Robust Particle Swarm Optimization for Active/Reactive and Reserve Scheduling in a Grid-connected Microgrid With Energy Storage Systems

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Robust Particle Swarm Optimization for Active/Reactive and Reserve Scheduling in a Grid-Connected Microgrid with Energy Storage Systems

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Abstract:

\textbf{Background:} In recent years, simultaneous participation in electrical energy and ancillary services markets has been very profitable for distributed energy resources (DERs). Moreover, the presence of renewable generations along with energy storage systems (ESS) is bringing a significant contribution to modern distribution systems. High penetration of non-predictable power sources in microgrids (MGs), due to the uncertainties of these products, increases the need for ancillary services and the management and coordination of these technologies combined with the ESSs.

\textbf{Results:} For the first time, this paper develops a robust particle swarm optimization model to handle the uncertain renewable power production involved in the joint active/reactive and reserve scheduling of a smart MG. The robust optimization approach has a medium priority compared to
deterministic and stochastic ones. The objective function utilized for the optimal joint active/reactive and reserve scheduling of an MG is defined as maximizing social welfare, which is accomplished based on a max-min optimization model. The robust optimal solution can be achieved in such a way that the maximizer at the outer level makes an optimal decision against the worst-case objective function, which is acquired based on the minimizer at the inner level considering the uncertainty neighborhood.

**Conclusions:** The effectiveness of the proposed method is examined on a 33-bus MG test system. Simulation results prove that the proposed RPSO model can help MG operators to reduce scheduling costs to obtain a higher social welfare. The consideration of more uncertainty in renewable energy resources production leads to higher operation costs, especially reserve costs. Integration of robustness against uncertainty in the joint active/reactive and reserve management in the smart MGs leads to a more robust operation at the expense of higher costs.

**Keywords:** Smart Micro-Grid, Uncertainty, Robust Optimization, Active/Reactive and Reserve Scheduling, Particle Swarm Optimization (PSO), Energy storage system, max-min operation model.

**Nomenclature:**

**Acronyms**
- DL: Dispatchable load
- ESS: Energy storage system
- UPN: Upstream network
- DER: Distributed energy resource
- RES: Renewable energy source
- SOC: State of charge

**Indices, Parameters and Variables**
- $b$ and $b'$: Bus indices from 1 to $Nb$
- $t$: Time index from 1 to $T$
- $m$: Block index in DL bid from 1 to $NB$
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DER_set$</td>
<td>The set of buses that have DER.</td>
</tr>
<tr>
<td>$DL_set$</td>
<td>The set of buses that contain DL.</td>
</tr>
<tr>
<td>$x$</td>
<td>The set of variables for outer maximization</td>
</tr>
<tr>
<td>$\hat{u}$</td>
<td>The set of variables for inner minimization</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Reactive cost function coefficient of the UPN</td>
</tr>
<tr>
<td>$\alpha_b, \beta_b, \gamma_b$</td>
<td>Cost function coefficients of producing active power by dispatchable DER at bus b.</td>
</tr>
<tr>
<td>$\lambda_b, \mu_b, \omega_b$</td>
<td>Cost function coefficients of providing reactive power by dispatchable DER at bus b.</td>
</tr>
<tr>
<td>$G_{bb'}, B_{bb'}$</td>
<td>The conductance and susceptance of the line between buses $b$ and $b'$.</td>
</tr>
<tr>
<td>$SW(t)$</td>
<td>The social welfare of base case at time $t$</td>
</tr>
<tr>
<td>$\bar{SW}(t)$</td>
<td>The social welfare of the worst-case at time $t$</td>
</tr>
<tr>
<td>$B_{DL_b}$</td>
<td>The benefit of dispatchable load at bus b</td>
</tr>
<tr>
<td>$C_{UPN}^{re}(t)$</td>
<td>The reserve cost of the UPN at time $t$</td>
</tr>
<tr>
<td>$C_{DER_b}^{re}(t)$</td>
<td>The reserve cost of DER at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$C_{A_{DER_b}}^A$</td>
<td>The active power cost of DER at bus $b$</td>
</tr>
<tr>
<td>$C_{UPN}^A$</td>
<td>The active power cost of the UPN</td>
</tr>
<tr>
<td>$C_{UPN}^R$</td>
<td>The reactive power cost of the UPN</td>
</tr>
<tr>
<td>$C_{B_{DER_b}}^B$</td>
<td>The active power cost of DER at bus $b$</td>
</tr>
<tr>
<td>$P_{T_{UPN}}(t)$</td>
<td>The reserve provided by the UPN at time $t$</td>
</tr>
<tr>
<td>$P_{T_{DER_b}}(t)$</td>
<td>The reserve provided by DER at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$p_{DL_b}^m$</td>
<td>The upper level of block $m$ in the bid of the DL at bus $b$</td>
</tr>
<tr>
<td>$p_{DL_b}^m$</td>
<td>The consumed active power of block $m$ in the bid of the DL at bus $b$</td>
</tr>
<tr>
<td>$P_{PV_b}(t)$</td>
<td>The forecasted active power of PV at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{WT_b}(t)$</td>
<td>The forecasted active power of WT at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{ch-ESS_b}(t)$</td>
<td>The charging power of ESS located at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{dis-ESS_b}(t)$</td>
<td>The discharging power of ESS located at bus $b$ at time $t$</td>
</tr>
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<td>$P_{UPN}(t)$</td>
<td>Active power purchased from the UPN at time $t$</td>
</tr>
<tr>
<td>$P_{DL_b}(t)$</td>
<td>Active power consumed by DL at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{DER_b}(t)$</td>
<td>Active power produced by DER at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{d_b}(t)$</td>
<td>Active demand at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$P_{d}(slack, t)$</td>
<td>Active demand at the slack bus at time $t$</td>
</tr>
<tr>
<td>$Q_{UPN}(t)$</td>
<td>Reactive power provided by the UPN at time $t$</td>
</tr>
<tr>
<td>$Q_{DER_b}(t)$</td>
<td>Reactive power produced by the DER at bus $b$ at time $t$</td>
</tr>
<tr>
<td>$Q_{d_b}(t)$</td>
<td>Reactive demand at bus $b$ at time $t$</td>
</tr>
</tbody>
</table>
$Q_d(slack,t)$
Reactive demand at slack bus at time $t$

$Q_{DL_b}(t)$
Reactive power demanded by DL at bus $b$ at time $t$

$p_{WT_b}(t)$
The produced active power of WT in the worst case at bus $b$ at time $t$

$p_{PV_b}(t)$
The produced active power of PV in the worst case at bus $b$ at time $t$

$p_{UPN}(t)$
The purchased active power from UPN in the worst case at time $t$

$p_{DL_b}(t)$
The consumed active power of DL in the worst case at bus $b$ at time $t$

$p_{DER_b}(t)$
The produced active power of DER in the worst case at bus $b$ at time $t$

$q_{UPN}(t)$
The provided reactive power by the UPN in the worst case at time $t$

$q_{DER_b}(t)$
The provided reactive power of DER in the worst case at bus $b$ at time $t$

$q_{DL_b}(t)$
The demanded reactive power of DL in the worst case at bus $b$ at time $t$

$\pi_{UPN}(t)$
The energy price of the UPN at time $t$

$\pi_{UPN}$
The reserve price of the UPN

$\pi_{DER_b}$
The reserve price of DER at bus $b$

$\pi_{DL_b}$
The offered price of block $m$ in the bid of the DL at bus $b$

$E_{ESS_b-min}$ and $E_{ESS_b-max}$
The minimum and maximum of the SOC of ESS at bus $b$

$Q_{UPN-max}$ and $Q_{UPN-min}$
The maximum and minimum of reactive power provided by the UPN

$Q_{DER_b-max}$ and $Q_{DER_b-min}$
The maximum and minimum of reactive power provided by DER at bus $b$

$p_{UPN-max}$
The maximum available active power of the UPN

$p_{ch-ESS_b-max}$ and $p_{dch-ESS_b-max}$
The maximum charge and discharge power of ESS at bus $b$

$p_{DER_b-max}$ and $p_{DER_b-min}$
The maximum and minimum producible active power by DER at bus $b$

$p_{PV_b-max}(t)$ and $p_{PV_b-min}(t)$
The upper and lower bounds of the uncertain PV production to form uncertainty neighborhood around $p_{PV_b}(t)$

$p_{WT_b-max}(t)$ and $p_{WT_b-min}(t)$
The upper and lower bounds of the uncertain WT production to form uncertainty neighborhood around $p_{WT_b}(t)$

$u_{ch-ESS_b}(t)$
A binary variable representing the charging status of ESS located at bus $b$ at time $t$

$u_{dch-ESS_b}(t)$
A binary variable representing the discharging status of ESS located at bus $b$ at time $t$

$E_{ESS_b-initial}$
The SOC initial level of ESS at bus $b$

$E_{ESS_b-final}$
The SOC final level of ESS at bus $b$

$SOC_{ESS_b}(t)$
The SOC of ESS at bus $b$ at time $t$

$\eta_{ch}$ and $\eta_{dch}$
The charging and discharging efficiency of the ESS

$V_b(t)$
The voltage at bus $b$ at time $t$

$\delta_b(t)$
Voltage angle at bus $b$ at time $t$

$p_b(t)$
The voltage at bus $b$ and time $t$ in the worst case
1. Introduction

In recent years, to hinder the destructive effects of environmental degradation and depletion of fossil fuel energy sources, distributed energy resources (DERs), including renewable energy sources (RESs), energy storage systems (ESSs), demand response (DR) programs and other policies with green characteristics are rapidly emerging in the world power industry [1]. However, the mismanagement of these resources in the network can create numerous challenges in the system operation, such as worsening the network reliability and flexibility, deteriorating the voltage profile of the feeders, and increasing power losses of the network [2-4]. In line with this concern, smart grids equipped with advanced communication infrastructure have been introduced to attain the best possible amalgamation of diverse energy resources in distribution systems [5]. Microgrid (MG) consists of generators of electrical energy (renewable generators, micro turbines, fuel cells, etc.), electrical loads and sometimes ESS, which can be a consumer or supplier of electricity with the coordination of its members [6]. Developing the DERs in the distribution systems, the power system can be divided into multi-MGs in which all these technologies are coordinately controlled by a central or local controller through wide-area monitoring systems [7]. Due to the high integration rate of the RESs such as photovoltaic (PV) cells and wind turbines (WTs) into the grid, the uncertain parameters will substantially increase in the distribution network. Variations during hours and rapid fluctuations within seconds-minute are defined as two types of the uncertainty of the WTs and PV panels. The rapid fluctuation refers to small-scale variations in output power of RES units while, the slow variations, which indicate the large-scale
changes in active power of renewables, is pertaining to the energy profile of these generations. The ESSs and demand response (DR) programs can successfully alleviate the influence of uncertainties and intermittencies of renewable generations in the electric power systems [8, 9]. The ESSs with small-capacity fast-response characteristics are appropriate for smoothing rapid fluctuations with low range, and the large-capacity ESSs with slow response are compatible with slow variations with large-scale extent [10]. Power loss reduction in electrical networks is another important ESS application, as reported in [11]. The grid power losses are generally minimized based on the optimal charging/discharging scheduling of the ESSs by shifting energy usage from off-peak periods to on-peak hours. Moreover, optimal operation of the ESS (i.e., charging-discharging management) reduces operational costs and improves voltage stability [12]. Many studies only have addressed the optimum active power scheduling of ESSs and have not investigated their reactive power capability to absorb or inject reactive power to the grid [13]. Active users equipped with a battery energy storage system (BESS) can contribute effectively to alleviate the overvoltage and undervoltage troubles, by exchanging a certain amount of active power with the grid [14]. The authors of [15] have developed a smart home energy management (HEM) strategy in which electric vehicles (EVs) and ESS can participate in reactive power compensation at the point of grid connection.

Considering the uncertainties in the problem modeling could affect the optimum decision variables and objective function value. Hence, uncertain parameters append an additional aspect to any global optimization search. Optimization problems by considering the uncertainties are typically dealt with using both stochastic [16-20] and robust models [21, 22]. Stochastic optimization requires the probability distributions of uncertain parameters along with numerous scenarios to address uncertainties and yield a guaranteed solution [23]. However, the probabilistic distribution
of some uncertain parameters is challenging to be acquired [24]. Accordingly, the stochastic model of the MG operation will be complicated, and needs to improve its computational burden. Alternatively, describing the lower and upper bounds for each uncertain parameter is usually easier than obtaining scenarios for the uncertain variables. A robust optimization approach finds an optimal decision that optimizes its performance in the worst case. This procedure is defined as a min-max or a max-min optimization. In the max-min one, the maximizer makes an optimal decision against the worst case obtained by the minimizer. This means that at any point in the solution space, an inner optimizer is applied to seek the minimum objective function value in the corresponding uncertainty intervals, while an outer optimizer is utilized to determine the maximum minimal objective function. While, in the min-max optimization, this trend will be reversed. In non-convex optimization problems under uncertainty, it is impossible to achieve an exactly desired solution. The robust optimization aims to find a robust solution where the worst-case solution still achieves a possible performance.

Under the restructuring of electric power systems, the utilization of DERs and DR programs is becoming the most beneficial way to provide ancillary services to comply with the grid code requirement and to improve the power quality [25-27]. Also, the distribution system operator (DSO) is willing to acquire the required ancillary services for the power system security from the active users. In the literature, the potentials of different DER technologies, ESS, and active loads in the smart distribution grid have been investigated in various fields. Currently, in most power systems, non-dispatchable DERs are not capable of participating in the ancillary services market. However, as reported in [28], a new framework is being defined, enabling non-dispatchable DERs to participate in such a market. Thus, equipping modern distribution networks with the DERs as new reactive power and reserve resources can lead to economic and technical benefits [26, 29].
The authors of [30] have proposed an energy management model based on the virtual power plant (VPP) scheme to examine the capability of the WTs and synchronous distributed generations in the coupled energy and reserve market. Authors of [31] have developed a stochastic optimal bidding framework of a renewable-based MG to contribute in the energy and reserve markets. In [32], stochastic-based programming has been introduced for optimal energy management of the renewable-based MG with a parking lot and DR programs to take part in both energy and reserve markets. In [27], the optimum active and reactive power management of the EVs has been proposed as an appropriate policy to alleviate the voltage drop and line overloading as well as to reduce network losses. In [33], a two-stage robust optimization is developed for active/reactive power dispatch in the active distribution systems using relaxed optimal power flow, and then a column-and-constraint generation algorithm has been employed to solve the robust optimization model.

Since the payment for reactive power support is much lower than that for electrical energy provision; the coupled active and reactive power market clearing is not convenient for the bulk power systems [34]. Furthermore, the computing burden and complexity in the coupled market are more significant than that of the separate energy and reactive power markets. However, in the active distribution networks and MGs, since the necessary electrical energy is provided locally by the DERs, the concurrent optimization of active-reactive power scheduling is utilized in many studies [18, 35-37].

In the literature, coupled energy/reserve management [29, 38, 39] and active/reactive power scheduling in the smart MGs have been addressed considering the problem's uncertainties. However, few papers have been reported on the concurrent active/reactive and reserve scheduling in the MG with the ESS. DERs such as diesel generators (DGs) along with WTs and PVs as RES
units are the new technologies that should be optimally managed in coordination with ESSs. Concisely, this paper presents a novel robust model to simultaneously address the active/reactive power and reserve scheduling in the grid-connected MG in the presence of ESS units and dispatchable loads to maximize social welfare (SW). In the proposed robust model (as a max-min optimization), a maximization level makes an optimal robust decision against the worst-case solution obtained by another minimization layer, which makes solving them more challenging than classical optimization problems. To the best of our knowledge, in this paper, it is the first time that a robust PSO [40] is implemented for optimal active/reactive and reserve management of the smart MGs. As stated in [41, 42], DC power flow is generally utilized to model the ESS management. However, DC power flow does not yield an accurate and realistic operation model of the network with ESS. Accordingly, AC power flow is implemented in this paper to analyze the network and ESSs in order to present a realistic and more practical model and results. The effectiveness of the proposed method is examined on a 33-bus MG test system.

Eventually, the main contributions of this paper are briefly summarized as follows:

- A robust model is proposed to simultaneously manage the active/reactive power and reserve scheduling in the grid-connected MG.
- A Robust particle swarm optimization (RPSO) model is presented to handle the uncertain renewable power production involved in the scheduling of a smart MG.
- The presence of the ESS and dispatchable loads are analyzed in the proposed model.

2. Problem formulation for robust scheduling of MG

Here, the proposed formulation for robust joint active/reactive and reserve scheduling of MG is presented based on a max-min optimization model.
2.1. Objective function

The objective function for the optimal joint active/reactive and reserve scheduling of an MG is defined as maximizing social welfare (SW), which is performed based on a max-min optimization problem. The robust optimal solution can be achieved in such a way that the maximizer at the outer level makes an optimal decision against the worst-case objective function, which is acquired based on the minimizer at the inner level considering the uncertainty neighborhood. Two operating cases are taken into consideration for this mathematical modeling as the base case and worst case. The base case refers to the situation that output power productions of RESs are assumed to be equal to their forecasted values. As mentioned, the worst-case represents the situation in which the power production of each RES equals to a particular value within its lower and upper bounds of uncertainty that lead to the minimum SW. The worst-case decision variables are distinguished by hat notation (i.e., \(\hat{\cdot}\)). The proposed objective function is formulated as follows:

\[
z = \max_x \min_u \sum_{t=1}^{T} \left[ SW(t) + \hat{SW}(t) - \left[ C_{UPN}^{re}(t) + C_{DER_b}^{re}(t) \right] \right]
\]

where, \(SW(t)\) and \(\hat{SW}(t)\) are defined as the difference between the total benefit of DLs and the operation costs of the MG in base and worst cases, respectively. The operation costs of the MG comprise the cost of procuring active/reactive power from the upstream network (UPN) along with the cost of producing active/reactive power by the DERs over the scheduling horizon.

\[
SW(t) = \sum_{b \in DLset} B_{DL_b} \left( P_{DL_b}(t) \right) - \left[ C_{UPN}^{A}(P_{UPN}(t)) + C_{UPN}^{R}(Q_{UPN}(t)) \right] + \sum_{b \in DERset} C_{DER_b}^{A} \left( P_{DER_b}(t) \right) + C_{DER_b}^{R} \left( Q_{DER_b}(t) \right)
\]
\[ S\hat{W}(t) = \sum_{b \in DLset} B_{DLb} \left( \hat{P}_{DLb}(t) \right) \]

\[- \left[ C_{UPN}^{A} \left( \hat{P}_{UPN}(t) \right) + C_{UPN}^{R} \left( \hat{Q}_{UPN}(t) \right) \right. \]

\[ + \sum_{b \in DERset} C_{DERb}^{A} \left( \hat{P}_{DERb}(t) \right) + C_{DERb}^{R} \left( \hat{Q}_{DERb}(t) \right) \]

In Eq. (1), \( x \) displays the decision variable set of outer maximization, which contains the active and reactive power of UPN, DERs, DLs and the provided reserve by UPN, DERs, as well as the charge/discharge power of ESSs. Inner minimization is performed over the decision variable set \( \hat{u} \) which consists of the active production of RESs. The variable sets \( x \) and \( \hat{u} \) are given in the following:

\[
x = \left\{ P_{UPN}(t), \hat{P}_{UPN}(t), P_{UPN}(t), P_{DERb}(t), \hat{P}_{DERb}(t), P_{DERb}(t), \right. \]

\[ P_{DLb}(t), \hat{P}_{DLb}(t), P_{ch-ESSb}(t), P_{dch-ESSb}(t), Q_{UPN}(t), \]

\[ \hat{Q}_{UPN}(t), Q_{DERb}(t), \hat{Q}_{DERb}(t), Q_{DLb}(t), \hat{Q}_{DLb}(t) \]

\[
\hat{u} = \{ \hat{P}_{PVb}(t), \hat{P}_{WTb}(t) \} \]

The costs of procuring active/reactive power from the UPN are given as follows [43]:

\[
C_{UPN}^{A}(P_{DAM}(t)) = P_{UPN}(t) \cdot \pi_{UPN}(t) \quad (6)
\]

\[
C_{UPN}^{R}(Q_{UPN}(t)) = \xi \cdot Q_{UPN}^{2}(t) \quad (7)
\]

Quadratic cost functions are taken into consideration for produced active/reactive power by a DER as displayed in Eqs. (8) and (9) [44]:

\[
C_{DERb}^{A} \left( P_{DERb}(t) \right) = \alpha_{b} + \beta_{b} \cdot P_{DERb}(t) + \gamma_{b} \cdot P_{DERb}^{2}(t) \quad (8)
\]

\[
C_{DERb}^{R} \left( Q_{DERb}(t) \right) = \lambda_{b} \left( Q_{DERb}(t) \right)^{2} + \mu_{b} \left( Q_{DERb}(t) \right) + \omega_{b} \quad (9)
\]

The payment for provided reserve by the UPN is expressed as follows:
The reserve cost of a DER is given as:

$$C_{UPN}^r(t) = Pr_{UPN}(t) \cdot \pi r_{UPN}$$

(10)

The reserve cost of a DER is given as:

$$C_{DER_b}^r(t) = Pr_{DER_b}(t) \cdot \pi r_{DER_b}$$

(11)

Fig. 1 shows an NB-block bid of a DL at bus b.

The bid function of a DL that corresponds to its benefit from energy consumption is calculated as follows:

$$0 \leq p_{DL_b}^m \leq P_{DL_b}^m - P_{DL_b}^{m-1} \quad m = 1, 2, \ldots, NB$$

(12)

$$P_{DL_b} = \sum_{m=1}^{NB} p_{DL_b}^m$$

(13)

$$B_{DR_b}(P_{DR_b}) = \sum_{m=1}^{NB} p_{DL_b}^m \cdot \pi_{DL_b}^m$$

(14)
2.2. Constraints

In the proposed formulation, a set of technical constraints that are expressed in the following should be satisfied.

The robust joint active/reactive and reserve scheduling of the MG has been examined regarding a set of various technical constraints, which are expressed in the following.

2.2.1. Constraints on ESS

The operational constraints of the ESS can be displayed by Eqs. (15) to (21). The active power charge and discharge of the ESS should be restricted by Eqs. (15) and (16). The concurrent charging and discharging of the ESS will be prevented by Eq. (17). The State of Charge (SOC) of the ESS should be controlled by defining a permissible level of energy as presented in Eq. (18). Eq. (19) indicates the initial SOC. The final level of SOC is satisfied by Eq. (20). The dynamic relationship of the ESS energy can be written as Eq. (21).

\begin{align*}
0 & \leq P_{ch-ESS_b}(t) \leq P_{ch-ESS_b-max} \times u_{ch-ESS_b}(t) \\
0 & \leq P_{dch-ESS_b}(t) \leq P_{dch-ESS_b-max} \times u_{dch-ESS_b}(t) \\
u_{ch-ESS_b}(t) + u_{dch-ESS_b}(t) & \leq 1 \\
E_{ESS_b-min} & \leq SOC_{ESS_b}(t) \leq E_{ESS_b-max} \\
SOC_{ESS_b}(0) & = E_{ESS_b-initiat} \\
SOC_{ESS_b}(T) & \geq E_{ESS_b-Final} \\
SOC_{ESS_b}(t) & = SOC_{ESS_b}(t-1) + \left( \eta_{ch} \times P_{ch-ESS_b}(t) - \frac{P_{dch-ESS_b}(t)}{\eta_{dch}} \right)
\end{align*}
2.2.2. Load flow constraints

\[
P_{PV_b}(t) + P_{WT_b}(t) + P_{DER_b}(t) + P_{dch-ESS_b}(t) - P_{ch-ESS_b}(t) - P_{DL_b}(t) - P_d(t) \\
= V_b(t) \sum_{b'=1 \atop b' \neq b}^{Nb} V_{b'}(t) \left( G_{bb'} \cos(\delta_b(t) - \delta_{b'}(t)) \right) \\
- B_{bb'} \sin(\delta_b(t) - \delta_{b'}(t)) \right) \tag{22}
\]

\[
Q_{DER_b}(t) - Q_{DL_b}(t) - Q_d(t) \\
= V_b(t) \sum_{b'=1 \atop b' \neq b}^{Nb} V_{b'}(t) \left( G_{bb'} \sin(\delta_b(t) - \delta_{b'}(t)) \right) \\
- B_{bb'} \cos(\delta_b(t) - \delta_{b'}(t)) \right) \tag{23}
\]

\[
\hat{P}_{PV_b}(t) + \hat{P}_{WT_b}(t) + \hat{P}_{DER_b}(t) + P_{dch-ESS_b}(t) - P_{ch-ESS_b}(t) - \hat{P}_{DL_b}(t) - \hat{P}_d(t) \\
= \hat{V}_b(t) \sum_{b'=1 \atop b' \neq b}^{Nb} \hat{V}_{b'}(t) \left( G_{bb'} \cos(\hat{\delta}_b(t) - \hat{\delta}_{b'}(t)) \right) \\
- B_{bb'} \sin(\hat{\delta}_b(t) - \hat{\delta}_{b'}(t)) \right) \tag{24}
\]

\[
\hat{Q}_{DER_b}(t) - \hat{Q}_{DL_b}(t) - \hat{Q}_d(t) \\
= \hat{V}_b(t) \sum_{b'=1 \atop b' \neq b}^{Nb} \hat{V}_{b'}(t) \left( G_{bb'} \sin(\hat{\delta}_b(t) - \hat{\delta}_{b'}(t)) \right) \\
- B_{bb'} \cos(\hat{\delta}_b(t) - \hat{\delta}_{b'}(t)) \right) \tag{25}
\]

Power flow equations of the slack bus can be expressed as follows:
\[ P_{UPN}(t) - P_d(slack,t) = \sum_{b'=1}^{Nb} V_{b'}(t) \left( G_{bb'} \cos(\delta_{b'}(t)) + B_{bb'} \sin(\delta_{b'}(t)) \right) \] (26)

\[ Q_{UPN}(t) - Q_d(slack,t) = \sum_{b'=1}^{Nb} V_{b'}(t) \left( G_{bb'} \sin(\delta_{b'}(t)) + B_{bb'} \cos(\delta_{b'}(t)) \right) \] (27)

\[ \hat{P}_{UPN}(t) - P_d(slack,t) = \sum_{b'=1}^{Nb} \hat{V}_{b'}(t) \left( G_{bb'} \cos(\hat{\delta}_{b'}(t)) + B_{bb'} \sin(\hat{\delta}_{b'}(t)) \right) \] (28)

\[ \hat{Q}_{UPN}(t) - Q_d(slack,t) = \sum_{b'=1}^{Nb} \hat{V}_{b'}(t) \left( G_{bb'} \sin(\hat{\delta}_{b'}(t)) + B_{bb'} \cos(\hat{\delta}_{b'}(t)) \right) \] (29)

2.2.3. DL Constraints

As a normal load behavior, it is assumed that DLs preserve a constant power factor (\(\cos(\varphi)\)) during the scheduling horizon.

\[ Q_{DLb}(t) = \tan(\varphi) \times P_{DLb}(t) \] (30)

\[ \hat{Q}_{DLb}(t) = \tan(\varphi) \times \hat{P}_{DLb}(t) \] (31)

2.2.4. The constraints on active and reactive powers exchanging with the UPN

\[ 0 \leq P_{UPN}(t) \leq P_{UPN-max} \] (32)

\[ Q_{UPN-min} \leq Q_{UPN}(t) \leq Q_{UPN-max} \] (33)

\[ 0 \leq \hat{P}_{UPN}(t) \leq P_{UPN-max} \] (34)

\[ Q_{UPN-min} \leq \hat{Q}_{UPN}(t) \leq Q_{UPN-max} \] (35)

2.2.5. Constraint on DERs

\[ P_{DERb-min} \leq P_{DERb}(t) \leq P_{DERb-max} \] (36)
\[ Q_{DER_b-min} \leq Q_{DER_b}(t) \leq Q_{DER_b-max} \]  
\[ P_{DER_b-min} \leq \hat{P}_{DER_b}(t) \leq P_{DER_b-max} \]  
\[ Q_{DER_b-min} \leq \hat{Q}_{DER_b}(t) \leq Q_{DER_b-max} \]  

2.2.6. Constraints on the reserve provided by the UPN and DERs

\[ \hat{P}_{UPN}(t) - P_{UPN}(t) \leq P_{UPN}(t) \]  
\[ \hat{P}_{DER_b}(t) - P_{DER_b}(t) \leq P_{DER_b}(t) \]  

2.2.7. Voltage constraint

\[ 0.90 \text{ p.u.} \leq V_b(t) \leq 1.10 \text{ p.u.} \]  
\[ 0.90 \text{ p.u.} \leq \hat{V}_b(t) \leq 1.10 \text{ p.u.} \]  

2.2.8. Constraints of the output power of the RESs

\[ P_{PV_b-min}(t) \leq \hat{P}_{PV_b}(t) \leq P_{PV_b-max}(t) \]  
\[ P_{WT_b-min}(t) \leq \hat{P}_{WT_b} \leq P_{WT_b-max}(t) \]  

3. Robust Particle Swarm Optimization (RPSO)

Here, the PSO approach is extended to an RPSO framework to tackle the max-min optimization problem. The proposed RPSO is accomplished based on an inner minimization search around any point visited by a particle to substitute the SW of the visited point with its worst value. By employing this method, all parts of the PSO formulation remains unchanged. Pseudo codes of the presented RPSO are given in the following.

```
1: Input data
2: For all particles (p=1, ..., Npop)
```
For all ESSs: Generating random feasible ESS schedule according to Eqs. (15-21).

End for

While convergence criteria have not been satisfied

For all particles (p=1, ..., Npop)

Scheduling the base condition based on solving bellow problem:

\[
z_1 = \max_{x_1} \sum_{t=1}^{T} SW(t)
\]

s.t.

ESS constraints: Eqs. (15-21)

Load flow constraints: Eqs. (22-23) and (26-27)

DL constraints: Eq. (30)

The UPN constraints: Eqs. (32-33)

DER constraints: Eqs. (36-37)

Voltage constraint: Eq. (42)

\[
\hat{u} = \min(\{\hat{P}_{PVb}(t), \hat{P}_{WTb}(t)\})
\]

according to Eqs. (44-45)

Scheduling for the worst case based on solving bellow problem:

\[
z_2 = \max_{x_2} \sum_{t=1}^{T} \tilde{SW}(t)
\]

s.t.

Load flow constraints: Eqs. (24-25) and (28-29)

DL constraints: Eq. (31)

The UPN constraints: Eqs. (34-35)

DER constraints: Eqs. (38-39)

Voltage constraint: Eq. (43)

Solving:
\[ z_3 = \max_{x_3} \sum_{t=1}^{T} -[C_{MG}^{re}(t) + C_{DERb}^{re}(t)] \]

s.t.

Reserve constraints: Eqs. (40-41)

12: \( z \leftarrow z1 + z2 + z3 \)
13: Updating position and velocity of all particles.
14: End while
15: Return robust solution

4. Simulation results

The proposed robust joint active/reactive and reserve scheduling in a grid-Connected MG is examined on a modified 33-bus MG as a case study displayed in Fig. 2. Two diesel generators (DGs), two ESSs along with four RESs, including two PVs and two WTs, are considered in this case study. Furthermore, it is supposed there is a DL located at bus 24.

Fig. 2. The modified 33-bus MG.
The hourly energy prices of the UPN reflecting the day-ahead prices are listed in Table 1. \( \xi \) as the only coefficient of the reactive cost function of the UPN are assumed to be 1\% of the hourly energy prices of the UPN.

**Table 1. The hourly prices of UPN**

<table>
<thead>
<tr>
<th>Hour</th>
<th>Price ($/MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.4</td>
</tr>
<tr>
<td>2</td>
<td>21.6</td>
</tr>
<tr>
<td>3</td>
<td>20.4</td>
</tr>
<tr>
<td>4</td>
<td>19.8</td>
</tr>
<tr>
<td>5</td>
<td>19.2</td>
</tr>
<tr>
<td>6</td>
<td>19.5</td>
</tr>
<tr>
<td>7</td>
<td>19.8</td>
</tr>
<tr>
<td>8</td>
<td>21</td>
</tr>
<tr>
<td>9</td>
<td>22.5</td>
</tr>
<tr>
<td>10</td>
<td>24</td>
</tr>
<tr>
<td>11</td>
<td>24.6</td>
</tr>
<tr>
<td>12</td>
<td>25.2</td>
</tr>
<tr>
<td>13</td>
<td>26.1</td>
</tr>
<tr>
<td>14</td>
<td>26.4</td>
</tr>
<tr>
<td>15</td>
<td>27.6</td>
</tr>
<tr>
<td>16</td>
<td>27.9</td>
</tr>
<tr>
<td>17</td>
<td>28.8</td>
</tr>
<tr>
<td>18</td>
<td>30</td>
</tr>
<tr>
<td>19</td>
<td>29.7</td>
</tr>
<tr>
<td>20</td>
<td>29.1</td>
</tr>
<tr>
<td>21</td>
<td>28.2</td>
</tr>
<tr>
<td>22</td>
<td>27.6</td>
</tr>
<tr>
<td>23</td>
<td>26.1</td>
</tr>
<tr>
<td>24</td>
<td>24.3</td>
</tr>
</tbody>
</table>

Table 2 provides the techno-economic data of the DGs, and the operational data of the ESSs are presented in Table 3. Table 4 represents the reserve prices offered by the UPN, the DGs and the DL.

**Table 2. Techno-economic data of the DGs.**

<table>
<thead>
<tr>
<th>Type</th>
<th>DG#1</th>
<th>DG#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus</td>
<td>6</td>
<td>28</td>
</tr>
</tbody>
</table>

The coefficients of the active cost function

| \( \gamma (\frac{\$}{MW^2h}) \) | 0 | 0 |
| \( \beta (\frac{\$}{MWh}) \)    | 2.1 | 2.3 |
| \( \alpha (\frac{\$}{h}) \)    | 21 | 22 |

The coefficients of the reactive cost function

| \( \lambda (\frac{\$}{MVar^2h}) \) | 0.21 | 0.23 |
| \( \mu (\frac{\$}{MVarh}) \)    | 0 | 0 |
| \( \omega (\frac{\$}{h}) \)    | 0 | 0 |
Max/min active/reactive power

<table>
<thead>
<tr>
<th></th>
<th>( P_{\text{max}} (kW) )</th>
<th>( P_{\text{min}} (kW) )</th>
<th>( Q_{\text{max}} (kVAr) )</th>
<th>( Q_{\text{min}} (kVAr) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{\text{max}} (kW) )</td>
<td>1000</td>
<td>0</td>
<td>500</td>
<td>-500</td>
</tr>
<tr>
<td>( P_{\text{min}} (kW) )</td>
<td>1000</td>
<td>0</td>
<td>500</td>
<td>-500</td>
</tr>
<tr>
<td>( Q_{\text{max}} (kVAr) )</td>
<td>500</td>
<td>0</td>
<td>500</td>
<td>-500</td>
</tr>
<tr>
<td>( Q_{\text{min}} (kVAr) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. The data of the ESSs.

<table>
<thead>
<tr>
<th>Bus</th>
<th>( P_{\text{ESS--dch--max}} (kW) )</th>
<th>( P_{\text{ESS--ch--max}} (kW) )</th>
<th>( E_{\text{ESS--max}} ) (kWh)</th>
<th>( E_{\text{ESS--min}} ) (kWh)</th>
<th>( E_{\text{ESS--initial}} ) (kWh)</th>
<th>( \eta_{\text{dch}} ) (%)</th>
<th>( \eta_{\text{ch}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>400</td>
<td>400</td>
<td>1200</td>
<td>240</td>
<td>720</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>22</td>
<td>650</td>
<td>650</td>
<td>1950</td>
<td>400</td>
<td>1170</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 4. The reserve prices of the UPN, the DGS and the DL.

<table>
<thead>
<tr>
<th>Reserve Provider</th>
<th>Bus</th>
<th>Price (($/MWh))</th>
</tr>
</thead>
<tbody>
<tr>
<td>UPN</td>
<td>-</td>
<td>30</td>
</tr>
<tr>
<td>DG#1</td>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>DG#2</td>
<td>28</td>
<td>22</td>
</tr>
<tr>
<td>DL</td>
<td>24</td>
<td>42</td>
</tr>
</tbody>
</table>

The hourly forecasted active power generations of PVs and WTs are delineated in Figs. 4 and 5, respectively.
The proposed method is examined on two scenarios as listed in Table 5. The simulation results are discussed in detail in the following. As observed, scenario S2 shows a higher range of uncertainty neighborhood around the forecasted power production of RESs.

### Table 5. The bounds of power production of RESs.

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$P_{PV_{b}-min}(t)$</th>
<th>$P_{PV_{b}-max}(t)$</th>
<th>$P_{WT_{b}-min}(t)$</th>
<th>$P_{WT_{b}-max}(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.9 $P_{PV_b}(t)$</td>
<td>1.1 $P_{PV_b}(t)$</td>
<td>0.9 $P_{WT_b}(t)$</td>
<td>1.1 $P_{WT_b}(t)$</td>
</tr>
<tr>
<td>S2</td>
<td>0.8 $P_{PV_b}(t)$</td>
<td>1.2 $P_{PV_b}(t)$</td>
<td>0.8 $P_{WT_b}(t)$</td>
<td>1.2 $P_{WT_b}(t)$</td>
</tr>
</tbody>
</table>

Fig. 5 illustrates the hourly charge and discharge power of the ESSs in scenario S1. The percentage of hourly load can also be plotted in Fig. 5. As expected, ESSs are charged during the low load period while in other hours, they are mostly discharged. Fig. 6 displays the SOC values of the ESSs in scenario S1. According to Fig. 6., the SOC initially increases and then decreases until the final SOC restriction for both ESSs is satisfied. Fig. 7 shows the hourly charge and discharge power of the ESSs in scenario S2, and their corresponding SOC values are depicted in Fig. 8. Similar to the obtained results in scenario S1, in low load period, the ESSs are charged, their SOC levels increase, and then they are discharged subsequently their SOC levels go down.
Fig. 5. The hourly charge and discharge power of the ESSs in scenario S1 and the hourly load percentage.

Fig. 6. The SOC of ESSs in scenario S1.
Figs. 7 and 8 demonstrate the scheduled reserve in scenarios S1 and S2, respectively. It can be seen that during the low load period, the required reserve is provided by the UPN. This is because the active power and reserve can be provided by the UPN at relatively low prices compared to the DGs in the low load period. In peak hours (i.e., hours 16, 17 and 18), because of the high prices of the electrical energy and reserve of the UPN, the DG at bus 28 presents most reserve. By carefully
examining Figs. 9 and 10, it can be observed that the reserve curves for scenarios S1 and S2 are similarly varied. Moreover, the hourly reserve values in S2 are higher than the ones in scenario S1, because a higher range of uncertainty neighborhood has been defined in the S2 compared to scenario S1. Hence, the higher reserve cost in scenario S2 can be expected (as demonstrated in Fig. 21).

![Fig. 9. The scheduled reserve in scenario S1.](image-url)
The hourly active power dispatch schedules of the UPN, DGs and DL in both scenarios S1 and S2 are provided in Figs. 11-14. Also, the hourly reactive power scheduling of these resources has been accomplished, as shown in Figs. 15-18. Since these figures express the optimal active/reactive dispatch values in the base case, which is the same for both scenarios, the hourly dispatch schedules of different resources are almost the same in both scenarios S1 and S2. Owing to the lower energy prices of the UPN in comparison with the DGs and DL in the low load period, active power is mainly provided by the UPN and the DGs generate more active power during other hours at which the energy prices of the UPN are relatively high.
Fig. 12. The scheduled active power of DG at bus 6 in scenarios S1 and S2.

Fig. 13. The scheduled active power of DG at bus 28 in scenarios S1 and S2.
Fig. 14. The hourly active power dispatch of DL at bus 24 in scenarios S1 and S2.

Fig. 15. The purchased reactive power from the UPN in scenarios S1 and S2.
The hourly voltage profiles of the network buses in scenarios S1 and S2 are plotted in Figs. 18 and 19, respectively. As can be observed, all voltages are within the permissible range. However, in peak hours, the voltage of buses with ESS, DG and RES is approaching the upper voltage limit. On the other side, buses 18 in S1 and 15 in S2 have the lowest voltages in the most hours.
Fig. 18. Hourly voltage profiles in scenario S1.

Fig. 19. Hourly voltage profiles in scenario S2.
Fig. 20 provides the SW values (i.e., $z$) with and without the ESSs as well as the different components of SW, i.e., $z_1$, $z_2$, and $z_3$, in both scenarios. From this figure, it can be seen the ESSs efficiently have improved the SW. Moreover, all components of the SW in scenario $S_2$ are less than the corresponding values in scenario $S_1$. This is due to that scenario $S_2$ takes a higher uncertain range into consideration for the output power production of the RESs. Thus, this situation leads to higher operating costs in the worst case and therefore, it will result in a decrease in the SW. Since considering a more uncertain production range of RESs imposes more cost, component $z_2$ has lower amount respect to component $z_1$ for both scenarios. Concerning the computed third component of SW, $z_3$, it can be inferred that more reserve cost should be paid by the system operator in scenario $S_2$.

![Fig. 20. The different components of the SW in scenarios S1 and S2.](image)

### 6. Conclusion

In this paper, a max-min optimization model to simultaneously manage the active/reactive power and reserve scheduling in the grid-connected MG in the presence of ESS units and dispatchable
loads has been presented to maximize social welfare. In the proposed robust problems, a maximization level makes an optimal robust decision against the worst-case solution obtained by another minimization layer, which makes solving them more challenging than classical optimization problems. The PSO approach is extended to a robust PSO framework to tackle the max-min optimization problem considering the uncertainty of wind speed and solar radiation. AC power flow is implemented in this paper to analyze the network and ESSs in order to present realistic and more practical models and results. The effectiveness of the proposed method has been evaluated on a 33-bus MG test system. In the simulation, two scenarios with different uncertainty ranges of RESs production have been regarded. The simulation results evidence that the ESSs are charged during low-cost off-peak hours and are discharged at on-peak high-cost hours. Moreover, they can reduce operational costs, and consequently, the social welfare will be significantly improved. The consideration of more uncertainty in RESs production leads to higher operation costs, especially reserve costs. It should be mentioned that integration of robustness against uncertainty in the joint active/reactive and reserve management in the smart MGs leads to a more robust operation at the expense of higher costs. The future research work will be addressed to utilize the EVs’ participation in the robust concurrent active/reactive power and reserve management.

**Abbreviations:**

DL: Dispatchable load  
ESS: Energy storage system  
UPN: Upstream network  
DER: Distributed energy resource  
RES: Renewable energy source  
SOC: State of charge
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Declarations

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Not applicable.

Consent for publication:
The authors have the authority to publish the research work in any publication.

Competing interests:
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Figure 6
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Figure 7

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Figure 8
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Figure 9
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