Modeling of spreading of the novel coronavirus based in the stochastic dynamic

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Modeling of spreading of the novel coronavirus based in the stochastic dynamic

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In this paper, one proposes a stochastic model based on Itô diffusion as mathematical model for time evolution of novel cases $N(t)$ of the SARS-CoV-2 (COVID-19) in each day t. I propose a correspondent stochastic differential equation (SDE) analogous to classical differential equation for epidemic growing for some diseases as smallpox and typhoid fever. Furthermore, we made an analysis using the Fokker-Planck equation giving an estimating of the novel cases in the day t as the mean half-width of the distribution $P(N,t)$ of novel cases. My results display that the model based on Itô diffusion fits well to the results supported by healthy Brazilian agencies due to large uncertainly in the official data generated by the low number of tests realized generating so a strong randomness in the official data.

I. INTRODUCTION

The current outbreak of coronavirus disease (COVID-19) has become a global crisis due to its quick and wide spread over the world. An understanding of the dynamics of the disease or forecasting its spread of it would greatly enhance the control and prevention.[1–9]

On the other hand, the use of models and mathematical methods for theoretical physicists to study the spread of contagious diseases goes back a least to some works by Daniel Bernoulli in XVIII century on smallpox,[14] where in nowadays, many mathematical models have been proposed and studied for many different diseases.[10–13]

Some diseases as the typhoid fever and also the COVID-19 are spread largely by carriers or individuals who can transmit the disease but who exhibit no overt symptoms. Let x and y denote the proportions of susceptible and carriers, respectively, in the population, and suppose that carriers are identified and removed from the population at a rate α, so that $dy/dt = -\alpha y$. Suppose, besides, that the disease spreads at a rate proportional to the product of x and y, thus $dx/dt = -\gamma xy$. We can determine by solution of the set of equations y at any time t subject to initial condition $y(0) = y_0$ and find the proportion of the population that escapes of the epidemic by finding the limiting value of x as $t \to \infty$ from the initial condition $x(0) = x_0$. For diseases as the smallpox that, once contracted and survived, confers a lifetime immunity, considering the cohort of individuals born in a given year and let $n(t)$ be the number of these individuals surviving a time t later. Let $x(t)$ be the number of members of this cohort who have not smallpox by year t and who are therefore still susceptible. Let α be the rate at which susceptible contract smallpox, and let ν be the rate at which people who contract smallpox die from the disease. Finally, let $\mu(t)$ be the death rate from all cases other than smallpox. Then dx/dt, the rate at which the number of susceptible declines, is given by $dx/dt = -\left[\alpha + \mu(t)\right] x$, where the first term of the right side is the rate of the susceptible to the smallpox, and the second term is the rate at which they die from due to other causes. Furthermore, $dn/dt = -\nu\alpha x - \mu(t)n$, where dn/dt is the death rate, and the two terms on the right side are the death rates due to all other causes, respectively. We make $z = x/n$ and obtain that z satisfies the equation $dz/dt = -\alpha z(1 - \nu z)$, $z(0) = 1$ that does not depend on $\mu(t)$. Bernoulli estimated that $\nu = \alpha = 1/8$. Using these values, we can determine the proportion of individuals who have not had smallpox yet.

If we allow for some randomness in some coefficients of the differential equation, we often obtain a more realistic mathematical model of the situation where the size of the population at time t and $\alpha(t)$ being the relative rate of growth at time t. It might happen that $\alpha(t)$ is not completely known, but subject to some random environmental effects, so that we have $\alpha(t) = r(t) +$ "noise", where we do not know the exact behaviour of the noise term, only its probability distribution. The function $r(t)$ is assumed to be non random.[15–19]

In this paper, one investigates the Itô diffusion model with white noise and nonlinear terms as a possible model for the spread of the COVID-19 in Brazil. The use of this type of analysis was employed by us in the study of the price dynamics of the financial market.[20–24] Due to uncertainly in the official data about the real cases numbers generated by the low number of tests made in the population generates a large randomness in the data and therefore, makes the use of the stochastic analysis greatly adequate to treat the spread of time evolution of the COVID-19. The plan of this paper is the following. In section II, we describe the stochastic model. In section III, we present the analytical results by Fokker-Planck equation. In section IV, we present our conclusions and final remarks.

II. MODEL

The behavior of the new cases number $N(t)$ of infected by coronavirus registered in the Brazil as function of time (days) registered from 15th March, 2020 is displayed in Fig. 1. From fit of least minimums squares to the set of officials data of the Brazilian ministry of health, we estimate the curve for the behavior of the novel cases of COVID-19 given by a polynomial of fourth order of t (solid-blue-line) in the Fig. 1 given as $g(t) = 1444.8 - 211.98t + 10.743t^2 - 0.19757t^3 + 0.0015721t^4$. The data are by May 22^{th}. The zig-zag behavior in the range of large t reflects in an increase of the uncertainty in the data and the population isolation conditions. For modeling of this behavior, we add a random term together with nonlinear terms in Eq. (1) with aim to simulate the effect of this uncertainty. The α and ν parameters and the nonlinear polynomial added comes from the Bernoulli model for the spreading of the smallpox with the value $\alpha = \nu = 1/8$ obtained by him. Both COVID-19 and smallpox once contracted, confers a lifetime immunity, considering a group of individuals in a given time t where we have a number of these individuals surviving a time t later. We propose then that the model for the new cases in each day t obeys to the following stochastic model with a drift term in the form of a standard Bernoulli model of disease spread and is driven by a Wiener process with normal distribution. The deterministic part can adjust the reported new cases data, in the average, using a high order polynomial.

$$dN(t) = [g(t) - \alpha N(t)(1 - \nu N(t))]\, dt + \beta dW, \tag{1}$$

where $g(t)$ is a polynomial of n degree ($n = 4$). We have an increase in the inclination of the curve from March 30^{th} which may be the effect of quarantine measures adopted. In the equation above, $W(t)$ is the Wiener process or Brownian motion. The equation above can be used to describe the behavior of a particle in Brownian motion under action of an polynomial potential of n degree.[25] Moreover, we have a dissipative force given by $-\gamma\dot{x}$ represented by the friction term in the Langevin's equation and an environment stochastic white noise $\zeta(t)$, that relates with the Wiener process $W(t)$ by $W(t) = \int_{t_0}^{t} \varepsilon(t')dt'$; $\langle \varepsilon(t) \rangle = 0$, $\langle \varepsilon(t)\varepsilon(t') \rangle = \delta\Gamma(t - t')$. Although the model was based on Brazilian data, it can apply to other countries as well, since $g(t)$ is the function that adjustments to the data of COVID-19 of the country with a smaller uncertainty and that after a time t, there is a larger distance between the real data and function of adjustment $g(t)$.

III. RESULTS

We perform a simulation of the model Eq. (1) with the term $\beta W(t)$, Wiener increment and an additive white noise of standard deviation $\sigma_w = \sqrt{\Delta t}$. We can write the Wiener increment as $\beta dW(t) \sim \sqrt{dt}\beta R_G$, where R_G is an aleatory generator number with Gaussian distribution of mean zero and variance $\sigma_w^2 = 1$. In Fig. 1, we plot the time series of the model Eq. (1)for a β value. We have the time series of the change of new cases number oscillating quickly as displayed.

From stochastic equation Eq. (1), we obtain the time development of an arbitrary function $f(X(t))$ by using of the Itô formula

$$f\left[X(t) + dX(t)\right] - f\left[X(t)\right] = \partial_x f[X(t)]\left\{[g(t') - \alpha X(t')(1 - \nu X(t'))]\, dt + \beta dW\right\} + \frac{\beta^2}{2}\partial_x^2 f[X(t)](dW)^2, \tag{2}$$

where higher order terms have been discarded, and $(dW(t))^2 = dt$. Taking the average of both sides in the equation above and defining $\gamma = \beta^2$, we obtain

$$\left\langle \frac{\partial f}{\partial t} \right\rangle = \left\langle \left[\frac{\partial f}{\partial x}\left\{[g(t) - \alpha x(1 - \nu x)]\, dt + \beta dW\right\} + \frac{\gamma}{2}\frac{\partial^2 f}{\partial x^2} \right] \right\rangle. \tag{3}$$

Using

$$\frac{d}{dt}\langle f(X(t))\rangle = \frac{d}{dt}\int_{-\infty}^{\infty} dx f(x)P(x,t) = \int_{-\infty}^{\infty} dx f(x)\frac{\partial}{\partial t}\left[P(x,t)\right] = \int_{-\infty}^{\infty} \frac{\partial f}{\partial x}\left\{[g(t) - \alpha x(1 - \nu x)]\right\} P(x,t)dx + \frac{\gamma}{2}\int_{-\infty}^{\infty} \frac{\partial^2 f}{\partial x^2} P(x,t)dx \tag{4}$$

we integrate by parts and discard surface terms to obtain

$$\int_{-\infty}^{\infty} dx f(x)\frac{\partial}{\partial t}\left[P(x,t)\right] = \int_{-\infty}^{\infty} f(x)\frac{\partial}{\partial x}\left\{[g(t) - \alpha x(1 - \nu x)]\, P(x,t)\right\} dx + \frac{\gamma}{2}\int_{-\infty}^{\infty} f(x)\frac{\partial^2}{\partial x^2}\left[P(x,t)\right] dx. \tag{5}$$

and hence

$$\frac{\partial}{\partial t}P(x,t) = -\frac{\partial}{\partial x}\left\{\left[g(t) - \alpha x(1-\nu x)\right]P(x,t)\right\} + \frac{\gamma}{2}\frac{\partial^2}{\partial x^2}P(x,t). \tag{6}$$

The associated Fokker-Planck equation to the above model is given by

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}\{\left[g(t) - \alpha x(1-\nu x)\right]P(x,t)\} + \frac{\gamma}{2}\frac{\partial^2[P(x,t)]}{\partial x^2}. \tag{7}$$

taking the Fourier transform of the Fokker-Planck equation, we can simultaneously guarantee the normalization of the probability density in which $P(x)$ is reasonably well behaved. We take the boundaries at infinity for $P(x,t)$ as $\lim_{x\to\infty} P(x,t) = 0$ and therefore $\partial_x P(x)$ being reasonably well behaved. As $\lim_{x\to\infty}\partial_x P(x,t) = 0$ thus, a nonzero current of probability at infinity will usually require that the terms in the equation above become infinite there[16]. We use the initial condition $P(x = x_0, t = 0) = P_0$.

For solving the Fokker-Plank equation above independent on time we make the power series expansion $P(x,t) = \sum_{n=0}^{\infty} a_n(t)x^n$ to obtain

$$\frac{\partial P}{\partial t} = \alpha(1-2\nu x)P - \left[g(t) - \alpha x(1-\nu x)\right]\frac{\partial P}{\partial x} + \frac{\gamma}{2}\frac{\partial^2 P}{\partial x^2}$$

$$\sum_{n=0}^{\infty}\left(\frac{da_n}{dt} + g(t)(n+1)a_{n+1}\right)x^n = \left[\alpha(1-\nu x) - \alpha\nu x\right]\sum_{n=0}^{\infty}a_n x^n + \alpha x(1-\nu x)\sum_{n=0}^{\infty}n a_n x^{n-1} + \frac{\gamma}{2}\sum_{n=0}^{\infty}n(n-1)a_n x^{n-2}. \tag{8}$$

We obtain the relations

$$\alpha\sum_{n=0}^{\infty}a_n x^n - 2\nu\alpha\sum_{n=1}^{\infty}a_{n-1}x^n + \sum_{n=1}^{\infty}n a_n x^n - \alpha\nu\sum_{n=2}^{\infty}(n-1)a_{n-1}x^n + \frac{\gamma}{2}\sum_{n=0}^{\infty}(n+1)(n+2)a_{n+2}x^n = k, \tag{9}$$

where k is a separation constant. For $k = 0$, one obtains the following recurrence relations

$$\begin{aligned} a_2 &= -\frac{\alpha}{\gamma}a_0 \\ a_3 &= 2\nu\alpha a_0 - a_1(1+\alpha) \\ a_4 &= \frac{\alpha}{3\gamma^2}\left(\frac{\alpha}{2} - 1\right)a_0 + \frac{\nu\alpha}{2\gamma}a_1 \end{aligned} \tag{10}$$

for $0 \le n \le 2$, and

$$a_{n+2} = \frac{2}{\gamma}\frac{\left[\alpha\nu(n+1)a_{n-1} - (\alpha+n)a_n\right]}{(n+2)(n+1)}, \tag{11}$$

for $n > 2$. Additionally, we have

$$\frac{da_n(t)}{dt} + g(t)(n+1)a_{n+1}(t) = k. \tag{12}$$

Therefore, we obtain $P(x)$ in the form

$$\begin{aligned} P(x) = \left\{1 - \frac{\alpha}{\gamma}x^2 + 2\nu\alpha x^3 + \frac{\alpha}{3\gamma^2}\left(\frac{\alpha}{2} - 1\right)x^4 - \frac{1}{10\gamma}\left[2\nu(3+\alpha) + \frac{2\nu\alpha}{\gamma}(\nu+\alpha)\right]x^5 \cdots\right\}a_0 \\ + \left\{x - (1+\alpha)x^3 + \frac{\nu\alpha}{2\gamma}x^4 + \frac{1}{10\gamma}(3+\alpha)(1+\alpha)x^5 + \cdots\right\}a_1. \end{aligned} \tag{13}$$

where the constants a_0 and a_1 are determined by the initial conditions $P(0,0) = P_0$ and $\partial_x P(x,0) = 0$ em $x = 0$. We obtain $a_0 = P_0$ and $a_1 = 0$. From the normalization condition, the second term in the density probability above must be zero and therefore, all coefficients a_1 must cancel. Therefore, we have

$$P(x,t) = P_0\left\{1 - \frac{\alpha}{\gamma}x^2 + 2\nu\alpha x^3 + \frac{\alpha}{3\gamma^2}\left(\frac{\alpha}{2} - 1\right)x^4 - \frac{1}{10\gamma}\left[2\nu(3+\alpha) + \frac{2\nu\alpha}{\gamma}(\nu+\alpha)\right]x^5 \cdots\right\} \tag{14}$$

To ensure the normalization of the probability density, P_0 must be non zero only within interval $-\varepsilon \leq x \leq \varepsilon$ and zero out it.

For $k \neq 0$, we have from Eq. (9) that $n = 0$ and $a_2 + \alpha a_0/\gamma = k$ and all a_n higher are zero. Consequently, we obtain from integration of the Eq. (12)

$$P(t) = \frac{\gamma}{\alpha} P_0 t[1 - a_2(t)] + a_0(0) - \int_0^t g(t')a_1(t')dt'. \tag{15}$$

We can obtain the n-th moments $m_n = \langle N^n \rangle = \int_{-\infty}^{\infty} N^n P(N,t)dx$, where the mean half-width of the probability distribution $\sigma = \sqrt{\langle N^2 \rangle - \langle N \rangle^2}$ gives an mean estimating of the new cases numbers in the day t. In general, we may introduce n-th order moments $\mu_n = \langle (x - m_1)^n \rangle$ about the mean or central moments, where we have the following relations: $c_1 = \mu_1$, $c_2 = \mu_2$, $c_3 = \mu_3$, $c_4 = \mu_4 - 3\mu_2^2$. Normalized measures often used, indicating a deviation from a Gaussian are the kurtosis λ_4, defined as $\lambda_4 = \mu_4/\sigma^4 - 3$. We obtain the mean half width of the distribution as function of time as given as

$$\sigma(t) = \left\{ \frac{[N(t)]^3}{3} \frac{\gamma}{\alpha} \left[P_0(1 - a_2(t)) + a_n(0) - \int_0^t g(t')a_1(t')dt' \right]^2 - \frac{[N(t)]^4}{4} \frac{\gamma^2}{\alpha^2} \left[P_0(1 - a_2(t)) + a_n(0) - \int_0^t g(t')a_1(t')dt' \right]^2 \right\}^{1/2}$$

Furthermore, we can have ($n = 0$) $da_0(t)/dt = 0$, either $a_0(t) = c$, $a_1 = k/2f(t)$ and $P(t) = P_0 t + c - k/2$, where we define $p_0 = c - k/2$. Therefore

$$\sigma(t) = (P_0 t + p_0) \sqrt{\frac{[N(t)]^3}{3} - \frac{[N(t)]^4}{4}}. \tag{16}$$

How the first cases are registered on March 15th ($t = 15$), we obtain $p_0 = -1.5$ an $P_0 = 0.1$ for the official results of the new cases numbers. In Fig. 2, we plot the mean half width as function of time t (days). From probability density of new cases $P(N,t)$ obtained from solution of the Fokker-Planck equation Eq. (7), we calculate the variance of the distribution where the standard deviation gives an estimating of novel cases number. The results adjust well to the official data of ministry of healthy within range considered. The difference between to real data can be due to the approach used. A treatment considering a multiplicative white noise or a real nonwhite noise approach $\zeta(t)$ (which approaches a delta-correlated noise) might give a better adjusts to the real data. Anyway, it seems to exists the possibility of an increase in the new cases number in the next weeks, even if the model results are affected by a large uncertainty. In Fig. 3, we display the behavior of the kurtosis (excess), $\lambda_4(t)$ as function of time. $\lambda_4(t)$ is calculated by numerically (solving Eq. (7)). The excess kurtosis relates to the deviation of the tail of the distribution, as compared to Gaussian. The negative value obtained for the kurtosis for all t values indicates that the shape of the distribution is near to the Wigner semicircle[18]. Furthermore, at range of large t where the kurtosis is nearest to zero we have that the distribution is nearest of a Gaussian ($\lambda_4 = 0$) what occurs for large values of t as expected.

IV. CONCLUSIONS

In Brief, we propose a model for the time evolution of the SARS-CoV-2 based in the Itô's diffusion. My results are compared with public data for forecasting of epidemic spreading of the novel coronavirus disease where due to uncertainty in the results generated by the low number of tests made in the populations and hence, to underreporting, which generates an uncertainly in the official results and consequently, becomes necessary, the addition of randomness in the differential equations for growing of the infected number. We can use a beyond white noise approach to try a better fit to the results, what can be made in a future work. In a general way, a polynomial of higher order in the stochastic differential equation presents local minimums and maximum beyond which the potential plummets to $-\infty$. It generates barriers separating stable from unstable regions modeling situations of crashes or crisis as for instance, the sudden break down of lock down conditions.

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Declarations:

Competing interests: I declare no competing interests.

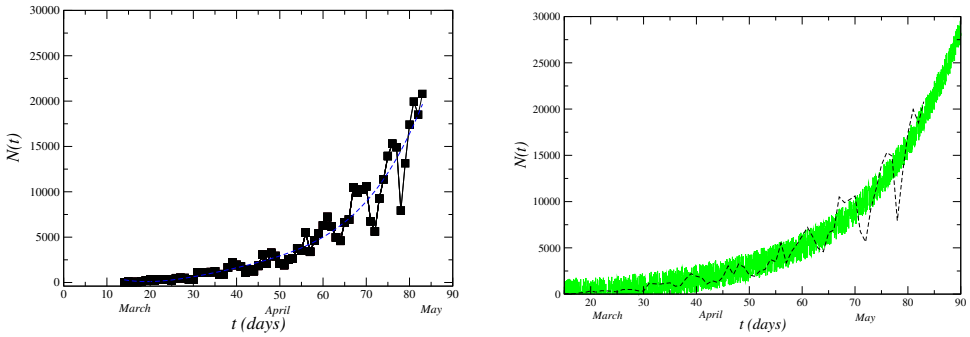

FIG. 1. (Left) Time evolution of new cases $N(t)$ in Brazil. The data was provided by the Brazilian ministry of health. The behavior zig-zag in the results is reflected by the stochastic term in Eq. (1). The dashed-blue-line in the figure is the least-squares fit $g(t)$ on results ($g(t)$ term) in Eq. (1). (Right) In the right, we plot the time series of the model Eq. (1)for $\alpha = \nu = 1/8$ and the noise amplitude $\Gamma = 1$ as well as $\beta = 1.0 \times 10^5$.

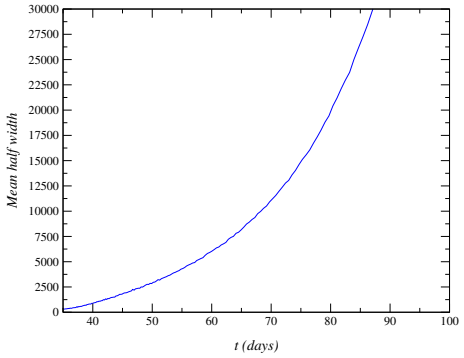

FIG. 2. Time evolution of the half width of the distribution σ with t. The half width gives an expectation of new cases in the day t.

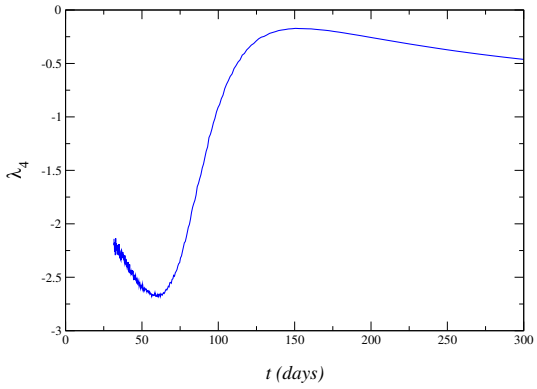

FIG. 3. Time evolution of the kurtosis $\lambda_4(t)$. The negative values obtained for all t values gives an estimating shape of distribution that becomes nearest of a Gaussian $\lambda_4 = 0$ at range of large t values since the firsts cases registered in Brazil on 15 th March 2020 displayed by the official data by the Brazilian ministry of Health.

Figures

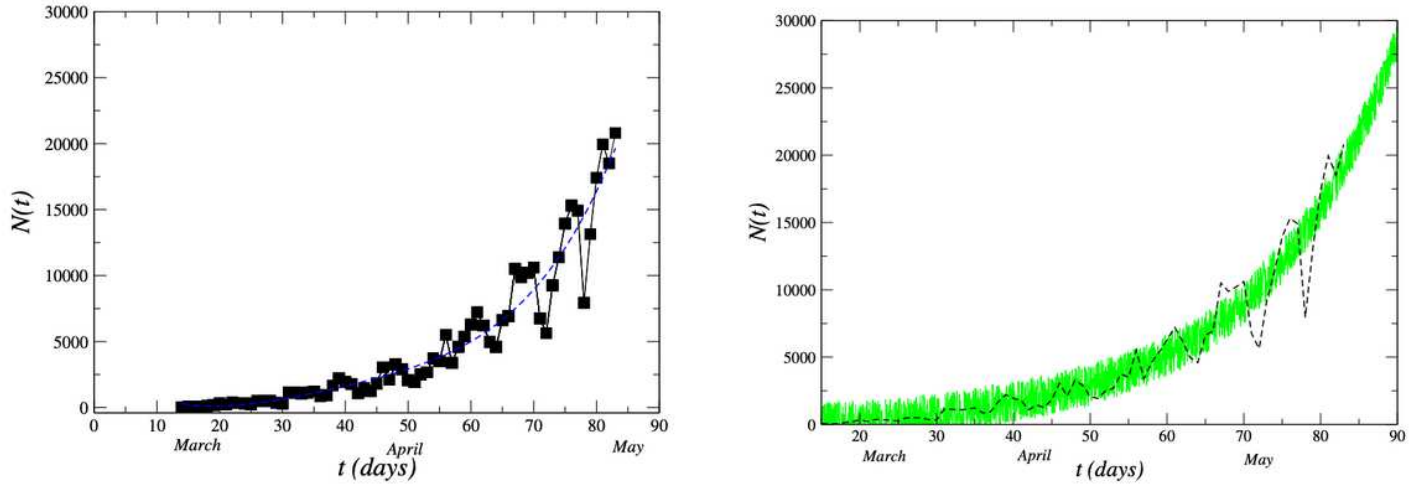

Figure 1

(Left) Time evolution of new cases N(t) in Brazil. The data was provided by the Brazilian ministry of health. The behavior zig-zag in the results is reflected by the stochastic term in Eq. (1). The dashed-blue-line in the figure is the leastsquares fit g(t) on results (g(t) term) in Eq. (1). (Right) In the right, we plot the time series of the model Eq. (1)for α = ν = 1/8 and the noise amplitude Γ = 1 as well as β = 1.0 × 105

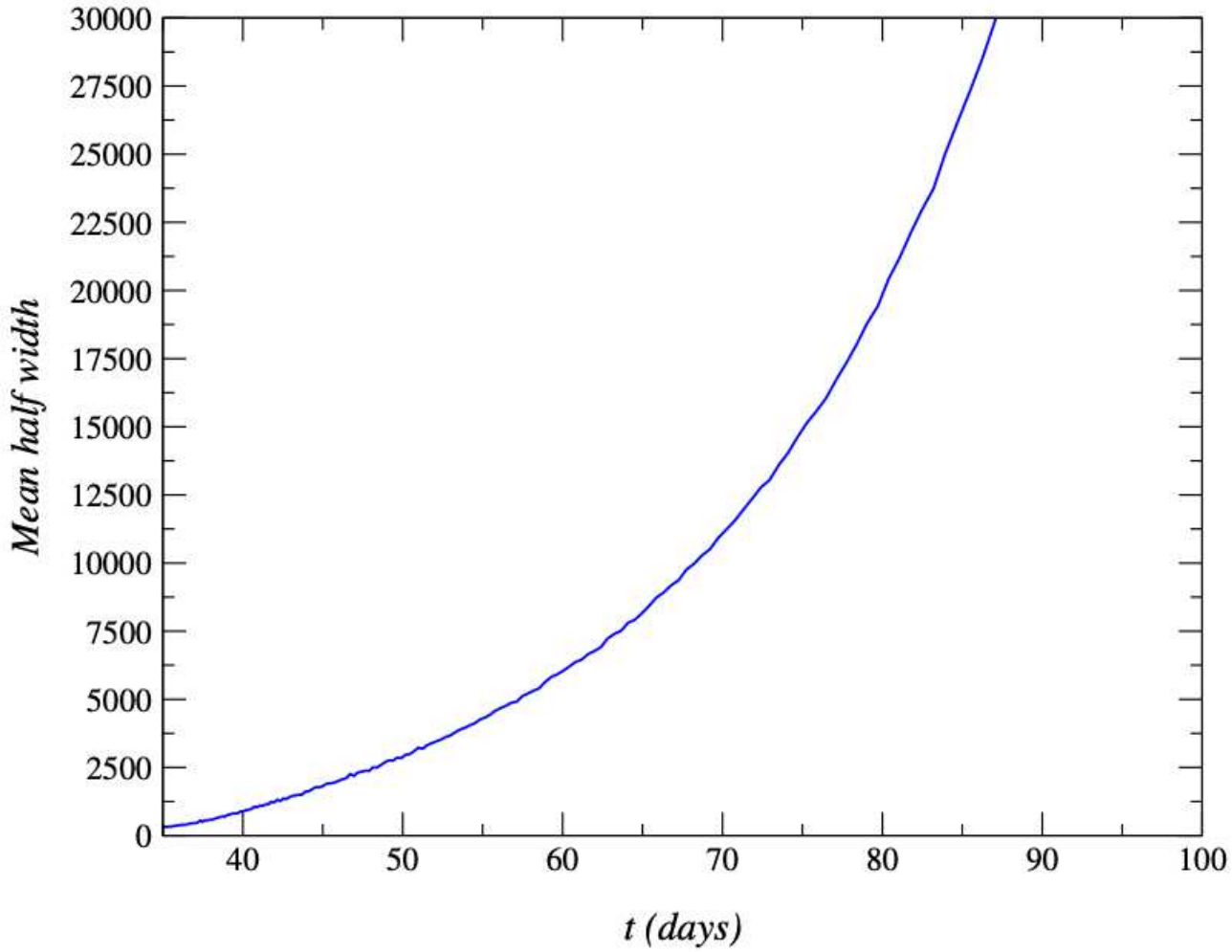

Figure 2

Time evolution of the half width of the distribution σ with t. The half width gives an expectation of new cases in the day t.

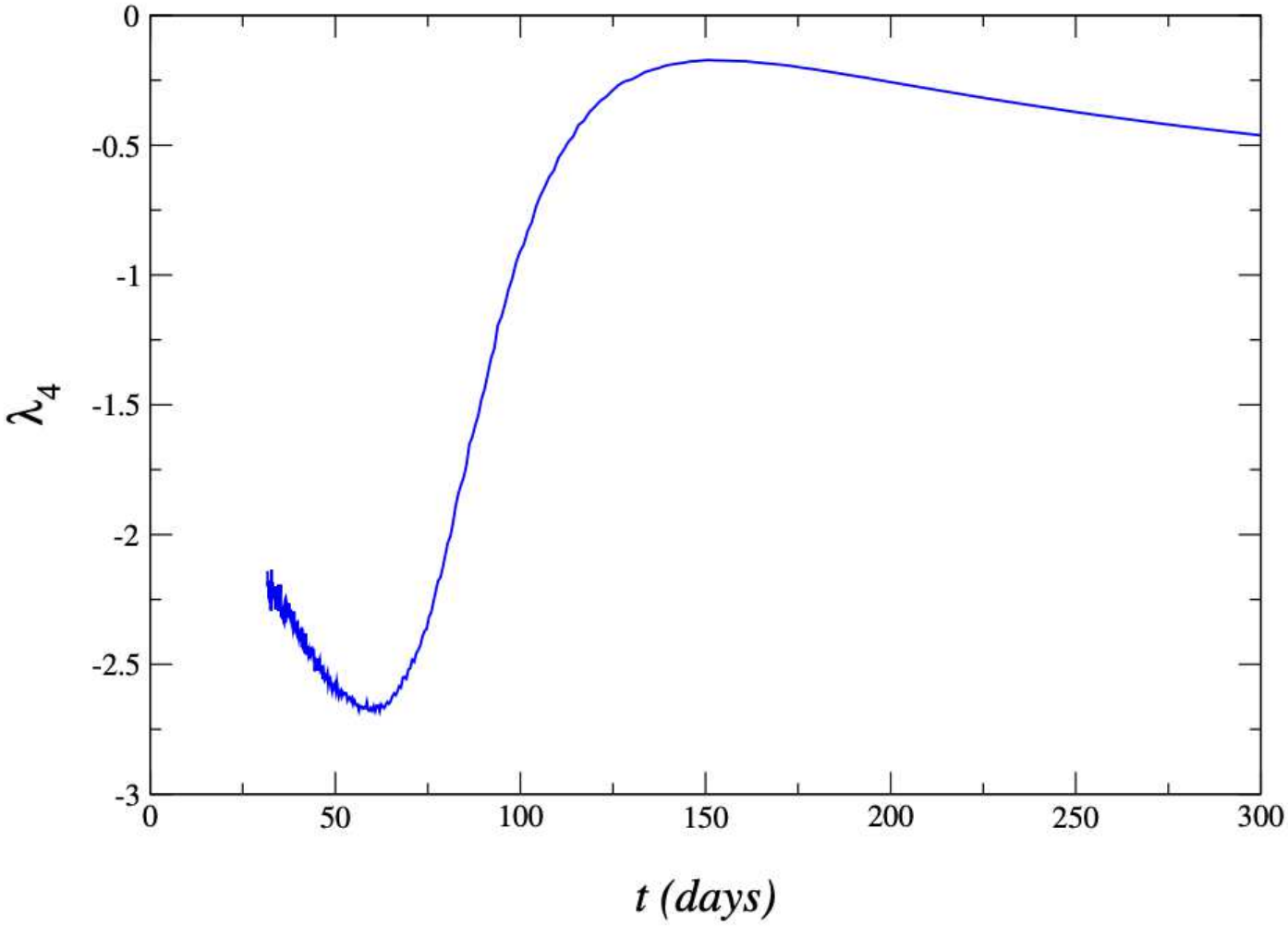

Figure 3

Time evolution of the kurtosis λ4(t). The negative values obtained for all t values gives an estimating shape of distribution that becomes nearest of a Gaussian λ4 = 0 at range of large t values since the firsts cases registered in Brazil on 15 th March 2020 displayed by the official data by the Brazilian ministry of Health