_{ij}(t) \in \{0,1\}$ $(j = 1,2,\cdots,n)$ to describe whether or not the j-th component of LSE $\hat{x}_i(t)$ is scheduled to access the reliable channel and thereby the corresponding scheduling matrix, essentially a diagonal matrix, can be defined as

$$H_i(t) = \mathrm{diag}\{\chi_{i1}(t), \chi_{i2}(t), \cdots, \chi_{in}(t)\}, \tag{4}$$

where variables $\chi_{ij}(t)$ $(j = 1,2,\cdots,n)$ satisfy $\sum_{j=1}^{n} \chi_{ij}(t) = r_i$ $(i \in \{1,2,\cdots,L\})$.

According to the above engineering scenario of sensor networks, the received LSE $\hat{x}_{r_i}(t)$ is denoted by

$$\hat{x}_{r_i}(t) = H_i(t)\hat{x}_i(t) + \beta_i(t)(I - H_i(t))\hat{x}_i(t). \tag{5}$$

In light of the received LSE, we construct the following typical fusion estimator

$$\hat{x}(t) = \sum_{i=1}^{L} \Omega_i(t)\hat{x}_{r_i}(t), \tag{6}$$

where $\Omega_i(t)$ satisfying the constraint $\sum_{i=1}^{L} \Omega_i(t) = I_n$ is the weighting matrix to be designed.

In what follows, for the sake of performance analysis and parameter design, let us introduce the local estimation error (LEE) and the fusion error (FE):

$$e_i(t) \triangleq x(t) - \hat{x}_i(t), \ \ e_F(t) \triangleq x(t) - \hat{x}(t).$$

Then, denoting $g_t(e_i(t)) = g(x(t)) - g(\hat{x}_i(t))$, one has the i-th estimation error dynamics

$$\begin{aligned} e_i(t+1) = &(A(t) - K_i(t)C_i(t))e_i(t) + F(t)e_i(t-\tau) \\ &+ B(t)g_t(e_i(t)) + \Gamma(t)w(t) - K_i(t)D_i(t)v_i(t). \end{aligned} \tag{7}$$

Furthermore, denoting

$$\begin{aligned} &\xi(t) = \mathrm{col}\{x(t), e_1(t), \cdots, e_L(t)\}, \\ &\bar{v}(t) = \mathrm{col}\{0, v_1(t), v_2(t), \cdots, v_L(t)\}, \bar{w}(t) = \mathrm{col}_{L+1}\{w(t)\} \\ &\tilde{g}_t(\xi(t)) = \mathrm{col}\{g(x(t)), g_t(e_1(t)), \cdots, g_t(e_L(t))\} \end{aligned}$$

from (1), (5), (6) and (7), one has the following fusion error dynamics

$$\begin{cases} \xi(t+1) = \bar{A}_t\xi(t) + \bar{F}_t\xi(t-\tau) + \bar{B}_t\tilde{g}_t(\xi(t)) \\ \qquad\qquad + \bar{\Gamma}_t\bar{w}(t) + \bar{D}_t\bar{v}(t), \\ \quad e_F(t) = \Psi_{0t}\xi(t), \end{cases} \tag{8}$$

where

$$\bar{A}_t = \text{diag}\{A(t), A(t) - K_1(t)C_1(t), \cdots, A(t) - K_L(t)C_L(t)\},$$
$$\bar{F}_t = \text{diag}_{L+1}\{F(t)\},\ \bar{B}_t = \text{diag}_{L+1}\{B(t)\},$$
$$\bar{\Gamma}_t = \text{diag}_{L+1}\{\Gamma(t)\},\ \bar{D}_t = \text{diag}\{0, \aleph_t\},$$
$$\aleph_t = \text{diag}\{-K_1(t)D_1(t), \cdots, -K_L(t)D_L(t)\},$$
$$\Psi_{0t} = \Big[\Psi_{0xt},\ \Psi_{01t},\ \Psi_{02t},\ \cdots,\ \Psi_{0Lt}\Big]$$

with

$$\Psi_{0xt} = \sum_{i=1}^{L}(1-\beta_i(t))\Omega_i(t)(I - H_i(t)),$$
$$\Psi_{0it} = \Omega_i(t)H_i(t) + \beta_i(t)\Omega_i(t)(I - H_i(t)),\ 1 \le i \le L.$$

Taking the constraint on $\Omega_i(t)$ into account, one further has

$$\Psi_{0L} = \Big(I - \sum_{i=1}^{L-1}\Omega_i(t)\Big)H_L(t) + \beta_L(t)\Big(I - \sum_{i=1}^{L-1}\Omega_i(t)\Big)(I - H_L(t)).$$

Based on a set of predetermined scheduling matrices $H_i(t)$, the goals of this paper are to design $K_i(t)$ in (3) and the weighting matrices $\Omega_i(t)$ $(i = 1, \cdots, L)$ in (6) under energy constraints to satisfy the following requirements:

R1) The local error dynamics (7) satisfies the following finite-horizon H_∞ index on $[0,\ N]$

$$\begin{aligned}\mathbb{E}\Big\{\sum_{t=0}^{N}\|e_i(t)\|^2\Big\} < \gamma_i^2 \sum_{t=0}^{N}\big(\|w(t)\|^2 + \|v_i(t)\|^2\big) \\ + \gamma_i^2\mathbb{E}\Big\{e_i^T(0)\bar{\mathscr{Q}}_1 e_i(0) \\ + \tau \max_{-\tau \le s < 0} e_i^T(s)\bar{\mathscr{Q}}_2 e_i(s)\Big\}\end{aligned} \tag{9}$$

for the given disturbance attenuation level γ_i and the weighted matrices $\bar{\mathscr{Q}}_1$ and $\bar{\mathscr{Q}}_2$, which are symmetrically positive definite;

R2) The fusion error dynamics (8) satisfies the following finite-horizon H_∞ index on $[0,\ N]$

$$\begin{aligned}\mathbb{E}\Big\{\sum_{t=0}^{N}\|e_F(t)\|^2\Big\} < \gamma_F^2 \sum_{t=0}^{N}\big(\|w(t)\|^2 + \|\bar{v}(t)\|^2\big) \\ + \gamma_F^2\mathbb{E}\Big\{e_F^T(0)\bar{\mathscr{Q}}_3 e_F(0) \\ + \tau \max_{-\tau \le s < 0} e_F^T(s)\bar{\mathscr{Q}}_4 e_F(s)\Big\}\end{aligned} \tag{10}$$

for the given disturbance attenuation level γ_F and the weighted matrices $\bar{\mathscr{Q}}_3$ and $\bar{\mathscr{Q}}_4$, which are symmetrically positive definite.

Remark 1 In this paper, general channels subject to packet loss are employed to carry out the information transmission with the purpose of increasing the service life of the battery. Furthermore, the fusion estimation issue facing the complexity of networked systems is investigated within a unified framework that evaluates the influence from energy constraint based on channel scheduling, nonlinear terms, time-delays as well as time-varying parameters. In comparison with existing results focusing on the optimization of energy cost, the main idea used in this paper is that, according to predetermined energy utilization schemes, a set of fusion weights for sensor networks are designed to achieve the desired local and fused H_∞ performance in a finite horizon.

3 Main Results

In this section, the H_∞ performance is analyzed to the dynamics of LEE (7) and the dynamics of FE (8) with the designed estimator. Then, the desired estimator gains are proposed in terms of the solution to certain matrix inequalities derived according to the obtained condition. First, let us give the following lemma that will be used in the proof of our main results in this paper.

Lemma 1 *[39] Let Ω_1, Ω_2 and Ω_3 be constant matrices where $\Omega_1 = \Omega_1^T$ and $0 < \Omega_2 = \Omega_2^T$, then $\Omega_1 + \Omega_3^T\Omega_2^{-1}\Omega_3 < 0$ if and only if*

$$\begin{bmatrix}\Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2\end{bmatrix} < 0, \quad \begin{bmatrix}-\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1\end{bmatrix} < 0.$$

For the simplify of the presentation, denote $A_{it} = A(t) - K_i(t)C_i(t)$, $F_t = F(t)$, $B_t = B(t)$, $\Gamma_t = \Gamma(t)$, and $D_{it} = -K_i(t)D_i(t)$.

Theorem 1 *Let the gains matrices $K_i(t)$, the fusion weighted matrices $\Omega_i(t)$, and weighted matrices $\bar{\mathscr{Q}}_i$ $(i = 1,2,3,4)$, and disturbance attenuation levels γ_F and γ_i $(i = 1,2,\cdots,L)$ be given. The dynamics of LEE (7) and the dynamics of FE (8) satisfy, respectively, the desired H_∞ performance constraints (9) and (10), if there exist positive scalars $\lambda_i(i = 0,1,\cdots,L)$ and positive definite matrices $(P_t,\ Q_t,\ R_t,\ T_t)$ $(t \in [0,\ N])$ with the initial conditions*

$$P_0 \le \gamma_i^2\bar{\mathscr{Q}}_1,\ \ Q_s \le \tau\gamma_i^2\bar{\mathscr{Q}}_2, \tag{11}$$

$$R_0 \le \gamma_F^2\bar{\mathscr{Q}}_3,\ \ T_s \le \tau\gamma_F^2\bar{\mathscr{Q}}_4 \tag{12}$$

for $s = -\tau, -\tau+1, \cdots, -1$ such that the following inequalities

$$\Pi_{1t} = \begin{bmatrix}\Upsilon_{11t} & A_{it}^T P_{t+1}F_t & \Upsilon_{13t} & A_{it}^T P_{t+1}\Gamma_t & A_{it}^T P_{t+1}D_{it} \\ * & \Upsilon_{22t} & F_t^T P_{t+1}B_t & F_t^T P_{t+1}\Gamma_t & F_t^T P_{t+1}D_{it} \\ * & * & \Upsilon_{33t} & B_t^T P_{t+1}\Gamma_t & B_t^T P_{t+1}D_{it} \\ * & * & * & \Upsilon_{44t} & \Gamma_t^T P_{t+1}D_{it} \\ * & * & * & * & \Upsilon_{55t}\end{bmatrix} < 0, \tag{13}$$

and

$$\Pi_{2t} = \begin{bmatrix}\Theta_{11t} & \bar{A}_t^T R_{t+1}\bar{F}_t & \Theta_{13t} & \bar{A}_t^T R_{t+1}\bar{\Gamma}_t & \bar{A}_t^T R_{t+1}\bar{D}_t \\ * & \Theta_{22t} & \bar{F}_t^T R_{t+1}\bar{B}_t & \bar{F}_t^T R_{t+1}\bar{\Gamma}_t & \bar{F}_t^T R_{t+1}\bar{D}_t \\ * & * & \Theta_{33t} & \bar{B}_t^T R_{t+1}\bar{\Gamma}_t & \bar{B}_t^T R_{t+1}\bar{D}_t \\ * & * & * & \Theta_{44t} & \bar{\Gamma}_t^T R_{t+1}\bar{D}_t \\ * & * & * & * & \Theta_{55t}\end{bmatrix} < 0$$

(14)

hold, where

$$
\begin{aligned}
&\Upsilon_{11t}=A_{it}^TP_{t+1}A_{it}-P_t+Q_t-\lambda_{it}\Phi_1^g+I,\\
&\Upsilon_{13t}=A_{it}^TP_{t+1}B_t+\lambda_{it}\Phi_2^{gT},\quad \Upsilon_{22t}=F_t^TP_{t+1}F_t-Q_{t-\tau},\\
&\Upsilon_{33t}=B_t^TP_{t+1}B_t-\lambda_{it}I,\quad \Upsilon_{44t}=\Gamma_t^TP_{t+1}\Gamma_t-\gamma_i^2I,\\
&\Upsilon_{55t}=D_{it}^TP_{t+1}D_{it}-\gamma_i^2I,\quad \Theta_{13t}=\bar{A}_t^TR_{t+1}\bar{B}_t+\lambda_{0t}\Phi_{20}^{gT},\\
&\Theta_{11t}=\bar{A}_t^TR_{t+1}\bar{A}_t-R_t+T_t-\lambda_{0t}\Phi_{10}^g+\chi_{0t},\\
&\Theta_{22t}=\bar{F}_t^TR_{t+1}\bar{F}_t-T_{t-\tau},\quad \Theta_{33t}=\bar{B}_t^TR_{t+1}\bar{B}_t-\lambda_{0t}I,\\
&\Theta_{44t}=\bar{\Gamma}_t^TR_{t+1}\bar{\Gamma}_t-\gamma_0^2I,\quad \Theta_{55t}=\bar{D}_t^TR_{t+1}\bar{D}_t-\gamma_0^2I\\
&\Phi_1^g=I\otimes Sym\{\frac{1}{2}\phi_1^{gT}\phi_2^g\},\quad \Phi_2^g=I\otimes(\phi_1^g+\phi_2^g\})/2\\
&\chi_{0t}=\bar{\Psi}_{0t}^T\bar{\Psi}_{0t}+\beta(1-\beta)\sum_{i=1}^{L}\tilde{\Psi}_{0it}^{iT}\tilde{\Psi}_{0it}^{i},\\
&\bar{\Psi}_{0t}=\left[\bar{\Psi}_{0xt}\ \bar{\Psi}_{01t}\ \bar{\Psi}_{02t}\ \cdots\ \bar{\Psi}_{0Lt}\right],\\
&\bar{\Psi}_{0xt}=\sum_{j=1}^{L-1}(1-\beta)\Omega_j(t)(I-H_j(t))\\
&\qquad +(1-\beta)(I-\sum_{j=1}^{L-1}\Omega_j(t))(I-H_L(t)),\\
&\bar{\Psi}_{0it}=\Omega_i(t)H_i(t)+\beta\Omega_i(t)(I-H_i(t)),\ 1\le i<L,\\
&\bar{\Psi}_{0Lt}=(I-\sum_{j=1}^{L-1}\Omega_j(t))H_L(t)+\beta(I-\sum_{j=1}^{L-1}\Omega_j(t))(I-H_L(t)),\\
&\tilde{\Psi}_{0it}^{i}=\big[\tilde{\Psi}_{0xt}^{i}\ \underbrace{0\ \cdots\ 0}_{i-1}\ \tilde{\Psi}_{0it}\ \ \underbrace{0\ \cdots\ 0}_{L-i}\big],\\
&\tilde{\Psi}_{0xt}^{i}=\Omega_i(t)(I-H_i(t)),\quad 1\le i<L,\\
&\tilde{\Psi}_{0it}=-\Omega_i(t)(I-H_i(t)),\quad 1\le i<L,\\
&\tilde{\Psi}_{0xt}^{L}=(I-\sum_{j=1}^{L-1}\Omega_j(t))(I-H_L(t)),\\
&\tilde{\Psi}_{0Lt}=-(I-\sum_{j=1}^{L-1}\Omega_j(t))(I-H_L(t)).
\end{aligned}
$$

Proof The proof will be divided into two parts: the first one is the analysis of finite-horizon H_∞ performance of LEEs and the other is the H_∞ performance analysis of FE. Now, let us deal with the first part. To this end, we first construct the following Lyapunov function for the ith dynamics of LEE (7):

$$V_e^i(t)=V_{e1}^i(t)+V_{e2}^i(t),\tag{15}$$

where

$$V_{e1}^i(t)=e_i^T(t)P_te_i(t),\ V_{e2}^i(t)=\sum\nolimits_{s=t-\tau}^{t-1}e_i^T(s)Q_se_i(s).$$

Calculating the differences of $V_{e1}^i(t)$ and $V_{e2}^i(t)$ along with the evolution of LEE (7), and then taking their mathematical expectation, one has

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_{e1}^i(t)\}\\
&=\mathbb{E}\{V_{e1}^i(t+1)-V_{e1}^i(t)\}\\
&=\mathbb{E}\{e_i^T(t+1)P_{t+1}e_i(t+1)-e_i^T(t)P_te_i(t)\}\\
&=\mathbb{E}\Big\{\big(A_{it}e_i(t)+F_te_i(t-\tau)+B_tg_t(e_i(t))+\Gamma_tw(t)\\
&\quad +D_{it}v_i(t)\big)^TP_{t+1}\big(A_{it}e_i(t)+F_te_i(t-\tau)+B_tg_t(e_i(t))\\
&\quad +\Gamma_tw(t)+D_{it}v_i(t)\big)-e_i^T(t)P_te_i(t)\Big\}\\
&=\mathbb{E}\Big\{e_i^T(t)A_{it}^TP_{t+1}A_{it}e_i(t)+e_i^T(t)A_{it}^TP_{t+1}F_te_i(t-\tau)\\
&\quad +e_i^T(t)A_{it}^TP_{t+1}B_tg_t(e_i(t))+e_i^T(t)A_{it}^TP_{t+1}\Gamma_tw(t)\\
&\quad +e_i^T(t)A_{it}^TP_{t+1}D_{it}v_i(t)+e_i^T(t-\tau)F_t^TP_{t+1}A_{it}e_i(t)\\
&\quad +e_i^T(t-\tau)F_t^TP_{t+1}F_te_i(t-\tau)+e_i^T(t-\tau)F_t^TP_{t+1}B_tg_t(e_i(t))\\
&\quad +e_i^T(t-\tau)F_t^TP_{t+1}\Gamma_tw(t)+e_i^T(t-\tau)F_t^TP_{t+1}D_{it}v_i(t)\\
&\quad +g_t^T(e_i(t))B_t^TP_{t+1}A_{it}e_i(t)+g_t^T(e_i(t))B_t^TP_{t+1}F_te_i(t-\tau)\\
&\quad +g_t^T(e_i(t))B_t^TP_{t+1}B_tg_t(e_i(t))+g_t^T(e_i(t))B_t^TP_{t+1}\Gamma_tw(t)\\
&\quad +g_t^T(e_i(t))B_t^TP_{t+1}D_{it}v_i(t)+w^T(t)\Gamma_t^TP_{t+1}A_{it}e_i(t)\\
&\quad +w^T(t)\Gamma_t^TP_{t+1}F_te_i(t-\tau)+w^T(t)\Gamma_t^TP_{t+1}B_tg_t(e_i(t))\\
&\quad +w^T(t)\Gamma_t^TP_{t+1}\Gamma_tw(t)+w^T(t)\Gamma_t^TP_{t+1}D_{it}v_i(t)\\
&\quad +v_i^T(t)D_{it}^TP_{t+1}A_{it}e_i(t)+v_i^T(t)D_{it}^TP_{t+1}F_te_i(t-\tau)\\
&\quad +v_i^T(t)D_{it}^TP_{t+1}B_tg_t(e_i(t))+v_i^T(t)D_{it}^TP_{t+1}\Gamma_tw(t)\\
&\quad +v_i^T(t)D_{it}^TP_{t+1}D_{it}v_i(t)\}-e_i^T(t)P_te_i(t)\Big\},
\end{aligned}\tag{16}
$$

and

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_{e2}^i(t)\}\\
&=\mathbb{E}\{V_{e2}^i(t+1)-V_{e2}^i(t)\}\\
&=\mathbb{E}\Big\{\sum\nolimits_{s=t+1-\tau}^{t}e_i^T(s)Q_se_i(s)-\sum\nolimits_{s=t-\tau}^{t-1}e_i^T(s)Q_se_i(s)\Big\}\\
&=\mathbb{E}\Big\{e_i^T(t)Q_te_i(t)+\sum\nolimits_{s=t+1-\tau}^{t-1}e_i^T(s)Q_se_i(s)\\
&\quad -\sum\nolimits_{s=t+1-\tau}^{t-1}e_i^T(s)Q_se_i(s)-e_i^T(t-\tau)Q_{t-\tau}e_i(t-\tau)\Big\}\\
&=\mathbb{E}\{e_i^T(t)Q_te_i(t)-e_i^T(t-\tau)Q_{t-\tau}e_i(t-\tau)\}.
\end{aligned}\tag{17}
$$

Denoting $\eta_i(t)=[\,e_i^T(t)\ e_i^T(t-\tau)\ g_t^T(e_i(t))\ w^T(t)\ v_i^T(t)\,]^T$ and then combining with (16) and (17) result in

$$
\begin{aligned}
\mathbb{E}\{\Delta V_e^i(t)\}&=\mathbb{E}\{V_e^i(t+1)-V_e^i(t)\}\\
&=\mathbb{E}\{\Delta V_{e1}^i(t)+\Delta V_{e2}^i(t)\}\\
&=\mathbb{E}\{\eta_i^T(t)\widetilde{\Pi}_{1t}\eta_i(t)\},
\end{aligned}\tag{18}
$$

where

$$\widetilde{\Xi}_{11t}=A_{it}^TP_{t+1}A_{it}-P_t+Q_t,\ \ \widetilde{\Xi}_{22t}=F_t^TP_{t+1}F_t-Q_{t-\tau},$$

$$
\widetilde{\Pi}_{1t}=\begin{bmatrix}
\widetilde{\Xi}_{11t} & A_{it}^TP_{t+1}F_t & A_{it}^TP_{t+1}B_t & A_{it}^TP_{t+1}\Gamma_t & A_{it}^TP_{t+1}D_{it}\\
* & \widetilde{\Xi}_{22t} & F_t^TP_{t+1}B_t & F_t^TP_{t+1}\Gamma_t & F_t^TP_{t+1}D_{it}\\
* & * & B_t^TP_{t+1}B_t & B_t^TP_{t+1}\Gamma_t & B_t^TP_{t+1}D_{it}\\
* & * & * & \Gamma_t^TP_{t+1}\Gamma_t & \Gamma_t^TP_{t+1}D_{it}\\
* & * & * & * & D_{it}^TP_{t+1}D_{it}
\end{bmatrix}.
$$

In what follows, to disclose the finite-horizon H_∞ performance of (9), let us introduce the cost function

$$J_1(t) = e_i^T(t+1)P_{t+1}e_i(t+1) - e_i^T(t)P_te_i(t) + \sum_{s=t+1-\tau}^{t} e_i^T(s)Q_se_i(s) - \sum_{s=t-\tau}^{t-1} e_i^T(s)Q_se_i(s).$$

On the other hand, it follows from (2) that

$$[g_t(e_i(t)) - (I\otimes\phi_1^g)e_i(t)]^T[g_t(e_i(t)) - (I\otimes\phi_2^g)e_i(t)] \le 0.$$

Taking such a constraint into account, one has

$$\mathbb{E}\{J_1(t)\} \le \mathbb{E}\{\eta_i^T(t)\widetilde{\Pi}_{1t}\eta_i(t) - \lambda_{it}[g_t(e_i(t)) - (I\otimes\phi_1^g)e_i(t)]^T[g_t(e_i(t)) - (I\otimes\phi_2^g)e_i(t)]\}.$$

Then, adding the zero term

$$\mathbb{E}\{\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2) - (\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2))\}$$

into the right side of the above inequality leads to

$$\begin{aligned}\mathbb{E}\{J_1(t)\} \le{}& \mathbb{E}\Big\{\eta_i^T(t)\widetilde{\Pi}_{1t}\eta_i(t) - \lambda_{it}[g_t(e_i(t)) - (I\otimes\phi_1^g)e_i(t)]^T[g_t(e_i(t)) - (I\otimes\phi_2^g)e_i(t)] \\ &+ \|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2)\Big\} \\ &- \mathbb{E}\Big\{\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2)\Big\} \\ ={}& \mathbb{E}\Big\{\eta_i^T(t)\Pi_{1t}\eta_i(t)\Big\} - \mathbb{E}\Big\{\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2)\Big\}, \end{aligned} \tag{19}$$

which means

$$\begin{aligned}\sum_{t=0}^{N}\mathbb{E}\{J_1(t)\} ={}& \sum_{t=0}^{N}\mathbb{E}\{\Delta V_e^i(t)\} = \mathbb{E}\{V_e^i(N+1)\} - \mathbb{E}\{V_e^i(0)\} \\ \le{}& \sum_{t=0}^{N}\mathbb{E}\Big\{\eta_i^T(t)\Pi_{1t}\eta_i(t)\Big\} - \sum_{t=0}^{N}\mathbb{E}\Big\{\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2)\Big\}. \end{aligned} \tag{20}$$

Noting the conditions (11) and (13), one cannot difficultly obtain from the above inequality that

$$\begin{aligned}&\mathbb{E}\Big\{\sum_{t=0}^{N}\Big(\|e_i(t)\|^2 - \gamma_i^2(\|w(t)\|^2+\|v_i(t)\|^2)\Big) - \gamma_i^2\big(e_i^T(0)\bar{\mathscr{Q}}_1e_i(0) + \tau\max_{-\tau\le s<0}e_i^T(s)\bar{\mathscr{Q}}_2e_i(s)\big)\Big\} \\ &\le \mathbb{E}\{V_e^i(0)\} - \mathbb{E}\{V_e^i(N+1)\} - \gamma_i^2\mathbb{E}\{e_i^T(0)\bar{\mathscr{Q}}_1e_i(0) + \tau\max_{-\tau\le s<0}e_i^T(s)\bar{\mathscr{Q}}_2e_i(s)\} \\ &\le \mathbb{E}\Big\{e_i^T(0)P_0e_i(0) + \sum_{s=-\tau}^{-1}e_i^T(s)Q_se_i(s)\Big\} - \gamma_i^2\mathbb{E}\{e_i^T(0)\bar{\mathscr{Q}}_1e_i(0) + \tau\max_{-\tau\le s<0}e_i^T(s)\bar{\mathscr{Q}}_2e_i(s)\} \\ &< 0, \end{aligned} \tag{21}$$

which implies the considered performance (9) is satisfied.

Up to now, we have analyzed the dynamical performance of LEEs, thereby lying in the position to deal with fusion performance. Similarly, let us employ the following Lyapunov function:

$$V_f(t) = V_{f1}(t) + V_{f2}(t), \tag{22}$$

where

$$V_{f1}(t) = \xi^T(t)R_t\xi(t),\ \ V_{f2}(t) = \sum_{s=t-\tau}^{t-1}\xi^T(s)T_s\xi(s).$$

Calculating the differences of $V_{f1}^i(t)$ and $V_{f2}^i(t)$ along with the evolution of FE (8), and then taking their mathematical expectation, one has

$$\begin{aligned}&\mathbb{E}\{\Delta V_{f1}(t)\} \\ &= \mathbb{E}\{V_{f1}(t+1) - V_{f1}(t)\} \\ &= \mathbb{E}\{\xi^T(t+1)R_{t+1}\xi(t+1) - \xi^T(t)R_t\xi(t)\} \\ &= \mathbb{E}\Big\{\big(\bar{A}_t\xi(t) + \bar{F}_t\xi(t-\tau) + \bar{B}_t\tilde{g}_t(\xi(t)) + \bar{\Gamma}_t\bar{w}(t) + \bar{D}_t\bar{v}(t)\big)^TR_{t+1}\big(\bar{A}_t\xi(t) + \bar{F}_t\xi(t-\tau) + \bar{B}_t\tilde{g}_t(\xi(t)) \\ &\quad + \bar{\Gamma}_t\bar{w}(t) + \bar{D}_t\bar{v}(t)\big) - \xi^T(t)R_t\xi(t)\Big\} \\ &= \mathbb{E}\Big\{\xi^T(t)\bar{A}_t^TR_{t+1}\bar{A}_t\xi(t) + \xi^T(t)\bar{A}_t^TR_{t+1}\bar{F}_t\xi(t-\tau) \\ &\quad + \xi^T(t)\bar{A}_t^TR_{t+1}\bar{B}_t\tilde{g}_t(\xi(t)) + \xi^T(t)\bar{A}_t^TR_{t+1}\bar{\Gamma}_t\bar{w}(t) \\ &\quad + \xi^T(t)\bar{A}_t^TR_{t+1}\bar{D}_t\bar{v}(t) + \xi^T(t-\tau)\bar{F}_t^TR_{t+1}\bar{A}_t\xi(t) \\ &\quad + \xi^T(t-\tau)\bar{F}_t^TR_{t+1}\bar{F}_t\xi(t-\tau) + \xi^T(t-\tau)\bar{F}_t^TR_{t+1}\bar{B}_t\tilde{g}_t(\xi(t)) \\ &\quad + \xi^T(t-\tau)\bar{F}_t^TR_{t+1}\bar{\Gamma}_t\bar{w}(t) + \xi^T(t-\tau)\bar{F}_t^TR_{t+1}\bar{D}_t\bar{v}(t) \\ &\quad + \tilde{g}_t^T(\xi(t))\bar{B}_t^TR_{t+1}\bar{A}_t\xi(t) + \tilde{g}_t^T(\xi(t))\bar{B}_t^TR_{t+1}\bar{F}_t\xi(t-\tau) \\ &\quad + \tilde{g}_t^T(\xi(t))\bar{B}_t^TR_{t+1}\bar{B}_t\tilde{g}_t(\xi(t)) + \tilde{g}_t^T(\xi(t))\bar{B}_t^TR_{t+1}\bar{\Gamma}_t\bar{w}(t) \\ &\quad + \tilde{g}_t^T(\xi(t))\bar{B}_t^TR_{t+1}\bar{D}_t\bar{v}(t) + w^T(t)\bar{\Gamma}_t^TR_{t+1}\bar{A}_t\xi(t) \\ &\quad + \bar{w}^T(t)\bar{\Gamma}_t^TR_{t+1}\bar{F}_t\xi(t-\tau) + \bar{w}^T(t)\bar{\Gamma}_t^TR_{t+1}\bar{B}_t\tilde{g}_t(\xi(t)) \\ &\quad + \bar{w}^T(t)\bar{\Gamma}_t^TR_{t+1}\bar{\Gamma}_t\bar{w}(t) + \bar{w}^T(t)\bar{\Gamma}_t^TR_{t+1}\bar{D}_t\bar{v}(t) \\ &\quad + \bar{v}^T(t)\bar{D}_t^TR_{t+1}\bar{A}_t\xi(t) + \bar{v}^T(t)\bar{D}_t^TR_{t+1}\bar{F}_t\xi(t-\tau) \\ &\quad + \bar{v}^T(t)\bar{D}_t^TR_{t+1}\bar{B}_t\tilde{g}_t(\xi(t)) + \bar{v}^T(t)\bar{D}_t^TR_{t+1}\bar{\Gamma}_t\bar{w}(t) \\ &\quad + \bar{v}^T(t)\bar{D}_t^TR_{t+1}\bar{D}_t\bar{v}(t)\} - \xi^T(t)R_t\xi(t)\Big\}, \end{aligned} \tag{23}$$

and

$$\begin{aligned}&\mathbb{E}\{\Delta V_{f2}(t)\} \\ &= \mathbb{E}\{V_{f2}(t+1) - V_{f2}(t)\} \\ &= \mathbb{E}\Big\{\sum_{s=t+1-\tau}^{t}\xi^T(s)T_s\xi(s) - \sum_{s=t-\tau}^{t-1}\xi^T(s)T_s\xi(s)\Big\} \\ &= \mathbb{E}\Big\{\xi^T(t)T_t\xi(t) + \sum_{s=t+1-\tau}^{t-1}\xi^T(s)T_s\xi(s) - \sum_{s=t+1-\tau}^{t-1}\xi^T(s)T_s\xi(s) - \xi^T(t-\tau)T_{t-\tau}\xi(t-\tau)\Big\}\end{aligned}$$

$$= \mathbb{E}\{\xi^T(t)T_t\xi(t) - \xi^T(t-\tau)T_{t-\tau}\xi(t-\tau)\}. \tag{24}$$

Denoting

$$\eta_0(t) = [\,\xi^T(t)\ \xi^T(t-\tau)\ \tilde{g}_t^T(\xi(t))\ \bar{w}^T(t)\ \bar{v}^T(t)\,]^T$$

and then synthesizing (23) and (24) lead to

$$\begin{aligned}\mathbb{E}\{\Delta V_f(t)\} &= \mathbb{E}\{V_f(t+1) - V_f(t)\}\\ &= \mathbb{E}\{\Delta V_{f1}(t) + \Delta V_{f2}(t)\}\\ &= \mathbb{E}\{\eta_0^T(t)\widetilde{\Pi}_{2t}\eta_0(t)\},\end{aligned} \tag{25}$$

where

$$\widetilde{\Theta}_{11t} = \bar{A}_t^T R_{t+1}\bar{A}_t - R_t + T_t,\ \ \widetilde{\Theta}_{22t} = \bar{F}_t^T R_{t+1}\bar{F}_t - T_{t-\tau},$$

$$\widetilde{\Pi}_{2t} = \begin{bmatrix} \widetilde{\Theta}_{11t} & \bar{A}_t^T R_{t+1}\bar{F}_t & \bar{A}_t^T R_{t+1}\bar{B}_t & \bar{A}_t^T R_{t+1}\bar{\Gamma}_t & \bar{A}_t^T R_{t+1}\bar{D}_t \\ * & \widetilde{\Theta}_{22t} & \bar{F}_t^T R_{t+1}\bar{B}_t & \bar{F}_t^T R_{t+1}\bar{\Gamma}_t & \bar{F}_t^T R_{t+1}\bar{D}_t \\ * & * & \bar{B}_t^T R_{t+1}\bar{B}_t & \bar{B}_t^T R_{t+1}\bar{\Gamma}_t & \bar{B}_t^T R_{t+1}\bar{D}_t \\ * & * & * & \bar{\Gamma}_t^T R_{t+1}\bar{\Gamma}_t & \bar{\Gamma}_t^T R_{t+1}\bar{D}_t \\ * & * & * & * & \bar{D}_t^T R_{t+1}\bar{D}_t \end{bmatrix}.$$

Similarly, one has the nonliner constraint

$$\lambda_{0t}[\tilde{g}_t(\xi(t)) - (I\otimes\phi_{10}^g)\xi(t)]^T[\tilde{g}_t(\xi(t)) - (I\otimes\phi_{20}^g)\xi(t)] \le 0,$$

and therefore further has the following condition

$$\begin{aligned}\mathbb{E}\{\Delta V_f(t)\} &\le \mathbb{E}\{\eta_0^T(t)\widetilde{\Pi}_{2t}\eta_0(t) - \lambda_{0t}[\tilde{g}_t(\xi(t))\\ &\quad - (I\otimes\phi_{10}^g)\xi(t)]^T[\tilde{g}_t(\xi(t)) - (I\otimes\phi_{20}^g)\xi(t)]\}\\ &\le \mathbb{E}\{\eta_0^T(t)\Pi_{2t}\eta_0(t)\}.\end{aligned} \tag{26}$$

In what follows, the cost function of H_∞ index can be rewritten by

$$\begin{aligned}J_2(t) &= e_F^T(t+1)R_{t+1}e_F(t+1) - e_F^T(t)R_te_F(t)\\ &\quad + \textstyle\sum_{s=t+1-\tau}^{t} e_F^T(s)T_se_F(s) - \sum_{s=t-\tau}^{t-1} e_F^T(s)T_se_F(s).\end{aligned}$$

It is not difficult to derive that

$$\begin{aligned}\sum_{t=0}^{N}\mathbb{E}\{J_2(t)\} &= \sum_{t=0}^{N}\mathbb{E}\{\Delta V_f(t)\}\\ &= \mathbb{E}\{V_f(N+1)\} - \mathbb{E}\{V_f(0)\}\\ &\le \mathbb{E}\Big\{\sum_{t=0}^{N}\eta_0^T(t)\Pi_{2t}\eta_0(t)\Big\} - \mathbb{E}\Big\{\sum_{t=0}^{N}\Big(\|e_F(t)\|^2\\ &\quad - \gamma_F^2\big(\|\bar{w}(t)\|^2 + \|\bar{v}(t)\|^2\big)\Big)\Big\},\end{aligned}$$

and

$$\begin{aligned}\mathbb{E}\Big\{\|e_F(t)\|^2\Big\} &= \mathbb{E}\Big\{(\Psi_0\xi(t))^T\Psi_0\xi(t)\Big\}\\ &= \mathbb{E}\Big\{\xi^T(t)\Psi_0^T\Psi_0\xi(t)\Big\}\\ &= \mathbb{E}\Big\{\xi^T(t)\Big(\bar{\Psi}_0^T\bar{\Psi}_0\\ &\quad + \beta(1-\beta)\sum_{i=1}^{L}\tilde{\Psi}_{0i}^{iT}\tilde{\Psi}_{0i}^{i}\Big)\xi(t)\Big\}.\end{aligned}$$

Noting the conditions (12) and (14), one cannot difficultly obtain that

$$\begin{aligned}&\mathbb{E}\Big\{\sum_{t=0}^{N}\Big(\|e_F(t)\|^2 - \gamma_F^2\big(\|\bar{w}(t)\|^2 + \|\bar{v}(t)\|^2\big)\Big)\\ &\quad - \gamma_F^2\big(e_F^T(0)\bar{\mathscr{Q}}_3e_F(0) + \tau\max_{-\tau\le s<0} e_F^T(s)\bar{\mathscr{Q}}_4e_F(s)\big)\Big\}\\ &\le \mathbb{E}\Big\{\sum_{t=0}^{N}\eta_0^T(t)\Pi_{2t}\eta_0(t)\Big\} + \mathbb{E}\Big\{e_F^T(0)R_0e_F(0)\\ &\quad + \sum_{s=-\tau}^{-1} e_F^T(s)T_se_F(s)\Big\} - \gamma_F^2\big(e_F^T(0)\bar{\mathscr{Q}}_3e_F(0)\\ &\quad + \tau\max_{-\tau\le s<0} e_F^T(s)\bar{\mathscr{Q}}_4e_F(s)\big)\Big\}\\ &< 0,\end{aligned} \tag{27}$$

which means that the finite-horizon H_∞ performance of fusion error is satisfied. The proof is complete.

In the above theorem, the finite-horizon H_∞ performance of both LEEs (7) and FE (8) has been handled with the help of the Lyapunov stability theorem. It should be pointed out that the developed conditions can only be utilized to perform the performance analysis and cannot be adopted to obtain the desired estimator gains and the fusion weights of sensor networks. Therefore, the subsequent issue core is to develop a suitable approach to obtain desired matrix parameters from the viewpoint of engineering applications of sensor networks.

Lemma 2 *Let parameters $\gamma_i > 0$ $(i = 1,\cdots,L)$ and weighted matrices $\bar{\mathscr{Q}}_1$ and $\bar{\mathscr{Q}}_2$ be given. The local dynamics of LEE (7) satisfies the H_∞ performance constraint (9) for all nonzero $w(t)$ and $v_i(t)$, if there exist positive definite matrices $P_t > 0$, $Q_t > 0$, matrices $\hat{K}_{it}$ and positive scalars $\lambda_{it}(i \in \{1,2,\cdots,L\})$ satisfying (11) and*

$$\begin{bmatrix} -P_{t+1} & \mathscr{M}_{12t} & \mathscr{M}_{13t} & \mathscr{M}_{14t} & \mathscr{M}_{15t} & \mathscr{M}_{16t} \\ * & \mathscr{M}_{22t} & 0 & \lambda_{it}\Phi_2^{gT} & 0 & 0 \\ * & * & -Q_{t-\tau} & 0 & 0 & 0 \\ * & * & * & -\lambda_{it}I & 0 & 0 \\ * & * & * & * & -\gamma_i^2 I & 0 \\ * & * & * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \tag{28}$$

where

$$\begin{cases}\mathscr{M}_{22t} = Q_t - P_t - \lambda_{it}\Phi_1^g + I,\\ \mathscr{M}_{12t} = P_{t+1}A(t) - \hat{K}_{it}C_i(t),\quad \mathscr{M}_{13t} = P_{t+1}F_t,\\ \mathscr{M}_{14t} = P_{t+1}B_t,\quad \mathscr{M}_{15t} = P_{t+1}\Gamma_t,\quad \mathscr{M}_{16t} = -\hat{K}_{it}D_i(t).\end{cases}$$

When the above inequality holds, the estimator gain $K_i(t)$ is can be determined by

$$K_i(t) = P_{t+1}^{-1}\hat{K}_{it}. \tag{29}$$

Proof According to the primary matrix operation, the inequality (13) is equivalent to

$$\begin{bmatrix} \mathscr{M}_{22t} & 0 & \lambda_{it}\Phi_2^{gT} & 0 & 0 \\ * & -Q_{t-\tau} & 0 & 0 & 0 \\ * & * & -\lambda_{it}I & 0 & 0 \\ * & * & * & -\gamma_i^2 I & 0 \\ * & * & * & * & -\gamma_i^2 I \end{bmatrix} + \Xi_{1t}^T P_{t+1} \Xi_{1t} < 0$$

where $\Xi_{1t} = \begin{bmatrix} A_{it} & F_t & B_t & \Gamma_t & D_{it} \end{bmatrix}$.

According to the Schur complement lemma, the above inequality holds if and only if

$$\begin{bmatrix} -P_{t+1} & P_{t+1}A_{it} & P_{t+1}F_t & P_{t+1}B_t & P_{t+1}\Gamma_t & P_{t+1}D_{it} \\ * & \mathscr{M}_{22t} & 0 & \lambda_{it}\Phi_2^{gT} & 0 & 0 \\ * & * & -Q_{t-\tau} & 0 & 0 & 0 \\ * & * & * & -\lambda_{it}I & 0 & 0 \\ * & * & * & * & -\gamma_i^2 I & 0 \\ * & * & * & * & * & -\gamma_i^2 I \end{bmatrix} < 0 \quad (30)$$

is true. Denoting $\hat{K}_{it} = P_{t+1}K_i(t)$, the inequality (30) can transform into

$$\begin{bmatrix} -P_{t+1} & \mathscr{M}_{12t} & \mathscr{M}_{13t} & \mathscr{M}_{14t} & \mathscr{M}_{15t} & \mathscr{M}_{16t} \\ * & \mathscr{M}_{22t} & 0 & \lambda_{it}\Phi_2^{gT} & 0 & 0 \\ * & * & -Q_{t-\tau} & 0 & 0 & 0 \\ * & * & * & -\lambda_{it}I & 0 & 0 \\ * & * & * & * & -\gamma_i^2 I & 0 \\ * & * & * & * & * & -\gamma_i^2 I \end{bmatrix} < 0.$$

Therefore, it can be drawn that this inequality will be right, if there exists a matrix $\hat{K}_{it}$ such that the inequality (28) holds. This completes the proof.

Lemma 3 *Let parameters $\gamma_F > 0$ and weighted matrices $\bar{\mathscr{Q}}_3$ and $\bar{\mathscr{Q}}_4$ be given. The dynamics of FE (8) satisfies the desired H_∞ performance constraint (10) for all nonzero $w(t)$ and $\bar{v}(t)$, if there exist positive definite matrices $R_t > 0$, $T_t > 0$ and positive scalars $\lambda_{0t} > 0$ and matrices $\Omega_i(t) (i = 1, \cdots, L)$ such that the condition (12) and the following linear matrix inequality hold:*

$$\begin{bmatrix} \Lambda_{11t} & 0 & \Lambda_{13t} & 0 & 0 & 0 & 0 \\ * & -R_{t+1} & \Lambda_{23t} & \Lambda_{24t} & \Lambda_{25t} & \Lambda_{26t} & \Lambda_{27t} \\ * & * & \Lambda_{33t} & 0 & \lambda_{0t}\Phi_2^{gT} & 0 & 0 \\ * & * & * & -T_{t-\tau} & 0 & 0 & 0 \\ * & * & * & * & -\lambda_{0t}I & 0 & 0 \\ * & * & * & * & * & -\gamma_F^2 I & 0 \\ * & * & * & * & * & * & -\gamma_F^2 I \end{bmatrix} < 0 \quad (31)$$

where

$$\begin{cases} \Lambda_{11t} = -I, \ \Lambda_{13t} = [E_{1t} \ \cdots \ E_{2t} \ \cdots \ E_{3t} \ \bar{\Psi}_{0t}]^T \\ E_{1t} = \sqrt{\beta(1-\beta)}\tilde{\Psi}_{01t}^1, \ E_{2t} = \sqrt{\beta(1-\beta)}\tilde{\Psi}_{0it}^i \\ E_{3t} = \sqrt{\beta(1-\beta)}\tilde{\Psi}_{0Lt}^L, \ \Lambda_{33t} = T_t - R_t - \lambda_{0t}\Phi_1^g \\ \Lambda_{23t} = R_{t+1}\bar{A}_t, \ \Lambda_{24t} = R_{t+1}\bar{F}_t, \ \Lambda_{25t} = R_{t+1}\bar{B}_t, \\ \Lambda_{26t} = R_{t+1}\bar{\Gamma}_t, \ \Lambda_{27t} = R_{t+1}\bar{D}_t. \end{cases}$$

Table 1 the desired fusion estimation $\hat{x}(t)$

Algorithm 1	
Step 1.	Given γ_F and γ_i and a group of variables $\chi_{ij}(t)$ $(i = 1, \cdots, L$, and $j = 1, \cdots, n)$;
Step 2.	Calculate the estimator gains $K_i(t)$ $(i = 1, \cdots, L)$ such that the inequality (28) holds;
Step 3.	Calculate the weighted matrices $\Omega_i(t)$ $(i = 1, \cdots, L)$ such that the inequality (31) holds;
Step 4.	Estimate LSEs $\hat{x}_i(t)(i = 1, \cdots, L)$ by substituting (29) into (3);
Step 5.	Collect the received LSEs $\hat{x}_{r_i}(t)(i = 1, \cdots, L)$ by (5);
Step 6.	Calculate the fusion estimation $\hat{x}(t)$ by (6).

Proof Similar to Lemma 2, the inequality (14) is equivalent to

$$\begin{bmatrix} \mathscr{N}_t & 0 & \lambda_{0t}\Phi_2^{gT} & 0 & 0 \\ * & -T_{t-\tau} & 0 & 0 & 0 \\ * & * & -\lambda_{0t}I & 0 & 0 \\ * & * & * & -\gamma_F^2 I & 0 \\ * & * & * & * & -\gamma_F^2 I \end{bmatrix} + \Xi_{2t}^T P_{t+1} \Xi_{2t} < 0 \quad (32)$$

where $\Xi_{2t} = \begin{bmatrix} \bar{A}_t & \bar{F}_t & \bar{B}_t & \bar{\Gamma}_t & \bar{D}_t \end{bmatrix}$ and $\mathscr{N}_t = T_t - R_t - \lambda_{0t}\Phi_1^g + \chi_{0t}$.

Applying the Schur complement lemma again, the above inequality holds if and only if

$$\begin{bmatrix} -R_{t+1} & R_{t+1}\bar{A}_t & R_{t+1}\bar{F}_t & R_{t+1}\bar{B}_t & R_{t+1}\bar{\Gamma}_t & R_{t+1}\bar{D}_t \\ * & \mathscr{N}_t & 0 & \lambda_{0t}\Phi_2^{gT} & 0 & 0 \\ * & * & -T_{t-\tau} & 0 & 0 & 0 \\ * & * & * & -\lambda_{0t}I & 0 & 0 \\ * & * & * & * & -\gamma_F^2 I & 0 \\ * & * & * & * & * & -\gamma_F^2 I \end{bmatrix} < 0. \quad (33)$$

Noting that $\mathscr{N}_t$ includes χ_{0t}, we adopt the Schur complement lemma again to conclude that the above inequality is true when the inequality (31) holds, which completes the proof.

Theorem 2 *Let parameters $\gamma_F > 0$ and γ_i $(i = 1, 2, \cdots, L)$ and weighted matrices $\bar{\mathscr{Q}}_i$ $(i = 1, 2, 3, 4)$ be given. The dynamics of LEE (7) and the dynamics of FE (8) satisfy, respectively, the desired H_∞ performance constraint (9) and (10), if there exist positive definite matrices P_t, Q_t, R_t, T_t and positive scalars $\lambda_{it} (i = 0, 1, \cdots, L)$ and $\hat{K}_{it}$ $(i = 1, \cdots, L)$ and matrices $\Omega_i(t)$ $(i = 1, \cdots, L)$ such that the conditions (11) and (12) in Theorem 1 and the linear matrix inequalities (28) and (31) in Lemmas 2 and 3 hold.*

Up to now, based on Theorem 2, the desired fusion estimation $\hat{x}(t)$ can be calculated by Algorithm 1.

Remark 2 It is worth noting that a set of analytical conditions on the predetermined finite-horizon H_∞ performance for the dynamics of both LEEs (7) and FE (8) is established in Theorem 1 with the help of constructed Lyapunov functions. Obviously, these conditions in this theorem is nonlinear on the filter gains $K_i(t)$, fusion weighted matrices $\Omega_i(t)$, and free matrices P_t. In other words, these matrix inequalities cannot be solved via existing tools, and hence the desired matrices $K_i(t)$ and $\Omega_i(t)$ cannot be obtained reliably. To overcome this shortage, the nonlinear conditions are transformed into linear matrix inequalities by resorting to the element matrix transformation combined with the famous Schur complement lemma, which gives rise to Lemma 2 and Lemma 3. In light of these two lemmas, Theorem 2 definitely provides a set of applicable conditions to perform the gain and weight design.

Remark 3 Note that Theorem 2 and the corresponding algorithm disclose that the established design scheme is a two-step process. Specifically, the condition (31) reflected the local H_∞ performance is independent of the fusion weight matrices of sensor networks, and therefore can be independently solved to catch the desired local estimator gain of each node. Such a feature avoids the coupling of gain matrices and weight matrices. The developed fusion scheme has the capability of deal with time-delays and energy constraints, reducing energy cost when communicating with FC, and therefore satisfies the requirement of practical engineering. Furthermore, the computation cost of the developed design scheme is related with linear matrix inequalities (LMIs) (28) and (31), and decision variables P_t, Q_t, R_t, T_t, $\Omega_i(t)$, λ_{it} and $\hat{K}_{it}$. Note that the standard LMI system has a polynomial-time complexity $O(M\mathscr{N}^3)$, where M is the total row size of LMIs and $\mathscr{N}$ is the total number of scalar decision variables. Similar to the analysis in [40], the computational complexity of the developed design scheme in Theorem 2 can be represented as $O(Nn^7L^7)$.

Remark 4 In the past few years, some interesting fusion algorithms have been developed in [7,11–13] based on Kalman filtering techniques for linear networked systems and in [1, 9,14,32] based on LMIs for linear/nonlinear networked systems. In comparison with these existing results, the novelty of this paper lies in that a unified framework of fusion estimation is established to effectively handle the complexity coming from energy constraints based on channel scheduling, nonlinear terms, time-delays as well as time-varying parameters. All information about the above factors are reflected in the developed matrix inequalities to affect their solvability.

4 Numerical Examples

In this section, a numerical example of sensor networks is presented to verify the effectiveness of the proposed methods. Consider the discrete time-varying nonlinear system with parameters

$$A = 1.25 * \begin{bmatrix} 0.72+0.1\sin(t) & 0.1 & 0.2 & 0.2 \\ 0.2 & 0.35 & 0.8-0.1\cos(t) & -0.1 \\ 0.1 & 0 & 0.1 & -0.5 \\ -0.1 & 0 & 0.2 & 0.8 \end{bmatrix},$$

and

$$F = \begin{bmatrix} 0.1 & 0 & 0.3 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0.2 & 0.2 & 0 & 0.01 \\ 0.1 & 0 & 0 & 0.01 \end{bmatrix}, B = \begin{bmatrix} 0.1 & 0 & 0.2 & 0 \\ 0.1 & 0.1 & 0 & 0 \\ 0 & 0.2 & 0.1 & 0.1 \\ 0.1 & 0 & 0.1 & 0.1 \end{bmatrix},$$

$$\Gamma = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.1 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}.$$

The nonlinear vector-valued functions $g(x(t))$ is taken as

$$g(x(t)) = \begin{bmatrix} -0.4x_1(t)+0.3x_2(t)+\tanh(0.3x_1(t)) \\ -0.1x_2(t)+0.2x_3(t) \\ 0.1x_1(t)+0.2x_3(t) \\ 0.2x_1(t)+0.1x_2(t)+0.2x_4(t)-\tanh(0.1x_4(t)) \end{bmatrix}.$$

Then, it is easy to see that the condition (2) can be met with

$$\phi_1^g = \begin{bmatrix} -0.4 & 0.3 & 0 & 0 \\ 0 & -0.1 & 0.2 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ 0.2 & 0.1 & 0 & 0.1 \end{bmatrix}, \phi_2^g = \begin{bmatrix} -0.2 & 0.3 & 0 & 0 \\ 0 & -0.1 & 0.2 & 0 \\ 0.1 & 0 & 0.2 & 0 \\ 0.2 & 0.1 & 0 & 0.2 \end{bmatrix}.$$

There are three sink nodes, and suppose that each sink node has sufficient processing capability to compute the LSE $\hat{x}_i(t)$ in this example. The measurement matrices are

$$C_1 = \begin{bmatrix} 0.75 & 0 & 0 & 0.75 \\ 0.75 & 0 & 0.75 & 0 \\ 0 & 0.75 & 0 & 0.75 \end{bmatrix}, C_2 = \begin{bmatrix} 0.65 & 0.65 & 0 & 0 \\ 0 & 0 & 0.65 & 0.65 \\ 0 & 0.65 & 0 & 0.65 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 0.7 & 0.7 & 0 & 0 \\ 0.7 & 0 & 0.7 & 0 \\ 0.7 & 0 & 0 & 0.7 \end{bmatrix},$$

and the measured noise matrices are

$$D_1 = \begin{bmatrix} 0.1 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0.1 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0.1 \end{bmatrix}.$$

The disturbance inputs are chosen as

$$w(t) = 0.1 \begin{bmatrix} \exp(-0.1t)\sin(t) \\ \exp(-0.2t)\sin(t) \\ \exp(-0.1t)\cos(t) \\ \exp(-0.1t)\sin(t) \end{bmatrix}, v_i(t) = 0.1 \begin{bmatrix} \cos(0.2t)/t \\ \cos(0.2t)/t \\ \cos(0.2t)/t \\ \cos(0.2t)/t \end{bmatrix}.$$

Table 2 Estimator parameters.

t	1	2	3	...
$K_1(t)$	$\begin{bmatrix} 0.7266 & 0.5841 & -0.2888 \\ -0.9551 & 1.3186 & 0.7077 \\ -0.4607 & 0.4347 & -0.3180 \\ 0.3952 & -0.3261 & 0.6355 \end{bmatrix}$	$\begin{bmatrix} 0.6004 & 0.4736 & -0.1145 \\ -1.0054 & 1.3938 & 0.7148 \\ -0.4797 & 0.3484 & -0.2272 \\ 0.4628 & -0.1959 & 0.4188 \end{bmatrix}$	$\begin{bmatrix} 0.5324 & 0.4297 & -0.0726 \\ -0.9435 & 1.3341 & 0.6598 \\ -0.4908 & 0.3674 & -0.2201 \\ 0.4911 & -0.1987 & 0.3984 \end{bmatrix}$	...
$K_2(t)$	$\begin{bmatrix} 1.2090 & 0.5953 & -0.6608 \\ 0.9507 & 1.1842 & -0.8627 \\ 0.3688 & -0.0774 & -0.6718 \\ -0.4129 & 0.5661 & 0.6540 \end{bmatrix}$	$\begin{bmatrix} 1.0912 & 0.6169 & -0.6055 \\ 0.9548 & 1.1602 & -0.8731 \\ 0.3650 & -0.1232 & -0.6524 \\ -0.4176 & 0.5792 & 0.6425 \end{bmatrix}$	$\begin{bmatrix} 0.9573 & 0.5945 & -0.5311 \\ 0.9429 & 1.1360 & -0.8622 \\ 0.3551 & -0.1326 & -0.6178 \\ -0.3800 & 0.6007 & 0.6016 \end{bmatrix}$	...
$K_3(t)$	$\begin{bmatrix} 0.4156 & 0.4427 & 0.4918 \\ 0.0433 & 1.1624 & -0.5564 \\ 0.3550 & 0.3308 & -0.7909 \\ -0.7293 & -0.0858 & 1.0289 \end{bmatrix}$	$\begin{bmatrix} 0.2956 & 0.4312 & 0.4637 \\ 0.1846 & 1.1740 & -0.6159 \\ 0.2320 & 0.3570 & -0.7466 \\ -0.5100 & -0.1334 & 0.9546 \end{bmatrix}$	$\begin{bmatrix} 0.2337 & 0.3839 & 0.4327 \\ 0.2271 & 1.1301 & -0.6139 \\ 0.2415 & 0.3419 & -0.7341 \\ -0.4952 & -0.1305 & 0.9529 \end{bmatrix}$	...

Table 3 Weight matrix parameters.

t	1	2	3	...
$\Omega_1(t)$	$\begin{bmatrix} 0.3337 & 0 & 0 & 0 \\ 0 & 0.3339 & 0 & 0 \\ 0 & 0 & 0.3321 & 0 \\ 0 & 0 & 0 & 0.3346 \end{bmatrix}$	$\begin{bmatrix} 0.3330 & 0 & 0 & 0 \\ 0 & 0.3334 & 0 & 0 \\ 0 & 0 & 0.3346 & 0 \\ 0 & 0 & 0 & 0.3337 \end{bmatrix}$	$\begin{bmatrix} 0.3333 & 0 & 0 & 0 \\ 0 & 0.3335 & 0 & 0 \\ 0 & 0 & 0.3330 & 0 \\ 0 & 0 & 0 & 0.3348 \end{bmatrix}$	...
$\Omega_2(t)$	$\begin{bmatrix} 0.3335 & 0 & 0 & 0 \\ 0 & 0.3393 & 0 & 0 \\ 0 & 0 & 0.3373 & 0 \\ 0 & 0 & 0 & 0.3321 \end{bmatrix}$	$\begin{bmatrix} 0.3331 & 0 & 0 & 0 \\ 0 & 0.3340 & 0 & 0 \\ 0 & 0 & 0.3332 & 0 \\ 0 & 0 & 0 & 0.3336 \end{bmatrix}$	$\begin{bmatrix} 0.3335 & 0 & 0 & 0 \\ 0 & 0.3332 & 0 & 0 \\ 0 & 0 & 0.3346 & 0 \\ 0 & 0 & 0 & 0.3330 \end{bmatrix}$	...
$\Omega_3(t)$	$\begin{bmatrix} 0.3328 & 0 & 0 & 0 \\ 0 & 0.3268 & 0 & 0 \\ 0 & 0 & 0.3307 & 0 \\ 0 & 0 & 0 & 0.3332 \end{bmatrix}$	$\begin{bmatrix} 0.3338 & 0 & 0 & 0 \\ 0 & 0.3326 & 0 & 0 \\ 0 & 0 & 0.3322 & 0 \\ 0 & 0 & 0 & 0.3327 \end{bmatrix}$	$\begin{bmatrix} 0.3332 & 0 & 0 & 0 \\ 0 & 0.3333 & 0 & 0 \\ 0 & 0 & 0.3324 & 0 \\ 0 & 0 & 0 & 0.3321 \end{bmatrix}$	...

Owing to the energy constraint, only one element of the LSE $\hat{x}_i(t)$ is admitted to be transmitted to FC via a reliable channel and others are transmitted via a general channel with data missing probability $1-\beta = 0.1$. The selection matrices $H_i(t)$ $(i = 1,2,3,4)$ are taken as

$$H_i(t) = \mathrm{diag}\{\vartheta_{s1}, \vartheta_{s2}, \vartheta_{s3}\},\ s = \mathrm{mod}(t-i,4)+1$$

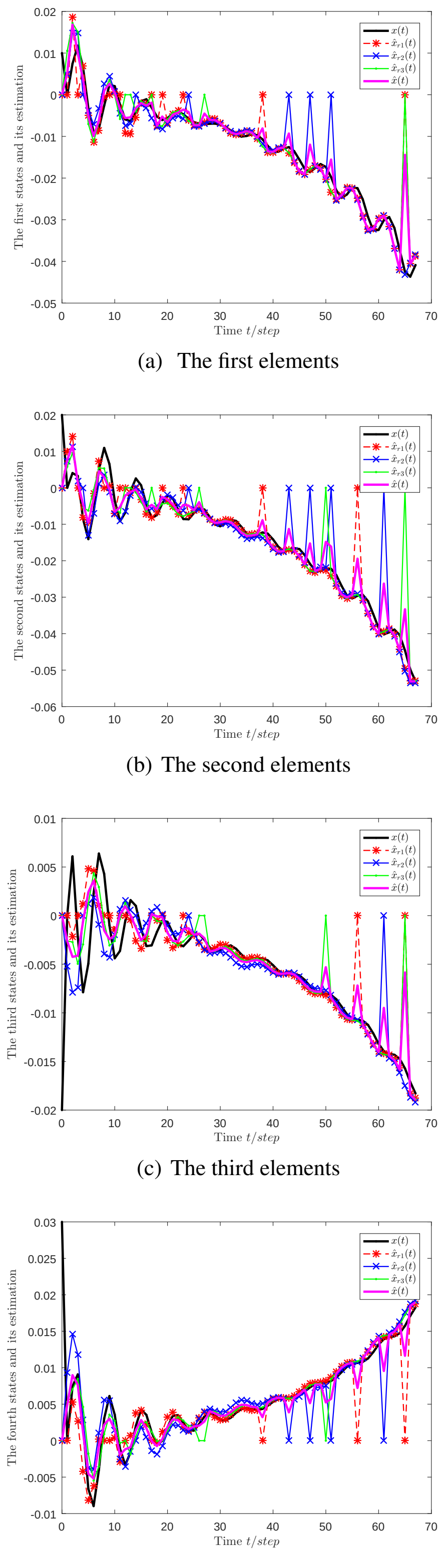
for any time instant t.

In the simulation, the delay parameter is taken as $\tau = 2$. When the disturbance attenuation levels are chosen as $\gamma_i = 5$ $(i = 1,2,3)$, using the LMI toolbox in MATLAB software (with the YALMIP 3.0, Version No. R20100813) to solve the linear matrix inequality (28), the estimator gain matrices $K_i(t)$ $(i = 1,\cdots,L)$ are shown in Table 2. When the disturbance attenuation level is chosen as $\gamma_F = 5$, using the LMI toolbox in MATLAB to solve the linear matrix inequality (31), the suboptimal weighting matrices $\Omega_i(t)$ $(i = 1,\cdots,L)$ are shown in Table 3.

The trajectories of state $x(t)$, received LSE $\hat{x}_{r_i}(t)$ and fusion estimation $\hat{x}(t)$ are plotted in Fig. 1 by resorting to the developed results in Theorem 1 and Theorem 2. It is not difficult to see from these four subgraphs that the state trajectories of the target dynamics can be well tracked in an ideal communication scenario, but emerge some fluctuations when general channels are subject to packet losses. Furthermore, fluctuation amplitudes of estimated states via local filters are obviously larger than that of fused states via the proposed fusion algorithm, which is further verified by Fig. 2 on estimation errors $e_{i,t}$ and fusion error $e_{F,t}$. For the convenience of comparison analysis, the fused trajectories via the obtained algorithm are further drawn in Fig. 4 for the cases with and without packet loss, and the instants of packet loss are plotted in Fig. 3. According to the above four figures, we can conclude that the performance of suboptimal fusion estimation is better than that of received LSE although they are degraded by the phenomenon of packet loss.

5 Conclusion

In this paper, the fusion estimation problem of sensor networks has been investigated for nonlinear time-varying systems with energy constraints, time-delays as well as packet loss. In this problem, because of the low energy cost in comparison with the case of reliable channels, general channels subject to packet loss have been employed to carry out the information transmission with the purpose of increasing the service life of the battery. It should be pointed out that the traditional fusion algorithm based on Kalman filtering techniques is difficult to effectively deal with the challenges brought by energy constraints, time-delays as well as nonlinear functions. Therefore, some local estimations are first obtained by using the designed Luenberger-type local estimator and then transmitted to a FC to obtain a desired fusion



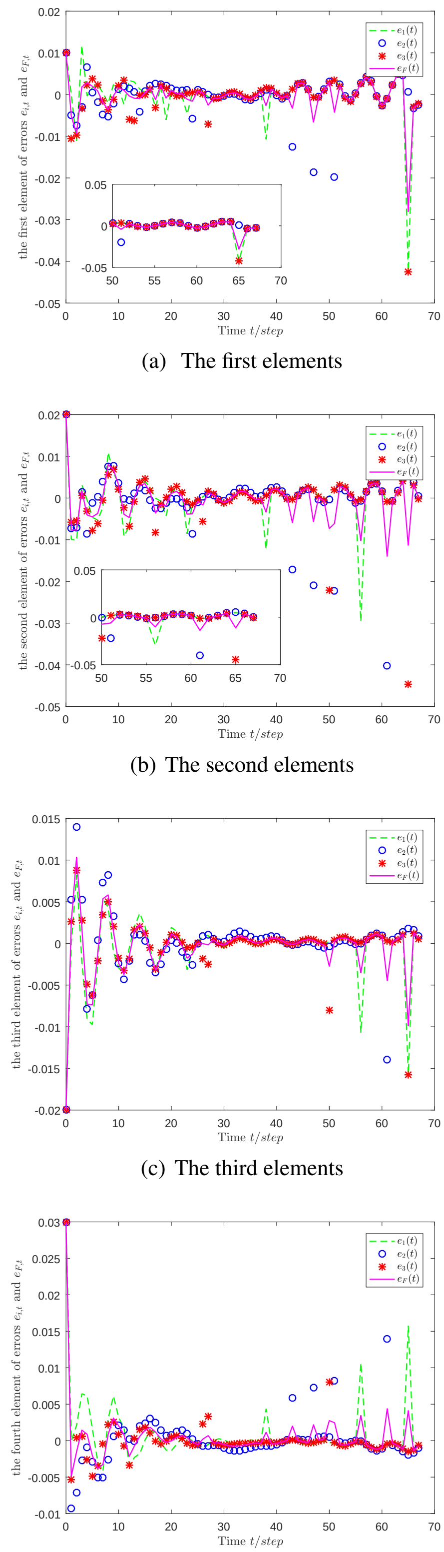
(a) The first elements

(b) The second elements

(c) The third elements

(d) The fourth elements

Fig. 1 Trajectories of state $x(t)$, LSE $\hat{x}_{r_i}(t)$ and fusion estimation $\hat{x}(t)$.



(a) The first elements

(b) The second elements

(c) The third elements

(d) The fourth elements

Fig. 2 Local estimation errors $e_{i,t}$ and fusion error $e_{F,t}$.

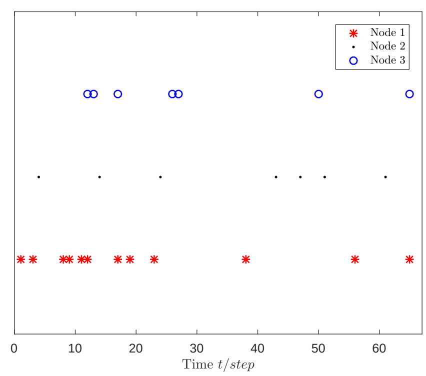
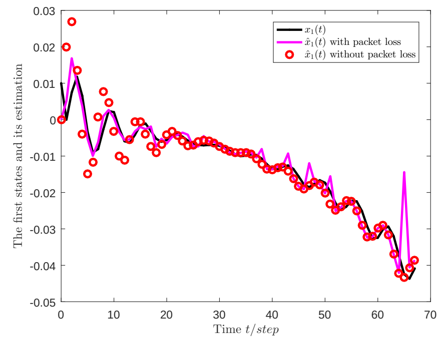
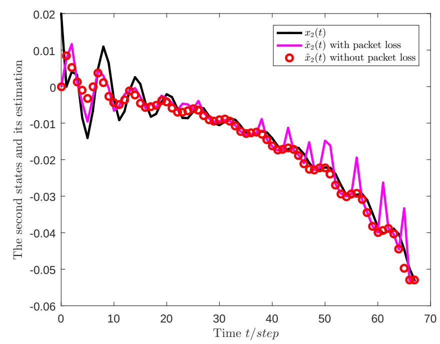


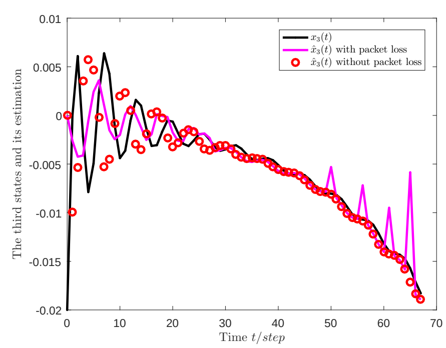
Fig. 3 Packet loss times.



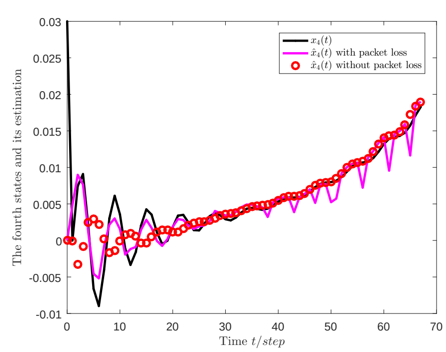
(a) The first elements



(b) The second elements



(c) The third elements



(d) The fourth elements

Fig. 4 Trajectories of state $x(t)$, fusion estimation $\hat{x}(t)$ with/without packet loss.

value. With the help of the Lyapunov stability theory, sufficient conditions are established to guarantee the predetermined local and fused H_∞ performances over a finite horizon. Furthermore, based on the established conditions, by means of the Schur complement lemma, the desired gains of local estimators and the suboptimal fusion weight matrices are obtained in light of the solution of linear matrix inequalities.

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Conflict of interest

The authors declare that they have no conflict of interest.

Availability of data and material

Not applicable.

Code availability

Not applicable.

Authors' contributions

Conceptualization: Derui Ding, Xiaojian Yi; Methodology: Guoliang Wei, Meiling Xie; Formal analysis and investigation: Meiling Xie, Xiaojian Yi; Writing - original draft preparation: Meiling Xie.

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Figures

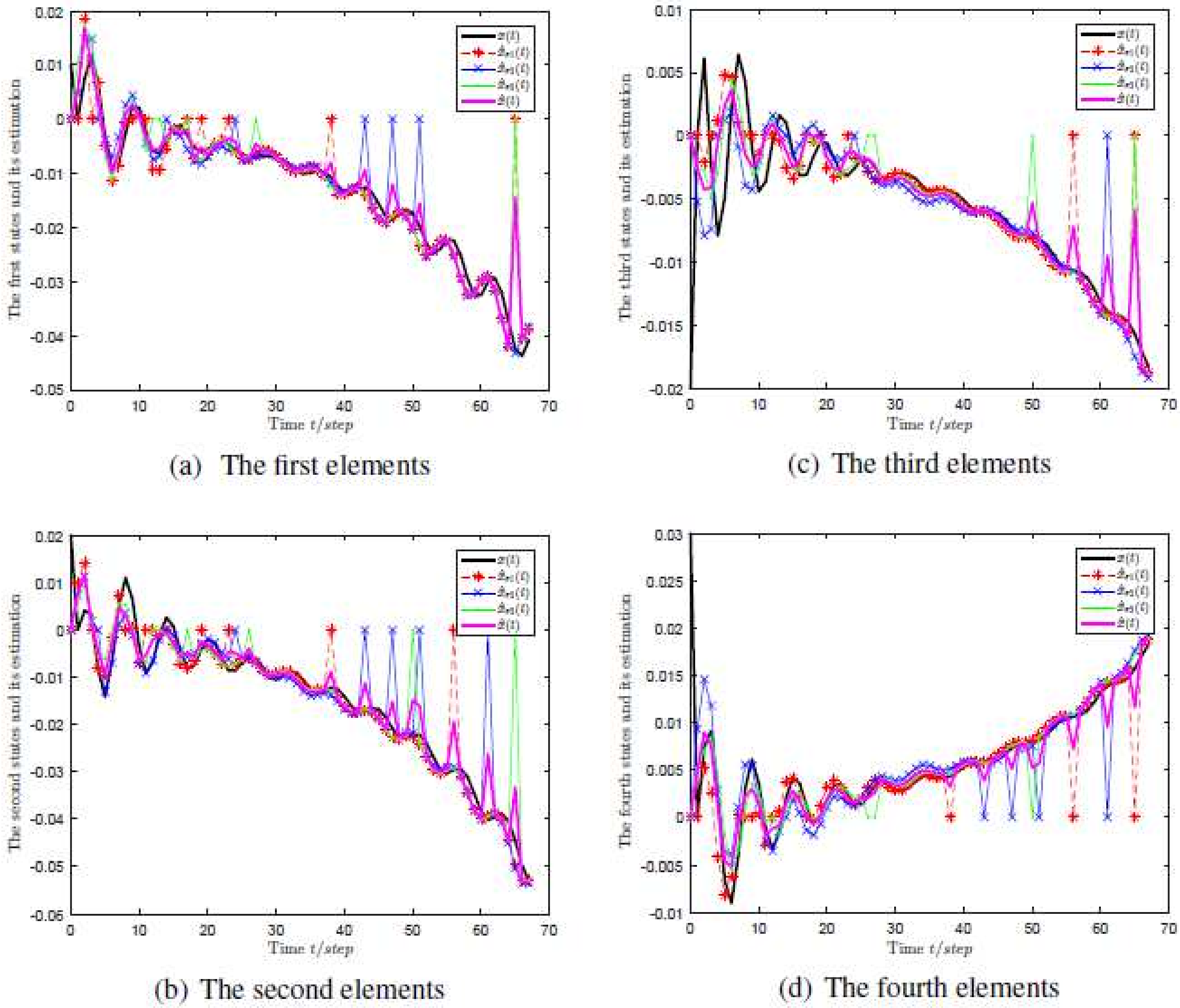

Figure 1

See the Manuscript Files section for the complete figure caption.

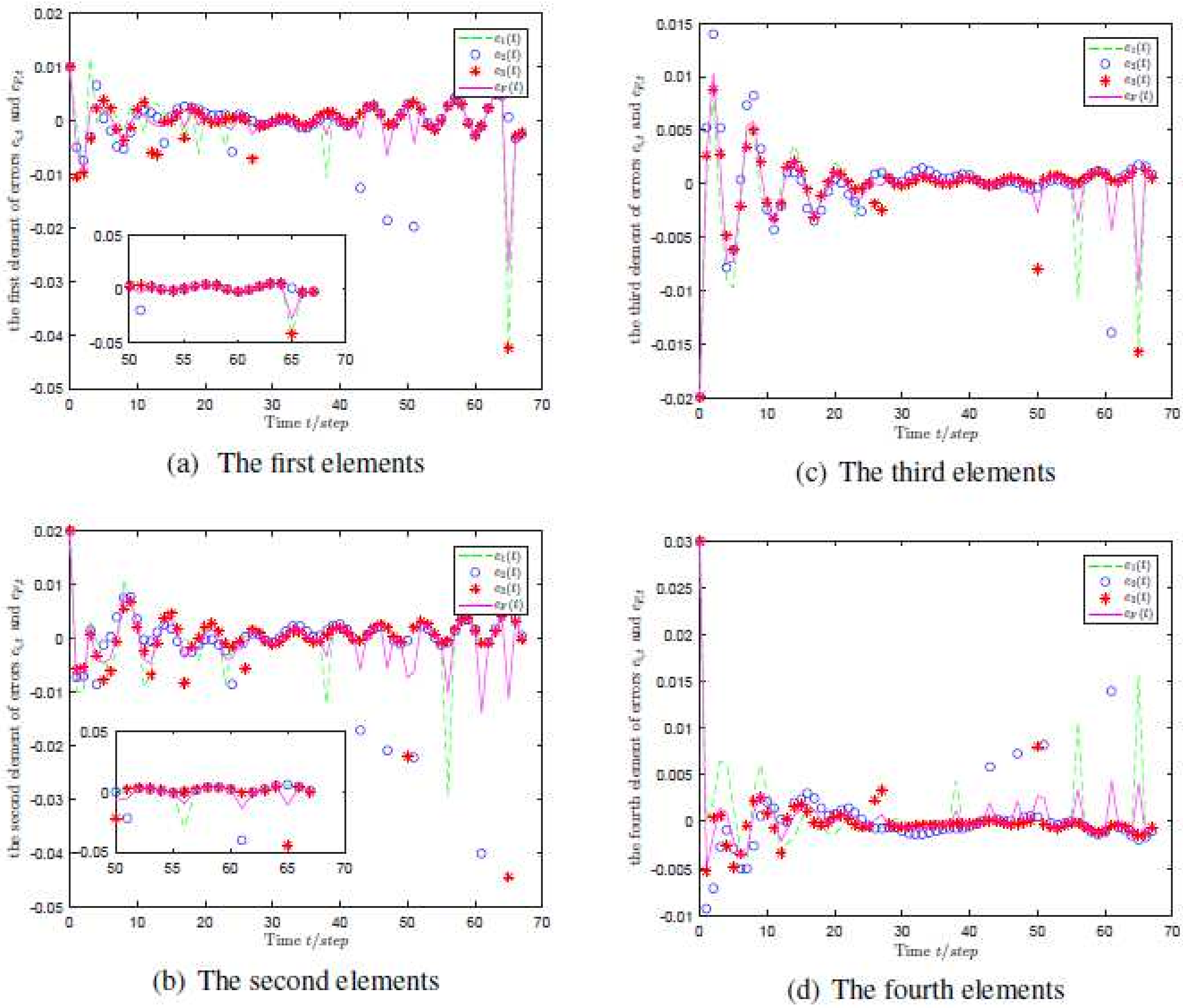

Figure 2

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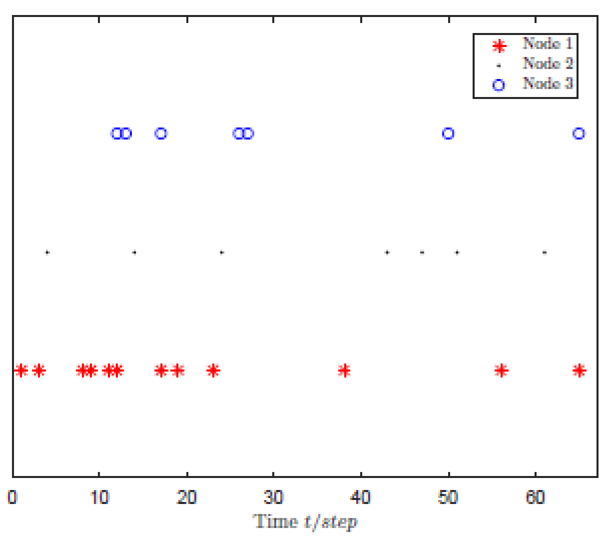
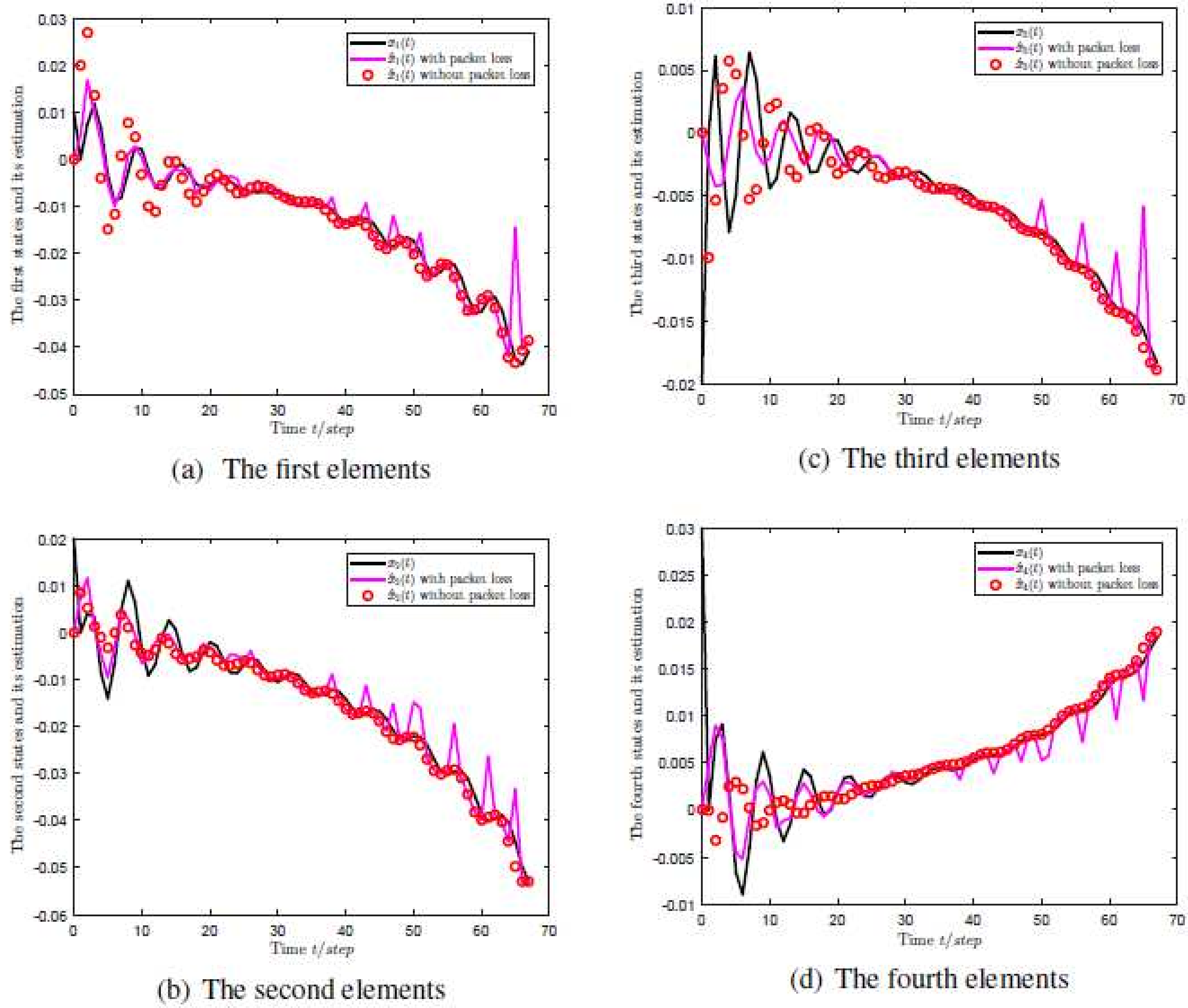


Figure 3

See the Manuscript Files section for the complete figure caption.



(a) The first elements

(c) The third elements

(b) The second elements

(d) The fourth elements

Figure 4

See the Manuscript Files section for the complete figure caption.