

# Solving the mystery of the walk-off soliton

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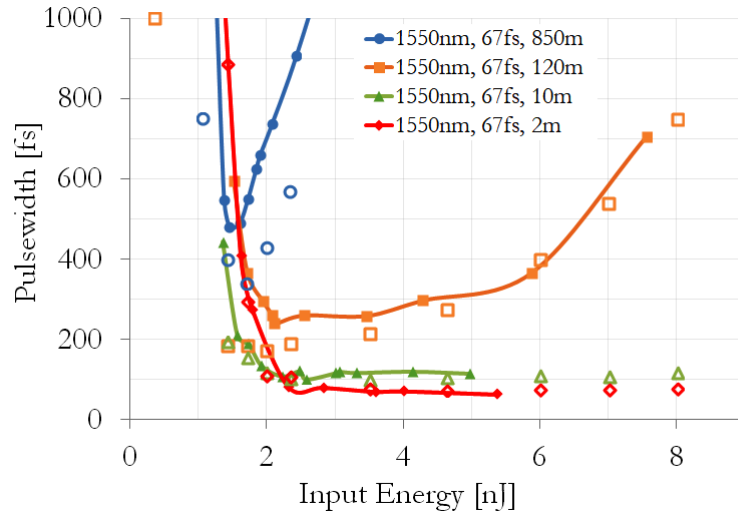
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## Supplementary information

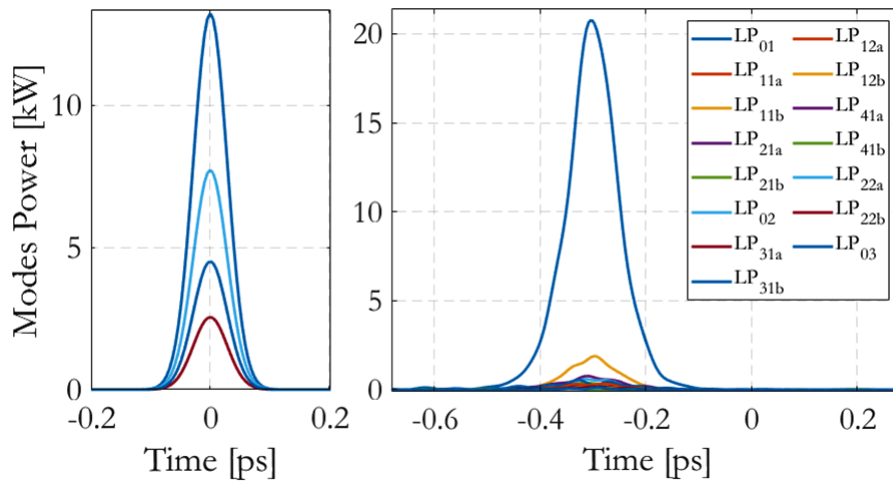
Experimental evidence with spans of GRIN fiber ranging from 2 m to 850 m (Supplementary Fig. 1), provided less stringent requirements for the optimal soliton energy as the fiber length reduces. At 850 m distance, 1550 nm and 67 fs input pulsewidth, a sharp input energy of 1.5 nJ is required to obtain a minimum pulsewidth of 550 fs at output; at 120 m, a minimum pulsewidth of 260 fs is measured for energy range between 2 nJ and 4 nJ; at 10 m and 2 m distance, pulsewidth remains minimum (110 fs and 60 fs respectively), for input energy larger than 2.5 nJ. Soliton pulsewidth increases with distance, as a consequence of the wavelength red-shift due to Raman SSFS, and the need to conserve the soliton energy condition  $E_1 = \lambda |\beta_2(\lambda)| w_e^2 / n_2 T_0$ , with  $T_0 = T_{FWHM} / 1.763$ ,  $n_2$  (m<sup>2</sup>/W) the nonlinear index coefficient,  $\beta_2(\lambda)$  the chromatic dispersion, and  $w_e$  the effective beam waist.

Numerical simulations (empty dots) substantially confirmed the experimental observations.



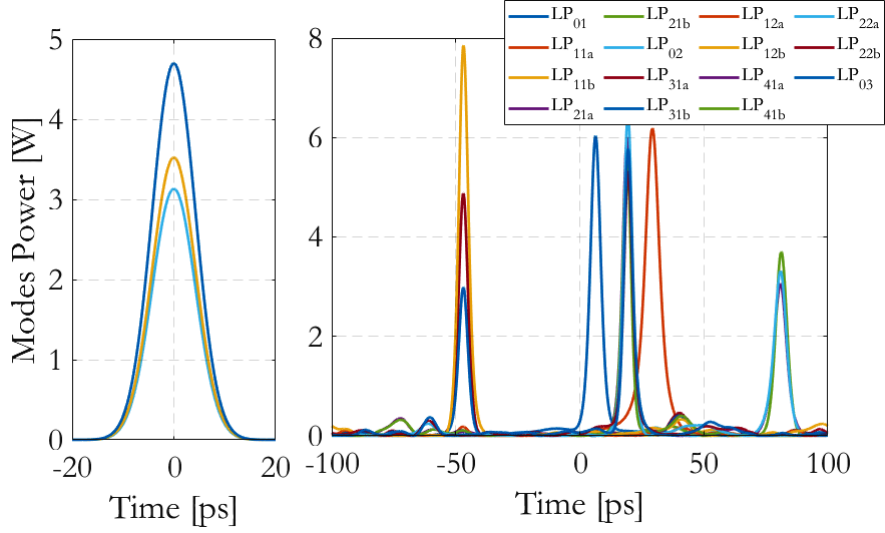
Supplementary Fig. 1 – Measured soliton pulsewidth vs. input energy, at 1550 nm wavelength, input pulsewidth 67 fs, input waist 15  $\mu\text{m}$ , and for GRIN fiber spans of length 2, 10, 120, 850 m.

In order to stress the generation of a monomodal soliton from several degenerate and non-degenerate modes, Supplementary Fig. 2 provides a simulation example of a 67 fs input pulse, 1550 nm, with input energy of 4 nJ. 15 modes are launched at input; the fraction of energy for the 3 axial modes is 24%, 13%, 8%; the remaining energy is distributed uniformly between the non-axial modes. After 20 m propagation, a spatiotemporal soliton has formed, with most of the energy transferred to the  $LP_{01}$  mode.



Supplementary Fig. 2 – Simulated ultra-short pulse mode evolution. Left: input pulse launched with 67 fs pulsewidth, 1550 nm wavelength, energy 4 nJ distributed on 15 modes. Right: Soliton propagated after 20 m of GRIN fiber.

What happens when larger pulses are launched? A spatiotemporal soliton is still possible? The numerical example of Supplementary Fig. 3, considers a 10 ps input pulse, 1550 nm, with energy 0.63 nJ. 15 modes are launched at input, divided into 5 groups of degenerate modes. Group energies were 0.05, 0.075, 0.1, 0.15, and 0.25 nJ respectively; energies within modes of each group are uniformly distributed. After 5 km of propagation in GRIN fiber, each of the 5 groups forms an independent spatiotemporal soliton. It could not be possible to produce a single spatiotemporal soliton.



Supplementary Fig. 3 – Simulated long pulse mode evolution. Left: input pulse with 10 ps pulsewidth, 1550 nm wavelength, input energy 0.62 nJ properly distributed on 15 modes. Right: Mode groups propagated after 5 km of GRIN fiber, each forming a spatiotemporal soliton.

In order to determine the mode content of the input laser beam and thus obtain the energy distribution over modes we performed the following mode decomposition procedure. The considered  $LP_{lm}$  modes are orthogonal and can be considered as a basis for laser beams. In this case, an input laser beam  $A(x, y)$  can be expressed as

$$A(x, y) = \sum_{l,m} \alpha_{lm} U_{lm}(x, y), \quad (\text{sup. 1})$$

where  $U_{lm}(x, y)$  is the spatial distribution of the mode  $LP_{lm}$ , and  $\alpha_{lm}$  is the decomposition coefficient, which determines the fraction of the laser beam energy contained in the mode  $LP_{lm}$ . Then, multiplying this expression from both sides by the conjugate spatial distribution of the considered mode and integrating over the x-y plane, we can obtain the decomposition coefficient:

$$\alpha_{lm} = \iint U_{lm}^*(x, y) A(x, y) dx dy. \quad (\text{sup. 2})$$