Improved nonlinear Burger creep model for soft rocks based on the fractional-order theory and considering viscoplastic deformation

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Improved nonlinear Burger creep model for soft rocks based on the fractional-order theory and considering viscoplastic deformation

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Abstract

The significance of creep behavior in soft rocks is crucial in rock engineering, particularly when ensuring the time-dependent stability of underground structures. This study proposed a new nonlinear creep constitutive model to represent the soft rock's creep behavior subjected to uniaxial and triaxial stress conditions. The Burger model was modified by substituting the traditional Newton dashpot with the fractional derivative Abel dashpot, and a viscoplastic body was introduced in series with the improved Burgers model to simulate the accelerating phase of rock creep. The model's efficacy was confirmed by fitting the parameters using creep test data from different soft rocks. The isochronous stress-strain curve approach was employed to calculate the long-term strength of rocks, and a sensitivity analysis was conducted to evaluate how the model parameters affect creep deformation. The high agreement between the predicted outcomes and the actual creep experimental data for salt, shale, and sandstone demonstrates the proposed model's accuracy and logic. These results indicate that the model reliably represents soft rocks' nonlinear creep characteristics and the whole creep process.

Article highlights

- A new creep constitutive model was proposed using the fractional derivative theory and introducing a viscoplastic body.
- The model's efficacy was confirmed through the creep test data from different soft rock types.
- Sensitivity analyses were conducted to evaluate the effect of the model’s parameters and stress level on creep strain.
Keywords
Fractional derivative, modified Burger model, soft rock, sensitivity analysis, three-dimensional creep model.

Statements and declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

Data availability Data will be supplied upon request.

Author contribution AT: Formal analysis, Proposing the model, Validation, Writing–original draft. ÁT: Methodology, Formal analysis, Writing–review & editing. PG: Formal analysis, Supervision, Writing–review & editing.

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1. Introduction

Understanding the creep behavior of rocks is a critical aspect of geomechanics, as it plays a vital role in assessing the long-term stability of underground spaces. Creep analysis finds practical application in various engineering projects, including underground energy storage, oil and gas well drilling, time-dependent stability of deep underground excavations, etc. (Ghadrdan et al. 2020; Tarifard et al. 2022). Given the wide-ranging applicability of this field, gaining a deeper understanding of the rock's creep behavior becomes crucial for establishing design and maintenance standards for underground facilities (Frenelus et al. 2022). The creep constitutive models, as one of the key topics in the time-dependent studies of rocks, have attracted significant research (Sharifzadeh et al. 2013). In previous decades, many constitutive models were proposed through empirical, component, and mechanism-based models, and scholars demonstrated that the creep behavior of rock shows significant nonlinear properties (Li et al. 2020b; Zhao et al. 2023).

As a fascinating method for solving challenging issues in mathematical physics modeling, the fractional calculus theory can more accurately represent the nonlinear creep properties of the material (Feng et al. 2021). Fractional derivative creep models have captured the interest of academics due to their substantial advantages, including having few parameters, simple form, and excellent applicability (Huang et al. 2021). Several researchers also considered that the accelerating creep phase involves the rapid development of micro-cracks and can be explained by introducing damage factors and viscoplastic elements (Zhou et al. 2023).

Deng et al. (2022) proposed a fractional creep model by modifying the Nishihara model and introducing a temperature-stress-damage component for coal and suggested a one- and three-dimensional creep model. Lyu et al. (2021) introduced the fractional derivative creep-damage (FDCD) model designed explicitly for salt rock. This model incorporated the Hooke body, Abel dashpot, and viscoplastic element, utilizing a modified Mohr-Coulomb criterion. Expanding the scope, Li et al. (2022b) proposed a 6-elements damage creep model for frozen sandstone; this model was constructed by changing the viscous component with a fractional derivative body and considering the temperature-damage-stress interaction. Xu et al. (2022) introduced a fractional damage model applicable to the creep behavior of rock surrounding circular tunnels. This model integrated the Abel dashpot, Hooke body, damaged Abel dashpot, and Hoek-Brown plastic element. They further provided a closed-form analytical solution using this model. Li et al. (2020a) also employed their novel fractional model to describe the long-term behavior of the Jinping II Hydropower tunnels excavated in marble. The tunnels' convergence strongly agreed with the analytical results obtained using the proposed model. Liu et al. (2021a) introduced an unsteady viscoplastic element coupled with the Maxwell body to explain the complete creep process and damage in swelling rock. They used the Abel dashpot instead of the Newton dashpot, creating the FVP model. Li et al. (2023) also presented a new creep model considering residual strength. They introduced a viscoelastic-plastic body and incorporated a residual strength correction factor. Li et al. (2022a) suggested a nonlinear creep model (the UCCM model) by studying the impact of water weakening damage on shale. They introduced the viscoplastic-damaged body and presumed that each component's damage law in the unloading creep phase is the same as the rise in time. Overall, these studies proposed various innovative models to understand better and characterize the creep behavior of different types of rocks under
specific conditions.

In this study, the initial step involved substituting the Newton dashpot with the fractional derivative Abel dashpot in the Kelvin body while introducing a nonlinear viscoplastic component to represent the tertiary creep phase of rocks. Subsequently, the proposed model was validated using creep test data from various rock types. The nonlinear least squares approach was employed to adopt the creep parameters specific to each rock type, and the isochronous stress-strain curve technique was utilized to assess the long-term strength of soft rocks. Lastly, a sensitivity analysis was conducted on both the model’s parameters and stress levels to evaluate the impact of these factors on creep deformation.

2. Establishment of new one- and three-dimensional creep constitutive model

The creep test results of various rocks show that the typical creep deformation under constant stress follows primary, secondary, and tertiary creep phases. As shown in Fig. 1, the initial stress generates immediate strain, and the creep deformation occurs by passing the time under constant stress. Previous experimental studies indicate that removing the stress at point A quickly reduces strain to point B, followed by an asymptotic return to zero at point C; therefore, region I can be categorized as viscoelastic. In addition, viscoplastic behavior can be classified as a permanent deformation that occurs by removing stress. The material-increasing microcracking causes the third phase of creep and reduces strength and stiffness before failing and losing all the material's capacity to withstand loads (Frenelus et al. 2022).

![Fig. 1 Typical creep deformation of rocks under constant stress](image-url)
In this research and to precisely describe all three phases of rock creep, the Burger model was modified, and the Newton dashpot was replaced with the Abel dashpot in the Kelvin body. In addition, the viscoplastic body was added in series with the modified Burger model to simulate the third phase of rock creep (Fig. 2).

The proposed model combines the modified Kelvin model, the Maxwell body, and the viscoplastic element. In a one-dimensional stress condition, the model's overall strain is:

$$
\varepsilon = \varepsilon_M + \varepsilon_{mK} + \varepsilon_{vp}
$$

(1)

Where $\varepsilon_M$, $\varepsilon_{mK}$, and $\varepsilon_{vp}$ are the Maxwell, modified Kelvin, and viscoplastic bodies strains, respectively.

For the Maxwell body:

$$
\varepsilon_M = \varepsilon_H + \varepsilon_N
$$

(2)

$$
\sigma = \sigma_H = \sigma_N
$$

(3)

$$
\sigma_H = E_1 \varepsilon_H
$$

(4)

$$
\sigma_N = \eta_1 \frac{d\varepsilon_N}{dt}
$$

(5)

$$
\varepsilon_M = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t
$$

(6)

Where $\varepsilon_H$ is the strain and $E_1$ is the elastic modulus of the Hook element, $\varepsilon_N$ is the strain and $\eta_1$ is the viscous coefficient of Newton element.

The Newton dashpot was replaced by the Abel dashpot within the modified Kelvin model to accurately represent the nonlinear behavior of rocks during the transient phase. The Abel dashpot is a fractional derivative representation of the Newton dashpot and is a typical example of a fractional calculus application. The Riemann-Liouville fractional calculus is a commonly employed definition of fractional calculus. The Riemann-Liouville fractional derivatives of $f(t)$ with an order $\gamma$ can be mathematically represented as:

Fig. 2 Schematic representation of the proposed model
\[ D^\gamma f(t) = \frac{1}{\Gamma(n-\gamma)} \left( \frac{d}{dt} \right)^n \int_0^t \frac{f(\xi)}{(t-\xi)^{\gamma-n+1}} d\xi \]  
\hspace{1cm} (7)

Where \( t \) is the time and \( \Gamma(\gamma) = \int_0^\infty e^{-\gamma} \cdot t^{\gamma-1} dt \) is the Gamma function.

The following describes Abel dashpot's constitutive equation:

\[ \sigma(t) = \eta^\gamma D^\gamma (\varepsilon_A(t)) \]  
\hspace{1cm} (8)

\( \eta^\gamma \) is the viscosity coefficient of Abel dashpot.

The Abel dashpot could be utilized to represent both materials between an ideal solid (in the case of \( \gamma = 0 \)) and an ideal fluid (when \( \gamma = 1 \)) (Zhou et al. 2011). By using the principles of the Riemann-Liouville operator and by taking the fractional integral calculation on each side of Eq. (8), the constitutive equation of Abel dashpot under constant stress conditions can be represented as:

\[ \varepsilon_A(t) = \frac{\sigma}{\eta^\gamma \Gamma(1+\gamma)} \frac{t^\gamma}{(0 \leq \gamma \leq 1)} \]  
\hspace{1cm} (9)

The equations of the modified Kelvin body can be expressed using the combined model theory:

\[ \varepsilon_{mK} = \varepsilon_H = \varepsilon_A \]  
\hspace{1cm} (10)

\[ \sigma = E_2 \varepsilon_{mK} + \eta^\gamma_2 D^\gamma (\varepsilon_{mK}) \]  
\hspace{1cm} (11)

\[ D^\gamma (\varepsilon_{mK}) + \frac{E_2}{\eta^\gamma_2} \varepsilon_{mK} = \frac{\sigma}{\eta^\gamma_2} \]  
\hspace{1cm} (12)

\[ a = \frac{E_2}{\eta^\gamma_2}, b = \frac{\sigma}{\eta^\gamma_2} \]  
\hspace{1cm} (13)

\[ D^\gamma (\varepsilon_{mK}) + a \varepsilon_{mK} = b \]  
\hspace{1cm} (14)

The connection between the Riemann-Liouville fractional derivative and the Caputo fractional derivative (\( ^C D^\gamma \)) according to the fractional calculus theory is expressed as:

\[ D^\gamma [f(t)] = ^C D^\gamma [f(t)] + \sum_{k=0}^{n-1} \frac{t^{\gamma-k} f^{(k)}}{\Gamma(k+1)} \]  
\hspace{1cm} (0 < \gamma < n)  
\hspace{1cm} (15)

Since \( D^\gamma [\varepsilon_{mK}] = ^C D^\gamma [\varepsilon_{mK}] \) is obtained from the initial condition \( \varepsilon_{mK} = 0 \), Eq. (14) can be rewritten as:

\[ ^C D^\gamma (\varepsilon_{mK}) + a \varepsilon_{mK} = b \]  
\hspace{1cm} (16)

With a Laplace transform applied to both sides of Eq. (16), we obtain:
\[ s'yE(s) + aE(s) = \frac{b}{s} \]  

\[ E(s) = \frac{b}{s(s'y + a)} \]  

(17)  

Using the inverse Laplace transform on Eq. (18), we get:

\[ \varepsilon_{mK} = b \int_{0}^{t} (t - s)^{y-1}E_{y',y}[\gamma(t - s)^{y}]ds \]  

(19)  

Where:

\[ E_{y',y}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(ky + y)} \]  

(20)  

Furthermore, it can be provided by:

\[ \varepsilon_{mK} = b \sum_{k=0}^{\infty} \frac{t^{y(1+k)}(-a)^{(k)}}{\gamma(1+k)\Gamma[(1+k)y]} \]  

(21)  

By replacing the value of \( a \) and \( b \):

\[ \varepsilon_{mK} = \frac{\sigma}{\eta_2} \sum_{k=0}^{\infty} \frac{t^{y(1+k)}(-\frac{E_2\gamma}{\eta_2})^{(k)}}{\gamma(1+k)\Gamma[(1+k)y]} \]  

(22)  

Once the stress surpasses the long-term strength threshold (\( \sigma_s \)), the consequent increase in creep strain leads to the occurrence of the third phase in rock creep. In this research, a nonlinear viscoplastic element that can describe the creep strain of the tertiary stage is adopted (Fig. 2). The viscoplastic component includes a plastic part representing the long-term strength and a viscous element characterizing the creep rate in the parallel combination. For the viscous element, it was assumed that the rock deformation is exponentially proportional to the time’s power function. The viscoplastic body’s creep equation is:

\[ \varepsilon_{vp} = \begin{cases} 0 & \sigma \leq \sigma_s \\ \frac{\sigma - \sigma_s}{\eta_3}(\exp(t^\alpha) - 1) & \sigma > \sigma_s \end{cases} \]  

(23)  

\( \alpha \) is the creep index that represents the accelerated creep rate of rock, and \( \eta_3 \) is the viscosity coefficient of viscoplastic body.

The constitutive equations of the proposed model in the uniaxial stress condition are derived by considering the strain components of the Maxwell, the modified Kelvin, and the viscoplastic bodies. These equations can be expressed as:
ε(𝑡) = \frac{\sigma}{E_1} t + \frac{\sigma}{\eta_2} t \sum_{k=0}^{\infty} t^{γ(1+k)} \left( - \frac{E_2}{\eta_2} \right)^{(k)} \frac{\Gamma((1+k)γ)}{(1+k)γ} \quad \sigma \leq \sigma_s \tag{24}

ε(𝑡) = \frac{\sigma}{E_1} + \frac{\sigma}{\eta_1} t + \frac{\sigma}{\eta_2} t \sum_{k=0}^{\infty} t^{γ(1+k)} \left( - \frac{E_2}{\eta_2} \right)^{(k)} \frac{\Gamma((1+k)γ)}{(1+k)γ} + \frac{\sigma - \sigma_s}{\eta_3} \left( \exp \left( t^α \right) \right) \quad \sigma > \sigma_s \tag{25}

Establishing three-dimensional creep constitutive equations is vital because rocks are frequently subject to complicated three-dimensional stress states in engineering (Xie et al. 2022). In a three-dimensional stress condition, the rock's stress tensor can be separated into two components: an effective spherical stress tensor (\(\sigma_m\)) which alters the volume of the object, and an effective deviator stress tensor (\(S_{ij}\)) which changes the shape of the object. \(\sigma_m\) and \(S_{ij}\) can be mathematically represented as (Yang et al. 2015):

\[
\sigma_m = \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) = \frac{1}{3} \sigma_{kk} \tag{26}
\]

\[
S_{ij} = \sigma_{ij} - \sigma_m \delta_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \tag{27}
\]

where \(\delta_{ij}\) is the Kronecker delta and \(\sigma_{kk}\) is the effective volume stress.

Additionally, the strain tensor can be separated into two parts: the partial strain tensor (\(\varepsilon_{ij}\)) and the spherical strain tensor (\(\varepsilon_m\)).

\[
\varepsilon_m = \frac{1}{3} (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) = \frac{1}{3} \varepsilon_{kk} \tag{28}
\]

\[
\varepsilon_{ij} = \varepsilon_{ij} - \varepsilon_m \delta_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \tag{29}
\]

For the elastic element under three-dimensional stress conditions:

\[
\sigma_m = 3K \varepsilon_m \tag{30}
\]

\[
S_{ij} = 2G \varepsilon_{ij} \tag{31}
\]

Where \(K\) is the bulk modulus, and \(G\) is the shear modulus. The elastic body's strain can be expressed as:

\[
\varepsilon_{ij} = \varepsilon_m + \varepsilon_{ij} \tag{32}
\]

\[
\varepsilon_{ij} = \frac{S_{ij}}{2\sigma_2} + \frac{\sigma_m}{3K_2} \delta_{ij} \tag{33}
\]

Where \(\sigma_m \delta_{ij}\) is the spherical stress tensor.
For the rheological behavior, it was assumed that the deviatoric stress tensor is the main factor and the spherical stress tensor has minimal effect. For the viscoplastic strain, the creep equation in the triaxial stress state can be expressed as:

$$\varepsilon_{vp} = \left( \frac{\exp(t^\alpha)}{\eta_3} \right) \frac{F}{F_0} \frac{\partial Q}{\partial \sigma_{ij}}$$

Where $Q$ is the plastic potential function, $F$ is the rock yield function, and $F_0$ is the yield function's initial value, typically considered 1 for calculation simplicity. $(F)$ indicates:

$$(F) = \begin{cases} 0 & F \leq 0 \\ F & F > 0 \end{cases}$$

When the corresponding flow rule is implemented, $F=Q$ (Liu et al. 2021b), and Eq. (34) becomes:

$$\varepsilon_{vp} = \left( \frac{(F)}{\eta_3} \right) \frac{\partial F}{\partial \sigma_{ij}} \left( \exp(t^\alpha) \right)$$

The adopted yield function is expressed as:

$$F = \sqrt{J_2} - \frac{\sigma_2}{\sqrt{3}}$$

Where, $J_2$ is the deviatoric stress's second invariant and is defined as:

$$J_2 = \frac{1}{6} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$

Assuming that the same confining pressure is used ($\sigma_2 = \sigma_3$)

$$\sqrt{J_2} = \frac{\sigma_1 - \sigma_3}{\sqrt{3}}$$

$$S_{ij} = \frac{2(\sigma_1 - \sigma_3)}{3}$$

$$\sigma_m = \frac{(\sigma_1 + 2\sigma_3)}{3}$$

The model's three-dimensional equations can be stated as follows:

$$\varepsilon(t) = \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 + 2\sigma_3}{9K_1} + \frac{\sigma_1 - \sigma_3}{3\eta_1} t + \frac{\sigma_1 - \sigma_3}{3\eta_2} \sum_{k=0}^\infty \frac{t^{y(1+k)}}{\Gamma[1+y(1+k)]} \frac{E_2^{(k)}(\eta_2)}{\eta_2}$$

$$\sigma_1 - \sigma_3 \leq \sigma_3$$

$$\varepsilon(t) = \frac{\sigma_1 - \sigma_3}{3G_1} + \frac{\sigma_1 + 2\sigma_3}{9K_1} + \frac{\sigma_1 - \sigma_3}{3\eta_1} t + \frac{\sigma_1 - \sigma_3}{3\eta_2} \sum_{k=0}^\infty \frac{t^{y(1+k)}}{\Gamma[1+y(1+k)]} \frac{E_2^{(k)}(\eta_2)}{\eta_2} \left( \exp(t^\alpha) \right)$$

$$\sigma_1 - \sigma_3 > \sigma_3$$

$$\sigma_1 - \sigma_3 > \sigma_3$$
3. Model validation

For assessing the efficacy of the suggested constitutive model, the creep test results of different types of rocks (salt, sandstone, and shale), (Zhang et al. 2019; Wu et al. 2020; Zhao et al. 2021), were selected to calculate the model parameters according to the nonlinear least square approach.

The isochronous stress-strain curve method was employed to specify the long-term strength of the rock ($\sigma_s$), which is considerably less than its normal instantaneous strength. This method, as a commonly utilized approach for determining $\sigma_s$, involves analyzing a series of creep curves at various stress conditions. Each isochronal curve is divided into two sections, the first section is roughly similar to a straight line, and the next part bends progressively. Based on the inflection points in the isochronal curves, $\sigma_s$ is calculated by drawing a line through the inflection points parallel to the transverse axis and intersecting the longitudinal axis. Fig. 3 shows the isochronal curves of different rocks.
After determining $\sigma_s$, the nonlinear least squares method was employed for each rock to derive the creep parameters. Table 1 displays the model's parameters obtained by fitting analysis, and Fig. 4 compares fitting results with experimental data.
Table 1 Calculated parameters for the proposed model

<table>
<thead>
<tr>
<th>Creep test</th>
<th>$\sigma$ (MPa)</th>
<th>$\gamma$ (GPa.h)</th>
<th>$\eta_3$ (GPa.h)</th>
<th>$E_1$ (GPa)</th>
<th>$E_2$ (GPa)</th>
<th>$\gamma$</th>
<th>$\eta_3$ (GPa.h)</th>
<th>$\alpha$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salt</td>
<td>18</td>
<td>0.39</td>
<td>2.33</td>
<td>0.45</td>
<td>0.72</td>
<td>0.42</td>
<td>0.96</td>
<td>0.72</td>
<td>98%</td>
</tr>
<tr>
<td>Sandstone</td>
<td>40</td>
<td>5.7</td>
<td>18.18</td>
<td>10.17</td>
<td>16.3</td>
<td>0.38</td>
<td>22.23</td>
<td>0.82</td>
<td>97%</td>
</tr>
<tr>
<td>Shale</td>
<td>58</td>
<td>2.9</td>
<td>33.14</td>
<td>4.35</td>
<td>64</td>
<td>0.31</td>
<td>32</td>
<td>0.98</td>
<td>96%</td>
</tr>
</tbody>
</table>

![Graph showing strain vs. time with fitted model parameters](image)
Fig. 4 The comparison of fitting results by a suggested model with experimental data, a) shale, b) salt, and c) sandstone.
The correlation between the proposed model and test results can be observed from Fig. 4 and Table 1, indicating the model's capability to accurately capture the creep behavior of different rocks, especially during the tertiary creep stage.

4. Parametric sensitivity analysis

In this part, a sensitivity analysis was conducted, and the influence mechanism of important parameters was assessed. For this purpose, we modified one of the parameters to perform a sensitivity study, focusing on its effect on creep properties. Meanwhile, the remaining parameters are held constant to isolate the influence of the single parameter under investigation.

The impact of axial stress levels on the creep strain of the tertiary phase was examined using the fitting data of sandstone. Axial stress values are considered 35, 40, 45, and 50 MPa, respectively.

![Fig. 5 Sensitivity analysis of creep strain to stress level at tertiary phase](image.png)

As seen in Fig. 5, when the applied stress is greater than $\sigma_s$, the higher the axial stress causes more considerable creep deformation and causes the accelerated creep stage to appear earlier. This evolution supports the hypothesis that entry into the tertiary stage is promoted by increasing differential stress, further supporting the validity of the model proposed in this research.

The effect of the fractional derivative order in the modified Kelvin body was assessed, as demonstrated in Fig. 6; it is observed that the creep deformation increases as the fractional derivative order increases. The variation in order
directly affects the extent of creep strain, highlighting the efficacy and versatility of fractional derivatives in assessing nonlinear rheological issues.

Fig. 6 The modified Kelvin body's fractional derivative order sensitivity analysis

Fig. 6 demonstrates that the creep strain also exhibits an increasing trend as the order of the fractional derivative rises. In this sensitivity analysis, when the $\gamma$ is smaller than 0.25, the creep strain enters a flat phase by passing the time, and the creep deformation grows rapidly when the $\gamma$ is higher than 0.25.

Fig. 7 illustrates that the viscoplastic body can accurately represent the rock's nonlinear creep acceleration phase. When the $\alpha$ value rises, the shale's associated creep rate also increases. For the shale, the creep deformation grows nonlinearly with time when $\alpha$ is larger than 0.5.
5. Discussion

The modeling of the creep behavior of rocks, especially weak and soft rocks, has received significant attention. After proposing the new one- and three-dimensional creep constitutive models, we verified the model with the experimental creep results. Different rock types with different test conditions were selected, the duration of creep tests varied from 15 mins to 300 hours, and the applied stress varied from 1.2 to 1.5 times of rocks’ long-term strength. Fig. 4 shows that the suggested model can successfully model the creep behavior of various soft rocks. The high correlation between the calculated results and test data especially for the third phase, is considerable in our study. Fig. 6 demonstrates that the various patterns can be obtained by modifying the derivative order values ($\gamma$). Our finding also indicates that the higher derivative order values typically result in more significant strain rates and higher creep strains. In addition, the high-stress level results in more considerable creep deformation and causes the accelerated creep stage to appear earlier. Li et al. (2021) assessed the sensitivity of the fractional derivative order, and Zhang et al. (2022) demonstrated the effect of applied stress on the rocks’ creep behavior. The comparison of Fig. 5 and 7 shows that for the viscoplastic body, the most sensitive parameter is $\alpha$ compared with the applied stress.

6. Conclusion

The new one- and three-dimensional creep constitutive models were presented with the help of the fractional-order derivative Kelvin body and the introduction of the viscoplastic element. The model's applicability and validity are confirmed using creep experiment results on different rock types. The followings are the main conclusions reached.
• Under the long-term strength stress, the viscoplastic component remains inactive in the proposed model, allowing for the simulation of the first and second creep stages. However, when the stress level surpasses the long-term strength threshold, the viscoplastic body becomes activated, enabling the model to accurately simulate the complete creep phases of rocks.

• The fitting results on the different types of soft rocks show a strong correlation demonstrating the proposed model's ability to capture the creep behavior, particularly the accelerated creep stage.

• Higher derivative order values typically result in larger strain rates and higher creep strains. By modifying the derivative order, different creep curve patterns can be obtained.

• The proposed fractional model can be reduced to the Burger model when the value of γ is set to 1, and the stress applied to the model is below the long-term strength.

• When the α value rises, the associated creep rate increases for the viscoplastic body. Investigating parameter sensitivity for the proposed model reveals that both the α parameter and the stress level significantly influence the creep strain at the accelerating phases.

• Compared to the experimental data and considering the high fitted correlation coefficient (above 96%), the suggested model can successfully represent the creep behavior of different soft rocks.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

Data availability Data will be supplied upon request.

Author contribution AT: Formal analysis, Proposing the model, Validation, Writing–original draft. ÁT: Methodology, Formal analysis, Writing–review & editing. PG: Formal analysis, Supervision, Writing–review & editing.

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