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Study on Algorithm of fault recording analysis combining its time-domain waveforms and phase-domain trajectories

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ABSTRACT When the period of a sinusoidal waveform is taken as a baseline, after sampling time of fault recording are transformed into polar angles and its instantaneous values are transformed into polar diameters, the recording is transformed from its time-domain waveform in a rectangular coordinate system into the phase-domain trajectory in a polar coordinate system. In the paper the recording is analyzed base on the combination of the time-domain waveform and the phase-domain trajectory of the fault recording to mine fault feature parameters for reformulating the recording, and then the reformulated recording is used to substitute the original one to achieve various fault analysis functions in power systems. A short-circuit current containing a dc component being taken as an example, the algorithm consisting of separating, reformulating and adjusting various components from the current waveform are researched. By the algorithm the programs are created in Matlab, after their running the post-adjustment reformulated waveforms with high accuracy are obtained, and their curves of absolute errors pre- and post-adjustment are plotted. The relational problems, such as the calculation of the phase angles of samples and their errors, the accuracy of the reformulated waveforms, etc., are discussed. The research results show that there is a high correlation between the reformulated waveform using the algorithm proposed in the paper and the original one for the fault recording with only dc components, and it can be used to substitute the origin one within an error range.

KEYWORDS fault recording; time-domain waveform; phase-domain trajectory; novel algorithm of fault recording analysis; correlation and errors between reformulated recording and its original one

1 Introduction

The functions and development of the various departments of national economy cannot be separated from the sustainable development of electrical energy. The untimely handling of faults in regional interconnected power systems may cause a large-scale blackout, and it is necessary to coordinate the equipment of protective relaying, and supervisory and control etc. for the safe and reliable operation of the systems [1][2]. In fault recording acquired by the digital devices such as fault recorders or microprocessor-based protection there contained the variation of electrical physical quantities in abnormal or fault states of power systems [3][4][5], which is applied to performance analysis of devices, fault detection, and protective setting [6] etc. In the fault recording, such as waveforms of currents and voltages, in addition to fundamental frequency components (FFCs) there are still dc, harmonic, and inter-harmonic components [7], etc. When fault recording is analyzed, the fault feature parameters (FFPs), containing the angular frequency, amplitude, and initial phase angle (IPA) of a FFC, the initial value and time constant of a decaying dc component (DDCC), the angular frequencies, amplitudes, and IPAs of each harmonic and inter-harmonic, can be used only after they are extracted from the waveforms.

In currently known recording analysis, there conducted mainly from the following three aspects: time-domain analysis, analysis combining time and frequency domain, and analysis using space vectors.

In the time-domain analysis of fault recording, there is a way where the eigenvalues of time-sequence features are calculated to identify the sections of single-phase-to-ground faults, and the distribution characteristics of the eigenvalues are applied to fault identification and classification by an improved K-means clustering algorithm [8]. There is also a way where fault detection and classification are completed by the voltage signals at the end of transmission lines based on a Euclidean distance measurement method [9]. And in literature [10] a criterion based on sequence components is generated by symmetrical components in fault currents and voltages to classify faults quickly without a threshold according to the different characteristics between fault and non-fault phases.

When fault recording is analyzed by combing time and frequency domain, there is a way where faults are identified by the method based on discrete S-transformation and fuzzy decision box according to the obvious differences of the alteration of the time-frequency amplitudes of currents caused by the faults of different distances and types [11]. And in literature [12] and [13] the discrete wavelet transformation (DWT) and back-propagation
neural network (BPNN) are applied to fault diagnosis, where the high-frequency components in fault currents are extracted by DWT, and the first-order coefficients detecting faults are studied to construct a BPNN-based decision algorithm.

In the analysis using space vectors, in literature [14] and [15] a method is proposed where fundamental positive sequence, negative sequence, and harmonic components are real-time detected, and three-phase instantaneous values are synthesized into one space vector to make it be synthesized by two rotating vector in an anticlockwise and clockwise direction and rotated in an appropriate angle, thus various components are separated from the fundamental component in real time and accurately.

In addition to the above-mentioned analysis methods, fault diagnosis is accurately achieved when fault analysis is combined with artificial intelligence (AI) algorithms, such as Kalman filter [16], expert system reasoning [17], Bayes estimation [18], petri nets [19], cluster analysis [20], and classical reasoning, etc. AI algorithms: neural network [21] and support vector machine [22] etc., fuzzy set theory [23], and rough set theory [24] etc. are also widely applied to the area of fault diagnosis using multi-source information fusion, and they are researched in a good prospect. However, in these studies there demanded a large number of data, and the computational processes are cumbersome. This increases the time cost and complexity of fault analysis and limits the depth and breadth of its application in power grids.

It is still not deep enough for all the present strategies of fault analysis to mine the characteristics of electrical physical quantities themselves. In this paper on the basis of the mapping relationship between time-domain waveforms (TDWs) and phase-domain trajectories (PDTs) of fault recording a fault analysis algorithm combing the TDWs and PDTs of fault recording is proposed.

2 Correspondence between TDWs of SCCs and their PDT

A short-circuit current (SCC) containing only a dc component is taken as an example, after its TDW and PDT are plotted, and the correspondence between them is discussed to study the method and process of fault recording analysis combining both of them.

2.1 The TDW and PDT of a SCC

The time-domain expression of A-phase current pre and post a three-phase short circuit in an infinite power supply system is shown as follows:

\[
i(t) = \begin{cases} 
\sin(100\pi t + 27.6^\circ), & 0 \leq t \leq 0.04s \\
4.36\sin[100\pi(t - 0.04) - 43.7^\circ] + 3.476e^{-(t-0.04)/0.05}, & t \geq 0.04s
\end{cases}
\]

In the SCC described by Equation (1), in addition to the power frequency fundamental component (PFFC) there is only the DDCC with the decay time constant (DTC) of 0.05s. It is supposed that the system frequency remains constantly 50Hz pre and post the short circuit, and the angular frequencies of the sinusoidal currents (SSCs) are all \(100\pi\) in the normal steady state (NSS) and the short-circuit steady state (SCSS).

In the software of Matlab, a polar coordinate system is created by function “polarplot(theta, rho)” and figures are plotted in it, where polar angle (PA) “theta” and polar diameter (PD) “rho” are two basic parameters. When \(\omega t\) is taken to be PA “theta” and \(i\) – PD “rho”, there are:

\[
\theta = \omega t, \rho = i
\]

After Equation (2) is substituted into Equation (1), the phase-domain expression of A-phase current is:

\[
\rho(\theta) = \begin{cases} 
\sin(\theta + 27.6^\circ), & 0 \leq \theta \leq 720^\circ \\
4.36\sin[(\theta - 720^\circ) - 43.7^\circ] + 3.476e^{-(\theta-720^\circ)/0.05}, & \theta \geq 720^\circ
\end{cases}
\]

In Equation (1) the phase angle of the SSC in the NSS (0 ≤ \(t\) ≤ 0.04s) is equal to 27.6° at \(t = 0s\). The current is started to be digitally sampled from this instant which is taken as a reference point (RP) (i.e., the 0-th sample point (SP)) of fault recording and \(\theta = 0^\circ\) (“RP” is described detailly in the 5th section), then the phase angle of the RP is equal to the IPA of the waveform of the SSC in the NSS and \(\phi_0 = 27.6^\circ\). The TDW and PDT in the time interval of 0s
~ 0.44s are plotted in Figure 1, where the SSC in the NSS (0 ≤ t ≤ 0.04s) and the SCC (t ≥ 0.04s) are indicated by the black and red solid lines respectively, and the DDCC and sinusoidal ac component (SACC) in the SCC – the green and blue dashed lines respectively.

Figure 1 The A-phase current pre and post a three-phase short circuit (0s ~ 0.44s): (a) the TDWs; (b) the PDTs

As can be seen from Figure 1(a), it is challenging to distinguish whether the dc component in the SCC still decays in the TDW at t = 0.2s. However, as can be seen from Figure 1(b), at this instant the red solid line and the blue dashed line in the PDT are still not completely overlapped, i.e., the SCC is still in the decaying process.

In order to compare the TDWs and PDTs pre and post the short circuit and in the SCSS, the TDWs in the time interval of 0s ~ 0.12s are shown in Figure 2, the PDTs in the time intervals of 0s ~ 0.06s and 0s ~ 0.08s – in Figure 3, and the TDWs and PDTs in the time interval of 1.96s ~ 2s – in Figure 4 in the SCSS.

Figure 2 The TDWs of the current pre and post the three-phase short circuit (0s ~ 0.12s)

In the TDWs of the time interval of 0 s ~ 0.12 s in Figure 2 there contained the SSC of 2 periods (black solid line) in the NSS pre the short circuit, the SCC of 4 periods (red solid line), the DDCC (green dashed line), and the SACC (blue dashed line) post the short circuit. The number and coordinate of each measure point (MP) are also marked in these figures, where a ~ j and o represent the number, and the values next to the signs of “X” and “Y” indicate the sampling time and currents respectively.
Figure 3 The PDTs of the current pre and post the three-phase short circuit: (a) 0s ~ 0.06s; (b) 0s ~ 0.08s

In the PDTs of the time interval of 0s ~ 0.06s in Figure 3(a) there contained the SSC of 2 periods (black solid-line circle) in the NSS pre the short circuit, the SCC of 1 period (red solid-line spiral), the DDCC (green dashed-line spiral) and the SACC (blue dashed-line circle) post the short circuit. There is one more period in the PDTs post the short circuit in Figure 3(b) than those in Figure 3(a), i.e., there contained the SSC of 2 periods in the NSS pre the short circuit, the SCC of 2 periods post the short circuit and its DDCC and SACC. The colors and styles of the lines in Figure 3(b) are completely the same as those in Figure 3(a).

MP a, b, c, d, g, h, and j in the PDTs shown in Figure 3(a) and MP c, d, e, and f in the PDTs shown in Figure 3(b) all are corresponded one-to-one with those in the TDWs shown in Figure 2 respectively. Point o and b are two separated ones in the TDWs shown in Figure 2, and their positions are different, but both of them are overlapped in the PDTs, thus point o is not marked. Similarly, point i and h are also two separated points in the TDWs shown in Figure 2 and their positions are different, while both of them are overlapped in the PDTs and point i is not marked.

“Theta” and “R” of each MP in the PDTs in Figure 3 indicate the PAs and PDs of the current at this point respectively, i.e., its phase angles and current values.

The TDWs and PDTs of the SCC in the time interval of 1.96s ~ 2s in the SCSS are shown in Figure 4, two MP k in the two figures are corresponded one-to-one with each other, and the colors and styles of the lines are the same as those in Figure 3.

As can be seen from Figure 4, the dc component in the SCSS has completely decayed to end, and the TDWs and PDTs of the SCC are all overlapped with those of the SSC in the SCSS respectively.
2.2 The correspondence between the TDW of the SCC and its PDT

The correspondence of the coordinate of each MP in the TDWs shown in Figure 2 with that in the PDTs shown in Figure 3, the correspondence of the coordinate of each MP in the TDWs shown in Figure 4(a) with that in the PDTs shown in Figure 4(b) are listed in Table 1 respectively.

Table 1 The comparison of the coordinate of each MP in the TDWs with that in PDTs

<table>
<thead>
<tr>
<th>MP</th>
<th>TDW (i(t))</th>
<th>PDT (\rho(\theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(o)</td>
<td>(X: t(s))</td>
<td>(Y: i)</td>
</tr>
<tr>
<td>(a)</td>
<td>0</td>
<td>0.463 3</td>
</tr>
<tr>
<td>(b)</td>
<td>0.003 45</td>
<td>7.36</td>
</tr>
<tr>
<td>(c)</td>
<td>0.047 3</td>
<td>-3.012</td>
</tr>
<tr>
<td>(d)</td>
<td>0.057 55</td>
<td>2.708</td>
</tr>
<tr>
<td>(e)</td>
<td>0.067 3</td>
<td>1311 / 311</td>
</tr>
<tr>
<td>(f)</td>
<td>0.077 3</td>
<td>-1.897</td>
</tr>
<tr>
<td>(g)</td>
<td>0.04</td>
<td>720 / 360</td>
</tr>
<tr>
<td>(h)</td>
<td>0.047 45</td>
<td>4.36</td>
</tr>
<tr>
<td>(i)</td>
<td>0.057 45</td>
<td>-4.36</td>
</tr>
<tr>
<td>(j)</td>
<td>0.04</td>
<td>3.476</td>
</tr>
<tr>
<td>(k)</td>
<td>1.967</td>
<td>4.36</td>
</tr>
</tbody>
</table>

* “\(\text{angle 1}^\circ\) / “\(\text{angle 2}^\circ\)”; “\(\text{angle 1}^\circ\)” is the phase angle of each MP based on the RP; “\(\text{angle 2}^\circ\)” is the value which “\(\text{angle 1}^\circ\)” is converted to a value between 0° and 360°.

** Position \(\theta_m\) is the angle of a MP between 0° and 360°. When the PD of a MP is negative, \(\theta_m\) is the angle of the point of positive PD which the MP is reflected to through the origin.

(1) Point \(o\): the RP in the TDWs and PDTs, IPA \(\varphi_0 = 27.6^\circ\) of the SCC in the NSS;
(2) Point \(a\): in the TDWs the 1st crest instant of the SCC in the NSS, in the PDTs the position of the 1st black solid-line circle;
(3) Point \(b\): in the TDWs the short-circuit instant, in the PDTs the start terminal of the maximum apple-shaped trajectory, i.e., the short-circuit reference point (SCRP);
(4) Point \(c\): in the TDWs the 1st crest instant of the SCC, in the PDTs the position of the maximum PD of the maximum apple-shaped trajectory;
(5) Point \(d\): in the TDWs the 1st trough instant of the SCC, in the PDTs the position of the maximum PD of the minimum balloon-shaped trajectory;
(6) point \(e\): in the TDWs the 2nd crest instant of the SCC, in the PDTs the position of the maximum PD of the 2nd largest apple-shaped trajectory;

Figure 4 The SCC in the time interval of 1.96s ∼ 2s: (a) the TDWs; (b) the PDTs
(7) Point \( f \): in the TDWs the 2nd trough instant of the SCC, in the PDTs the position of the maximum PD of the 2nd smallest balloon shaped trajectory;

(8) Point \( g \): in the TDWs the initial instant of the SSC in the SCSS, in the PDTs the start terminal of the blue dashed-line circle;

(9) Point \( h \): in the TDWs the 1st crest instant of the SSC in the SCSS, in the PDTs the position of the PD of the diameter of the blue-dashed line circle, and this diameter is through the origin;

(10) Point \( i \): in the TDWs the 1st trough instant of the SSC in the SCSS, it is overlapped with point \( h \) in the PDTs;

(11) Point \( j \): in the TDWs the initial instant of the DDCC, in the PDTs the start terminal of the spiral trajectory of the green dashed-line;

(12) Point \( k \): in the TDWs one of the crest instants of the SSC in the SCSS when the DDCC decays to end, in the PDTs the position of the PD of the diameter of the red solid-line circle, and this diameter is through the origin.

In either TDW or PDT the SCC in the SCSS have been already overlapped with its SSC in the SCSS.

2.3 The changing rules of the PDT of the SCC

In the PDTs shown in Figure 3(a), the black solid-line circle containing point \( a \) and \( b \) represents the PDT of the SSC in the NSS. Point \( b \) is corresponded to the short-circuit instant, and it is also the start terminal of the apple-shaped trajectory. Due to the DDCC in the SCC shown in the figure is greater than zero, the 1st crest instant located above the time axis is corresponded with the position of the maximum PD of the maximum apple-shaped trajectory, and the 1st trough instant located below the time axis – the position of the maximum PD of the minimum balloon-shaped trajectory. When the dc component decays, the “apple” becomes gradually smaller and the “balloon” – larger. The 1st largest apple trajectory and the 1st smallest balloon trajectory are shown in Figure 3(a), and the 2nd larger apple trajectory and the 2nd smaller balloon trajectory – in the PDT in Figure 3(b).

The TDWs of the SCC and its components shown in Figure 2 being compared, the changing rules of the PDTs shown in Figure 3 and 4 are as follows.

(1) The PDT of the SCC

When the dc component decays, the apple-shaped trajectory becomes smaller and simultaneously the balloon-shaped trajectory becomes larger, and both of the two trajectories approach the SSC circle in the SCSS. These two trajectories are overlapped and stabilize at the position of this circle, as shown by the overlapped red solid-line circle and blue dashed-line circle in the PDTs shown in Figure 4(b).

(2) The PDT of the SACC in the SCC

The PDT of the SACC in the SCC is the circle started from the short-circuit instant (point \( g \)) and is also the PDT of the SSC in the SCSS, i.e., the blue dashed-line circle shown in the PDTs in Figure 3.

(3) The PDT of the DDCC in the SCC

The PDT of the DDCC in the SCC is the spiral started from the short-circuit instant (point \( j \)) and decays when revolving around the origin, one circle every power frequency period (PFP). The PD decays to zero in the SCSS, and the decaying speed depends on the time constant. It is the green dashed-line spiral shown in the PDTs in Figure 3.

This proves, after the PDT of a SCC is formed by an appropriate way, the following short-circuit FFPs can be obtained from the analysis combining the PDT with the TDW of the SCC: the amplitude and phase angle of the SSC in the NSS, the short-circuit initial phase angle (SCIPA) (i.e., the IPA of the SACC at the short-circuit instant), the current at the short-circuit instant, the initial value and time constant of the DDCC, the amplitude of the SSC in the SCSS, etc. The original TDW can be thus reformulated by these features.

2.4 Analysis ways of fault recording

Due to the complexity of both faults themselves and fault recording, it is very difficult to analyze them both quickly and accurately, and the choice of ways and methods depends on the purpose of the analysis. If the fault recording is analyzed to find fault causes, determine fault types and identify the changes of system operation modes, which are conducive to developing measures to improve the safe and stable operation of power systems, the demand for real-time analysis is not high, while the demand for the accuracy of FFP is high. Offline analysis is suitable for
In particular, when the voltage and current waveforms are recorded by DFRs in such states as NSSs, transient states, SCSSs, and post-removal of short circuits, etc., it is also accessible that periodic and non-periodic components are separated from a SCC using the characteristics of circular and spiral PDTs, and other components in it such as harmonics etc. are considered further.

The function of fault recording is also included in such devices as microcomputer relay protection and automation, and it is mainly for the purpose of identifying fault types and determining fault distances in order to decide whether to send a tripping signal to the tripping circuit. In this case the real-time demand for the analysis of fault recording is relatively high, while the accuracy demand for the FFPs is not as high as that in offline analysis, as long as it is met. Therefore, online analysis is necessary to be applied.

Whether offline or online analysis is applied to fault recording, there are inevitably errors between the reformulated TDW and the original waveform. Therefore, it is necessary that a reformulated waveform which meets certain accuracy demand is used as the analysis result.

3 Offline analysis of fault recording

3.1 Calculating the TDW from the circular PDT of the SSC in the NSS

The PDT of a sinusoidal function is circular, and its diameter is equal to the amplitude of the sinusoidal waveform (SSW). The relationship between the circular position $\theta_m$ (i.e., the PA of the diameter through the origin) and the IPA $\phi_0$ of the waveform is determined by the following equation:

$$\phi_0 + \theta_m = 90°$$

It is uncomplicated to obtain the amplitude and IPA of the corresponded sinusoidal TDW from the diameter (through the origin) of the circular PDT and its position. When they are calculated an attention should be paid that when the PD of this diameter is positive, it is more convenient to convert its PA to the value between 0° and 360° before it is used. If the PD of this diameter is negative, it should be firstly reflected through the origin to the position where its PD is positive, and then its PA is converted to the convenient value between 0° and 360°.

For instance, in the PDT of the SSC in the NSS pre the short circuit shown in Figure 3(a), point $a$ is the position of the diameter (through the origin) of the PDT circle, its PD and PA are 1 and 62.1° respectively. From that the PD is equal to 1 the amplitude of the SSC is obtained to be 1. The PA of the diameter (through the origin) of the PDT circle is also the position of the circle $\theta_m = 62.1°$, and the calculated value of the IPA of the SSC is obtained by Equation (3) to be:

$$\phi_{0C} = 90° - 62.1° = 27.9°$$

The PA and PD (their values are already listed in Table 1) of RP $a$ and SCRP $b$ are found from the PDT shown in Figure 3(a), and the duration of the SSC in the NSS is calculated to be 0.04s. The frequency of the SSC in the NSS has been calculated from the sampling time and currents of the SPs before the PDT is plotted and it is supposed to be 50 Hz, and the expression of the initial reformulated TDW of this SSC is:

$$i_{ph}(t) = \sin (100\pi t + 27.9°), \ 0 \leq t \leq 0.04 \text{s}$$

Equation (4) being compared with the original function shown in Equation (1), calculated IPA $\phi_{0C} = 27.9°$ obtained from the PDT circle is not equal to actual IPA $\phi_0 = 27.6°$, and this results in errors between the initial reformulated TDW and the original one. The curve of absolute errors calculated by each SP is shown in Figure 5(a), where the maximum absolute error ($e_{max} = -0.005236$) is marked.
3.2 The correlation analysis between the reformulated SSW and its original one in the NSS

After such features as the amplitude and IPA of the sinusoidal TDW from the circular PDT are obtained, the similarity between the reformulated TDW and the original one is required to be measured using indexes. Obviously, it is not appropriate to use the curve of absolute errors shown in Figure 5(a).

The similarity or interdependency between two signals is measured by a correlation coefficient and a relative error. In a certain time interval of, e.g., \( t_1 \sim t_2 \), when signal \( y(t) \) is used to approximate signal \( x(t) \), the similarity or interdependency of the two signal waveforms is measured by correlation coefficient \( c_{xy} \), which is calculated by Equation (5):

\[
c_{xy} = \frac{\int_{t_1}^{t_2} x(t)y(t)dt}{\sqrt{\int_{t_1}^{t_2} x^2(t)dt} \sqrt{\int_{t_1}^{t_2} y^2(t)dt}}
\]

In this time interval when signal \( y(t) \) is used to approximate signal \( x(t) \) the relative error is:

\[
\varepsilon = 1 - c_{xy}^2
\]

Actual IPA \( \phi_0 \) of the original SSW is unknown pre its determination. In order to obtain calculated IPA \( \phi_{0C} \) with a tiny error from actual IPA \( \phi_0 \), the sample window (SW) of \( \Delta \phi \) is taken as the adjustment range of a phase angle (the definition and calculation of a SW are detailly in the 5th section), and actual IPA \( \phi_0 \) is located in the following interval centered around calculated IPA \( \phi_{0C} \):

\[
[\phi_{0C} - \Delta \phi, \phi_{0C} + \Delta \phi]
\]

The above interval is taken as an initial interval, calculated IPA \( \phi_{0C} \) – an initial value for the adjustment, and the IPA meeting an accuracy demand will be found in it using a dichotomy method. The accuracy demand is expressed by correlation coefficient \( c_M \) between the reformulated waveform and the original one, and when the coefficient calculated by Equation (5) meets the following condition:

\[
c_{xy} \geq c_M
\]

the adjustment process is ended. The middle value of the present interval is exactly the calculation result meeting the accuracy demand.

The algorithm and process of the adjustment of a phase angle are shown in Figure 6.
The calculated IPA obtained from the PDT circle shown in Figure 3(a) is \( \phi_{0C} = 27.9^\circ \). When the adjustment range is taken to be the value of SW \( \Delta \phi \), which is 0.9°, actual IPA 27.6° is located in interval \([27.0^\circ, 28.8^\circ]\). Calculated IPA 27.9° is taken as the initial value, and the adjustment program of the IPA is created using the algorithm shown in Figure 6. The accuracy demand is taken as that the correlation coefficient is not less than \( c_M = 1 - 10^{-10} \). The data in the adjustment process are listed in Table 2.

**Table 2** The adjustment process of the IPA of the SSW in the NSS

<table>
<thead>
<tr>
<th>No.</th>
<th>Adjustment intervals of the IPA (°)</th>
<th>Calculated IPA ( \phi_{0C} ) (°)</th>
<th>Error of IPA (°)</th>
<th>Correlation coefficient ( c_{xy} )</th>
<th>*Relative error ( \varepsilon )</th>
<th>**Maximum absolute error ( \varepsilon_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.562 5 5</td>
<td>27.675</td>
<td>0.01875</td>
<td>0.999999946453970</td>
<td>1.07092×10^{-7}</td>
<td>0.000327245</td>
</tr>
<tr>
<td>5</td>
<td>27.590 625 27.61875</td>
<td>27.64875</td>
<td>0.0046875</td>
<td>0.999999996653373</td>
<td>6.96325×10^{-9}</td>
<td>-8.18112×10^{-5}</td>
</tr>
<tr>
<td>7</td>
<td>27.59765625</td>
<td>27.6017188</td>
<td>0.001171875</td>
<td>0.99999999790836</td>
<td>0.002617937</td>
<td>0.002617937</td>
</tr>
<tr>
<td>8</td>
<td>27.59765625</td>
<td>27.6017188</td>
<td>0.000585938</td>
<td>0.9999999947709</td>
<td>1.04582×10^{-10}</td>
<td>-1.02264×10^{-8}</td>
</tr>
</tbody>
</table>

* Relative error \( \varepsilon \) is calculated by Equation (6).

** Maximum absolute error \( \varepsilon_{\text{max}} \) is the maximum among the absolute errors between the reformulated waveform and the original one (see Figure 5).

As can be seen from Table 2, when \( n = 9 \), the correlation coefficient between the reformulated function and the original one has exceeded \( c_M = 1 - 10^{-10} \). The adjustment result of the IPA is 27.599 414 06°, which is smaller than actual IPA 27.6° by 0.000 585 938°. The reformulated sinusoidal function meeting the accuracy demand is changed from Equation (4) to be:
The curve of the absolute errors between the reformulated sinusoidal function and the original one is shown in Figure 5(b), where the maximum absolute error $e_{\text{max}} = -1.023 \times 10^{-5}$ of the reformulated SSW post-adjustment is marked.

### 3.3 Calculating the sinusoidal TDW from the circular PDT of the SSC in the SCSS

Following the same method as analyzing the SSC in the NSS, the amplitude and the calculated SCIPA of the SSW in time interval $1.96s \sim 2s$ in the SCSS are obtained from the PD (4.36) and PA (134°) of point $k$ in the PDT in Figure 4(b) and Table 1, which are 4.36 and $\phi_{\text{calc}} = -44^\circ$ respectively. When MP $o$ is taken as a RP, the expression of the initial reformulated function of the SSW in this time interval is:

$$i_p(t) = 4.36 \sin \left[100\pi \left(t - 1.96\right) - 44^\circ\right], \quad 1.96s \leq t \leq 2s \quad (7)$$

As can be seen with reference to the expression of the original waveform, which is Equation (1), there is an error between calculated SCIPA $\phi_{\text{calc}} = -44^\circ$ and actual SCIPA $\phi_{\text{act}} = -43.7^\circ$. The curve of the absolute errors between the initial reformulated SSW and the original one caused by the error is shown in Figure 7(a), where the maximum absolute error ($e_{\text{max}} = 0.02283$) is marked.

### 3.4 The correlation analysis between the reformulated SSW and its original one in the SCSS

When IPA $-44^\circ$ is taken as the initial value and the adjustment range is taken to be the value of SW $\Delta\phi$, which is $0.9^\circ$, actual IPA $-43.7^\circ$ is located in the initial interval of $[-44.9^\circ, -43.1^\circ]$ of the phase angle adjustment. Following the same way as the phase angle adjustment of the SSC in the NSS, the accuracy demand is taken as that the correlation coefficient is not less than $c_M = 1 - 10^{-10} (0.999 999 999 9)$, and the initial reformulated waveform of the SCC in the SCSS is adjusted using the algorithm shown in Figure 6, the data in the adjustment process are listed in Table 3.

#### Table 3 The adjustment process of the IPA of the SSW in the SCSS

<table>
<thead>
<tr>
<th>No.</th>
<th>Adjustment interval of the IPA $^\circ$</th>
<th>Calculated IPA $\phi_{\text{calc}}$ $^\circ$</th>
<th>Error of IPA $^\circ$</th>
<th>Correlation coefficient $c_y$</th>
<th>*Relative error $e$</th>
<th>**Maximum absolute error $e_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-44.9</td>
<td>-43.1</td>
<td>-44</td>
<td>-0.3</td>
<td>0.999986292247427</td>
<td>2.74153$\times 10^{-5}$</td>
</tr>
<tr>
<td>1</td>
<td>-44</td>
<td>-43.1</td>
<td>-43.55</td>
<td>-0.05</td>
<td>0.999996573055985</td>
<td>6.85388$\times 10^{-6}$</td>
</tr>
<tr>
<td>2</td>
<td>-44</td>
<td>-43.55</td>
<td>-43.75</td>
<td>-0.075</td>
<td>0.99999143636296</td>
<td>1.71347$\times 10^{-6}$</td>
</tr>
<tr>
<td>3</td>
<td>-43.775</td>
<td>-43.55</td>
<td>-43.6625</td>
<td>0.0375</td>
<td>0.99999785185885</td>
<td>4.28368$\times 10^{-7}$</td>
</tr>
<tr>
<td>4</td>
<td>-43.775</td>
<td>-43.6625</td>
<td>-43.71875</td>
<td>-0.01875</td>
<td>0.99999946453970</td>
<td>1.07092$\times 10^{-7}$</td>
</tr>
<tr>
<td>5</td>
<td>-43.71875</td>
<td>-43.6625</td>
<td>-43.690625</td>
<td>0.009375</td>
<td>0.99999986613492</td>
<td>2.673$\times 10^{-8}$</td>
</tr>
<tr>
<td>6</td>
<td>-43.71875</td>
<td>-43.690625</td>
<td>-43.7046875</td>
<td>-0.0046875</td>
<td>0.99999996653373</td>
<td>6.69325$\times 10^{-9}$</td>
</tr>
<tr>
<td>7</td>
<td>-43.7046875</td>
<td>-43.690625</td>
<td>-43.69765625</td>
<td>0.00234375</td>
<td>0.99999999163343</td>
<td>1.6731$\times 10^{-9}$</td>
</tr>
<tr>
<td>8</td>
<td>-43.7046875</td>
<td>-43.69765625</td>
<td>-43.7017188</td>
<td>-0.001171875</td>
<td>0.99999999790836</td>
<td>4.18328$\times 10^{-10}$</td>
</tr>
<tr>
<td>9</td>
<td>-43.7017188</td>
<td>-43.69765625</td>
<td>-43.69941406</td>
<td>0.000585938</td>
<td>0.99999999947709</td>
<td>1.04582$\times 10^{-10}$</td>
</tr>
</tbody>
</table>

Figure 7 The curves of absolute errors of the reformulated function pre and post the IPA adjustment of the SSC in the SCSS:

(a) pre-adjustment; (b) post-adjustment
As can be seen from Table 3, when \( n = 9 \), the correlation coefficient between the reformulated function and the original one has exceeded \( c_M = 1 - 10^{-10} \) \((0.9999999999)\) and meets the accuracy demand. The adjustment result of the IPA is -43.699 414 06°, which is larger than actual IPA -43.7° by 0.000 585 938°. The reformulated sinusoidal function meeting the accuracy demand is changed from Equation (7) to be:

\[
i_p(t) = 4.36\sin\left[100\pi \left(t - 1.96\right) - 43.69941406^\circ\right], \ 1.96 s \leq t \leq 2 s
\]  

(8)

The curve of the absolute errors between the reformulated sinusoidal function and the original one is shown in Figure 7(b), where the maximum absolute error \( e_{max} = 4.459 \times 10^{-5} \) of the reformulated SSW post-adjustment is marked.

**3.5 Calculating the DDCC from the TDW of the SSC**

Equation (8) represents the sinusoidal ac current in the SCSS, from which the expression of the SACC in the time interval of 0.04 s ~ 0.44 s in the transient process is directly written to be:

\[
i_p(t) = 4.36\sin\left[100\pi \left(t - 0.04\right) - 43.69941406^\circ\right]
\]  

(9)

After the SAAC shown by Equation (9) is subtracted from the SCC shown by Equation (1), the initial time-domain expression of the DDCC is obtained to be:

\[
i_{ap}(t) = 3.476e^{-\left(-0.04/0.05\right)} - 1.022658222 \times 10^{-5} \cos\left[100\pi \left(t - 0.04\right) - 43.69970703^\circ\right]
\]  

(10)

Its waveform is shown in Figure 8.

![Figure 8 The DDCC in the SCC](image)

In Equation (9) which is the expression of the SAAC in the SSC reformulated from the sinusoidal ac current in the SCSS, the IPA is -43.699 414 06°, and the IPA of the original SAAC in Equation (1) is -43.7°. The error between these two IPAs causes the errors between the DDCC shown in Equation (10) and the original one in Equation (1), i.e., the waveform shown in Figure 8 is not an accurate expression of the DDCC. From Equation (10) the errors are as follows:

\[
\Delta i_{ap}(t) = -1.022658222 \times 10^{-5} \cos\left[100\pi \left(t - 0.04\right) - 43.69970703^\circ\right]
\]

Therefore, such small errors cannot be clearly observed in the waveform shown in Figure 8.

When the initial value and time constant of the DDCC are expressed as \( M \) and \( T_a \) respectively, its TDW is expressed as:

\[
i_{ap}(t) = Me^{-\left(-t/0.04\right)/T_a}, \ 0.04 s \leq t \leq 0.44 s
\]  

(11)

When any two points are taken in the waveform (e.g., point \( m(t_m, i_m) \) and \( n(t_n, i_n) \)) in Figure 8, the two
coordinates are substituted into Equation (11), the expressions in the time interval of 0.04 s ~ 0.44 s in the transient
process are:

\[
\begin{align*}
  i_m &= Me^{-(t_m - 0.04)/T_a} \\
  i_n &= Me^{-(t_n - 0.04)/T_a}
\end{align*}
\]

This set of equations are solved to be:

\[
\begin{align*}
  T_a &= \frac{(t_n - t_m)}{(\ln i_m - \ln i_n)} \\
  M &= i_m e^{(t_m - 0.04)/T_a}
\end{align*}
\] (12)

After time constant \( T_a \) and initial value \( M \) calculated by Equation (12) are substituted back into Equation (11),
the expression of the initial reformulated TDW of the DDCC in the SCC is then obtained.

3.6 The correlation analysis between the reformulated waveform of the DDCC and its original one

There are errors between the waveform of the DDCC shown in Equation (10) or Figure 8 and the original one in
Equation (1), while Equation (11) and (12) are calculated from the two coordinates in Figure 8, thus there must also
be errors between the TDW of the DDCC reformulated by them and the original one in Equation (1). In order to
measure the correlation between these two waveforms and improve the accuracy of the reformulated one, from the
DDCC shown in Figure 8 a certain segment is taken as an adjustment object according to time intervals, after time
constant \( T_a \) and initial value \( M \) are calculated by the coordinates of the two boundary points of the segment and
Equation (12), the DDCC is reformulated. When the accuracy demand between the reformulated waveform and the
original one is met, the adjustment process is completed.

When the initial instant where the dc component starts to decay is taken as a start terminal and the instant where
it decays to just less than or equal to 1% of the initial value \( M \) – an end terminal, the waveform to be adjusted of this
segment is described as an initial adjustment object. The duration from the start terminal to the end one is described
as an initial time interval, which is denoted by \([t_m, t_n]\), i.e., point \( m \) is the start terminal where the dc component
decays, and point \( n \) is the terminal where it decays to just less than or equal to 1% of the initial value \( M \). The
waveform of the dc component shown in Figure 8 is taken as an example, there are:

\( i_m = M, i_n \leq 1\%M \)

Correlation coefficient \( c_M \) between the reformulated waveform and the original one (i.e., the DDCC in Equation
(1)) is taken as an accuracy demand. Time constant \( T_a \) and decaying initial value \( M \) of the initial adjustment object
are calculated by Equation (12) using the coordinates of boundary point \( m(t_m, i_m) \) and \( n(t_n, i_n) \) of initial time interval
\([t_m, t_n]\), and then they are substituted back into Equation (11) to calculate correlation coefficient \( c_{xy} \) between the
reformulated waveform and the original one. When condition \( c_{xy} \geq c_M \) is satisfied, it means that the reformulated
DDCC meets the accuracy demand, and the adjustment process of the waveform is completed. When condition \( c_{xy} \geq c_M \)
is not satisfied, it means that the reformulated DDCC calculated from the coordinates of the two boundary points
of initial time interval \([t_m, t_n]\) does not meet the accuracy demand. The waveform of the initial adjustment object is
dichotomized into two segments by the time interval, the first one is taken as a new adjustment object, after its \( T_a \)
and \( M \) are calculated, the DDCC is reformulate, and the correlation coefficient between the reformulated waveform
and the original one is calculated to determine whether its accuracy meets the demand. The adjustment process is
detailed in the block chart of the program shown in Figure 9.
Calculate the original data of a DDCC, find decay initial value $M$, sampling time $t_{m}$ when $i_{m} = M$, set accuracy $c_{M}$.

Calculate $i_{n} = 1\%M$, sample time $t_{s}$, set initial time interval $[t_{m}, t_{n}]$, let the start and end terminal $t_{s} = t_{m}$, $t_{e} = t_{n}$.

Substitute the initial end terminal back: $t_{e} = t_{e}$.

Use the coordinates of point $m(t_{m}, i_{m})$ and $n(t_{n}, i_{n})$ to calculate $T_{a}$ and $M$ of the waveform in time interval $[t_{m}, t_{n}]$ and reformulate the DDCC.

Calculate correlation coefficient $c_{xy}$ between the reformulated waveform and the original one in time interval $[t_{m}, t_{n}]$.

Output $T_{a}$, $M$, correlation coefficient $c_{xy}$, relative errors, the curves of the absolute errors, save $T_{a}$ and $M$ of the maximum correlation coefficient.

Use calculated values $T_{a}$ and $M$ of the waveform in this time interval to reformulate the DDCC of initial interval $[t_{s}, t_{e}]$, calculate correlation coefficient $c_{xy}$ between the reformulated waveform and the original one.

Calculate midpoint $t_{p}$ of time interval $[t_{m}, t_{n}]$, set $t_{e} = t_{p}$.

Extend the length of the adjustment interval to one power frequency period, i.e., $t_{n} = t_{m} + 0.02$.

$t_{n} - t_{m} \leq 0.02s$?

$c_{xy} \geq c_{M}$?

$Y$

$t_{n} > t_{e}$?

$Y$

$N$

$t_{n} - t_{m} \leq 0.02s$?

$Y$

Use the calculated values $T_{a}$ and $M$ of the maximum correlation coefficient in all time intervals as the final results, reformulate the post-adjustment DDCC.

$N$

$t_{m} = t_{n}$

$N$

$t_{m} = t_{s}$?

$N$

$Y$

$Y$

$N$

$Y$

$N$

$Y$

$N$

Figure 9 The block chart of the program achieving the adjustment algorithm of the DDCC.
In the adjustment algorithm of the DDCC shown in Figure 9, after $T_a$ and $M$ of each segment are calculated and the waveform of this segment is reformulated, whether the accuracy meets the demand or not, the initial adjustment object is reformulated using $T_a$ and $M$ and its accuracy is determined. If the accuracy of the reformulated waveform meets the demand, there is no need to continue the subsequent adjustment process, this reformulated waveform and other parameters and errors are output immediately.

It should be noted that in the adjustment process of the DDCC, if the duration of a certain segment is equal to one PFP, regardless of whether the accuracy of the reformulated waveform meets the demand or not, there is no need to continue the subsequent adjustment of the waveform of this time interval. If all time intervals after the adjustment is completed are equal to or less than one PFP and all the reformulated waveforms do not meet the accuracy demand, $T_a$ and $M$ of the segment with the largest correlation coefficient are taken to reformulate the DDCC, and it is exactly the final result of the adjustment.

Two points of $m$ and $n$ are taken in the DDCC waveform shown in Figure 8. Point $m$ is the short-circuit instant, which is:

$$t_m = 0.04 \text{s}, \quad i_m = 3.475 \text{967 764}$$

The value of 3.475 967 764 is actually the initial value of the DDCC, i.e., $M = 3.475 \text{967 764}$. As can be seen from Figure 8, when the DC component decays to less than 1% of the initial value $M$, the decaying is nearly completed. Point $n$ is taken to be the one where the DC component is less than or equal to 1% of $M$, there are:

$$t_n = 0.270 \text{35 s}, \quad i_n = 0.034 \text{7}$$

The time constant and initial value are calculated by Equation (12) using the two coordinates $(t_m, i_m)$ and $(t_n, i_n)$ of point $m$ and $n$, there are:

$$T_a = 0.050 \text{011 177 313 940 1 s, } M = 3.475 \text{967 764}$$

They are substituted into Equation (11), the time-domain expression of the reformulated DDCC is obtained as:

$$i_{ap}(t) = 3.475967764 e^{-\frac{(t-0.04)}{0.0500111773139401}}, \quad 0.04 \leq t \leq 0.44 \text{s}$$

When $T_a = 0.050 \text{011 177 313 940 1 s and } M = 3.475 \text{967 764}$ are taken as the initial values and interval $0.04 \text{ s} \sim 0.270 \text{35 s}$ – the initial time interval of the adjustment, a program is created using the algorithm shown in Figure 9 to adjust the reformulated DDCC shown in Equation (13). The accuracy demand is taken as that the correlation coefficient is not less than $c_M = 1 - 10^{-10} (0.999 999 999 9)$, and the data in the adjustment process of the DDCC achieved using the algorithm shown in Figure 9 are listed in Table 4.

<table>
<thead>
<tr>
<th>No.</th>
<th>Time interval (s)</th>
<th>Initial value $M$</th>
<th>Calculated time constant $T_a$ (s)</th>
<th>Error of time constant $\Delta T_a$ (s)</th>
<th>Correlation coefficient $c_{xy}$</th>
<th>Relative error $\varepsilon$</th>
<th>Maximum absolute error $e_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.04</td>
<td>0.27035</td>
<td>3.475967764</td>
<td>0.0500111773139401</td>
<td>1.11773×10^{-4}</td>
<td>0.999999993807655</td>
<td>1.23847×10^{-8}</td>
</tr>
<tr>
<td>1</td>
<td>0.04</td>
<td>0.0688</td>
<td>3.475967764</td>
<td>0.0500003072262776</td>
<td>3.07226×10^{-7}</td>
<td>0.9999999998369</td>
<td>3.26295×10^{-12}</td>
</tr>
</tbody>
</table>

As can be seen from Table 4, the time constant and decay initial value of the DDCC meeting the accuracy demand are respectively as follows:

$$T_a = 0.050 \text{000 307 226 277 6 s, } M = 3.475 \text{967 764}$$

The expression of the reformulated TDW of the DDCC is:

$$i'_{ap}(t) = 3.475967764 e^{-\frac{(t-0.04)}{0.0500003072262776}}, \quad 0.04 \leq t \leq 0.44 \text{s}$$

Equation (13) and (14) represent the function expressions of the DDCC pre- and post-adjustment, and the curves of the absolute errors between either of them and the expression in Equation (1) are shown in Figure 10(a) and (b) respectively, where the maximum absolute errors are marked.
3.7 The correlation analysis between the reformulated waveform of the fault recording and its original one

When the accuracy demands for the reformulated waveforms of the SSC in the NSS and each component in the SCC are the same, for example, the correlation coefficient between the reformulated waveform and the original one is demanded to be greater than or equal to $1 - 10^{-10}$ ($0.9999999999999$), the reformulated waveform meeting the demand is obtained by the values in Table 2, 3, and 4 to be:

$$i'(t) = \begin{cases} 
\sin(100\pi t + 27.59941406^\circ), & 0 \leq t \leq 0.04s \\
4.36\sin\left[100\pi (t - 0.04) - 43.69941406^\circ\right] + 3.475967764e^{-(t-0.04)/0.050000037226776}, & t \geq 0.04s 
\end{cases}$$

(15)

The correlation coefficient between the waveform represented by Equation (15) and the one by Equation (1) is $0.999999999999951432$ post calculation, and the relative error is $4.85678164352521 \times 10^{-11}$. The curve of the absolute errors is shown in Figure 11, where the maximum absolute error is marked ($6.393 \times 10^{-5}$).

4 Online analysis of fault recording

In the devices of microprocessor-based protection, especially ones requiring quick action, the real-time demand for fault recording analysis is high, and the fault waveform that can be used for analysis is often shorter than that in offline analysis. For example, the action time of ordinary quick protection is up to 0.12s [25], the fault transient process in the recorded waveform lasts for only 6 periods. In order to meet the speediness requirement for protection, in the online analysis the SACC and DDCC have to be separated from the SCC quickly based on the waveforms of only 6 periods.

The online analysis of the SSW in the NSS and the adjustment process of the reformulated waveform are
completely the same as that in offline analysis. There are DDCCs and SACCs in SCCs. Due to the lack of the SSW in the SCSS, the way of separating the SACC from the SCC before analyzing the dc component is very difficult to conduct smoothly in the online analysis of fault recording in quick protection.

4.1 Calculating the DDCC and SACC in the SCC (40ms)

As can be seen from Figure 3(b), the DDCC can be calculated using the magnitudes of the decrease of the “apple shape” and the increase of the “balloon shape” and their phase angles in the PDTs. In the SCC shown in Figure 1(a) there are only one SACC and one DDCC, and the expression of its TDW is:

\[
i_k(t) = i_p(t) + i_{ap}(t) = A_k \sin \left[100\pi (t - 0.04) + \varphi_{ao}\right] + Me^{-(t-0.04)/T_a}, \ t \geq 0.04 \text{s}
\]  

MP\( c \) and \( e \) in the PDT shown in Figure 3(b) are the positions of the maximum PDs and their PAs of the largest and second largest “apple shapes” respectively, and MP\( d \) and \( f \) are the positions of the maximum PDs and their PAs of the smallest and second smallest “balloon shapes” respectively. They are corresponded to the MPs of the same names in the TDW shown in Figure 2, and their parameters are already listed in Table 1. After the coordinates of these MPs in the TDW are substituted into Equation (15), the following equations are obtained:

\[
\begin{align*}
A_k \sin \left[0.0473(t - 0.04) + \varphi_{ao}\right] + Me^{-(0.0473-0.04)/T_a} &= 7.36 \\
A_k \sin \left[0.0673(t - 0.04) + \varphi_{ao}\right] + Me^{-(0.0673-0.04)/T_a} &= 6.37 \\
A_k \sin \left[0.0573(t - 0.04) + \varphi_{ao}\right] + Me^{-(0.0573-0.04)/T_a} &= -1.897 \\
A_k \sin \left[0.0773(t - 0.04) + \varphi_{ao}\right] + Me^{-(0.0773-0.04)/T_a} &= -2.708
\end{align*}
\]

As can be seen from Figure 2, the interval between point \( c \) and \( e \) and that between point \( d \) and \( f \) are all one PFP, therefore the sinusoidal components of the first two equations are equal and the ones of the second two equations are equal. The above equations are simplified to be:

\[
\begin{align*}
M \left(e^{-0.0073/T_a} - e^{-0.0273/T_a}\right) &= 0.99 \\
M \left(e^{-0.0173/T_a} - e^{-0.0373/T_a}\right) &= 0.811
\end{align*}
\]

Equation (17) is solved, and there are:

\[T_a = 0.050 141 175 233 319 0 \text{ s}, \ \ M = 3.481 507 286 669 61\]

After the two values are substituted into Equation (16), the TDW expression of the DDCC is obtained as:

\[
i'_{ap}(t) = 3.481507 286669 61e^{-(t-0.04)/0.05014117523319}, \ t \geq 0.04 \text{s}
\]

After the DDCC shown in Equation (18) is subtracted from the SCC in Equation (16), the waveform of the SACC shown in Figure 12(a) is obtained. After RP\( t = 0 \) is taken into account, the PDT is shown in Figure 12(b). The TDW of 6 periods post the short circuit is only shown in Figure 12(a), and the PDT circle corresponding to the first half period of the TDW – in Figure 12(b).
Following the same way as that in the offline analysis, from PA 854.1° and PD 4.354 of the PDT circle shown in Figure 12(b) the time-domain expression of the SACC in the SCC is immediately obtained to be:

\[ i_p'(t) = 4.354 \sin \left[ 100\pi \left( t - 0.04 \right) - 44.1^\circ \right], \quad t \geq 0.04 \text{s} \quad (19) \]

The DDCC is calculated using the two periods post the short circuit, which means that at least 40ms are needed to obtain the DDCC and SACC in the SCC by this means.

4.2 Estimating the SACC in the SCC (20ms)

As can be seen from Figure 1(b), Figure 3, and Figure 4(b), the DDCC in the SCC impacts on only the current values, but not the phase angles of the SACC. To improve the speediness of the protection, the SACC can be estimated in one period using the PDT.

As shown in Figure 3(a), the phase angles of MP\textsubscript{c} and \textsubscript{d} approach that of MP\textsubscript{h}, and the PDs and PAs of each point are already listed in Table 1. The average of the positions of MP\textsubscript{c} (131.4°) and \textsubscript{d} (131°) is taken as the position of the PDT circle of the SACC, which is

\[ \theta_{mk} = \frac{131.4^\circ + 131^\circ}{2} = 131.2^\circ \]

The IPA of the SACC in the SCC is obtained from relationship Equation (3) between the IPA of the TDW and the position of the PDT circle, which is:

\[ \varphi_0k = 90^\circ - 131.2^\circ = -41.2^\circ \]

The amplitude of the SACC can be estimated by the average of the absolute values of the PD of MP\textsubscript{c} (7.36) and that of MP\textsubscript{d} (-1.897), which is:

\[ A_k = \frac{7.36 + 1.897}{2} = 4.6285 \]

And the SACC in the SCC is:

\[ i_p''(t) = 4.6285 \sin \left[ 100\pi \left( t - 0.04 \right) - 41.2^\circ \right], \quad t \geq 0.04 \text{s} \quad (20) \]

Being compared with Equation (1) and (19), the errors of the SACC expressed in Equation (20) are obviously much larger.

4.3 The correlation analysis between the reformulated SCC waveform and the original one

The TDW expression of the reformulated SCC is obtained from Equation (18) and (19), which is:

\[ i_p''(t) = i_p'(t) + i_{ap}'(t) = 4.354 \sin \left[ 100\pi \left( t - 0.04 \right) - 44.1^\circ \right] + 3.48150728666961e^{-\left( t-0.04 \right)/0.050141175223319}, \quad t \geq 0.04 \text{s} \quad (21) \]

The correlation coefficient and relative error between the reformulated SCC waveform expressed by Equation (21) and the original one by Equation (1) are respectively calculated by Equation (5) and (6), which are:

\[ c_{xy} = 0.9999980063830419, \quad \varepsilon = 3.98719417112892 \times 10^{-5} \]

The curve of the absolute errors of the reformulated SCC is shown in Figure 13, where the maximum absolute error in the first two periods (40ms) of the waveform is marked. It can be seen that the similarity between the reformulated waveform using the first two periods (40ms) of the SCC and the original one is also quite high.
5 Errors of phase angles of SPs and accuracy of reformulated waveforms

5.1 Reference point and its selection

When an SSC waveform is digitally sampled and its PDT is plotted, the PD and PA of each SP are respectively the current value and phase angle of this point. In order to ensure the correspondence between the TDW and PDT of one same sinusoidal function, it is necessary to determine a starting SP (or SP 0), the time of which in the TDW and the PA of which in the PDT are all equal to zero, i.e., $t = 0$ and $\theta = 0^\circ$. This point is described as a RP. The phase angle of a RP is equal to the IPA of the TDW.

The time of each SP in digital waveforms is the recorded one in DFRs, and any instant may be selected as a RP, e.g., the first SP following a zero-crossing point (ZCP) from negative to positive. Once the phase angle (i.e., IPA) of the RP (i.e., SP0) is determined, the phase angles of following SP1, SP2, ... etc. are all calculated from the IPA and the phase angle corresponded to sample period $T_s$.

Obviously, when a certain specific fault recording is analyzed, only one RP can be selected, otherwise there caused confusion during its processing, and correct results are not obtained from it.

5.2 Phase window and sample window

After an SSW is digitally sampled, the phase angle corresponded to the time interval between any two sample points is defined as a phase window (PW). The phase angle corresponded to one PFP is $360^\circ$ (or $2\pi$ radians), and it is described as a power frequency PW, or a power frequency window (PFW) for short; the phase angle corresponded to half a PFP is $180^\circ$ (or $\pi$ radians), and it is described as a half power frequency PW, or a half power frequency window (HPFW) for short. The phase angle corresponded to one sample period (i.e., the time interval between two adjacent SPs) in one PFP is described as a sample phase window, or a sample window (SW) for short.

The system frequency and sample frequency are expressed as $f_0$ and $f_s$ respectively, and the number of the SPs in a PFP is $f_s / f_0$. A SW being expressed as $\Delta\varphi$, since a PFW is $360^\circ$, the value of SW $\Delta\varphi$ is calculated by the following equation:

$$\Delta\varphi = \frac{360^\circ}{f_s / f_0} = 360^\circ \times \frac{f_0}{f_s}$$

The sizes of a PFW and a HPFW are $360^\circ$ and $180^\circ$ respectively, and both the values are fixed. However, as can be seen from Equation (22), the size of a SW varies with the changes of the system frequency and the sample frequency, i.e., any change in the system frequency or sample frequency impacts on its size. When the sample frequency remains unchanged, the size of the SW is needed to be constantly adjusted with the change of the system frequency.

5.3 Calculation of phase angles of RP and SPs

As shown in Figure 14, a ZCP of a SSC from negative to positive is $(t_{00}', 0)$, where $t_{00}'$ is the instant of the ZCP.
Current $i_{-1}$ of the previous SP($t_{-1}$, $i_{-1}$) is negative, current $i_0$ of subsequent SP($t_0$, $i_0$) is positive, and $t_0 - t_{-1}$ is sample period $T_s$. Point ($t_0$, $i_0$) is the first positive SP following the ZCP from negative to positive, and it may be taken as a RP (the 0-th SP).

Figure 14 (a) Determining the phase angle of a RP using linear interpolation method; (b) The estimation of the calculated error of the phase angle of SPs

A straight line is formed after point ($t_{-1}$, $i_{-1}$) is connected with RP($t_0$, $i_0$), as shown by the green line in Figure 14(a). When sample frequency $f_s$ is much greater than system frequency $f_0$, the intersection of the line with the time axis is approximated as the zero-crossing instant expressed as $t(0)$. Its value is calculated using linear interpolation method:

$$t(0) = \frac{t_0 - t_{-1}}{i_0 - i_{-1}} (i(0) - i_{-1}) + t_{-1}$$  \hspace{1cm} (23)

where, $i(0)$ —— the value of the current corresponded to approximate zero-crossing instant $t(0)$, $i(0) \approx 0$.

The actual value of IPA $\varphi_0$ is equal to the phase angle of RP($t_0$, $i_0$), and its computed value $\varphi_{0C}$ is obtained from Figure 14(a) as:

$$\varphi_{0C} = \varphi(0) + \Delta \varphi \cdot \frac{t_0 - t(0)}{t_0 - t_{-1}} = \varphi(0) + \Delta \varphi \left( t_0 - t(0) \right) f_s$$ \hspace{1cm} (24)

where, $\varphi(0)$ —— The phase angle corresponded to approximate zero-crossing instant $t(0)$, $\varphi(0) \approx 0^\circ$.

The calculated value of the phase angle of $n$-th SP($t_n$, $i_n$) $\varphi_{nC}$ is calculated by the following equation:

$$\varphi_{nC} = \varphi_{0C} + n \Delta \varphi$$ \hspace{1cm} (25)

Since instant $t(0)$ of the ZCP calculated by Equation (23) is approximate, there is an error between it and its actual instant $t(0)'$, therefore, there are also errors between $\varphi_{0C}$ and $\varphi_{nC}$ calculated on this basis and their actual values respectively. As can be seen from Figure 14(a), the error value is smaller if sample frequency $f_s$ is much greater than system frequency $f_0$.

5.4 The adjustment of SWs and the calibration of PDTs

That whether the size of a SW is accurate or not is the prerequisite for ensuring that each HPFW in an SSW is corresponded exactly with a complete circle in its PDT. In other words, the angular frequency of the PDT is necessary to be the same as that of its TDW. Only when this condition is met can the TDW be reformulated from the parameters of each MP in the PDT. As can be seen from Equation (22), which is the definition of a SW, when sample frequency $f_s$ is fixed, SW $\Delta \varphi$ varies with system frequency $f_0$. To ensure that the angular frequency of the PDT is equal to that of the corresponded TDW and keep their periods to be matched, and reduce the errors between the TDW reformulated from its PDT and the original one, it is necessary to adjust SW $\Delta \varphi$ with the variation of the system frequency and to calibrate its PDT.

The period and frequency of an SSW are easy to be calculated from one of the ZCPs of the SSW, and the methods are quite mature, which is no longer detailed in the paper.
5.5 The calculation error of the phase angle of SPs and its reduction

When the first SP next to the ZCP from the negative to positive one is chosen as a RP, the phase angle of each SP is calculated using the method shown in Figure 14(a). However, the ZCP calculated using linear interpolation method is approximate, which results in the errors between the phase angles of the RP and other SPs and their actual ones. As can be seen from Figure 5, a tiny error of the IPA of an SSW may cause the large errors between the reformulated waveform and the original one. The method for estimating the calculated error of the phase angle of the SPs is shown in Figure 14(b).

The calculated and actual values of the instant of one of the ZCPs in an SSW are expressed as $t_{(0)}$ and $t'_{(0)}$ respectively, the calculated and actual values of the phase angle of the ZCP – $\phi_{(0)}$ and $\phi'_{(0)}$ respectively, and the calculated and actual values of the phase angle of the RP – $\phi_0$ and $\phi'_0$ respectively. As can be seen in Figure 14(b), since $\phi'_{(0)} = 0$, error $e_\phi$ between calculated phase angle $\phi_0$ of the RP and its actual one $\phi'_0$ is exactly equal to the calculated phase angle $\phi_{(0)}$ of the ZCP. The percentage of error $e_\phi$ is calculated by the following equation:

$$e_\phi \% = \frac{\phi_{(0)} - \phi'_0}{\Delta \phi} \times 100\% = \frac{360^\circ \times (t_{(0)} - t'_{(0)}) \times f_0}{\Delta \phi} \times 100\%$$  (26)

When the sampled data from the digital simulation of an SSW are taken as an example and the system frequency and sample frequency are taken as $f_0 = 50$ Hz and $f_s = 20 000$ Hz respectively, the SW calculated by Equation (22) is $\Delta \phi = 0.9^\circ$. Actual $t'_{(0)}$ and calculated $t_{(0)}$ of a certain ZCP in a waveform are 0.001 564 187 961 238 12 s and 0.004 674 597 260 738 6 s respectively. After they are substituted into Equation (26), the calculated error of the phase angle of SPs is obtained as:

$$e_\phi \% = \frac{360^\circ \times (0.004 674 597 260 738 6 - 0.004 674 559 726 073 8) \times 50}{0.9^\circ} \times 100\% = 0.075 069 329 6\%$$

The phase angle of each SP in the SSW calculated using linear interpolation method is approximate, thus the formed PDT is not exactly correspondent with its TDW, and then the SSW reformulated from the PDT is not accurate. In order to reduce the errors of the reformulated SSW, calculated IPA $\phi_{0C}$ is taken as the center of interval $[\phi_{0C} - \Delta \phi, \phi_{0C} + \Delta \phi]$ and is adjusted in it to make the correlation coefficient between the reformulated TDW and the original one is maximum. The method and process of the adjustment are detailed in the above-mentioned offline analysis of the fault recording.

5.6 The accuracy of the reformulated waveform

To measure the similarity between the reformulated waveform and the original one, the correlation coefficient is calculated by Equation (5), and the larger this coefficient, the smaller the error of the reformulated waveform. The maximum coefficient is 1, which means that the original waveform is completely restored from the reformulated one. However, due to the interference in the digital waveforms sampled in practice, the complete restoration of the original SCC waveform is not conducive to the analysis of the fault recording. Therefore, the reformulated SCC is only needed to meet a certain accuracy demand, which may be decided by the application scenarios of the results of fault recording analysis.

6 Conclusion

When reading various literature of fault recording analysis, we found that there is little research related to the PDTs of electrical physical quantities in the steady and transient states and the fault analysis using the PDTs. In this paper an algorithm of fault recording analysis combining TDWs and PDTs are proposed, and it is applied in an SCC example to calculating its FFPs, the correlations between the reformulated waveforms of the SACC and DDCC and their original ones; the algorithms of adjusting the initial reformulated waveforms using the correlation are proposed and the programs are created, the curves of the absolute errors of the reformulated SCC pre and post its adjustment are plotted after the running results are output by the programs.
The research results show that the accuracy of the reformulated fault recording is related to the features of the fault recording itself, the purposes of analyzing the fault recording and the accuracy demands for the analysis. When there contained the waveforms of a fault steady state in transient processes of fault recording and analysis results are applied to finding fault causes, determining fault types and fault distances, and identifying system operation modes, etc., the real-time demand for the recording analysis is not high, and the so-called offline analysis may be used. Due to the fact that in fault recording there contained a fault steady-state waveform, after it is separated from a transient process the remaining components are then analyzed. There is a high correlation between the reformulated waveform and its original one using this method, and the correlation coefficient between the two waveforms is quite high and approaches 1.

If there contained only a fault transient waveform in a transient process and the analysis results are applied to determining the types, phases and distances of faults and then for protection, the real-time demand for the recording analysis is very high, and the so-called online analysis must be used. Due to the fact that a fault steady-state waveform is not obtained from fault recording, the method that a SACC is calculated first and separated then from the recording to analyze the remaining components cannot be used. The correlation between the reformulated waveform and the original one is inevitably lower than that in offline analysis.

Regardless of which analysis way will be used, there is a high correlation between the reformulated fault recording and the original one using the method researched in this paper and the reformulated waveform can be used to substitute approximately the original one to achieve certain functions in power system analysis, e.g., static equivalence and dynamic equivalence, etc. In addition to this, the errors of phase angles when a waveform is mapped from the time domain to the phase domain and the accuracy of the reformulated waveform are also analyzed and discussed in the paper.

There are the following innovations in the research direction of fault recording analysis in the paper.

1. The analysis algorithms of a sinusoidal ac waveform, a decaying dc and a waveform containing only a DDCC combing TDWs and PDTs are proposed, and the programs are created to be used to analyze a SCC example.

2. The problems such as the error of a SP phase angle, a correlation coefficient, the curves of absolute errors, etc. related to the accuracy of reformulated waveforms are discussed and solved.

3. The concept of the analysis ways of fault recording, i.e., offline and online analysis, are proposed to apply to the scenarios such as different recorded waveforms, the purposes of recording analysis and the accuracy demand for recording analysis etc. respectively.

The algorithms of fault recording analysis presented in the paper are not complex, and there are not various time-consuming calculations, thus they are more reliable and understandable than other algorithms and easy to implement and spread in practice.

At present, the process analyzing and studying the PDTs of fault recording and their application is still in an initial stage. In order to establish a theoretical basis and due to the length limit of an article, the constructed waveform is used in this paper, thus there is still a large gap between the waveform and the sampled one in practice even simulated one. Either simulated or fault recording from practical is necessary to be the study object in our subsequent research. Therefore, there is still a long way to go in this direction. Let us make great efforts to work and look forward to it together.

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Reference


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Data availability

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

Contributions

Qun Ge: Methodology and Writing.
Lu Ren: Software and Data analysis.
Jia li: Validation and Investigation.

Competing interests

The authors declare no competing interests.
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