COVERAGE VERSUS RESPONSE TIME OBJECTIVES IN AMBULANCE LOCATION

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Abstract:

Background: This paper deals with the location of emergency medical stations in a large-scale area. Operations research tools are used to design a new infrastructure that reflects demographic changes.

Methods: A bi-criteria mathematical programming model is proposed. The criteria include the accessibility of high-priority patients in a short time limit and response time of all patients. The model is compared with the p-median model with a single response time objective and with a hierarchical pq-median model that considers different vehicle types. A detailed computer simulation model is used to evaluate the solutions.

Results: The methodology is verified in the conditions of the Slovak Republic using real historical data on ambulance trips. Considerable improvements regarding the average response time, coverage of population and coverage of high-risk patients can be achieved by redistributing current stations. In a large-scale area, the coverage objective does not work well. It does not outperform the response time objective that enables saving more lives. The best results are achieved by the hierarchical median-type model.

Conclusions: The resulting distribution of ambulances significantly improves the accessibility of urgent health care to patients.

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Emergency medical service (EMS) is an inseparable component of health care systems in many countries. Its main role is to provide first medical aid to patients in emergency situations. The organization of the EMS system substantially effects patients’ chances of survival and recovery. Therefore planning EMS at all levels (strategic, tactical and operational) represents a challenging problem that is still topical in the constantly changing socio-economic environment.

In the past two decades, an increasing demand for EMS service worldwide has been reported. Population’s ageing has been identified as the key factor of this phenomenon [1, 2]. Elderly people suffer from chronic diseases and mental or physical dysfunctions. They are subject to the risk of sudden worsening of their medical conditions and injuries caused by falls. Also the risk of life-threatening emergency events, such as the stroke, severe respiratory difficulties, and cardiac arrest, increases with age. As a result, elderly people require EMS at a higher rate than younger people do. Although the elderly do not constitute a large part of the whole population, their share in EMS demand is significant. For example, the study [3] analyzes the situation in Bavaria, which is the largest German federal state. In 2012 people aged 75 years and over constituted about 9% of the total population but accounted for 33% of all emergency cases. Lowthian et al. [1] state that in 2008 the proportion of Melbourne’s population aged 85 years and over was 1.6% but the proportion of emergency transportations accounted for this group was 13.6%.
The available census and EMS data proves that the Slovak Republic follows this trend. The demographic trend elicits the need for changes in the EMS infrastructure so that the EMS system can operate better – save more lives, reduce permanent disablement, and improve the outcome of patients. The organization of EMS, including the distribution of ambulance base stations across the country, was established in 2010, thus 10 years ago. It probably does not reflect the current needs. The responsiveness of the system could be improved by better distribution of the stations so that they are closer to the locations where emergencies may occur. The discussion about the need for the system reorganization due to demographic changes has started also in other countries, for example in Slovenia [2].

In this paper we focus on locating ambulance stations. The purpose of this work is to identify optimal ambulance locations satisfying increasing EMS demand conditioned by age. Two location models are used. Firstly, the problem is formulated as a bi-criteria location problem to maximize performance level, and then to find optimal locations minimizing response time. Performance level is defined as the volume of high-priority demand that will (probably) get the service in a pre-specified amount of time. The solution of this model is then compared with the solution of the well-known $p$-median model. Finally, a hierarchical version of the $p$-median model is applied to reflect two levels of emergency services.

**Outline of the paper.** Section 2 presents selected related research. In Section 3, specifications of the region under consideration are given, together with demand modelling and model assumptions. The section also presents a mathematical programming model for optimal location of ambulance stations and a computer simulation model that is used for the evaluation of the existing and proposed distributions of the stations. The following Section 3 reports and discusses experimental results. Section 4 concludes by summarizing the main contribution and disclosing the potential for improvement.
2 RELEVANT LITERATURE

This literature review is concerned with successful location models in EMS. Special attention is paid to the models dealing with different categories of patients and multi-objective models. Moreover, computer simulation in EMS infrastructure optimization is reviewed.

Optimization problems arising at emergency care pathway are surveyed in [4]. Regarding EMS location problems, the authors focus on the models incorporating equity and uncertainty. Valuable for our research is especially Section 6 of the paper on using computer simulation for evaluation of the performance of EMS systems and validation of optimization models.

A survey on recent research in healthcare facility location is supplied by Ahmadi-Javid et al. [5]. The study reveals that the maximal covering location problem (MCLP) is widely used to study location of emergency facilities. The problem allows for numerous variations and extensions, the most popular of which is the maximum expected coverage location problem (MEXCLP). The MEXCLP seeks to maximize the expected covered demand supposing an ambulance being busy with a certain probability and operating independently from other ambulances.

McLay [6] enhances the MEXCLP considering two different types of emergency vehicles and three patient classes. Calls are classified as Priority 1, 2, 3, where Priority 1 calls are life-threatening, Priority 2 calls may be life-threatening and Priority 3 calls are not life-threatening. The objective is to maximize the total number of expected Priority 1 calls in a specified amount of time. The probabilities of vehicles being busy are the same for all candidate locations and are calculated by the hypercube queuing model. Knight et al. [7] deal with multiple classes of heterogeneous patients. Patients differ by medical conditions, so they have different urgency levels. The authors use the maximal expected survival location model
with a different survival function for each patient class. The objective is to maximize the overall expected survival probability across all patient types. The utilization of ambulances is again calculated by queuing theory. A similar problem is solved by McCormack and Coates [8]. The authors use a genetic algorithm with an integrated simulation model to decide on base station location and vehicle fleet allocation. The objective is maximization of the overall expected survival probability for high-priority calls divided into two groups – cardiac arrests and other life-threatening calls. Leknes et al. [9] modify the maximal expected survival location model by Knight et al. [7]. The service time depends on the distance from a station to a demand zone, the distance from the scene to a hospital, the drop-off time and the probability of the transportation to a hospital. This way the model reflects the heterogeneity of the demand zones in the solved region. Three severity levels of calls are applied. In [10] the authors experiment with the model introduced in [9] to propose the location of ambulance stations and allocation of ambulances to these stations under three scenarios. Two of them are associated with the changes in the organization of EMS, the third case study is concerned with time varying demand. Hammami and Lebali [11] propose a location-allocation model that decides on the locations of the stations, the number of ambulances to be assigned to each of them, and demand allocation to the stations. The objective function is the total system cost including the cost of opening the stations, the cost of allocating ambulances to the opened stations and the transportation costs. Also in this study, multiple types of vehicles and multiple types of patients are considered.

The models maximizing the overall expected survival probability across all patient types [7, 8, 9] are in fact multi-objective models, where individual objectives for each patient class are combined into a single objective using the scalarization method. The main drawback of this method is how to set weights of individual objectives to make the model produce good results acceptable in practice. Another approach to cope with multiple objectives is goal
programming. Alsalloum and Rand [12] optimize locations of a pre-defined number of ambulances. The objective function consists of two goals. The former is to maximize the expected coverage, and the latter is to reduce the spare capacities of located ambulances.

Aboueljinane et al. [13] supply an overview of the literature on simulation models applied to emergency medical service operations. The review covers the time period from 1969 to 2013. Computer simulation is identified as a useful tool for the analysis and improvement of EMS since it allows us to model the system in a high degree of detail that is not possible when using other methods such as mathematical programming or queuing theory. Most simulation studies support decisions on the base stations to open and the number of ambulances to assign to each opened station. Aringhieri et al. [14] compare the current station locations in Milan (Italy) with locations proposed by the capacitated version of the location set covering model. Several scenarios with different ambulance speeds, the number of ambulances and dispatch protocols were evaluated by simulation. A trace-driven simulation approach was used, which means the model accepts a stream of actual call data as input. In contrast to self-driven simulation models, trace-driven simulation does not need the estimation of probability models describing the time and spatial distribution of calls and duration of service times. On the other hand, it has some shortcomings [15]: this approach requires a large amount of historical data; one has to handle erroneous records in the database of interventions; the existing data does not represent the future, so the simulation model cannot be used for mid-time and long-time planning when the demand volume will increase.

Recently published research [16] evaluates three different deployment strategies by a trace-driven simulation model using Mecklenburg County (US) EMS data. The simulation model is not very realistic since it uses constant values for ambulances’ speed, on-scene and drop-off times. The travel times are calculated using the Manhattan distance. Study [17] compares four coverage-based models for station location and ambulance allocation via
discrete event simulation model. The experiments include a case study of Instanbul and randomly generated instances. Also this simulation model uses constant travel speed and drop-off times. Every patient is supposed to be transported to a hospital. Moreover, ambulances cannot be dispatched to another call while they return to their original station.

We conclude this literature review by a recursive optimization-simulation approach for the ambulance location and dispatching problem [18]. The method iterates through two steps. First, an optimal location of ambulances and dispatching strategy is proposed by mathematical programming using an initial estimation of ambulances’ busy fraction. The model is a variant of the MEXCLP. Then the system with optimal infrastructure is assessed by computer simulation resulting in an updated busy fraction (equal for all ambulances) that inputs the mathematical model in the following iteration. The process is repeated until convergence is achieved. Convergence is measured by busy fraction and the location vector, respectively. The most inspiring issue for our research is the conclusion of the paper where the authors emphasise the necessity of using heterogeneous busy fractions especially in large case studies.

From the presented literature review one can make the following conclusions:

The simulation models published in the literature oversimplify the real operation due to the lack of operational data or for the sake of shorter computer processing time. Some common simplifications were mentioned with the references. However, location decisions are of strategic nature with long-term consequences and are associated with considerable investment costs. So they are worth of some extra time spent to careful assessment of the proposed infrastructural changes. In our opinion, the simulation model should be as realistic as possible. It should accurately capture all sub-processes of the service. The parameters of the model should be derived from real operation of the system. Of course, better model
requires more computing time, but computing time does not matter in strategic planning, the outcome of the approach is more important. A similar conclusion is derived in [18].

3 MATERIALS AND METHODS

3.1 Description of the region of interest

In the Slovak Republic, EMS is organized at a state level. It means that the number of emergency ambulances and locations of their base stations are defined for the whole country by the regulations issued by the Ministry of health care. Currently, there are 273 stations uniformly distributed over the whole state territory. Every station is occupied by a single ambulance and its crew. The intention for the distribution of the stations was to be able to reach 95% of patients within 15 min or less after the emergency call, regardless the patient’s condition or the character of the area (urban or rural). The stations are deployed in 211 towns and villages. Larger towns have multiple stations. The regulations define just the town where a station should be, not its precise geographical location. A provider who gets the license to operate a given station chooses a suitable building, and so determines its address. The Slovak system works in a Franco-German style, where the ambulance crew is qualified to provide on-site medical care. There are two types of ambulances. Most of them provide basic life support (BLS; Slovak abbreviation RZP) and have only a paramedic and a rescue driver on board. About one third of ambulances are well-equipped advanced life support units (ALS; Slovak abbreviation RLP). An ALS crew consists of an emergency physician, a paramedic and a driver. The staff is capable of performing additional life-saving procedures, e.g. inserting breathing tubes. The closest available ambulance to the emergency site is always dispatched regardless of its type. If it is a BLS ambulance and the incident is life-threatening, then the closest available ALS ambulance is dispatched concurrently. The rationale is that
any medical treatment is better than waiting for a doctor without a professional intervention.

In 2019, both types of ambulances served together 558,695 calls [19].

In this paper we focus on the relocation of the current stations. We do not want to change the number of stations because adding stations would be unacceptable due to economic reasons and closing some stations would worsen the accessibility of urgent health care. Our aim is to relocate some existing stations to other potential locations hoping that the new distribution will shorten response times.

3.2 Modelling demand

The first task in optimization of the station locations is to define the demand zones where potential patients live. We decided to identify the demand zones with the territorial units used in the census, namely for two reasons. The first one is that we face an emergency system whose infrastructure is spread over a large-scale area (specifically, the whole state territory) populated by millions of people (population of Slovakia in April 2020 is 5,457,926). Inhabitants, i.e. potential patients, have to be aggregated in a limited number of units, so that the resulting location model can be solved by common computational resources with limited memory and in an acceptable amount of processing time. The division of the country into smaller demand zones (e.g. by a rectangular grid) would result in an intractable location problem due to a huge volume of input data and an enormous number of variables. Thus our demand zones correspond to villages and towns. The two largest cities (the capital Bratislava with 438,610 inhabitants and Košice with 238,484 inhabitants) are administratively divided into boroughs (17 boroughs in Bratislava and 22 boroughs in Košice) that are regarded as separate demand zones.

The demand in particular zones can be estimated in several ways: from real data on EMS calls [20, 21], from the population in the given demand zone [22, 23], or from EMS interventions per 1,000 population and population structure [24]. The first way is possible if
EMS statistics for all demand zones under consideration are available. The second way is a rough estimation that need not correlate with a real number of patients, since the demand for EMS is influenced by the population’s age structure that varies in a large-scale area, as we will demonstrate later on. The result is that the solution could not be optimal for real demand, and a so called surrogation error might arise [25]. Since historical data on ambulance interventions in every municipality was not available to us, we decided to predict EMS cases according to the third way, using a sample of patient data provided us by Falck Záchranná, a.s., which was the largest EMS provider in Slovakia in the last decade, and publicly available demographic data on population’s size and age structure. This way of demand-modelling results in a more realistic solution.

Falck Záchranná a.s. supplied us with depersonalized data on 149,474 patients served in the year 2015. Therefore demographic data we used for demand estimation is also for 2015. The 2015 population data published by the Statistical Office of the Slovak Republic reveals that people aged 65 years and over constitute 14.45% of the population. However, the population’s age structure is not homogenous throughout the state. To get a better idea about the age of people in different regions of Slovakia, we calculate an aging index for each territorial unit as the ratio of inhabitants who are at least 65 years old over inhabitants below the age of 65. The index varies a lot among municipalities (min = 0.012, max = 1.333, median = 0.177, mean = 0.189, sd = 0.083). At the district level the differences are not so conspicuous (min = 0.096, max = 0.263, median = 0.171, mean = 0.171, sd = 0.030) but their graphical presentation is more readable, and it illustrates the distribution of elderly people across the country (Fig. 1). The regions in the north with low index have a high birth rate. The highest index is in the central part of two largest cities Bratislava and Košice, where elderly people are in majority.
To calculate the share of elderly people in emergency dispatches, we use the Falck sample data. This dataset contains information about the time and date of each incident, the patient’s age, the initial medical diagnosis, and time stamps of the whole EMS trip. The data suggests that patients aged 65 years and over required 42.34% of the interventions.

Combining Falck data with publicly available statistics reported by the Operation Centre of the EMS of the Slovak Republic [26] and the population statistics published by the Statistical Office of the Slovak Republic we can calculate the rates of emergency interventions for various age groups according to Eq. (1):

\[
rate_k = \frac{D_{\text{Falck}}}{\text{Pop}_k \cdot \text{Falck}_{\text{total}}} \times 1000
\]

where \(rate_k\) is the one-year number of emergency cases per 1,000 persons in age group \(k\), \(Falck_k\) is the number of patients in age group \(k\) in the Falck dataset, \(Falck_{\text{total}}\) is the total number of patients in the dataset, \(D\) is the total number of ambulance dispatches reported by
the Operation Centre of the EMS of the Slovak Republic for the year 2015, and $Pop_k$ is the number of inhabitants in age group $k$.

Within each age group we can further distinguish two groups of patients according to their initial medical diagnoses. The most severe diagnoses are denoted as the First Hour Quintet (FHQ), and they include: chest pain, severe trauma, stroke, severe respiratory difficulties, and cardiac arrest. Although the international definition of FHQ does not list unconsciousness, it is also a life-threatening condition. Therefore, after a consultation with emergency physicians, we decided to include it in FHQ. The FHQ conditions require immediate rescuing. If a call is recognized as a FHQ call, it gets the highest priority because every minute of delay in the response reduces patient’s chance of survival. The FHQ patients account for 26.51% of all patients in the Falck dataset.

The analysis of EMS data reveals that the overall rates as well as FHQ rates increase with age (Fig. 2). The Spearman correlation is $\rho = 0.95$ for overall rate and $\rho = 0.96$ for FHQ rate. The dependency curve has an exponential shape, with the acceleration from the age of 65 years.

![Figure 2: Emergency incident rates increase with age (Slovakia, 2015)](image-url)
For the modelling purposes we will distinguish three age categories: (i) children in the age of 0-14 who have the lowest emergency incident rates (see Fig. 2), (ii) teens and nonelderly adults aged 15-64, and (iii) elderly people aged 65 years and over who call EMS the most frequently. The emergency incident rates for these categories are shown in Table 1. Based on the age structure and the rates we can estimate the annual number of EMS patients in municipality $j$ according to Eq. (2):

$$b_j = \sum_{k=1}^{3} rate_k pop_{kj}$$

where $rate_k$ is the one-year number of emergency cases per 1,000 persons in age group $k$, and $pop_{kj}$ is the number of inhabitants in age group $k$ in municipality $j$. Similarly, the annual number of high-risk patients $b_j^{FHQ}$ can be calculated using the rates of FHQ incidents.

<table>
<thead>
<tr>
<th>Age group</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-14</td>
<td>26.09</td>
</tr>
<tr>
<td>15-64</td>
<td>69.12</td>
</tr>
<tr>
<td>65+</td>
<td>267.07</td>
</tr>
</tbody>
</table>

Table 1: Emergency incident rates

3.3 The bi-criteria mathematical programming model with coverage and response time objectives

In planning EMS infrastructure, we do not consider investment costs associated with the redeployment of the stations. They are not extremely high because the ambulance can be housed in a fabricated building. Rather we use such optimization criteria that reflect the main
goal of the EMS system – to save as many people as possible. Since this output cannot be measured when designing the system, surrogate optimization criteria are formulated instead. We propose a bi-criteria model to maximize the expected coverage of high-priority FHQ patients and to minimize response time for all potential patients.

Candidate locations where stations can be placed are all municipalities and other villages that are not self-governing units but are the seats of stations today. There are 2,934 candidate locations in Slovakia and 2,928 of them are self-governing municipalities. The towns, boroughs and villages are represented by the nodes on the road network that are closest to the centre of the municipality. This way the calculation of travel times can be based on real network distances. The digital road network was downloaded from the OpenStreetMap database [27], which is a freely available source of geographical data. The travel times are related to deterministic speed of vehicles that depends on the quality of the road, its location inside or outside built-up area, the type of the movement (whether the ambulance drives at standard, or all possible speed with lights and sirens), and rush hours. The average ambulance speeds with regard to the road category and day time were reported in [22].

In the first stage we do not take the type of ambulances into account. To get a realistic solution and simplify the model, we do not allow all stations to change their current position. To decide which stations must remain where they are we apply two rules-of-thumb. First, we suppose that ambulances in large towns are fully engaged. Therefore if the expected number of patients in a town exceeds the capacity of all ambulances currently stationed there, we do not allow them to change their positions. The capacity of an ambulance was set to 1811 interventions per year which is the average number of patients served by one ambulance 2015 [26]. This rule leads to 71 stations that cannot be relocated. They are denoted as fixed and are not subject to the optimization. The demand volume in the corresponding municipalities is reduced by the total number of patients served by fixed stations. As the second rule, we
respect previous managerial decisions about multiple stations in a town where the estimated number of patients is less than the capacity of a station. In such a case our model leaves one of the stations in the place and seeks for better locations of the other stations. The preserved stations are not fully engaged by local residents in this case. Therefore the mathematical model fixes their positions but allows other municipalities to be assigned to their service area. There are nine stations fixed according to this rule.

A demand point is covered if an ambulance reaches it in a time standard. The desired service standard was set with regard to critical patients who are in life-threatening conditions and every minute delay in response time dramatically worsens their outcomes. These patients should be reached within 8 minutes, which is a widely accepted standard in most European countries for critical patients [28]. Thus assuming one minute pre-trip delay, we set the travelling time limit $T_{max}$ to the value of 7 minutes. Using this time standard we define the neighbourhood of a municipality. The neighbourhood consists of all candidate locations which are at most $T_{max}$ minutes far away.

To formalize the model, we introduce the following notation.

**Sets and indices**

$I$: set of candidate locations; $|I| = 2,934$

$I_1$: set of fixed candidate location, where the ambulances are not fully engaged; $|I_1| = 9$

$J$: set of demand points (all municipalities); $|J| = 2,928$

$i \in I$: candidate location

$j \in J$: demand point

$k$: index corresponding to the number of stations

$N_j = \{i \in I: t_{ij} \leq T_{max}\}$: set of candidate locations in the neighbourhood of demand point $j$
Parameters

$p$  number of stations to be sited; $p = 273$
$q_j$  probability of an ambulance in the neighbourhood of demand point $j$ being unavailable

$T^{max}$  the desired service standard; $T^{max} = 7$

$b_j$  the annual number of EMS patients in municipality $j$ reduced by the capacity of the fixed stations

$b_j^{FHQ}$  the annual number of FHQ patients in municipality $j$

$t_{ij}$  shortest travel time from candidate location $i$ to demand point $j$

$st_i$  the number of fixed stations in candidate location $i$

$n_j = |N_j|$  the number of candidate locations in the neighbourhood of demand point $j$

Decision variables

$x_i = \begin{cases} 1, & \text{if a station is located at site } i \\ 0, & \text{otherwise} \end{cases}$

$y_{jk} = \begin{cases} 1, & \text{if demand point } j \text{ is covered by at least } k \text{ stations} \\ 0, & \text{otherwise} \end{cases}$

$z_{ij} = \begin{cases} 1, & \text{if demand point } j \text{ is served by the station located at site } i \\ 0, & \text{otherwise} \end{cases}$

The following model is a mathematical programming formulation of the bi-criteria MEXCLP-$p$MP location model.

$maximize \quad f = \sum_{j \in J} \sum_{k=1}^{n_j} b_j^{FHQ} (1 - q_j) q_j^{k-1} y_{jk}$  \hspace{1cm} (3)

$minimize \quad g = \sum_{i \in I} \sum_{j \in J} t_{ij} b_j z_{ij}$  \hspace{1cm} (4)
subject to

\[ \sum_{i \in N_j} (x_i + s_t_i) \geq \sum_{k=1}^{n_j} y_{jk} \quad \text{for } j \in J \] (5)

\[ \sum_{i \in I} z_{ij} = 1 \quad \text{for } j \in J \] (6)

\[ z_{ij} \leq x_i \quad \text{for } i \in I - I_1, j \in J \] (7)

\[ z_{ij} \leq 1 \quad \text{for } i \in I_1, j \in J \] (8)

\[ \sum_{i \in I} (x_i + s_t_i) = p \] (9)

\[ x_i \in \{0,1\} \quad \text{for } i \in I - I_1, j \in J \] (10)

\[ y_{jk} \in \{0,1\} \quad \text{for } j \in J, k = 1, \ldots, n_j \] (11)

\[ z_{ij} \in \{0,1\} \quad \text{for } i \in I, j \in J \] (12)

The objective function (3) maximizes the expected coverage of critical patients taking into account possible unavailability of ambulances. The term \( b_j^{PHQ} (1 - q_j) q_j^{k-1} \) represents the increase in expected coverage of municipality \( j \) brought about by \( k \)th station. According to Eq. (5), sitting multiple stations in the neighbourhood of municipality \( j \) enables multiple variables \( y_{jk} \) take the value of one and account for the increase in coverage. The objective function (4) minimizes the total travel time needed by the ambulances to reach all patients. The average travel time is equal to the total travel time divided by the number of all patients.

Constraints (6) assign every municipality \( j \) to the service area of exactly one station \( i \). Constraints (7) ensure that if a municipality \( j \) is assigned to a node \( i \), then a station will be opened at the node \( i \). Constraints (8) allow a municipality \( j \) to be served from a fixed station that is not fully engaged. Constraint (9) limits the number of located stations to their current amount. The obligatory constraints (10)–(12) specify the definition domains of the variables.

To solve the bi-criteria model, we used the lexicographic method. First, the single criteria model (3), (5), (9)–(11) is solved. The model maximizes the expected coverage of high-
priority patients. Let us denote its optimal objective value as $f^*$. Then the weighted $p$-median problem (4), (6)–(12) with additional constraint (13) is solved.

$$\sum_{j \in J} \sum_{k=1}^{n_j} b_j F^{HQ} (1 - q_j) d_j^{k-1} y_{jk} \geq f^*$$

Constraint (13) assures that the expected coverage of most critical patients will not worsen when minimising the average response time for all patients.

The probability of an ambulance in the neighbourhood of a municipality being unavailable is estimated using computer simulation of the EMS system. The probability is calculated as the average workload of potential ambulances in the neighbourhood. However, the workload depends on the distribution of the ambulances and therefore it is de facto the output of the model. Since we need it as an input parameter, it must be estimated a priori. Initially, the workload is estimated using the current station location. If there is at least one ambulance currently operating in a candidate location, then the probability of this candidate is calculated as the average workload of currently operating ambulances. If the candidate does not have a station today, then its workload is set to the average workload of the stations that are in the 30 minute neighbourhood of the candidate. The optimized distribution of the stations is submitted to simulation to obtain workload for the second run of the model. The process is repeated until convergence is achieved. Convergence is measured by ambulance distribution. When the locations in two successive solutions are (almost) identical, then the process stops.

The solution of the model defines the municipalities where the stations will be deployed (at most one station in a municipality). This output is merged with the pre-processed fixed locations, resulting in multiple stations in more populated towns and boroughs. However, at this moment we do not have specific addresses, but multiple stations are regarded as located in the single (central) node of the municipality. The seats of the stations inside a given
municipality are determined afterwards, using a rule-of-thumb. We proceed from the existing locations. The addresses of fixed stations are preserved. A new station, if there is one, is placed at the municipality’s central node on the road network. If one or more stations out of multiple existing stations are removed, they are selected randomly.

The model does not distinguish the types of emergency units. However, their distribution, especially the locations of ALS ambulances affect the efficiency of the system because an ALS ambulance is always dispatched to the high-priority call. We distribute ambulances among the optimized station locations a-posteriori in the following way: first, we retain the type of fixed stations that are disregarded in the optimization, and also the type of those stations whose positions were not changed by the optimization model. As regards the relocated stations, firstly we place ALS ambulances close to their current positions that are mainly in hospitals. The reason is that we try to respect managerial decisions made in the past to propose a solution that would be acceptable by decision makers. After allocation of ALS ambulances, the remaining stations are assigned by BLS ambulances.

The structure of the MEXCLP model (3), (5), (9)–(11) makes it easy to solve by a general-purpose solver. We used the solver Gurobi Optimizer 8.1.1. The computing time was about 1 sec on a common PC. The weighted $p$-median problem (4), (6)–(10), (12) is known to be NP-hard. Despite its complexity, an efficient solution method was developed [29]. However, constraint (13) makes the problem more complicated and disables using this exact method. Instead, an approximation algorithm has to be used. We chose the kernel search method. Kernel search is a recently developed matheuristic that has been successfully applied for solving mixed integer linear problems (MILPs) with binary variables [30, 31]. In principle, it is a decomposition method that in sequence solves sub-problems of the original MILP problem. A sub-problem consists of a subset of decision variables. The subset contains only promising variables (a kernel) and a small subset of the remaining variables. The sub-
1 problems are solved using a general-purpose MILP solver as a black-box. We implemented
2 the method in Java language in combination with the solver Gurobi Optimizer 8.1.1 library.

3 3.4 The hierarchical model minimizing response time

4 To cope with the two-tiered EMS system that works in Slovakia and in many other countries,
5 it is desirable to design an optimization model where different types of EMS units are taken
6 into account. The EMS system with two vehicle types can be viewed as a hierarchical facility
7 system. Using the classification by Şahin and Süral [32], we face a multi-flow, nested, and
8 non-coherent system. If the objective is to minimize the total distance (or travel time,
9 respectively) from demand zones to the closest ALS and BLS stations, then the hierarchical
10 $pq$-median problem is to be solved. We propose a modification of the $pq$-median model by
11 Serra and ReVelle [33].
12
13 In addition to location variables $x_i$ that decide on location of stations regardless of their
14 type, we need another set of variables that model the decisions on locating only ALS stations:
15
16 $u_i = \begin{cases} 1, & \text{if an ALS station is located at site } i \\ 0, & \text{otherwise} \end{cases}$
17
18 Service areas of the ALS stations are modelled using the following allocation variables:
19
20 $v_{ij} = \begin{cases} 1, & \text{if demand point } j \text{ is served by the ALS station located at site } i \\ 0, & \text{otherwise} \end{cases}$
21
22 The lower level of the hierarchical $pMP$ model consists of the objective function (4) and
23 constraints (6)–(10) and (12). It decides on location of stations regardless of their type and
24 creates their service areas. The upper level of the model decides which stations opened in the
25 lower level will house ALS ambulances:
minimize \[
\sum_{i \in I} \sum_{j \in J} t_{ij} b_{ij} v_{ij}
\]  \hspace{1cm} (14)

subject to \[
\sum_{i \in I} v_{ij} = 1 \quad \text{for } j \in J
\]  \hspace{1cm} (15)

\[
v_{ij} \leq u_i \quad \text{for } i \in I, \ j \in J
\]  \hspace{1cm} (16)

\[
u_i \leq st_i + x_i \quad \text{for } i \in I, \ j \in J
\]  \hspace{1cm} (17)

\[
\sum_{i \in I} u_i = r
\]  \hspace{1cm} (18)

\[
u_i \in \{0,1\} \quad \text{for } i \in I, \ j \in J
\]  \hspace{1cm} (19)

\[
v_{ij} \in \{0,1\} \quad \text{for } i \in I, \ j \in J
\]  \hspace{1cm} (20)

The objective function (14) minimizes the total travel time needed by the ALS ambulances to reach all patients. Constraints (15) assign every municipality \(j\) to the service area of exactly one ALS station \(i\). Constraints (16) say that a municipality \(j\) can be assigned only to an opened ALS station. Constraints (17) allow an ALS ambulance to be allocated only to a station opened at the lower level of hierarchy. Constraint (18) limits the number of located ALS stations to their current amount \(r\). The remaining constraints (19) and (20) specify binary variables.

The ALS ambulances will be allocated to those fixed or relocated stations, for which \(u_i = 1\). The remaining stations will house a BLS ambulance.

### 3.5 Computer simulation model

A detailed computer simulation model was developed [22]. Its purpose in this study is twofold: (i) to estimate the workload of ambulances as the input for the mathematical programming model, and (ii) to evaluate the performance of the EMS system with the infrastructure proposed by the model. The computer simulation models the reality on a less abstract level than a mathematical programming model does, therefore it provides us with
better idea of the performance of the projected system. It calculates such quantitative indicators that cannot be derived from a mathematical programming model itself.

We implemented a self-driven, agent-based simulation model using AnyLogic simulation software. The model is developed on the Java simulation core. We implemented a library of classes and functions in Java for the simulation support.

The model was calibrated using the data sources as follow:

1. publicly available statistics published by the Operation Centre of the EMS of the Slovak Republic;
2. a sample of patient data provided by Falck Záchranná a.s.;
3. LandScan data on population distribution;
4. OpenStreetMap data on the road network;
5. historical data on the average ambulance speed with regard to the road category and day time provided by Falck Záchranná a.s.

The dataset obtained from Falck Záchranná a.s. allows us to extract important knowledge. First of all, the time distribution of calls can be revealed. With regard to the seasons and weekdays, we did not observe statistically significant differences in the number of calls. However, the call rates change significantly during a day. We can observe two peaks, one between 9 and 11 am and the other one between 5 and 9 pm. So the arrival of calls is modelled as a non-homogeneous Poisson process with the arrival rate varying depending on the time of day.

The spatial distribution of patients is modelled using the LandScan database [34]. LandScan data represents an ambient population (average presence of people over 24 hours). A grid cell corresponds to an area of 30”×30” (arc-seconds) in the WGS84 geographical coordinate system. The territory of the Slovak Republic is covered by 70,324 grid elements. The call that has been generated by the Poisson process is assigned to a grid cell with a
probability that is proportional to its population. Inside the grid cell, the call is assigned
randomly to a node on the road network.

The model captures all important processes presented in the management of emergency
patients including precise modelling of the distribution of processing times.

The main features are the following:

• As to demand modelling, we take into account three important characteristics: the
arrival distribution, the geographical distribution and the priority of calls.

• The model of the service time comprises all phases of the ambulance trip – the journey
to a patient, treatment of the patient at the site of the incident, transportation to a
hospital, drop-off time in the hospital, and the journey back to the base station.

• The movement of an ambulance respects the underlying road network.

• The on-scene time is modelled using a probability distribution that depends on the
patient’s diagnosis and crew’s qualification.

• The probability of the transportation of a patient to the hospital depends on the type of
the intervening crew. The real data shows that 77% of the patients treated by a
paramedic team and 51% of the patients treated by a physician are transported to a
hospital. If a patient has to be transported to a hospital, then the closest hospital
complying with their condition and age is chosen (for example, there are hospitals
specialized in cardiovascular diseases or children’s hospitals).

• In the hospital, the rescue team hand over the patient to the hospital staff, then they
may spend some time cleaning and resupplying the vehicle. The time needed to
perform these tasks is called drop-off time. The probability distribution of the drop-off
time is modelled separately for every hospital. In most cases, the Erlang distribution
fits well. The average drop-off time ranges from 7.1 to 36.2 min.
• After leaving the hospital, the ambulance is available to respond to another call. It means that the ambulance can be dispatched to another call on its way home. The logic of ambulance dispatching approximates very well the rules adopted in practice. For example, an ambulance can be dispatched to another rescue while it is returning to its home station. In the simulation model it means that it is possible to change the destination of the ambulance while it is moving along the road.

• Secondary transports are modelled as well, since they reduce the availability of the ambulances. A secondary transport is a planned activity where an ambulance does not respond to an emergency call but transports a patient or medical material between two hospitals.

These features represent a significant improvement in comparison to other simulation models reported in the literature.

The model was verified using the following techniques recommended also in [13]:

• Animation to graphically visualize the movements of vehicles through the road network to check whether the rescue process and the chosen routes are as expected. During the rescue process, the colour of the vehicle changes to reflect its current state (movement to a patient, stay at the scene, transport of the patient to a hospital, return back to the base station).

• Face validity by consulting EMS specialists who evaluated the model’s conception and output behaviour compared to the real-world system.

• Traces to track the movement of vehicles and occurrence of every event in the model (call arrival, vehicle assignment, destination hospital selection etc.) so as to validate the correctness of the model logic.
• Sensitivity analysis by performing a comprehensive set of simulation experiments with different values of input parameters (e.g. arrival rate or hospitals with emergency departments) to determine if the model’s output is as expected.

4 RESULTS AND DISCUSSION

The proposed simulation model was used to evaluate the current locations of emergency stations, as well as the optimized locations proposed by mathematical models. The solutions of the MEXCLP-pMP model (3)–(12) and the pMP model (4), (6)–(10), (12) were further processed using a heuristic procedure for distribution of the ALS and BLS ambulances. The hierarchical model (4), (6)–(10), (12), (14)–(20) encompasses different vehicle types by itself.

The output of the simulation model includes the following performance indicators:

1. average response time, since it has been monitored by the Operation Centre of the EMS of the Slovak Republic;
2. percentage of calls responded to within 15 min, because a 15-minute response time is regarded as standard in Slovakia;
3. number of municipalities with the average response time longer than 15 min;
4. average response time for the high-priority (FHQ) calls and the percentage of these calls responded to within 8 min;
5. average ambulance workload and its variation.

The results of the simulation experiments are summarised in Table 1. The simulation experiment for one set of station locations consisted of 10 replications. One replication simulated 91 days of EMS performance. For response times, the mean values from 10
replications and 95% confidence intervals are reported. For coverage indicators, the mean values are given. The best values of the indicators are displayed in bold.

Table 2: Performance indicators for the current and optimized locations

<table>
<thead>
<tr>
<th>Indicator</th>
<th>Current locations (June 2017)</th>
<th>MEXCLP-pMP</th>
<th>pMP</th>
<th>Hierarchical pMP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Response time for all patients (min)</strong></td>
<td>11.52</td>
<td>10.75</td>
<td>10.56</td>
<td><strong>10.55</strong></td>
</tr>
<tr>
<td></td>
<td>(11.50; 11.54)</td>
<td>(10.73; 10.77)</td>
<td>(10.55; 10.57)</td>
<td>(10.52; 10.58)</td>
</tr>
<tr>
<td>% of calls responded to within 15 min</td>
<td>75.26</td>
<td>79.97</td>
<td>80.28</td>
<td><strong>80.33</strong></td>
</tr>
<tr>
<td><strong>Number of municipalities with the average response time longer than 15 min</strong></td>
<td>868</td>
<td>676</td>
<td><strong>600</strong></td>
<td>601</td>
</tr>
<tr>
<td><strong>Response time for high-priority patients (min)</strong></td>
<td>11.37</td>
<td>10.62</td>
<td>10.48</td>
<td><strong>10.44</strong></td>
</tr>
<tr>
<td></td>
<td>(11.34; 11.40)</td>
<td>(10.59; 10.65)</td>
<td>(10.45; 10.52)</td>
<td>(10.40; 10.49)</td>
</tr>
<tr>
<td>% of high-priority calls responded to within 8 min</td>
<td>38.84</td>
<td>43.75</td>
<td>44.23</td>
<td><strong>44.36</strong></td>
</tr>
<tr>
<td>Average ambulance workload (%)</td>
<td>31.98</td>
<td>31.89</td>
<td>31.84</td>
<td><strong>31.78</strong></td>
</tr>
<tr>
<td>Coefficient of variation of ambulance workload</td>
<td>0.29</td>
<td><strong>0.24</strong></td>
<td>0.24</td>
<td><strong>0.24</strong></td>
</tr>
</tbody>
</table>

The computer simulation of the current system revealed that the system is short of the target to reach 95% of patients within 15 min. The real accessibility within this time limit is only 75.26%. 868 municipalities (almost 30%) have the average response time longer than 15 min (Fig. 3). The Slovak system also exhibits poor performance regarding the 8-minute response-time standard for the high-priority calls. Only 38.84% of critical patients are reached within 8 min, which is far less than the EU average of 66.9% [27]. The average ambulance workload is 32.98%, which corresponds to other EMS systems worldwide where ambulances are typically busy at least 30% of the time [20].

From the rest of the table we can observe that the reorganization of the system has a positive effect on the performance. Both MEXCLP-pMP and pMP models relocate approximately 78% of the stations (150 and 151, respectively). Regardless of the ambulance allocation, the mathematical programming models reduce significantly the overall average
response time, as well as response time for the most critical patients (their confidence intervals do not overlap with the confidence intervals for status quo). In parallel with reducing the response time, the accessibility within a given time threshold is increasing.

As regards the two policies of allocation of ALS ambulances, the hierarchical model that incorporates the decisions on particular ambulance types achieves better results than the models where the type of the stations is defined in a post-optimization process. The most important improvement is in the accessibility of the critical patients. In comparison to the current state, the average response time of them is reduced by 56 seconds. It may seem that one minute is not too much, but one has to realize that for a person who is in a life-threatening condition, such as a cardiac arrest, the line between life and death is very thin, and every second matters. Cardiac arrest and unconsciousness are the most frequent diagnoses of those patients who die before or during the rescue operation. From the Falck sample data on these patients we can derive the survival probability as a function of response time \( t \). The survival probability function is as follows [22]:

\[
s(t) = \frac{1}{1 + \exp(-2.04492 + 0.045427t)}
\]

From the sample data and the total number of patients reported by the Operation Centre we can estimate that in 2019 there were 26,003 most-critical patients in Slovakia. The reduction of the average response time by 56 seconds means that the survival probability increases by 0.61%. As a result, by 142 more patients could be saved. We think this is a significant improvement since every life matters.

Regardless of the model and ALS allocation policy, relocating the stations improves the accessibility mainly in the densely populated western part of the country. The most successful hierarchical \( p \)MP model reduces the overall number of municipalities with the
average response time greater than 15 min by 267 (31%) (Fig. 4). The model also generates
the smallest ambulance workload and thus increases the probability that the closest
ambulance will be available when needed. At the same time, ambulance workload is
distributed more evenly (coefficient of variation is less than at present).

Figure 3: Municipalities with the average response time longer than 15 min, current station location

Figure 4: Municipalities with the average response time longer than 15 min after optimization by the
hierarchical pMP model
5 CONCLUSIONS

Different techniques from operations research are used to support decision making on location of EMS stations. A bi-criteria mathematical programming model is proposed. The criteria include the accessibility of high-priority patients and response time of all patients. The model is compared to the $p$-median model with a single response time objective. A detailed computer simulation model is used to evaluate the solutions. The main findings are as follow:

1. Both mathematical models improve current distribution of the stations.
2. A single response time objective produces better results.
3. The distribution of ambulances with a physician on board should copy the present distribution proposed by medical experts.

The reasons of worse performance of the coverage objective may be in the imprecise estimation of busy fractions of ambulances. Busy fractions of candidate locations that do not have a station today are set at the average value of the stations in the neighbourhood. Probably it is too optimistic value for many candidate locations. Covering models in general do not differentiate between different locations within the same response time threshold. So it may happen that the model decides to place some stations to small villages near large towns.

From the covering objective’s point of view, all patients in the town are considered perfectly satisfied with the service. But the real workload of ambulances would be enormous. For example, let the village, whose demand is 1, is 5 minutes away from the town with the call volume of 100. If the ambulance was located in the village, its workload by travelling to the
scene would be 500 minutes versus 5 minute, if the station was located in the town. So we do not recommend using coverage criteria in a large-scale region.

The simulation model itself is not able to suggest the best station locations, however, it is useful in evaluating various scenarios that include not only the number and distribution of the EMS stations but also such factors as the types of ambulances, destination hospitals, or dispatch policies. To get credible output, the model must capture all processes at emergency care pathway including reliable distributions of processing times. In the future, we will elaborate demographic prognoses for particular regions of the country and incorporate them into the model. Then the simulation will allow us to predict the future performance of the EMS system, and to identify the resources necessary for ensuring a satisfactory system quality.

**List of abbreviations**

14. ALS  Advanced life support  
15. BLS  Basic life support  
16. EMS  Emergency medical service  
17. FHQ  First hour quintet  
18. MEXCLP  Maximum expected coverage location problem  
19. MILP  Mixed integer linear problem  
20. MLCP  Maximal covering location problem  
21. pMP  \( p \)-median problem

**Declarations**

24. Ethics approval and consent to participate

25. Not applicable.
Consent for publication

Not applicable.

Availability of data and materials

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Competing interests

The authors declare that they have no competing interests.

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Authors’ contributions

LJ: Conception, design of the study, methodology, data analysis, drafting the article

PJ: Data analysis, simulation model, drafting the article

MK: Data analysis, mathematical model, revising the article

FZ: Data analysis, mathematical model, revising the article

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