Multi-Soliton Solutions of the N-Component Nonlinear Schrodinger Equations via Riemann-Hilbert Approach*

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Multi-soliton solutions of the N-component nonlinear Schrödinger equations via Riemann-Hilbert approach *

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Abstract: In this paper, we utilize the Riemann-Hilbert approach to discuss multi-soliton solutions of the N-component nonlinear Schrödinger equations. Firstly, by transformed Lax pair, we construct the matrix valued functions $P_{1,2}$ that satisfy the analyticity and normalization and the corresponding jump matrix can be determined. Then, in the reflectionless case, we get the multi-soliton solutions $q_l(l = 1, \cdots, N)$ of the N-component nonlinear Schrödinger equations, which are related to spectral parameters η. Particularly, the 2-soliton solutions q_1, q_2 and q_3 of the three-component nonlinear Schrödinger equations are given and the corresponding 2-soliton diagrams are drawn.

Keywords: N-component NLS equations; Lax pair; Riemann-Hilbert approach; Multi-soliton solutions.

MSC codes: 35Q51; 35Q15; 37K10

1 Introduction

Riemann-Hilbert (RH) problem is the 21st question that Hilbert mentioned at the International Congress of Mathematicians in Paris [1], which means the boundary value problem of matrix valued functions on the complex plane. Then it has been developed into a powerful analytical tool to solve a large class of pure and applied mathematics, called Riemann-Hilbert approach, which can widely used in initial boundary value problem [2]- [8], asymptotic of orthogonal polynomials [9], Bäcklund transformation [10]- [11] and long-time asymptotics [12]- [14]. Afterwards, It was found that the RH method can be used to obtain the soliton solutions of integral equations by inverse scattering theory [15]- [22]. In recent years, Wazwaz AM solved multiple soliton solutions of the equations [23]- [26]. Then, RH approach is generalized to get multi-soliton solutions of multi-dimensional equations [27]- [31]. In this paper, we mainly discuss multi-soliton solutions of the N-component nonlinear Schrödinger (NLS) equations by RH approach.

The N-component NLS equations [32] take the form

$$iq_{lt} + \frac{1}{2}q_{lxx} + \sum_{l=1}^{N} |q_l|^2 q_l = 0, \quad l = 1, ..., N. \tag{1.1}$$

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where $q_l = q_l(t, x)(l = 1, ..., N)$ are complex functions and the subscripts mean the partial derivatives.

The process of this paper is given as follows. In section 2, we transform Lax pair to construct the RH problem for the N-component NLS equations. In section 3, the multi-soliton solutions for the N-component NLS equations are obtained, which are relevant to the spectral parameters. Then, the 2-soliton solutions of the three-component NLS equations are given and the corresponding 2-soliton graphs are drawn. In section 4, we give the conclusion.

2 Riemann-Hilbert problem

Based on the Eq. (1.1), we have the Lax pair

$$\begin{cases} \Phi_x + i\eta\sigma\Phi = iQ\Phi, \\ \Phi_t + i\eta^2\sigma\Phi = (i\eta Q + \frac{1}{2}(i\sigma Q^2 - \sigma Q_x)\Phi, \end{cases} \tag{2.1}$$

where

$$Q = \begin{pmatrix} 0 & q_1^* & q_2^* & \cdots & q_N^* \\ q_1 & 0 & 0 & \cdots & 0 \\ q_2 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ q_N & 0 & 0 & \cdots & 0 \end{pmatrix}, \sigma = \begin{pmatrix} -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{pmatrix}, \tag{2.2}$$

η is the spectral parameter. Then, we get the Jost solution of Lax pair (2.1) with asymptotic form by

$$\Phi \sim e^{-i\eta\sigma x - i\eta^2\sigma t}, \qquad |x| \to \infty. \tag{2.3}$$

In order to facilitate calculation, we define a matrix function $\Psi = \Psi(x, t; \eta)$. Let

$$\Phi = \Psi e^{-i\eta\sigma x - i\eta^2\sigma t}, \tag{2.4}$$

then

$$\Psi \to I, \qquad |x| \to \infty. \tag{2.5}$$

The Lax paix (2.1) can be rewritten as

$$\begin{cases} \Psi_x + i\eta[\sigma, \Psi] = U_1\Psi, \\ \Psi_t + i\eta^2[\sigma, \Psi] = U_2\Psi, \end{cases} \tag{2.6}$$

where $U_1 = iQ$, $U_2 = i\eta Q + \frac{1}{2}(i\sigma Q^2 - \sigma Q_x)$. Then the two Volterra integral equations can be expressed as

$$\begin{aligned} \Psi_1(t, x; \eta) &= I + \int_{-\infty}^{x} e^{-i\eta(x-x')\sigma} U_1 \Psi_1 e^{i\eta(x-x')\sigma} dx', \\ \Psi_2(t, x; \eta) &= I - \int_{x}^{+\infty} e^{-i\eta(x-x')\sigma} U_1 \Psi_2 e^{i\eta(x-x')\sigma} dx'. \end{aligned} \tag{2.7}$$

By calculation, we can know

$$e^{-i\eta(x-x')\sigma}U_1e^{i\eta(x-x')\sigma} = \begin{pmatrix} 0 & iq_1^*e^{2i\eta(x-x')} & \cdots & iq_N^*e^{2i\eta(x-x')} \\ iq_1e^{-2i\eta(x-x')} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ iq_Ne^{-2i\eta(x-x')} & 0 & \cdots & 0 \end{pmatrix}. \tag{2.8}$$

Let $\Psi_1 = ([\Psi_1]_1, [\Psi_1]_2, \ldots, [\Psi_1]_{N+1})$ and $\Psi_2 = ([\Psi_2]_1, [\Psi_2]_2, \cdots [\Psi_2]_{N+1}))$, it can be got that $[\Psi_1]_1$ is analytic in $\mathcal{C}^-$, $[\Psi_1]_2, \cdots, [\Psi_1]_{N+1}$ are analytic in $\mathcal{C}^+$. $[\Psi_2]_1$ is analytic in $\mathcal{C}^+$, $[\Psi_2]_2, \cdots, [\Psi_2]_{N+1}$ are analytic in $\mathcal{C}^-$. We can rewrite $\Psi_{1,2}$ as follows

$$\Psi_1 = ([\Psi_1]_1, [\Psi_1]_2, \cdots, [\Psi_1]_{N+1}) = (\Psi_1^-, \Psi_1^+, \cdots, \Psi_1^+), \tag{2.9}$$

$$\Psi_2 = ([\Psi_2]_1, [\Psi_2]_2, \cdots, [\Psi_2]_{N+1}) = (\Psi_2^+, \Psi_2^-, \cdots, \Psi_2^-). \tag{2.10}$$

Based on the properties of $\Psi_{1,2}$ and tr Q=0, we can know that $\det \Psi_{1,2}$ are independent for all x. By the asymptotic conditions $\Psi_{1,2} \to I$ at $|x| \to \infty$, we know

$$\det \Psi_{1,2} = 1. \tag{2.11}$$

Therefore, $\Psi_{1,2}$ are linearly related by a spectral matrix $S(\eta) = (s_{kj}(\eta))_{(N+1)\times(N+1)}$, which can be expressed as

$$\Psi_1 E = \Psi_2 E S(\eta), \qquad E = e^{-i\eta\sigma x}. \tag{2.12}$$

From above, it is obvious to get

$$\det S(\eta) = 1. \tag{2.13}$$

Taking the inverse of both sides of Eq. (2.12), we can obtain

$$\Psi_1^{-1} = ES(\eta)^{-1}E^{-1}\Psi_2^{-1}. \tag{2.14}$$

Applying Eq. (2.14) and the analytic properties of column vectors of $\Psi_{1,2}$, we can get the analytic properties of $\Psi_{1,2}^{-1}$, that is

$$\Psi_1^{-1} = \begin{pmatrix} (\Psi_1^{-1})^1 \\ (\Psi_1^{-1})^2 \\ \vdots \\ (\Psi_1^{-1})^{N+1} \end{pmatrix} = \begin{pmatrix} \hat{\Psi}_1^+ \\ \hat{\Psi}_1^- \\ \vdots \\ \hat{\Psi}_1^- \end{pmatrix}, \Psi_2^{-1} = \begin{pmatrix} (\Psi_2^{-1})^1 \\ (\Psi_2^{-1})^2 \\ \vdots \\ (\Psi_2^{-1})^{N+1} \end{pmatrix} = \begin{pmatrix} \hat{\Psi}_2^- \\ \hat{\Psi}_2^+ \\ \vdots \\ \hat{\Psi}_2^+ \end{pmatrix} \tag{2.15}$$

In order to construct a matrix RH problem, we should determine the two matrix functions

$$\begin{cases} P_1 = ([\Psi_2]_1, [\Psi_1]_2, \cdots, [\Psi_1]_{N+1}) = (\Psi_2^+, \Psi_1^+, \cdots, \Psi_1^+), \quad \eta \in \mathcal{C}^+, \\ P_2 = \begin{pmatrix} (\Psi_2^{-1})^1 \\ (\Psi_1^{-1})^2 \\ \vdots \\ (\Psi_1^{-1})^{N+1} \end{pmatrix} = \begin{pmatrix} \hat{\Psi}_2^- \\ \hat{\Psi}_1^- \\ \vdots \\ \hat{\Psi}_1^- \end{pmatrix}, \quad \eta \in \mathcal{C}^-. \end{cases} \tag{2.16}$$

Let

$$B_1=\begin{pmatrix}0&0&\cdots&0\\0&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1\end{pmatrix},B_2=\begin{pmatrix}1&0&\cdots&0\\0&0&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&0\end{pmatrix},\tag{2.17}$$

$P_{1,2}$ can be rewritten as

$$P_1=\Psi_1B_1+\Psi_2B_2=\Psi_1\begin{pmatrix}r_{11}&0&\cdots&0\\e^{-2i\eta x}r_{21}&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\e^{-2i\eta x}r_{N+1,1}&0&\cdots&1\end{pmatrix},\tag{2.18}$$

$$P_2=B_1\Psi_1^{-1}+B_2\Psi_2^{-1}=\begin{pmatrix}s_{11}&e^{2i\eta x}s_{12}&\cdots&e^{2i\eta x}s_{1,N+1}\\0&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&1\end{pmatrix}\Psi_1^{-1},\tag{2.19}$$

where $R(\eta)=S^{-1}(\eta)=(r_{kj}(\eta))_{(N+1)\times(N+1)}$. We can study the asymptotic expansion of P_1

$$P_1=P_1^{(0)}+\frac{P_1^{(1)}}{\eta}+\frac{P_1^{(2)}}{\eta^2}+O(\eta^{-3}).\tag{2.20}$$

Submitting (2.20) into the first equation of (2.6) and comparing the corresponding coefficients of η, we obtain

$$\begin{aligned}O(\eta^1):i[\sigma,P_1^{(0)}]=0,\\O(\eta^0):P_{1,x}^{(0)}+i[\sigma,P_1^{(1)}]=U_1P_1^{(0)}.\end{aligned}\tag{2.21}$$

We can get

$$P_1\to I,\quad \eta\in\mathcal{C}^+\to\infty,\tag{2.22}$$

in the same way,

$$P_2\to I,\quad \eta\in\mathcal{C}^-\to\infty.\tag{2.23}$$

By the above calculation, we can propose the RH problem of the N-component NLS equations

- $P_1(t,x;\eta)$ is analytic in $\mathcal{C}^+$, $P_2(t,x;\eta)$ is analytic in $\mathcal{C}^-$.
- $P_2(t,x;\eta)P_1(t,x;\eta)=G(t,x;\eta),\quad \eta\in\mathcal{R}$,
- $P_{1,2}(t,x;\eta)\to I,\quad as\ \eta\to\infty$,

where the jump matrix

$$G(t,x;\eta)=\begin{pmatrix}1&e^{2i\eta x}s_{12}&\cdots&e^{2i\eta x}s_{1,N+1}\\e^{-2i\eta x}r_{21}&1&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\e^{-2i\eta x}r_{N+1,1}&0&\cdots&1\end{pmatrix}.\tag{2.24}$$

3 Multi-soliton solutions of the N-component NLS equations

3.1 Multi-soliton solutions

In this section, we can get the multi-soliton solutions of Eq. (1.1). Firstly, we should derive the properties of the zeros of $\det P_{1,2}(\eta)$. On the basis of Eq. (2.18) and Eq. (2.19), we can get

$$\begin{aligned} \det P_1(\eta) &= r_{11}(\eta), \quad \eta \in \mathcal{C}^+, \\ \det P_2(\eta) &= s_{11}(\eta), \quad \eta \in \mathcal{C}^-, \end{aligned} \tag{3.1}$$

that is to say, the zeros of $\det P_1(\eta)$ is the zeros of $r_{11}(\eta)$ and the zeros of $\det P_2(\eta)$ is the zeros of $s_{11}(\eta)$. It is easy to know Q is Hermite matrix, that is

$$Q^\dagger = Q, \tag{3.2}$$

where † means conjugate transpose. Because of Eq. (2.18)-Eq. (2.19) and $\Psi_{1,2}^\dagger(\eta^*) = \Psi_{1,2}^{-1}(\eta)$, we can get the relationship holds

$$P_2^\dagger(\eta^*) = P_1(\eta). \tag{3.3}$$

Then, it follows from the scattering relation (2.12) that

$$S^\dagger(\eta^*) = S^{-1}(\eta), \quad s_{11}^*(\eta^*) = r_{11}(\eta). \tag{3.4}$$

The points $\eta_j (j = 1, 2, ..., N)$ are the zeros of $\det P_1(\eta)$ in $\mathcal{C}^+$, $\eta_j^* (j = 1, 2, ..., N)$ are the zeros of $\det P_2(\eta)$ in $\mathcal{C}^-$. Due to $\det P_1(\eta_j) = \det P_2^*(\eta_j^*)$, we suppose non-zero column vectors ν_j and non-zero row vectors ν_j^* are the solutions of the following linear equations, respectively,

$$\begin{aligned} P_1(\eta_j)\nu_j(\eta_j) &= 0, \\ \nu_j^*(\eta_j^*)P_2(\eta_j^*) &= 0. \end{aligned} \tag{3.5}$$

In fact, the scattering data for solving the RH problem (2.24) is composed of the discrete scattering data $\{\eta_j, \eta_j^*, \nu_j, \nu_j^*\}$ and the continuous scattering data $\{s_{21}, s_{31}, \cdots, s_{N+1,1}\}$. Because of Eq. (3.3)-Eq. (3.5), we get

$$\nu_j^* = \nu_j^\dagger. \tag{3.6}$$

Differentiate the Eq. (3.5) with respect to x and consider Lair pair (2.6) and Eq. (3.6), we obtain

$$\begin{cases} \nu_j = e^{-i\eta_j\sigma x - i\eta_j^2\sigma t}\nu_{j,0}, \\ \nu_j^* = \nu_{j,0}^\dagger e^{i\eta_j^*\sigma x + i\eta_j^{*2}\sigma t}. \end{cases} \tag{3.7}$$

where ν_{j0} are the (N+1)-dimensional constant column vectors, then we can get multi-soliton solutions for Eq. (1.1) with the reflectionless case. We can define the matrix $M = (M_{kj})_{(N+1)\times(N+1)}$

$$M_{kj} = \frac{\nu_k^*\nu_j}{\eta_j - \eta_k^*}. \tag{3.8}$$

Based on the canonical normalization condition (2.16), the RH problem has the unique solution

$$\begin{aligned} P_1(\eta) &= I - \sum_{k,j=1}^{N} \frac{\nu_k \nu_j^* (M^{-1})_{kj}}{\eta - \eta_j^*}, \\ P_2(\eta) &= I + \sum_{k,j=1}^{N} \frac{\nu_k \nu_j^* (M^{-1})_{kj}}{\eta - \eta_k}. \end{aligned} \tag{3.9}$$

Then, we can expand P_1 as follows

$$P_1 = P_1^{(0)} + \frac{P_1^{(1)}}{\eta} + \frac{P_1^{(2)}}{\eta^2} + O(\eta^{-3}). \tag{3.10}$$

Putting the asymptotic expansion (3.10) into Eq. (2.6), we have

$$i[\sigma, P_1^{(1)}] = iQ. \tag{3.11}$$

Hence, the potential functions $q_l (l = 1, 2, \cdots, N)$ can be expressed as

$$q_l = 2(P_1^{(1)})_{l+1,1}, \quad l = 1, 2, \cdots, N, \tag{3.12}$$

where $(P_1^{(1)})_{l+1,1} (l = 1, 2, \cdots, N)$ are the $(l+1,1)$ entry of matrix $P_1^{(1)}$, which can be got by Eq. (3.9), that is

$$P_1^{(1)} = - \sum_{k,j=1}^{N} \nu_k \nu_j^* (M^{-1})_{kj}. \tag{3.13}$$

Substituting Eq. (3.7) into Eq. (3.13), a general multi-soliton solutions for the N-component NLS equations can be shown as

$$q_l = -2 \sum_{k,j=1}^{N} \nu_{k,l+1} \nu_{j,1}^* (M^{-1})_{kj}, \quad l = 1, 2, \cdots, N, \tag{3.14}$$

where $\nu_k = (\nu_{k1}, \nu_{k2}, \cdots, \nu_{k,N+1})^T$ and $\nu_j^* = (\nu_{j1}^*, \nu_{j2}^*, \cdots, \nu_{j,N+1}^*) (k, j = 1, 2, \cdots, N)$ are defined by Eq. (3.7).

3.2 2-soliton solutions

Particularly, we can study the case of $N = 3$, the three-component NLS equations can be written as

$$iq_{lt} + \frac{1}{2} q_{lxx} + \sum_{l=1}^{3} |q_l|^2 q_l = 0, \quad l = 1, 2, 3. \tag{3.15}$$

We can obtain the soliton solutions for this particular case. Assume $\nu_{1,0} = (\alpha_1, \beta_1, \gamma_1, \epsilon_1)^T$, $\nu_{2,0} = (\alpha_2, \beta_2, \gamma_2, \epsilon_2)^T$, and let $\xi_1 = -i\eta_1 x - i\eta_1^2 t$, $\xi_2 = -i\eta_2 x - i\eta_2^2 t$, $\eta_1 = a_1 + ib_1$, $\eta_2 = a_2 + ib_2$. we can get 2-soliton solutions as follows

$$q_1 = \frac{-2(\beta_1 \alpha_1^* e^{\xi_1 - \xi_1^*} m_{22} - \beta_1 \alpha_2^* e^{\xi_1 - \xi_2^*} m_{12} - \beta_2 \alpha_1^* e^{\xi_2 - \xi_1^*} m_{21} + \beta_2 \alpha_2^* e^{\xi_2 - \xi_2^*} m_{11})}{m_{11} m_{22} - m_{12} m_{21}} \tag{3.16}$$

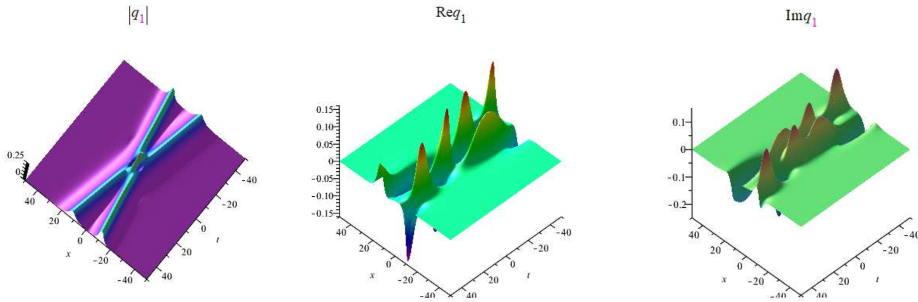

Figure 1: Evolution plot of the 2-soliton $|q_1|$, $\mathrm{Re}q_1$ and $\mathrm{Im}q_1$.

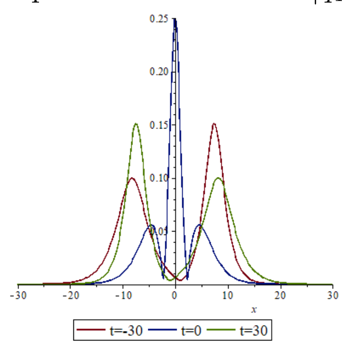

Figure 2: The soliton along the x axis with different t in panel.

$$q_2 = \frac{-2(\gamma_1\alpha_1^* e^{\xi_1-\xi_1^*} m_{22} - \gamma_1\alpha_2^* e^{\xi_1-\xi_2^*} m_{12} - \gamma_2\alpha_1^* e^{\xi_2-\xi_1^*} m_{21} + \gamma_2\alpha_2^* e^{\xi_2-\xi_2^*} m_{11})}{m_{11}m_{22} - m_{12}m_{21}} \tag{3.17}$$

$$q_3 = \frac{-2(\epsilon_1\alpha_1^* e^{\xi_1-\xi_1^*} m_{22} - \epsilon_1\alpha_2^* e^{\xi_1-\xi_2^*} m_{12} - \epsilon_2\alpha_1^* e^{\xi_2-\xi_1^*} m_{21} + \epsilon_2\alpha_2^* e^{\xi_2-\xi_2^*} m_{11})}{m_{11}m_{22} - m_{12}m_{21}} \tag{3.18}$$

where

$$m_{11} = \frac{\alpha_1^*\alpha_1 e^{-\xi_1^*-\xi_1} + \beta_1^*\beta_1 e^{\xi_1^*+\xi_1} + \gamma_1^*\gamma_1 e^{\xi_1^*+\xi_1} + \epsilon_1^*\epsilon_1 e^{\xi_1^*+\xi_1}}{\eta_1 - \eta_1^*}, \tag{3.19}$$

$$m_{12} = \frac{\alpha_1^*\alpha_2 e^{-\xi_1^*-\xi_2} + \beta_1^*\beta_2 e^{\xi_1^*+\xi_2} + \gamma_1^*\gamma_2 e^{\xi_1^*+\xi_2} + \epsilon_1^*\epsilon_2 e^{\xi_1^*+\xi_2}}{\eta_2 - \eta_1^*}, \tag{3.20}$$

$$m_{21} = \frac{\alpha_2^*\alpha_1 e^{-\xi_2^*-\xi_1} + \beta_2^*\beta_1 e^{\xi_2^*+\xi_1} + \gamma_2^*\gamma_1 e^{\xi_2^*+\xi_1} + \epsilon_2^*\epsilon_1 e^{\xi_2^*+\xi_1}}{\eta_1 - \eta_2^*}, \tag{3.21}$$

$$m_{22} = \frac{\alpha_2^*\alpha_2 e^{-\xi_2^*-\xi_2} + \beta_2^*\beta_2 e^{\xi_2^*+\xi_2} + \gamma_2^*\gamma_2 e^{\xi_2^*+\xi_2} + \epsilon_2^*\epsilon_2 e^{\xi_2^*+\xi_2}}{\eta_2 - \eta_2^*}, \tag{3.22}$$

By selecting appropriate values for the parameters as $\alpha_1 = \alpha_2 = 1$, $\beta_1 = \beta_2 = \frac{1}{4}$, $\gamma_1 = \gamma_2 = \frac{\sqrt{59}}{8}$, $\epsilon_1 = \epsilon_2 = \frac{1}{8}$, $a_1 = -0.1$, $b_1 = 0.2$, $a_2 = 0.2$, $b_2 = 0.3$, the three-dimensional plots and x-curves of solutions are shown in Fig 1-Fig 6.

4 Conclusion

In general, we investigate the multi-soliton solutions of the N-component NLS equations. By the Volterra equations, the corresponding analytic properties can be got. Then, we define P_1

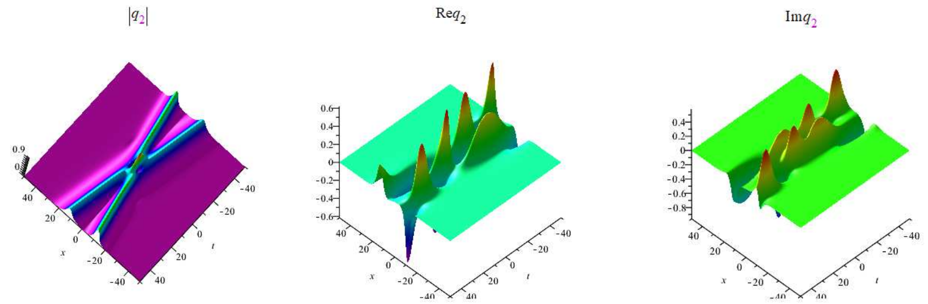

Figure 3: Evolution plot of the 2-soliton $|q_2|$, Req_2 and Imq_2.

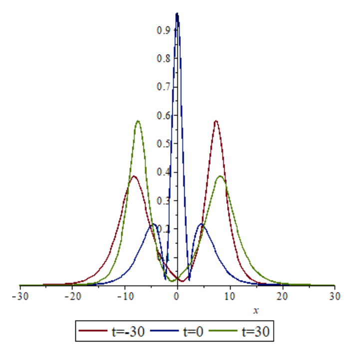

Figure 4: The soliton along the x axis with different t in panel.

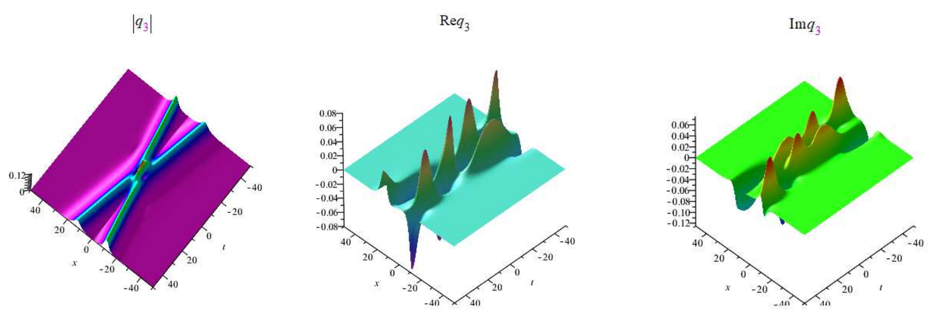

Figure 5: Evolution plot of the 2-soliton $|q_3|$, Req_3 and Imq_3.

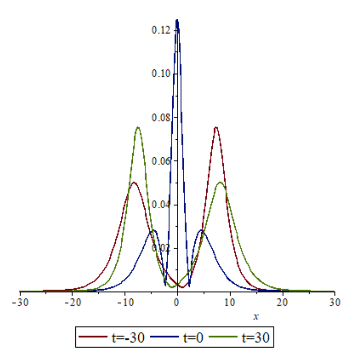

Figure 6: The soliton along the x axis with different t in panel.

and P_2 to construct the RH problem. In reflectionless case, we expand P_1 and submit it into the equation to obtain multi-soliton solutions. Can we use this method to obtain the multi-soliton solutions for other meaningful equations and which kinds of equations can be solved by this method. These questions that we can continue studying in the future.

References

[1] Hilbert D. Mathematics problem. Gott.Nachr. (1990) 253-297.

[2] Bikbaev RF. Soliton generation for initial-boundary-value problems. Phys.Rev.Lett. 68 (1992) 3117-3120.

[3] Jenkins R, Liu J, Perry P and Sulem C. Soliton resolution for the derivative nonlinear Schrödinger equation. Commun.Math.Phys. 363 (2018) 1003-1049.

[4] Xu J and Fan EG. The three-wave equation on the half-line. Phy.Lett.A. 378 (2014) 26-33.

[5] Hu BB, Zhang L and Zhang N. On the Riemann-Hilbert problem for the mixed Chen-Lee-Liu derivative nonlinear Schrödinger equation. J.Comp.Appl.Math. 390 (2021) 113393.

[6] Hu BB, Zhang L and Xia TC. On the Riemann-Hilbert problem of a generalized derivative nonlinear Schrödinger equation. Commun.Theor. Phys. 73 (2021) 015002.

[7] Monvel A, Shepelsky BDC. Riemann-Hilbert approach for the Camassa-Holm equation on the line. CR.Math. 343 (2006) 627-632.

[8] Hu BB, Zhang L, Xia TC and Zhang N. On the Riemann-Hilbert problem of the Kundu equation. Appl.Math.Comput. 381 (2020) 125262.

[9] Wong R. Asymptotics of orthogonal polynomials via the Riemann-Hilbert approach. Acta.Math.Sci. 29 (2009) 1005-1034.

[10] Liu LC, Tian B and Qin B. Bäcklund transformation, superposition formulae and N-soliton solutions for the perturbed Korteweg-de Vries equation. Commun.Nonlinear Sci. 17 (2012) 2394-2402.

[11] Zhang Y and Chen DY. Bäcklund transformation and soliton solutions for the shallow water waves equation. Chaos Soliton.Fract. 20 (2004) 343-351.

[12] Wang DS and Wang XL. Long-time asymptotics and the bright N-soliton solutions of the Kundu-Eckhaus equation via the Riemann-Hilbert approach. Nonlinear Anal-Real. 41 (2018) 334-361.

[13] Geng XG, Wang KD and Chen MM. Long-time asymptotics for the Spin-1 Gross-Pitaevskii equation. Commun.Math.Phys. (2021) 1-27.

[14] Cheng QY and Fan EG. Long-time asymptotics for a mixed nonlinear Schrödinger equation with the Schwartz initial data. J.Math.Anal.Appl. 489 (2020) 124188.

[15] Geng XG and Wu JP. Riemann-Hilbert approach and N-soliton solutions for a generalized Sasa-Satsuma equation. Wave Motion. 60 (2016) 62-72.

[16] Ma WX. The inverse scattering transform and soliton solutions of a combined modified Korteweg-de Vries equation. J.Math.Anal.Appl. 471 (2019) 796-811.

[17] Chen XT, Zhang Y, Liang JL and Wang R. The N-soliton solutions for the matrix modified Korteweg-de Vries equation via the Riemann-Hilbert approach. Eur.Phys.J.Plus. 135 (2020) 574.

[18] Wu JP. Integrability aspects and multi-soliton solutions of a new coupled Gerdjikov-Ivanov derivative nonlinear Schrödinger equation. Nonlinear Dynam. 96 (2019) 789-800.

[19] Xu T and Chen Y. Riemann-Hilbert approach of the coupled nonisospectral Gross-Pitaevskii system and its multi-component generalization. Commun.Nonlinear Sci. 57 (2017) 276-289.

[20] Li J and Xia TC. N-soliton solutions for the nonlocal Fokas-Lenells equation via RHP. Appl.Math.Lett. (2020) 113.

[21] Ma WX. Riemann-Hilbert problems and N N mathContainer Loading Mathjax-soliton solutions for a coupled mKdV system. J.Geome.Phys. 132 (2018) 45-54.

[22] Wen L, Zhang N and Fan EG. N-soliton solution of the Kundu-type equation via Riemann-Hilbert approach. Acta.Math.Sci. 40 (2020) 113-126.

[23] Wazwaz AM. The extended tanh method for new solitons solutions for many forms of the fifth-order KdV equations. Appl.Math.Comput. 184 (2007) 1002-1014.

[24] Wazwaz AM. Multiple soliton solutions and multiple singular soliton solutions of the modified KdV equation with first-order correction. Phys.Scripta. 82 (2010) 055006.

[25] Wazwaz AM. Multiple soliton solutions and multiple complex soliton solutions for two distinct Boussinesq equations. Nonlinear Dynam. 85 (2016) 731-737.

[26] Wazwaz AM. Two-mode fifth-order KdV equations: necessary conditions for multiple-soliton solutions to exist. Nonlinear Dynam. 87 (2017) 1685-1691.

[27] Kang ZZ, Xia TC and Ma WX. Riemann-Hilbert approach and N-soliton solution for an eighth-order nonlinear Schrödinger equation in an optical fiber. Adv.Differ.Equ-NY. (2019) 188.

[28] Yang JJ, Tian SF, Peng WQ and Zhang TT. The N-coupled higher-order nonlinear Schrödinger equation: Riemann-Hilbert problem and multi-soliton solutions. Math.Meth.Appl.Sci. (2019) 1-15.

[29] Li PR. The solvability and explicit solutions of singular integral-differential equations of non-normal type via Riemann-Hilbert problem. J.Comput.Appl.Math. 374 (2020) 112759.

[30] Li J and Xia TC. A Riemann-Hilbert approach to the Kundu-nonlinear Schrödinger equaction and its multi-component generalization. J.Math.Anal.Appl. 500 (2021) 125109.

[31] Wei HY, Fan EG and Guo HD. Riemann-Hilbert approach and nonlinear dynamics of the coupled higher-order nonlinear Schrödinger equation in the birefringent or two-mode fiber. Nonlinear Dynam. (2021) 1-12.

[32] Ling LM, Zhao LC and Guo BL. Darboux transformation and multi-dark soliton for N-component nonlinear Schrödinger equations. Nonlinearity. 28 (2015) 3243-3261.

Figures

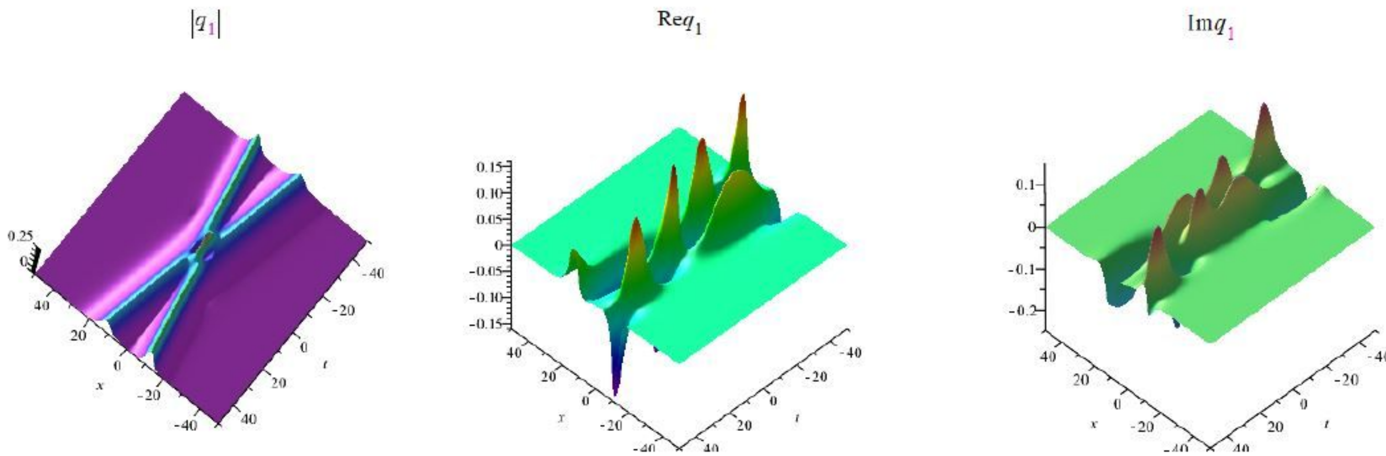

Figure 1

Evolution plot of the 2-soliton |q1|, Req1 and Imq1.

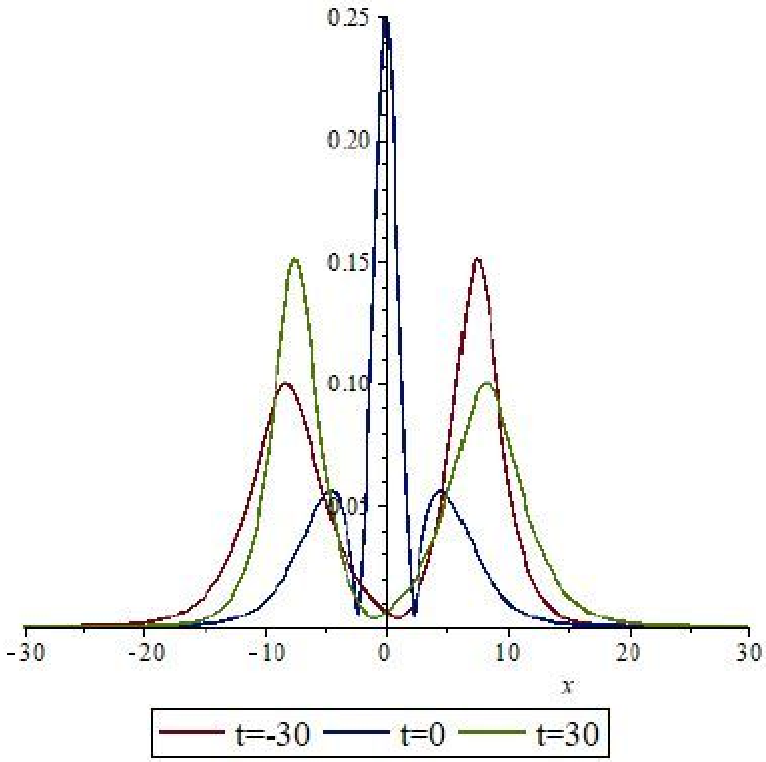

Figure 2

The soliton along the x axis with different t in panel.

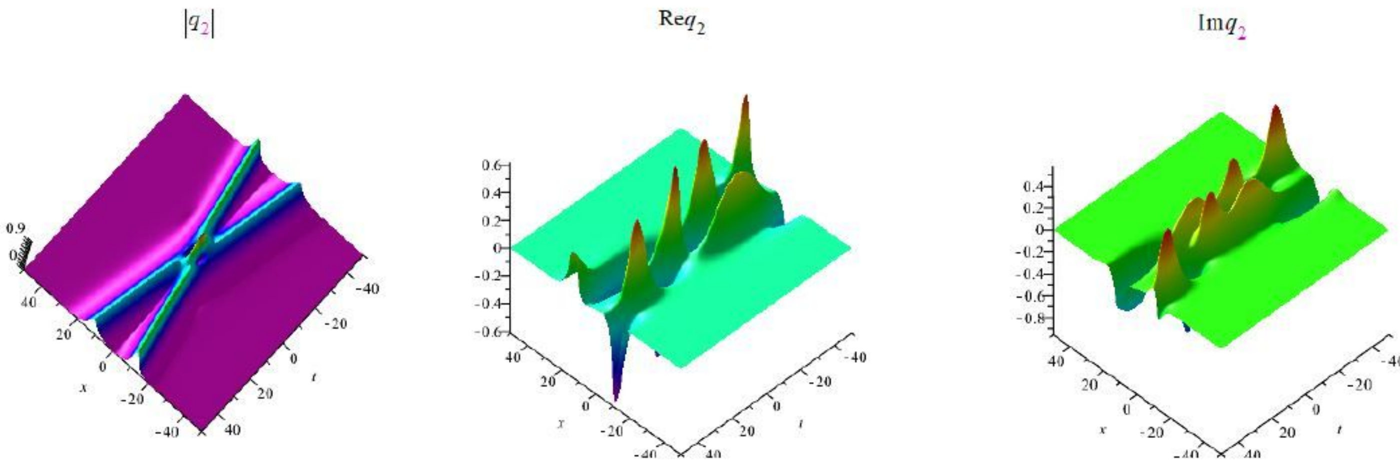

Figure 3

Evolution plot of the 2-soliton |q2|, Req2 and Imq2.

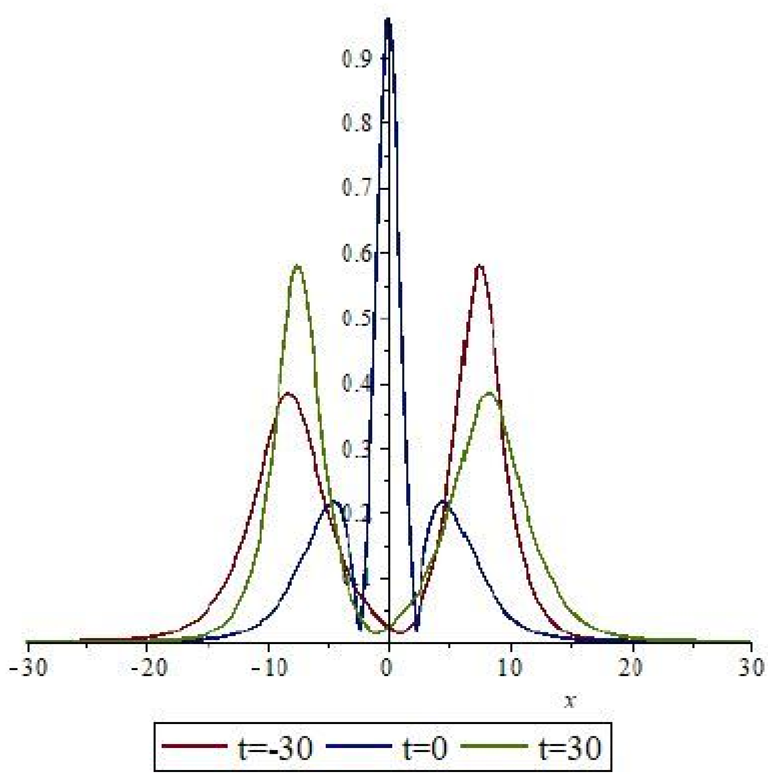

Figure 4

The soliton along the x axis with different t in panel.

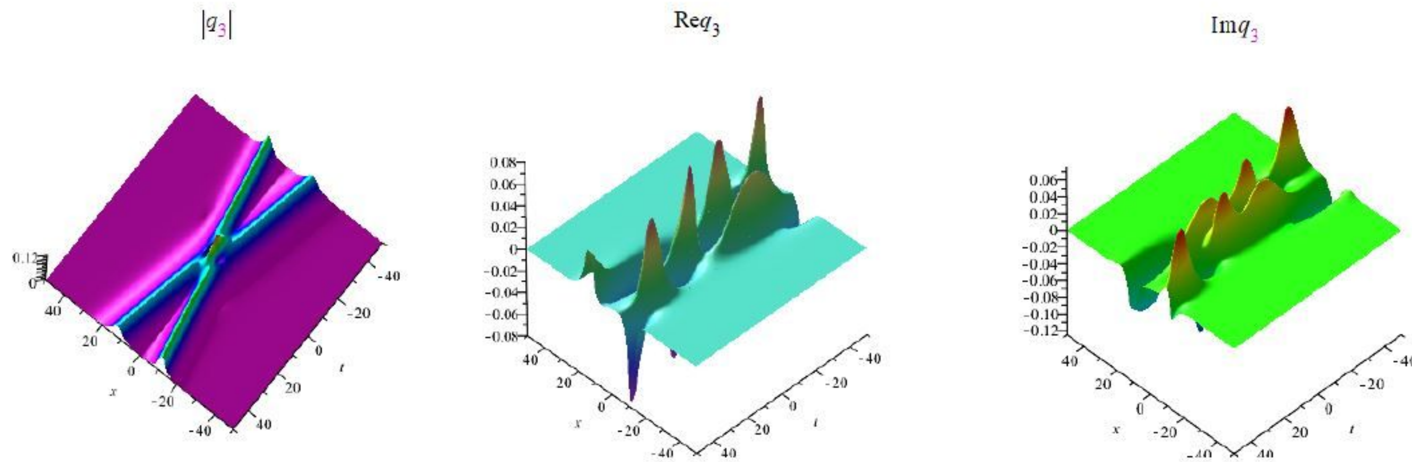

Figure 5

Evolution plot of the 2-soliton |q3|, Req3 and Imq3.

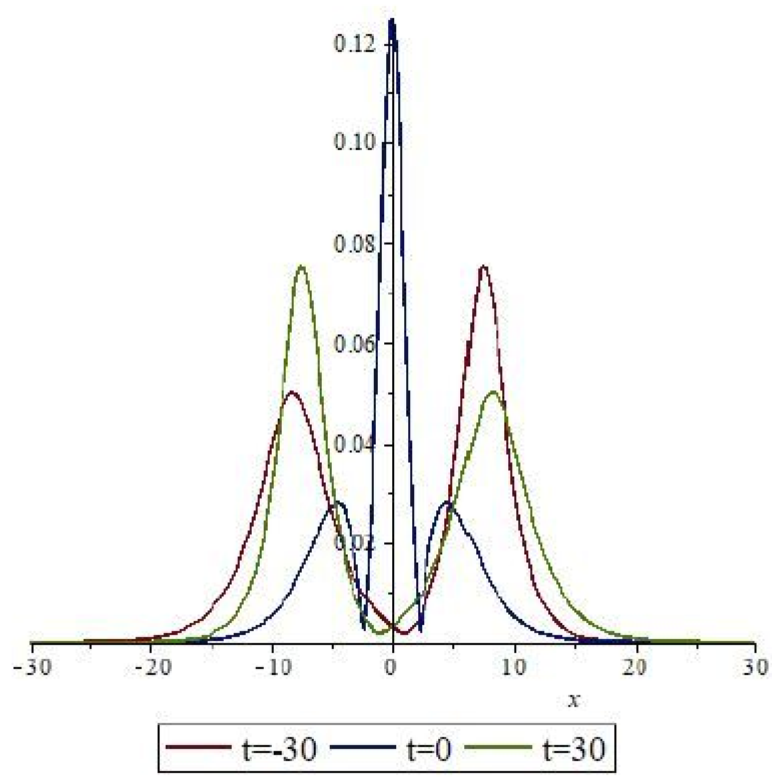

Figure 6

The soliton along the x axis with different t in panel.