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Complex Fermatean Fuzzy Partitioned Maclaurin Symmetric Mean operators and their Application to Hostel Site Selection

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Abstract

In relation to multi-criteria group decision making (MCGDM) problems where the evaluated value of each criteria is represented as complex Fermatean fuzzy sets, a novel approach to MCGDM is formulated. Firstly, a new aggregation operator, termed the partitioned Maclaurin symmetric mean (PMSM) operator, is introduced to address situations where criteria are partitioned into different parts, and there exist interrelationships among multiple criteria within the same part, while criteria in different parts are independent. The desirable properties of the PMSM operator are thoroughly investigated. To mitigate the adverse effects of unreasonable evaluation values of criteria on the aggregated result, complex Fermatean fuzzy power partitioned Maclaurin symmetric mean (CFFPPMSM) operators are further proposed. These operators combine the PMSM operator with the power average (PA) operator within complex Fermatean fuzzy sets. The weight of the criteria is taken unknown and find using the statistical method. In conclusion, a numerical instance is furnished to exemplify the application of the proposed approach in determining the optimal location for a university hostel, and a comparative analysis is conducted to demonstrate the advantages of the proposed methodology in handling complex Fermatean fuzzy MCGDM problems.

Index Terms: Fermatean Fuzzy set, Complex Fermatean Fuzzy set, Partitioned Maclaurin symmetric mean, power partitioned Maclaurin symmetric, Multiple criteria group decision making.

1 Introduction

MAGDM holds significant importance in real-life scenarios, involving the selection of the best alternative from a set of possibilities. In these problems, decision-makers typically express their evaluations of criteria and alternatives using fuzzy sets (FS). Moreover, Atanassove [1] expanded the concept of fuzzy sets (FS) to include an intuitionistic fuzzy set (IFS), which incorporates the non-membership degree. IFS proves to be a more dependable framework for effectively representing human opinions. Essentially, an IFS is characterized by membership and non-membership grades whose sum is less than or equal to one. Furthermore, Yager [2] introduced the concept of ordered weighted aggregation of intuitionistic fuzzy sets (IFS). Several researchers have suggested various information types, such as similarity measures Szmidt and Kacprzyk [3], distance measures Wang and Xin [4, 5], entropy measures Szmidt and Kacprzyk [6], Burillo and Bustince [7], and score and score function Garg [8]. Yager [9] introduced a class of sets called -rung orthopair fuzzy sets, where
the sum of the th power of the support for and the qth power of the support against is bounded by one. Yager observed that as L increases, the range of acceptable orthopairs expands, providing users with greater flexibility in expressing their beliefs about membership grades. Senapati and Yager [10] further explored -rung orthopair fuzzy sets when \( q = 3 \), considering them as Fermatean fuzzy sets (FFSs). Senapati and Yager [10] established fundamental operations for Fermatean fuzzy sets (FFSs) and introduced score functions and accuracy functions for FFSs. They extended the technique of order preference by similarity to ideal solution (TOPSIS) to address the multi-criteria decision-making (MCDM) problem with Fermatean fuzzy information. Furthermore, in Senapati and Yager [11], they introduced novel operations, including subtraction, division, and Fermatean arithmetic mean operations, specifically designed for FFSs. Additionally, they applied the Fermatean fuzzy weighted product model to solve multi-criteria decision-making problems.

The concept of complex fuzzy sets (CFS) was introduced by Romat et al. [12], while the notion of complex intuitionistic fuzzy sets (CIFS) was initiated by Alkouri and Salleh [13]. CIFS is a generalization of CFS, encompassing complex non-membership values. CIFS proves to be more effective than CFS in describing uncertain and unpredictable information during real decision-making. Recently, Garg and Rani [14] proposed a robust correlation coefficient measure for CIFS and explored its applications in decision-making. They also developed generalized complex intuitionistic fuzzy aggregation operators and applied them to multicriteria decision-making Garg and Rani [14]. Additionally, Rani and Garg [15] introduced complex intuitionistic fuzzy power aggregation operators and investigated their applications in multicriteria decision-making. Moreover, Rani and Garg [16] developed distance measures between CIFS and explored their applications in the decision-making process. Chinnadurai et al. [17] presented the concept of complex cubic intuitionistic fuzzy sets (CCIFS) along with its properties. They also introduced complex cubic interval-valued intuitionistic fuzzy sets (CCIVIFS) through a case study. Akram et al. [18] developed the notion of complex Pythagorean fuzzy Yager aggregation operator and applied it to real-life scenarios. Chinnadurai et al. [17] discussed the idea of complex interval-valued Pythagorean fuzzy sets (CIVPFS) and its properties. Ullah et al. [19] investigated the properties of complex Pythagorean fuzzy sets (CPFS) and discussed its application in decision-making with a pattern recognition example. In Chinnadurai et al. [20], the concept of complex Fermatean fuzzy sets (CFFS) was introduced, and its properties were established. The study aims to highlight the limitations of existing complex theories and demonstrate the superiority of CFS. CFFS is proposed to handle information that cannot be effectively processed using complex fuzzy sets (CFS), complex intuitionistic fuzzy sets (CIFS), and complex Pythagorean fuzzy sets (CPFS).

Aggregation operators hold a significant role in multi-attribute group decision making (MAGDM), particularly those that consider the interrelationships among attributes. Based on the type of relationship between attributes, aggregation operators can be categorized into two groups. The first group assumes that each attribute is related to other attributes, exemplified by operators like the power average (PA) operator by Yger [21] and power geometric (PG) operator [22]. These operators allow attributes to be aggregated to support and reinforce each other. However, the PA and PG operators only account for the relationship by assigning weights to each attribute, without directly reflecting the interrelationship structure among them. Consequently, Yager [2] extended the BM [23] to account for the interrelationship between any two attributes. Xia et al. [24] further generalized the classical BM and introduced the generalized weighted BM (GWBDM), enabling the measurement of interrelationship among any three arguments. Zhang et al. [25] also defined the dual generalized weighted BM (DGWBM) operator. To capture the interrelationship among multiple attributes, Detemple and Robertson [26] explored the MSM [27] operator in MAGDM.
The MSM assumes that each argument is related to other k-1 arguments, and the parameter k can be adjusted by the decision maker, providing a flexible approach. Due to this adaptability, the MSM has been effectively utilized to address various MAGDM problems [28, 29]. The operators mentioned above assume that each attribute is related to others in MAGDM. However, in reality, interrelationships may not exist among all attributes. Therefore, the second group of operators focuses on situations where some attributes are related, while others have no interrelationship. One representative operator for such scenarios is the partitioned Bonferroni mean (PBM) operator [30]. The PBM considers situations where the arguments are partitioned into several parts, and the arguments within the same part are related to each other. Similarly, Liu et al. [31] extended the Heronian mean (HM) to the partitioned Heronian mean (PHM). Both the PBM and PHM operators find extensive application in the decision-making process [32,33].

We initiate by introducing the PMSM operator along with its corresponding mathematical formulation. Additionally, we explore favorable characteristics and distinct scenarios involving the PMSM. Notably, we unveil that by manipulating the parameters of the PMSM, various established operators can be derived. Expanding on this, we enhance the PMSM within the context of complex fermatean fuzzy set (CFFS), resulting in the introduction of the complex fermatean fuzzy partitioned Maclaurin symmetric mean (CFFPMSM) operator and the complex fermatean fuzzy weighted partitioned Maclaurin symmetric mean (CFFWPMSM) operator. These novel operators are designed to effectively manage complex fermatean fuzzy information. To mitigate the adverse impact of irrational assessments on decision outcomes, we synergize the PMSM and PA methodologies. This synergy gives rise to the CFFPPMSM operator and its weighted counterpart, known as the complex fermatean fuzzy weighted power partitioned Maclaurin symmetric mean (CFFWPMSM) operator. Lastly, we introduce an innovative approach centered around the CFFWPMSM operator to address challenges posed by complex fermatean fuzzy MAGDM problems. To demonstrate the efficacy of our proposed approach, we present a numerical example in which we select the best location for the hostel and conduct a comparative analysis, highlighting the distinct advantages of our approach. The paper’s notable contributions can be outlined as follows:

- To handle the division of attributes into segments with interrelationships, propose the PMSM operator within the framework of CFFS.
- To leverage both PMSM and PA within CFFS, we introduce the CFFPPMSM and its weighted variant, aiming not only to utilize PMSM’s benefits but also to mitigate the adverse impact of unfounded arguments on the aggregation outcome.
- To select the best location for hostel establish an MCDM strategy depending on the created CFFWPMSM operator.
- To conduct a case study and a comparative analysis to show the value and superiority of the conventional technique.

The subsequent structure of this paper is as outlined: Section 2 presents certain fundamental notions. In Section 3, we establish the definitions for the PMSM, the CFFPMSM, and the CFFWPPMSM. Section 4, Based on the PA and PMSM operators, we put forward the CFFPPMSM and CFFWPMSM. A statistical variance to find the weight of the criteria is define in section 5. Section 6, introduces a novel approach for complex fermatean fuzzy MCGDM that relies on CFFWPMSM. In Section 7, a numerical illustration regarding optimal hostel location showcases the effectiveness and merits of the suggested methodology. In Section 8, By evaluating their ranking outcomes and
conducting a validity assessment, we compare our approach with established methods, while the final section provides a concise overview of the paper’s content.

2 Some Basic Concepts

In this section, we allocate some basic concepts related to complex fermatean fuzzy set and partitioned maclaurin symmetric mean.

**Definition 1** [34] Let $T$ be a domain of discourse. Then, a Fermatean Fuzzy set (FFS) $\mathcal{G}$ on $T$ is described as:

$$\mathcal{G} = \{(t, R_F (t), L_F (t)) | t \in T\},$$

where $R_F$ and $L_F$ are the functions which show the degree of membership and non-membership of the element $t$ in the set $\mathcal{G}$ and assigns values within the range of $[0, 1]$. Including the condition

$$0 \leq (R_F)^3 + (L_F)^3 \leq 1.$$

For any FFS $\mathcal{G}$ and $t \in T$. The degree of indeterminacy of $t$ to $\mathcal{G}$ is defined as:

$$\pi_F (t) = \sqrt[3]{1 - (R_F)^3 - (L_F)^3}.$$

for the sake of simplicity, we will refer to the symbol $\mathcal{G} = (R_F, L_F)$ as Fermatean Fuzzy number (FFN).

**Definition 2** [34] Let $\mathcal{G}_1 = (R_{F1}, L_{F1})$, and $\mathcal{G}_2 = (R_{F2}, L_{F2})$ be two FFNs and $\kappa > 0$, then

1. $\mathcal{G}_1 \cap \mathcal{G}_2 = (\min (R_{F1}, R_{F2}), \max (L_{F1}, L_{F2}));$
2. $\mathcal{G}_1 \cup \mathcal{G}_2 = (\max (R_{F1}, R_{F2}), \min (L_{F1}, L_{F2}));$
3. $\mathcal{G}^\kappa = (L_F, R_F);$
4. $\mathcal{G}_1 \oplus \mathcal{G}_2 = \left(\sqrt[3]{(R_{F1})^3 + (R_{F2})^3 - (R_{F1})^3 (R_{F2})^3}, (L_{F1}) (L_{F2})\right);$
5. $\mathcal{G}_1 \otimes \mathcal{G}_2 = \left((R_{F1}) (R_{F2}), \sqrt[3]{(L_{F1})^3 + (L_{F2})^3 - (L_{F1})^3 (L_{F2})^3}\right);$
6. $\kappa \mathcal{G} = \left(\sqrt[3]{1 - (1 - R_F^3)^\kappa}, L_F^\kappa\right);$
7. $\mathcal{G}^\kappa = \left(R_F^\kappa, \sqrt[3]{1 - (1 - L_F^3)^\kappa}\right);$

**Definition 3** [35] Let $T$ be a domain of discourse. Then, a complex fermatean fuzzy set (CFFS) $\mathcal{Y}$ on $T$ is described as:

$$\mathcal{Y} = \left(t', R(t'), L(t') | t' \in T\right),$$

such that $R(t') = s_1' = u + iv$ and $L(t') = s_2' = u + iv$ provided that $0 \leq |s_1|^3 + |s_2|^3 \leq 1$ or...
\( R(t') = \gamma(t')e^{2\pi \phi_\gamma(t')} \) and \( \Sigma(t') = \theta(t')e^{2\pi \phi_\theta(t')} \).

Satisfying the condition \( 0 \leq (\theta(t'))^3 + (\phi(t'))^3 \leq 1 \) and \( 0 \leq (\phi_\gamma(t'))^3 + (\theta(t'))^3 \leq 1 \).

**Definition 4** Let \( T \) be a domain of discourse and all \( t' \in T \) then, hesitancy degree define as \( D = \zeta(t')e^{2\pi \phi_\zeta(t')} \) such that,

\[
\zeta(t') = \sqrt[3]{\frac{1 - (\gamma(t'))^3 - (\theta(t'))^3}{1 - (\phi_\gamma(t'))^3 - (\phi_\theta(t'))^3}}.
\]

Furthermore \( \mathcal{J} = (\gamma.e^{2\pi \phi_\gamma}, \theta.e^{2\pi \phi_\theta}) \) is called the complex fermatean fuzzy set (CFFs).

**Definition 5** [35] Let \( \mathcal{J} = (\gamma.e^{2\pi \phi_\gamma}, \theta.e^{2\pi \phi_\theta}), \mathcal{J}_1 = (\gamma_1.e^{2\pi \phi_{\gamma_1}}, \theta_1.e^{2\pi \phi_{\theta_1}}) \) and \( \mathcal{J}_2 = (\gamma_2.e^{2\pi \phi_{\gamma_2}}, \theta_2.e^{2\pi \phi_{\theta_2}}) \) be three CFFs and \( \kappa > 0 \), then the basic operation are defined as:

1. \( \mathcal{J}_1 \cap \mathcal{J}_2 = (\min(\gamma_1, \gamma_2) . e^{2\pi \min(\phi_{\gamma_1}, \phi_{\gamma_2})}, \max(\theta_1, \theta_2) . e^{2\pi \max(\phi_{\theta_1}, \phi_{\theta_2})}) \);
2. \( \mathcal{J}_1 \cup \mathcal{J}_2 = (\max(\gamma_1, \gamma_2) . e^{2\pi \max(\phi_{\gamma_1}, \phi_{\gamma_2})}, \min(\theta_1, \theta_2) . e^{2\pi \min(\phi_{\theta_1}, \phi_{\theta_2})}) \);
3. \( \mathcal{J}^c = (\theta.e^{2\pi \phi_\theta}, \gamma.e^{2\pi \phi_\gamma}) \);
4. \( \mathcal{J}_1 \oplus \mathcal{J}_2 = (\sqrt[3]{\gamma_1^3 + \gamma_2^3 - \gamma_1 \gamma_2} . e^{2\pi \left(\sqrt[3]{\gamma_1^2 + \gamma_2^2 - \gamma_1 \gamma_2} - \phi_{\gamma_1} + \phi_{\gamma_2} - \phi_{\gamma_1} - \phi_{\gamma_2}\right)}, (\theta_1 \theta_2) . e^{2\pi \phi_{\theta_1} + \phi_{\theta_2}}) \);
5. \( \mathcal{J}_1 \oplus \mathcal{J}_2 = (\gamma_1 \gamma_2) . e^{2\pi (\phi_{\gamma_1} + \phi_{\gamma_2})}, \frac{1}{2}(\theta_1^2 + \theta_2^2 - \theta_1 \theta_2) . e^{\pi (\phi_{\theta_1} + \phi_{\theta_2} - \phi_{\theta_1} - \phi_{\theta_2})}) \);
6. \( \kappa.\mathcal{J} = (\gamma_1 \gamma_2) . e^{2\pi (\phi_{\gamma_1} + \phi_{\gamma_2})}, \frac{1}{2}(\theta_1^2 + \theta_2^2 - \theta_1 \theta_2) . e^{\pi (\phi_{\theta_1} + \phi_{\theta_2} - \phi_{\theta_1} - \phi_{\theta_2})}) \);
7. \( \mathcal{J}^\kappa = (\gamma_1^{\kappa} . e^{2\pi (\phi_{\gamma_1}^\kappa)}, \frac{1}{2}(\theta_1^2 + \theta_2^2 - \theta_1 \theta_2) . e^{\pi (\phi_{\theta_1}^\kappa + \phi_{\theta_2}^\kappa)}) \).

**Definition 6** [36] For a CFFs \( \mathcal{J} = (\gamma.e^{2\pi \phi_\gamma}, \theta.e^{2\pi \phi_\theta}) \), the score function and accuracy function are defined as:

\[
S_c(\mathcal{J}) = \frac{1}{2} \left( (\gamma^3 - \theta^3) + (\phi_{\gamma}^3 - \phi_{\theta}^3) \right),
\]

\[
A_c(\mathcal{J}) = \frac{1}{2} \left( (\gamma^3 + \theta^3) + (\phi_{\gamma}^3 + \phi_{\theta}^3) \right).
\]

**Definition 7** [37] Let \( \mathcal{J}_{kl} = (\gamma_{kl}e^{2\pi \phi_{\gamma_{kl}}}, \theta_{kl}e^{2\pi \phi_{\theta_{kl}}}) \) and \( \mathcal{J}_{km} = (\gamma_{km}e^{2\pi \phi_{\gamma_{km}}}, \theta_{km}e^{2\pi \phi_{\theta_{km}}}) \) \((l, m = 1, ..., n)\) be two CFFs. Then, the Hamming distance is defined as:

\[
d(\mathcal{J}_{kl}, \mathcal{J}_{km}) = \frac{1}{4} \left( |\gamma_{kl} - \gamma_{km}| + |\theta_{kl} - \theta_{km}| + |\phi_{\gamma_{kl}} - \phi_{\gamma_{km}}| + |\phi_{\theta_{kl}} - \phi_{\theta_{km}}| \right).
\]
Definition 8 [21] Let $\mathcal{J}_{kl}$ and $\mathcal{J}_{km}$ ($l, m = 1, \ldots, n$) be two CFFs, then their supports and total supports is defined as:

$$\text{supp} (\mathcal{J}_{kl}, \mathcal{J}_{km}) = 1 - d (\mathcal{J}_{kl}, \mathcal{J}_{km}),$$ (6)

$$T (\mathcal{J}_{kl}) = \sum_{l=1, l \neq m}^{n} \text{Supp} (\mathcal{J}_{kl}, \mathcal{J}_{km}).$$ (7)

$\text{Supp} (\mathcal{J}_{kl}, \mathcal{J}_{km})$ is denoted as the support degree for $\mathcal{J}_{kl}$ from $\mathcal{J}_{km}$, which satisfies following properties:

1. $\text{Supp} (\mathcal{J}_{kl}, \mathcal{J}_{km}) \in [0, 1]$;
2. $\text{Supp} (\mathcal{J}_{kl}, \mathcal{J}_{km}) = \text{Supp} (\mathcal{J}_{km}, \mathcal{J}_{kl})$;
3. $\text{Supp} (\mathcal{J}_{kl}, \mathcal{J}_{km}) \geq \text{Supp} (\mathcal{J}_{kp}, \mathcal{J}_{kr})$, if $|\mathcal{J}_{kl}, \mathcal{J}_{km}| \leq |\mathcal{J}_{kp}, \mathcal{J}_{kr}|$.

3 Proposed CFFPMSM operator and CFFWPMMSM operator

In this section, we implement the PMSM operator into the complex fermatean fuzzy model in order to develop a clear two synthetic operators, CFFPMSM and CFFWPMMSM:

3.1 CFFPMSM aggregation operators

Definition 9 Let $\mathcal{J}_i$ ($1, 2, \ldots, n$) be a range of $n$ CFFs. Then, CFFPMSM operator is characterized as

$$\text{CFFPMSM}^{(\rho_1, \rho_2, \ldots, \rho_t)} (\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{1}{t} \sum_{\gamma=1}^{t} \left( \frac{1}{u_{\mathcal{J}_i}} \right) \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_1}} \right)^{\frac{1}{\rho_1}} \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_2}} \right)^{\frac{1}{\rho_2}} \cdots \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_t}} \right)^{\frac{1}{\rho_t}},$$ (8)

The parameter $\rho$, $\rho = 1, 2, \ldots, o_\gamma$, $o_\gamma$ denotes maximum number of criteria in category $r_\gamma$, $t$ represents the total number of criteria, and $(k_1, k_2, \ldots, k_\rho)$ encompasses all possible $\rho$-tuples of $(1, 2, \ldots, o_\gamma)$, where $C_{\mathcal{J}_i}^{\rho} = \frac{\rho!}{\rho!(o_\gamma - \rho)!}$.

Theorem 1 Let $\mathcal{J}_i$ ($1, 2, \ldots, n$) be the collection of complex fermatean fuzzy set. The aggregated result of formula citation is still CFFs, characterized as below:

$$\text{CFFPMSM}^{(\rho_1, \rho_2, \ldots, \rho_t)} (\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{1}{t} \sum_{\gamma=1}^{t} \left( \frac{1}{u_{\mathcal{J}_i}} \right) \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_1}} \right)^{\frac{1}{\rho_1}} \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_2}} \right)^{\frac{1}{\rho_2}} \cdots \left( \frac{1}{C_{\mathcal{J}_i}^{\rho_t}} \right)^{\frac{1}{\rho_t}}.$$. (9)
\[ \oplus_{l=1}^{\rho} C_{\mathbb{I}} = \left\{ \prod_{l=1}^{\rho} u_{il} e^{i2\pi (\prod_{l=1}^{\rho} u_{il})} \left( 1 - \prod_{l=1}^{\rho} (1 - v_{il}^3) \right)^{\frac{1}{3}} \cdot \epsilon \left( 1 - \prod_{l=1}^{\rho} (1 - v_{il}^3) \right)^{\frac{1}{3}} \right\} \]

\[ \oplus_{i_1 < \ldots < i_k < o_Y} \left( \oplus_{l=1}^{\rho} C_{\mathbb{I}} \right) = \left\{ \prod_{i_1 < \ldots < i_k < o_Y} \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right) \right)^{\frac{1}{3}} \cdot \epsilon \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right) \right)^{\frac{1}{3}} \right\} \]

\[ \oplus_{\gamma=1}^{t} \left( \oplus_{i_1 < \ldots < i_k < o_Y} \left( \oplus_{l=1}^{\rho} C_{\mathbb{I}} \right) \right)^{\frac{1}{3}} = \left\{ \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{i_1 < \ldots < i_k < o_Y} \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right) \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \cdot \epsilon \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right) \right)^{\frac{1}{3}} \right\} \]

\[ \epsilon \left( 1 - \left( \prod_{i_1 < \ldots < i_k < o_Y} \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right) \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \]
\[
\frac{1}{t} \otimes_{\gamma=1}^{t} \left( \frac{\otimes_{1 < i_1 < \ldots < i_k < 0, \gamma} (\otimes_{l=1}^{n} 3 \mathfrak{d})}{C^{\rho}_{\omega_{\gamma}}} \right)^{\frac{1}{\gamma}} = \\
\left\{ \begin{array}{c}
\left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \end{array} \right\},
\right.
\]

This completes the verification. In the following, we state and prove certain desired results based on Theorem 1.

**Theorem 2 (Idempotency)** If the given, CFFS \( \mathfrak{d}_i (i = 1, 2, 3, \ldots, n) \) are equal i.e.

\[
CFFPM^{(\rho)} (\mathfrak{d}_1, \mathfrak{d}_2, \ldots, \mathfrak{d}_n) = \mathfrak{d}_i.
\]

**Proof.** In accordance with Theorem 1, we have

\[
CFFPM^{(\rho_1, \rho_2, \ldots, \rho_n)} (\mathfrak{d}_1, \mathfrak{d}_2, \ldots, \mathfrak{d}_n) = \\
\left\{ \begin{array}{c}
\left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \end{array} \right\},
\right.
\]

\[
= \\
\left\{ \begin{array}{c}
\left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \\
\left( \prod_{\gamma=1}^{t} \left( 1 - \left( \prod_{1 < i_1 < \ldots < i_k < 0, \gamma} \left( 1 - \frac{1}{\prod_{l=1}^{n} \rho^{3 u_{\gamma, l}}} \right)^{\frac{1}{\gamma}} \right) \right)^{\frac{1}{\gamma}} \right)^{\frac{1}{\gamma}} \end{array} \right\},
\right.
\]

8
\[
\begin{align*}
= & \left\{ \left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - \left( u_{\gamma}^3 \right)^{\frac{1}{3}} \right) \right) \right)^{\frac{1}{3}} \cdot e^{i2\pi \left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - u_{\gamma}^3 \right) \right) \right)^{\frac{1}{3}}} \right\}, \\
= & \left\{ \left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - \left( -v_{\gamma}^3 \right)^{\frac{1}{3}} \right) \right) \right)^{\frac{1}{3}} \cdot e^{i2\pi \left( 1 - \left( \prod_{\gamma=1}^{t} \left( 1 - (-v_{\gamma}^3)^{\frac{1}{3}} \right) \right) \right)^{\frac{1}{3}}} \right\} = \left( u, e^{i2\pi u}, v, e^{i2\pi v} \right).
\end{align*}
\]

This complete the proof. ■

**Theorem 3 (Monotonicity)** let \( \mathcal{J}_i (i = 1, 2, 3, \ldots, n) \) be CFFs where \( \mathcal{J}_i = (u_{3i}, e^{i2\pi u_{3i}}, v_{3i}, e^{i2\pi v_{3i}}) \) and let \( \mathcal{J}_i' (i = 1, 2, 3, \ldots, n) \) where \( \mathcal{J}_i' = (u_{3i}', e^{i2\pi u_{3i}'}, v_{3i}', e^{i2\pi v_{3i}'}) \), which meet the condition \( u_i' \geq u_i \) and \( v_i' \leq v_i \) for all \( i = 1, 2, \ldots, n \), then

\[
CFFPMSM^\rho (\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) \leq CFFPMSM^\rho (\mathcal{J}_1', \mathcal{J}_2', \ldots, \mathcal{J}_n).
\]  

**Proof.**

We can capture that \( \rho \geq 1 \) and \( C_{\phi}^0 \geq 1 \) easily. Firstly, let us consider the membership part. Because \( u_i \leq u_i' \) for all \( i \), Thus we have

\[
\prod_{l=1}^{\rho} u_{il} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \leq \prod_{l=1}^{\rho} u_{il}' e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il}' \right)^{\frac{1}{3}}},
\]

\[
\implies 1 - \left( \prod_{l=1}^{\rho} u_{il} \right)^{\frac{3}{\rho}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \geq 1 - \left( \prod_{l=1}^{\rho} u_{il}' \right)^{\frac{3}{\rho}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il}' \right)^{\frac{1}{3}}}.
\]

\[
\implies 1 - \left( \prod_{1<i_1<\ldots<i_{\phi}} \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right)^{\frac{3}{\rho}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \right) \right) \cdot e^{i2\pi \left( \prod_{1<i_1<\ldots<i_{\phi}} \left( 1 - \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}}}
\]

\[
\leq 1 - \left( \prod_{1<i_1<\ldots<i_{\phi}} \left( 1 - \left( \prod_{l=1}^{\rho} u_{il} \right)^{\frac{3}{\rho}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \right) \right) \cdot e^{i2\pi \left( \prod_{1<i_1<\ldots<i_{\phi}} \left( 1 - \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}} e^{i2\pi \left( \prod_{i=1}^{\rho} u_{il} \right)^{\frac{1}{3}}} \right) \right)^{\frac{1}{3}}}.
\]
which shows that, \( u_i.e^{i2\pi u_i} \leq u_i'.e^{i2\pi u_i'} \) so we can get that \( (u_i.e^{i2\pi u_i})^3 \leq (u_i'.e^{i2\pi u_i'})^3 \).

Similarly, if \( v_i.e^{i2\pi v_i} \geq v_i'.e^{i2\pi v_i'} \) we can obtain \( (v_i.e^{i2\pi v_i})^3 \geq (v_i'.e^{i2\pi v_i'})^3 \).

If \( (\mu_i.e^{i2\pi \mu_i})^3 < (\mu_i'.e^{i2\pi \mu_i'})^3 \) and \( (\nu_i.e^{i2\pi \nu_i})^3 \geq (\nu_i'.e^{i2\pi \nu_i'})^3 \) then,

\[
CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) < CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1', \tilde{\beta}_2', ..., \tilde{\beta}_n').
\]

If \( (\mu_i.e^{i2\pi \mu_i})^3 = (\mu_i'.e^{i2\pi \mu_i'})^3 \) and \( (\nu_i.e^{i2\pi \nu_i})^3 > (\nu_i'.e^{i2\pi \nu_i'})^3 \) then,

\[
CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) < CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1', \tilde{\beta}_2', ..., \tilde{\beta}_n').
\]

If \( (\mu_i.e^{i2\pi \mu_i})^3 = (\mu_i'.e^{i2\pi \mu_i'})^3 \) and \( (\nu_i.e^{i2\pi \nu_i})^3 = (\nu_i'.e^{i2\pi \nu_i'})^3 \) then,

\[
CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) = CFFPMSM^{(\rho_1, \rho_2, ..., \rho_n)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n).
\]

This complete the proof. ■

**Theorem 4 (Boundedness)** for a given CFFs suppose that \( \tilde{\beta}^– = \min_i \tilde{\beta}_i \) and \( \tilde{\beta}^+ = \max_i \tilde{\beta}_i \) and \( \tilde{\beta} = (\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) \), then

\[
\tilde{\beta}^– \leq CFFPMSM^{(\rho)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) \leq \tilde{\beta}^+.
\]

**Proof.**

As given that \( \tilde{\beta}^– = \min_i \tilde{\beta}_i \leq \tilde{\beta}_i \), from Theorem 2 and 3, we can write

\[
\tilde{\beta}^– = CFFPMSM^{(\rho)}(\tilde{\beta}^–, \tilde{\beta}^–, ..., \tilde{\beta}^–) \leq CFFPMSM^{(\rho)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n),
\]

same as above

\[
CFFPMSM^{(\rho)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) \leq CFFPMSM^{(\rho)}(\tilde{\beta}^+, \tilde{\beta}^+, ..., \tilde{\beta}^+) = \tilde{\beta}^+.
\]

Thus, we have

\[
\tilde{\beta}^– \leq CFFPMSM^{(\rho)}(\tilde{\beta}_1, \tilde{\beta}_2, ..., \tilde{\beta}_n) \leq \tilde{\beta}^+.
\]

■
Theorem 5 For given range of CFFEs \( \mathcal{J}_i = (i = 1, 2, ..., n) \), \( l = 1, 2, ..., \min_Y o_Y \). In situations where there is no partition among argument sets and the interrelationship among arguments shares the same types, specifically when the cardinality \( |t_1| \) equals \( n \) and the parameter \( \rho_1 \) equals \( \rho \) (where \( \rho \) can range from \( 1 \) to \( o_Y \)), we proceed to examine specific instances of CFFPMSM. These instances involve selecting particular values for the parameter \( \rho \) under the aforementioned conditions.

Case 1: Now if \( t = 1 \) and \( \rho = 1 \) then the definition of CFFPMSM operator, we have

\[
CFFPMSM^{(1)}(\mathcal{J}_1, \mathcal{J}_2, ..., \mathcal{J}_n) = \left\{ 1 - \left( \prod_{r=1}^{i} \left( 1 - \left( \prod_{1<i_1<o_Y} \left( 1 - \left( \prod_{l=1}^{i} u_{il} \right)^3 \right) \right)^{ \frac{1}{3} \frac{1}{\rho_Y}} e \right) \right) \right\} .
\]

\[
= \left\{ 1 - \left( \prod_{i_1=1}^{o_Y} \left( 1 - \left( v_{i_1} \right)^3 \right) \right)^{ \frac{1}{3} \frac{1}{\rho_Y}} e \right\} \cdot e^{i2\pi \left( \prod_{i_1=1}^{o_Y} \left( 1 - \left( u_{i_1} \right)^3 \right) \right)^{ \frac{1}{3} \frac{1}{\rho_Y}}}.
\]

Which is reduced to complex q-rung orthopair fuzzy weighted average mean operator.
Case 2: Now if $t = 1$ and $\rho = 2$ then the definition of CFFPMSM operator, we have

\[
\text{CFFPMSM}^{(2)}(\tilde{J}_1, \tilde{J}_2, \ldots, \tilde{J}_n) = \left\{
\begin{array}{l}
1 - \left( \prod_{\gamma = 1}^\tau \left( 1 - \left( 1 - \left( \prod_{i < j \leq \nu} (1 - \left( \prod_{l=1}^2 u_{il} \right)^3 \right) \frac{1}{\nu_{i\gamma} (i\gamma - 1)} \right) \right) \right) \right) \frac{1}{\nu_{i\gamma} (i\gamma - 1)} \frac{1}{2} \frac{1}{2} \\
i2\pi e^{-i} \left( \prod_{\gamma = 1}^\tau \left( 1 - \left( 1 - \left( \prod_{i < j \leq \nu} (1 - \left( \prod_{l=1}^2 u_{il} \right)^3 \right) \frac{1}{\nu_{i\gamma} (i\gamma - 1)} \right) \right) \right) \right) \frac{1}{\nu_{i\gamma} (i\gamma - 1)} \frac{1}{2} \frac{1}{2} \\
\end{array}\right. 
\]

= \left\{
\begin{array}{l}
1 - \left( \prod_{i_1, i_2 = 1, i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^2 u_{i_1 l} \right)^3 \right) \frac{1}{\nu_{i_1 \gamma} (i_1 \gamma - 1)} \right) \frac{1}{2} \frac{1}{2} \\
i2\pi e^{-i} \left( \prod_{i_1, i_2 = 1, i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^2 u_{i_1 l} \right)^3 \right) \frac{1}{\nu_{i_1 \gamma} (i_1 \gamma - 1)} \right) \frac{1}{2} \frac{1}{2} \\
\end{array}\right. 
\]

= \left\{
\begin{array}{l}
1 - \left( \prod_{i_1, i_2 = 1, i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^2 u_{i_1 l} \right)^3 \right) \frac{1}{\nu_{i_1 \gamma} (i_1 \gamma - 1)} \right) \frac{1}{2} \frac{1}{2} \\
i2\pi e^{-i} \left( \prod_{i_1, i_2 = 1, i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^2 u_{i_1 l} \right)^3 \right) \frac{1}{\nu_{i_1 \gamma} (i_1 \gamma - 1)} \right) \frac{1}{2} \frac{1}{2} \\
\end{array}\right. 
\]
Which is reduces to the complex q-rung orthopair fuzzy Bonferroni mean operator.

**Case 3:** Now if \( t = 1 \) and \( \rho = n \) then the definition of CFFPMSM operator, we have

\[
CFFPMSM^{(n)}(\mathfrak{J}_1, \mathfrak{J}_2, \ldots, \mathfrak{J}_n) =
\left\{ \begin{array}{l}
\left( 1 - \left( \prod_{i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^{n} u_{i_1 l} \right)^3 \right) \right) \right)^\frac{1}{t} \cdot \frac{i 2\pi}{e} \left( 1 - \left( \prod_{i_1 \neq i_2} \left( 1 - \left( \prod_{l=1}^{n} u_{i_1 l} \right)^3 \right) \right) \right)^\frac{1}{t} \\
\left( 1 - \left( 1 - \left( \prod_{l=1}^{n} u_{i_1 l} \right)^3 \right) \right)^\frac{1}{t} \cdot \frac{i 2\pi}{e} \left( 1 - \left( 1 - \left( \prod_{l=1}^{n} u_{i_1 l} \right)^3 \right) \right)^\frac{1}{t} \\
\left( 1 - \left( 1 - \left( \prod_{l=1}^{n} v_{i_1 l}^3 \right) \right) \right)^\frac{1}{t} \cdot \frac{i 2\pi}{e} \left( 1 - \left( 1 - \left( \prod_{l=1}^{n} v_{i_1 l}^3 \right) \right) \right)^\frac{1}{t} \\
\left( 1 - \left( 1 - \left( \prod_{l=1}^{n} v_{i_1 l}^3 \right) \right) \right)^\frac{1}{t} \cdot \frac{i 2\pi}{e} \left( 1 - \left( 1 - \left( \prod_{l=1}^{n} v_{i_1 l}^3 \right) \right) \right)^\frac{1}{t} \\
\end{array} \right.
\]

\[ (14) \]

which is a special case of the complex q-rung orthopair fuzzy weighted geometric operator.
3.2 CFFWPMSM aggregation operators

In this section we present the CFFWPMSM operator. It is considered that all the attributes have the same importance. But their weights are not equal in applicable decision-making. Then it is important to consider that each attribute has its own weight. Let the weight of each attribute \( J_i (i = 1, 2, ..., n) \) is \( w_i \) that fulfills \( 0 \leq w_i \leq 1 \). Thus the developed CFFWPMSM operator for CFFE is characterized as follow:

**Definition 10** let \( J_i (i = 1, 2, ..., n) \) be the course set of \( m \) CFFs. Then, CFFWPMSM operator is characterized as

\[
CFFWPMSM^{(\rho_1, \rho_2, ..., \rho_1)} (J_1, J_2, ..., J_n) = \frac{1}{t} \oplus_{Y=1}^{t} \left( \frac{\sum_{1 \leq k_1 < ... < k_l \leq o_Y} (\sum_{i=1}^{o_Y} (\sum_{j=1}^{J_i}))}{C_{o_Y}^{o_Y}} \right)^{\frac{1}{t}},
\]

where \( t \) denotes the number of categories, \( \rho \) is a parameter, \( \rho = 1, 2, ..., o_Y, o_Y \) denotes the number of criteria in category \( t_Y \), the binomial coefficient whose expression is \( C_{o_Y}^{o_Y} = \frac{o_Y!}{\rho!(o_Y-\rho)!} \) and \( w_i \geq 0 \).

**Theorem 6** For given CFFE sets \( J_i (i = 1, 2, ..., n) \). The aggregated result of formula citation is still CFEs, characterized as below:

\[
CFFWPMSM^{(\rho_1, \rho_2, ..., \rho_1)} (J_1, J_2, ..., J_n) = \left\{ \begin{array}{l}
\frac{1 - \frac{t}{Y=1} \left( 1 - \prod_{i_1 < i_2 < ... < i_l} \left( \frac{1 - \rho}{l=1} \left( 1 - (1 - (1 - w_i^{\rho}))^{o_Y} \left( \frac{1}{C_{o_Y}^{o_Y}} \right)^{\frac{1}{t}} \right) \right) \right)}{e^{t}} \\
\frac{1 - \prod_{i_1 < i_2 < ... < i_l} \left( \frac{1 - \rho}{l=1} \left( 1 - (1 - (1 - w_i^{\rho}))^{o_Y} \left( \frac{1}{C_{o_Y}^{o_Y}} \right)^{\frac{1}{t}} \right) \right) \right)}{e^{t}} \\
\frac{1 - \prod_{i_1 < i_2 < ... < i_l} \left( \frac{1 - \rho}{l=1} \left( 1 - (1 - (1 - w_i^{\rho}))^{o_Y} \left( \frac{1}{C_{o_Y}^{o_Y}} \right)^{\frac{1}{t}} \right) \right) \right)}{e^{t}} \\
\end{array} \right\}.
\]

**Proof.** Based on the lines of Theorem 1, one can easily prove it. 

**Theorem 7** (Idempotency) If the domain of discourse of given CFFs \( J_i (i = 1, 2, ..., n) \) are same i.e., Then

\[
CFFWPMSM^{(\rho_1, \rho_2, ..., \rho_1)} (J_1, J_2, ..., J_n) = J.
\]

**Theorem 8** (Monotonicity) let \( J_i (i = 1, 2, ..., n) \) be CFFs where \( J_i = (w_i, e^{i2\pi u}, v_i, e^{i2\pi v}) \), and let \( J_i' (i = 1, 2, ..., n) \) where \( J_i' = (w_i', e^{i2\pi u'}, v_i', e^{i2\pi v'}) \), which meet the condition \( u_i' \geq u_i \) and \( v_i' \leq v_i \) for all \( i = 1, 2, ..., n \), then

\[
CFFWPMSM^{(\rho)} (J_1, J_2, ..., J_n) \leq CFFWPMSM^{(\rho)} (J_1', J_2', ..., J_n).
\]
Theorem 9 (Boundedness) for a given CFFs suppose that $\bar{J} = \min_i \bar{J}_i$ and $\bar{J} = \max_i \bar{J}_i$ and $(i=1,2,\ldots,n)$, then
\[
\bar{J} = CFWPM\rho M^{(\rho)}(\bar{J}_1, \bar{J}_2, \ldots, \bar{J}_n) \leq \bar{J}.
\]

4 Presented CFFPPMSM Operator and CFFWPPMSM operator

In practical decision-making processes, the evaluator’s limited time and varying knowledge can lead to the assignment of overly high or low evaluation values for attributes. To mitigate the adverse effects of unjustifiable arguments on the aggregation results, the calculation of support measures between arguments is crucial. Hence, we introduce the CFFPPMSM (complex Fermatean fuzzy power partitioned Maclaurin symmetric mean) and the CFFWPPMSM (complex Fermatean fuzzy weighted power partitioned Maclaurin symmetric mean) operators, which effectively utilize PMSM and PA to address this concern.

4.1 CFFPPMSM aggregation operators

Definition 11 Let $\bar{J}_1, \bar{J}_2, \ldots, \bar{J}_n$ be a range of n CFFEs. Then, CFFPMMS operator is characterized as
\[
CFFPPMSM^{(\rho_1, \rho_2, \ldots, \rho_t)}(\bar{J}_1, \bar{J}_2, \ldots, \bar{J}_n) = \frac{\sum_{l=1}^{t} \left( \bar{J}_{l_1} \otimes_{l=1}^{t} \left( \prod_{1 \leq k_1 < \ldots < k_l \leq o_{\gamma} \otimes_{l=1}^{\rho} \frac{n(1+T(\bar{J}_{k_l}))}{\sum_{l=1}^{n}(1+T(\bar{J}_{k_l}))} \otimes_{l=1}^{\rho} \bar{J}_{k_l} \right) \right)^{\frac{1}{\rho}}}.
\]

The parameter $\rho$, $\rho = 1,2,\ldots,o_{\gamma}$, $o_{\gamma}$ denotes maximum number of criteria in category $t_{\gamma}$, $t$ represents the total number of criteria, and $(k_1, k_2, \ldots, k_\rho)$ encompasses all possible $\rho$-tuples of $(1,2,\ldots,o_{\gamma})$, where $C_{\rho} = \frac{o_{\gamma}}{\rho(o_{\gamma} - \rho)!}$. Meanwhile, the $T(\bar{J}_{k_1}) = \sum_{l=1,k\neq1}^{n} Supp(\bar{J}_{k_1}, \bar{J}_{k_m})$ and the $Supp(\bar{J}_{k_1}, \bar{J}_{k_m}) = 1 - d(\bar{J}_{k_1}, \bar{J}_{k_m})$ is the support of $\bar{J}_{k_1}$ and $\bar{J}_{k_m}$ which satisfies the following properties:

(i) $Supp(\bar{J}_{k_1}, \bar{J}_{k_m}) \in [0,1]$;

(ii) $Supp(\bar{J}_{k_1}, \bar{J}_{k_m}) > Supp(\bar{J}_{k_1}, \bar{J}_{k_m})$;

(iii) $d(\bar{J}_{k_1}, \bar{J}_{k_m}) < d(\bar{J}_{k_1}, \bar{J}_{k_m})$; the $d(\bar{J}_{k_1}, \bar{J}_{k_m})$ is the distance of CFFNs.

To enhance the clarity and readability of Equation (21), we define
\[
W_{k_1} = \frac{(1 + T(\bar{J}_{k_1}))}{\sum_{l=1}^{n}(1 + T(\bar{J}_{k_l}))},
\]

15
where $\mathcal{W}_k = (\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n)$. The $\mathcal{W}$ is called as the power weighting vector which satisfies $\mathcal{W}_k \in [0, 1]$. Therefore Equation (21) can be expressed as follows:

$$
CFFPMSM^{(p_1, p_2, ..., p_t)}(\mathfrak{J}_1, \mathfrak{J}_2, ..., \mathfrak{J}_n) = \frac{1}{t} \sum_{i=1}^{t} \left( \frac{\prod_{1 \leq k_1 < ... < k_t \leq \rho_v} (1 - \left( 1 - (1 - v_i^{3\eta} n \mathcal{W}_{k_i}) \right)^{\frac{1}{2}}} \right)^{\frac{1}{2}}.
$$

(23)

**Theorem 10** For given CFFs $\mathfrak{J}_i$ $(i = 1, 2, ..., n)$. The aggregated result of formula citation is still CFFEs, characterized as below:

$$
CFFPPMSM^{(p_1, p_2, ..., p_t)}(\mathfrak{J}_1, \mathfrak{J}_2, ..., \mathfrak{J}_n) = \\
\left\{ \begin{array}{l}
1 - \prod_{1 \leq k_1 < ... < k_t \leq \rho_v} \left( 1 - \left( \prod_{1 \leq i_k \leq \rho_v} \left( \prod_{1 \leq l \leq t} \left( 1 - \left( 1 - (1 - v_i^{3\eta} n \mathcal{W}_{l}) \right)^{\frac{1}{2}} \right) \right) \right) \right)
\end{array} \right\}
$$

(24)

**Proof.**
The proof of this theorem is similar to Theorem 1, so it is omitted here. ■

**Theorem 11 (Idempotency)** If the domain of discourse of given CFFs $\mathfrak{J}_i$ $(i = 1, 2, ..., n)$ are same i.e., $\mathfrak{J}_i = \mathfrak{J}$. Then

$$
CFFPPMSM^{(p_1, p_2, ..., p_t)}(\mathfrak{J}_1, \mathfrak{J}_2, ..., \mathfrak{J}_n) = \mathfrak{J}.
$$

(25)

**Proof.**
Since all CFFs $\mathfrak{J}_k (k = 1, 2, ..., m)$ are equal to $\mathfrak{J} = (u.e^{i2\pi v}, v.e^{i2\pi v})$, we can get $Supp(\mathfrak{J}_k, \mathfrak{J}_m) =$
1 for k, l = 1, 2, . . . , n. Based on Equation (22), we can get \( W_{kl} = \frac{1}{n} (l = 1, 2, ..., n) \), then

\[
C_{FFPPSM}(p_1, p_2, ..., p_t) (\mathfrak{J}_1, \mathfrak{J}_2, ..., \mathfrak{J}_n) = \left\{ \begin{array}{l}
\left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( 1 - (1-u^3)^v \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 4} \left( 1 - \prod_{i_1<i_2<...<i_Y} \left( 1 - \left( 1 - (1-u^3)^v \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right) \right\}^{1 \over 2} \cdot e^{i2\pi \left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( 1 - (1-u^3)^v \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 2}} .
\end{array} \right.
\]

\[
= \left\{ \begin{array}{l}
\left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( (u^3)^v \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 2} \cdot e^{i2\pi \left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( (u^3)^v \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 2}} .
\end{array} \right.
\]

\[
= \left\{ \begin{array}{l}
\left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( (1-v^3)^u \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 2} \cdot e^{i2\pi \left( 1 - \prod_{Y=1}^{t} \left( 1 - \left( (1-v^3)^u \right) \frac{1}{C_{\mathfrak{J}_Y}} \right) \right)^{1 \over 2}} .
\end{array} \right.
\]

Which complete the proof.\[\blacksquare\]
Theorem 12 (Boundedness) for a given CFFs suppose that $\mathcal{J}^- = \min_i \mathcal{J}_i$ and $\mathcal{J}^+ = \max_i \mathcal{J}_i$ and 
$(i=1,2,...,n)$, then

$$\mathcal{J}^- \leq \text{CFFPPSM}^{(o)}(\mathcal{J}_1, \mathcal{J}_2,..., \mathcal{J}_n) \leq \mathcal{J}^+.$$  \hfill (26)

Proof.

\[
n\mathcal{W}_{k_i} \otimes \mathcal{J}_{k_i} = \left\{ \left( 1 - (u^{\mathcal{J}_{k_i}}_{k_i} - 1)^{n\mathcal{W}_{k_i}} \right)^{\frac{1}{2}} \cdot e^{i2\pi \left( 1 - (u^{\mathcal{J}_{k_i}}_{k_i} - 1)^{n\mathcal{W}_{k_i}} \right)^{\frac{1}{2}}} \right\}
\]

\[
\geq \left\{ \left( 1 - (u^{-\mathcal{J}_{k_i}}_{k_i})^{n\mathcal{W}_{k_i}} \right)^{\frac{1}{2}} \cdot e^{i2\pi \left( 1 - (u^{-\mathcal{J}_{k_i}}_{k_i})^{n\mathcal{W}_{k_i}} \right)^{\frac{1}{2}}} \right\}
\]

\[
\otimes_{l=1}^{\rho} (n\mathcal{W}_{k_i} \otimes \mathcal{J}_{k_l}) =
\]

\[
\left\{ \left( \prod_{\gamma=1}^{t} \left( 1 - (u^{\mathcal{J}_{k_l}}_{k_l} - 1)^{n\mathcal{W}_{k_l}} \right)^{\frac{1}{2}} \cdot e^{i2\pi \left( \prod_{\gamma=1}^{t} \left( 1 - (u^{\mathcal{J}_{k_l}}_{k_l} - 1)^{n\mathcal{W}_{k_l}} \right)^{\frac{1}{2}} \right) \right) \right\}
\]

\[
\geq \left\{ \left( \prod_{\gamma=1}^{t} \left( 1 - (v^{-\mathcal{J}_{k_l}}_{k_l})^{3n\mathcal{W}_{k_l}} \right)^{\frac{1}{2}} \cdot e^{i2\pi \left( \prod_{\gamma=1}^{t} \left( 1 - (v^{-\mathcal{J}_{k_l}}_{k_l})^{3n\mathcal{W}_{k_l}} \right)^{\frac{1}{2}} \right) \right) \right\}
\]
\[
\frac{1}{t} \sum_{Y=1}^{t} \left( \prod_{1 \leq k_1 < \ldots < k_Y \leq \alpha_Y} \left( \prod_{i=1}^{t} \left( 1 - \prod_{l=1}^{\rho} \left( 1 - (1 - \rho_i^3 n W_{k_l}) \right) \right) \right)^{\frac{1}{\rho}} \right)^{\frac{1}{Y}}
\]

\[
\left( \prod_{Y=1}^{t} \left( 1 - \prod_{i=1}^{t} \prod_{l=1}^{\rho} \left( 1 - (1 - (u_i^3)^n W_{k_l}) \right) \right) \right)^{\frac{1}{\rho}} \}
\]

Similarly, we can easily prove that CFFPPMSM\((\rho_1, \rho_2, \ldots, \rho_t)\) \((J_1, J_2, \ldots, J_n) \leq \tilde{J}^+\).

Therefore, we can obtain \(\tilde{J}^- \leq \text{CFFPPPMSM}\((\rho_1, \rho_2, \ldots, \rho_t)\) \((J_1, J_2, \ldots, J_n) \leq \tilde{J}^+\).

### 4.2 CFFWPPMSM aggregation operators

In this section we present the CFFWPPMSM operator. It is consider that all the attribute have the same importance. But their weights are not equal in applicable decision-making. Then it is important to consider that each attribute have its own weight. Let the weight of each attribute \(J_i (i = 1, 2, \ldots, n)\) is \(w_i\) that fulfills \(0 \leq w_{k_l} \leq 1\). Thus the developed CFFWPPMSM operator for CFFs is characterized as follow.

**Definition 12** Let \(J_i (i = 1, 2, \ldots, n)\) be a range of \(n\) CFFs. Then, CFFPMSM operator is characterized as
\[
CFFWPMSM^{(\rho_1, \rho_2, \ldots, \rho_t)}(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{1}{t} \sum_{\gamma=1}^{t} \left( \bigotimes_{1 \leq k_1 \leq \ldots \leq k_l \leq o} \left( \left( \frac{n \mathcal{W}_{k_l} (1 + T(\mathcal{J}_{k_l}))}{\sum_{l=1}^{n} \mathcal{W}_{k_l} (1 + T(\mathcal{J}_{k_l}))} \bigotimes \mathcal{J}_{k_l} \right) \right) \right) ^{\frac{1}{t}}.
\]

To enhance the clarity and readability of Equation (27), we define
\[
\mathcal{W}_{k_l} = \frac{\mathcal{W}_{k_l} (1 + T(\mathcal{J}_{k_l}))}{\sum_{l=1}^{n} \mathcal{W}_{k_l} (1 + T(\mathcal{J}_{k_l}))},
\]
where \(\mathcal{W}_k = (\mathcal{W}_1, \mathcal{W}_2, \ldots, \mathcal{W}_n)\). The \(\mathcal{W}\) is called as the power weighting vector which satisfies \(\mathcal{W}_k \in [0, 1]\). Therefore Equation (27) can be expressed as follows:
\[
CFFWPMSM^{(\rho_1, \rho_2, \ldots, \rho_t)}(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \frac{1}{t} \sum_{\gamma=1}^{t} \left( \bigotimes_{1 \leq k_1 \leq \ldots \leq k_l \leq o} \left( \left( \mathcal{W}_{k_l} \bigotimes \mathcal{J}_{k_l} \right) \right) \right) ^{\frac{1}{t}}.
\]

**Theorem 13** For given CFFs \(\mathcal{J}_i\) (1, 2, ..., \(n\)). The aggregated result of formula citation is still CFFEs, characterized as below:
\[
CFFWPMSM^{(\rho_1, \rho_2, \ldots, \rho_t)}(\mathcal{J}_1, \mathcal{J}_2, \ldots, \mathcal{J}_n) = \left\{ \begin{array}{l}
\left( 1 - \prod_{\gamma=1}^{t} \left( 1 - \left( 1 - \prod_{i_1 < i_2 < \ldots < i_{o}} (1 - \prod_{l=1}^{\rho} (1 - (1 - u_{i_l}^3)^{n \mathcal{W}_{k_l}})) \bigotimes \mathcal{J}_{k_l}^{\frac{1}{t}} \right) \right) \right) ^{\frac{1}{t}}.
\end{array} \right.
\]
\[
\frac{1}{t} \sum_{\gamma=1}^{t} \left( 1 - \prod_{i_1 < i_2 < \ldots < i_{o}} (1 - \prod_{l=1}^{\rho} \left( (1 - (1 - u_{i_l}^3)^{n \mathcal{W}_{k_l}}) \bigotimes \mathcal{J}_{k_l}^{\frac{1}{t}} \right) \right) \right) ^{\frac{1}{t}}
\]

**Proof.**
The proof of this theorem is similar to Theorem 1, so it is omitted here. 

5 Statistical Variance Method on CFFs

In this section, a novel extension to the SV method is introduced within the context of CFFs. According to Liu et al. [38], conventional approaches for criteria weight calculation, such as AHP
(analytical hierarchy process) [38,39], entropy measures [40], and optimization models [41], result in weight values that are deemed unreasonable and involve computationally complex procedures. Additionally, optimization models rely on partial attribute information for weight calculation, making them inappropriate for situations where complete weight value information is entirely unknown.

To address these challenges, Liu et al. [38] proposed the utilization of the SV method for criteria weight calculation. The SV method quantifies the degree to which a set of preference information deviates from the average preference information. The underlying rationale for employing the SV method in criteria weight calculation lies in its ability to capture decision-makers’ hesitation or confusion concerning each criteria. When there is significant variation in preference information for a specific criteria, indicating higher levels of confusion, that attribute is assigned greater relative importance (weight). Since the primary objective of decision-making is to arrive at rational choices while appropriately accounting for uncertainty, the SV method is adopted due to its ability to mimic this tendency (for further details, refer to [42]). The authors claimed several advantages of the SV method: (i) simplicity and straightforwardness, (ii) consideration of all data points rather than selective distribution determination as in other statistical methods, and (iii) Kao [42] demonstrated that the SV method better captures uncertainty by assigning higher relative importance to attributes with increased confusion or hesitation during preference elicitation.

Inspired by the effectiveness of the SV method, this paper introduces an extension of the SV method within the CFFs context for criteria weight calculation. The following section outlines the systematic procedure for calculating criteria weights:

**Step 1:** Create a matrix for assessment with dimensions of $m \times n$, where $m$ signifies the alternatives and $n$ indicates the count of criteria.

**Step 2:** Compute the accuracy of each CFFN by applying Equation (4). Derive the average value for each criteria and employ it for the computation of variance.

$$\sigma_i^2 = \frac{\sum_{k=1}^{m} (Ac(J_{kl}) - Av(J_{kl}))^2}{m - 1}. \tag{31}$$

Here, $Ac(J_{kl})$ represents the accuracy of the CFFN corresponding to the $k$th alternative and $l$th criteria, while $Av(J_{kl})$ denotes the average value.

**Step 3:** Perform the normalization of the variance to obtain the weight of each attribute.

$$W_l = \frac{\sigma_i^2}{\sum_i \sigma_i^2}. \tag{32}$$

The weight of the $l$th attribute, denoted as $W_l$, falls within the range of $0 \leq W_l \leq 1$.

### 6 Technique for MCDM with CFFEs

To address MCDM problems, a novel approach is introduced, utilizing the CFWPPMSM operator.

In a standard scenario of Multiple Criteria Decision Making (MCDM), the aim is to choose the most preferable option from a set of alternatives $E = \{E_1, E_2, ..., E_m\}$, taking into account a range of criteria $S = \{S_1, S_2, ..., S_n\}$. The criteria weight vector $W = \{W_1, W_2, ..., W_n\}$, adheres to
the condition $\mathfrak{W}_l \in [0, 1]$ (where $l = 1, 2, \ldots, n$), and the summation reflects the significance of attribute $\mathfrak{S}_l$ (where $l = 1, 2, \ldots, n$) in the decision-making process. Assume that the attributes are categorized into $d$ distinct classes $t_1, t_2, \ldots, t_d$, where in there exists a correlation among the attributes within each class $t_h$ (where $h = 1, 2, \ldots, o_y$), while attributes from different classes are unrelated. Considering the presence of uncertainty in MCDM problems, the evaluation of alternative $\mathfrak{E}_k$’s (where $k = 1, 2, \ldots, m$) performance concerning attribute $\mathfrak{S}_l$ (where $l = 1, 2, \ldots, n$), is expressed in the form of a CFFN and consolidated in the decision matrix.

In order to identify the optimal alternative, an algorithm leveraging the CFFWPPMSM operator is presented. The algorithm comprises the following essential stages:

**Step 1** Evaluation Matrix: Consider the mentioned situation, the formulation of the MCDM problem can be initiated by creating an evaluation matrix.

$$A_{m \times n} = \begin{pmatrix} \mathfrak{J}_{11} & \mathfrak{J}_{12} & \cdots & \mathfrak{J}_{1n} \\ \mathfrak{J}_{21} & \mathfrak{J}_{22} & \cdots & \mathfrak{J}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathfrak{J}_{m1} & \mathfrak{J}_{m2} & \cdots & \mathfrak{J}_{mn} \end{pmatrix}. \quad (33)$$

**Step 2** Normalization: Rearrange the decision matrix $A = (\mathfrak{J}_{kl})_{m \times n}$ to $A' = (\mathfrak{J}'_{kl})_{m \times n}$ in accordance with the provided formulation.

$$\mathfrak{J}'_{kl} = \begin{cases} \mathfrak{J}_{kl}, & \mathfrak{S}_l \text{ is a benefit criterion,} \\
(\mathfrak{J}_{kl})^*, & \mathfrak{S}_l \text{ is cost criteria.} \end{cases} \quad (34)$$

**Step 3** Criteria weights: Compute the weights of the criteria, denoted as $\mathfrak{W}_l$ (where $l = 1, 2, \ldots, n$), utilizing the methodology outlined in Section (5).

**Step 4** Supports: Derive the $\text{Supp}(\mathfrak{J}'_{kl}, \mathfrak{J}'_{km})$ by applying the equation provided below:

$$\text{Supp}(\mathfrak{J}'_{kl}, \mathfrak{J}'_{km}) = 1 - d(\mathfrak{J}'_{kl}, \mathfrak{J}'_{km}) ; k = 1, 2, \ldots, m; l = 1, 2, \ldots, n; \quad (35)$$

where $d(\mathfrak{J}'_{kl}, \mathfrak{J}'_{km})$ denotes the distance measure between $\mathfrak{J}'_{kl}$ and $\mathfrak{J}'_{km}$ (5).

**Step 5** Total support: Determine the total support allocated to the following equation by performing the computation:

$$T(\mathfrak{J}'_{kl}) = \sum_{l=1}^{n} \text{Supp}(\mathfrak{J}'_{kl}, \mathfrak{J}'_{km}). \quad (36)$$

**Step 6** Evaluation value weights: Obtain $\mathfrak{W}_{kl}$ under CFF $\mathfrak{J}'_{kl}$.

$$\mathfrak{W}_{kl} = \frac{\mathfrak{W}_k \left( 1 + T(\mathfrak{J}'_{kl}) \right)}{\sum_{l=1}^{n} \mathfrak{W}_l \left( 1 + T(\mathfrak{J}_l) \right)} ; k = 1, 2, \ldots, m, \quad (37)$$

where $\mathfrak{W}_k$ is the weight of criteria which can be find in according to section (5). $\mathfrak{W}_k$ is the weight of power aggregation operator which can be find by equation (25).
**Step 7** Aggregation: Apply CFWPMSM operator to aggregate the values $J'_k$.

$$
\text{CFWPMSM}(\rho_1, \rho_2, \ldots, \rho_n) \left( J_1, J_2, \ldots, J_n \right) =
\left\{
\begin{array}{l}
\frac{t}{e} \left( 1 - \prod_{i=1}^{t} \left( 1 - \prod_{i_1 < i_2 < \ldots < i_v} \left( 1 - \rho \prod_{i=1}^{\rho} \left( 1 - \left( 1 - u_{i_j}^{n W_{i_j}} \right) \right) \right) \right) \right)
\end{array}\right.
\right.

\text{Step 8} \text{ Score values: compute the score value of each aggregated value } J_k \text{ } (k = m).

\text{Step 9} \text{ Order relation: Based on the score values, acquire the order relation of alternatives and select the best one.}

**7 Illustrative example**

The practical prime example of Hostel site selection is described in this part, followed by an essential for optimizing.

**7.1 Example**

Quaid-i-Azam University (QAU) is a renowned public research university located in Islamabad, the capital city of Pakistan. It is named after Muhammad Ali Jinnah, the founder of Pakistan, who is also known as Quaid-i-Azam (Great Leader). Quaid-i-Azam University was established on July 22, 1965, under the University of Islamabad Ordinance. Initially, it started as a postgraduate institution with only four departments: Economics, Statistics, Mathematics, and Psychology. The aim was to provide quality education and research opportunities to students in various disciplines. In 1971, due to the growing number of students and the need for expanded facilities, the university was relocated to its current campus at the foothills of Margalla Hills in Islamabad. The new campus provided a larger area and a scenic environment conducive to learning. Over the years, Quaid-i-Azam University expanded its academic programs and established several new faculties and departments. Today, it offers undergraduate, graduate, and doctoral programs in various fields, including Natural Sciences, Social Sciences, Biological Sciences, Pharmacy, International Relations, and Computer Science. Quaid-i-Azam University has gained recognition for its research contributions. Faculty members and students have published numerous research papers in national and international journals. The
The university also publishes its research journal, "The Pakistan Development Review," which focuses on economic research. Quaid-i-Azam University has consistently been recognized as one of the top universities in Pakistan. It has received accolades for its academic excellence and research output. The university has also achieved high rankings in international university rankings, further enhancing its reputation. Quaid-i-Azam University has established collaborations and partnerships with various international institutions and organizations. These collaborations facilitate faculty and student exchanges, joint research projects, and academic collaborations, promoting a global outlook and enriching the academic experience. Quaid-i-Azam University continues to focus on enhancing its academic programs, research capabilities, and infrastructure. It aims to further strengthen its position as a leading research university in Pakistan and maintain its commitment to providing quality education and research opportunities. Quaid-i-Azam University has played a vital role in shaping the intellectual landscape of Pakistan. With its rich history and commitment to academic excellence, it continues to contribute to the development of knowledge and society in Pakistan and beyond. QAU has undertaken initiatives to construct new hostels to cater to the increasing student population and ensure adequate residential facilities. Although the availability and specific details of new hostels may vary over time, the university administration has expressed plans to expand the hostel capacity. The construction of new hostels aims to accommodate the growing number of students and provide them with comfortable living arrangements conducive to academic pursuits. These hostels are typically equipped with modern facilities and amenities to meet the students' needs. It's important to note that specific information about the number and names of new hostels, as well as their completion timelines, may vary. For the most accurate and up-to-date information about existing and new hostels at Quaid-i-Azam University, it is recommended to visit the official website of the university or contact the relevant university authorities. Now, let's explore the application of complex fermatean fuzzy set in the context of hostel site selection at QAU. When selecting a suitable location for a hostel, several factors need to be considered.

\( \mathcal{G}_1 \) **Proximity to Campus**: The distance between the hostel and the university campus is a crucial factor. Students prefer a convenient location that reduces commuting time and provides easy access to academic facilities. \( \mathcal{G}_2 \) **Safety and Security**: The safety and security of the hostel and its surroundings are paramount. Factors such as crime rate, availability of security personnel, and presence of security measures like CCTV cameras and access control systems should be assessed. \( \mathcal{G}_3 \) **Amenities**: The availability of amenities such as study areas, recreational facilities, internet connectivity, dining options, and laundry services significantly impacts the quality of student life in a hostel. \( \mathcal{G}_4 \) **Affordability**: The cost of accommodation is a vital consideration for students. It is essential to evaluate the pricing structure, including room rates and additional expenses, to ensure it aligns with students' budgets. \( \mathcal{G}_5 \) **Surrounding Infrastructure**: The availability of essential facilities nearby, such as public transportation, medical facilities, grocery stores, and recreational spots, adds convenience and enhances the overall appeal of a hostel. These five attributes are separated into two groups \( \mathcal{I}_1 \) and \( \mathcal{I}_2 \) such that \( \mathcal{I}_1 = \{ \mathcal{G}_1, \mathcal{G}_3 \} \) and \( \mathcal{I}_2 = \{ \mathcal{G}_2, \mathcal{G}_4, \mathcal{G}_5 \} \), then any two criteria in each category are related to one another, with \( l_1 = 2 \) and \( l_2 = 3 \). And there will be four alternative location for the hostel \( \mathcal{E}_i (i = 1, 2, 3, 4) \). Imam bari place \( \mathcal{E}_1 \), Barakoh place \( \mathcal{E}_2 \), Hostel city chatta bakhtawar \( \mathcal{E}_3 \), Inside university \( \mathcal{E}_4 \).

Finally, based on the desirability scores, a ranking or recommendation can be generated, highlighting the top choices for hostel site selection at QAU. This approach allows decision-makers to incorporate subjective judgments, preferences, and uncertainties into the decision-making process effectively.

By leveraging the principles of complex fermatean fuzzy set decision-making, the process of
selecting a suitable hosted site at QAU can be more systematic, transparent, and inclusive of various factors important to students’ well-being and convenience.

The following lists the stages that make up the proposed method.

**Step 1:** The construction of the CFF evaluation matrix

<table>
<thead>
<tr>
<th>G₁</th>
<th>G₂</th>
<th>G₃</th>
<th>G₄</th>
<th>G₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>e₁</td>
<td>(0.5e^{12πi/6}, 0.3e^{12πi/6})</td>
<td>(0.8e^{12πi/6}, 0.6e^{12πi/6})</td>
<td>(0.8e^{12πi/6}, 0.2e^{12πi/6})</td>
<td>(0.5e^{12πi/6}, 0.6e^{12πi/6})</td>
</tr>
<tr>
<td>e₂</td>
<td>(0.7e^{2πi/6}, 0.6e^{2πi/6})</td>
<td>(0.3e^{2πi/6}, 0.8e^{2πi/6})</td>
<td>(0.6e^{2πi/6}, 0.5e^{2πi/6})</td>
<td>(0.7e^{2πi/6}, 0.4e^{2πi/6})</td>
</tr>
<tr>
<td>e₃</td>
<td>(0.3e^{2πi/6}, 0.9e^{2πi/6})</td>
<td>(0.4e^{2πi/6}, 0.6e^{2πi/6})</td>
<td>(0.7e^{2πi/6}, 0.3e^{2πi/6})</td>
<td>(0.5e^{2πi/6}, 0.8e^{2πi/6})</td>
</tr>
<tr>
<td>e₄</td>
<td>(0.8e^{2πi/6}, 0.6e^{2πi/6})</td>
<td>(0.6e^{2πi/6}, 0.8e^{2πi/6})</td>
<td>(0.8e^{2πi/6}, 0.5e^{2πi/6})</td>
<td>(0.5e^{2πi/6}, 0.4e^{2πi/6})</td>
</tr>
</tbody>
</table>

**Step 2:** Construction of normalized matrix

<table>
<thead>
<tr>
<th>G₁</th>
<th>G₂</th>
<th>G₃</th>
<th>G₄</th>
<th>G₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>e₁</td>
<td>(0.3e^{12πi/6}, 0.5e^{12πi/6})</td>
<td>(0.8e^{12πi/6}, 0.6e^{12πi/6})</td>
<td>(0.8e^{12πi/6}, 0.2e^{12πi/6})</td>
<td>(0.6e^{12πi/6}, 0.5e^{12πi/6})</td>
</tr>
<tr>
<td>e₂</td>
<td>(0.6e^{12πi/6}, 0.7e^{12πi/6})</td>
<td>(0.3e^{12πi/6}, 0.8e^{12πi/6})</td>
<td>(0.6e^{12πi/6}, 0.5e^{12πi/6})</td>
<td>(0.7e^{12πi/6}, 0.4e^{12πi/6})</td>
</tr>
<tr>
<td>e₃</td>
<td>(0.9e^{12πi/6}, 0.3e^{12πi/6})</td>
<td>(0.4e^{12πi/6}, 0.6e^{12πi/6})</td>
<td>(0.7e^{12πi/6}, 0.3e^{12πi/6})</td>
<td>(0.5e^{12πi/6}, 0.2e^{12πi/6})</td>
</tr>
<tr>
<td>e₄</td>
<td>(0.6e^{12πi/6}, 0.8e^{12πi/6})</td>
<td>(0.6e^{12πi/6}, 0.8e^{12πi/6})</td>
<td>(0.8e^{12πi/6}, 0.5e^{12πi/6})</td>
<td>(0.5e^{12πi/6}, 0.4e^{12πi/6})</td>
</tr>
</tbody>
</table>

**Step 3:** calculate the criteria weight \( W_i \) where \( i = 1, 2, 3, 4, 5 \) by using Eq.(31-32),

\[
W_i = (0.1584, 0.3088, 0.1628, 0.2828, 0.08723).
\]

**Step 4**-5: Calculate the \( \text{Supp}\left(\tilde{J}_k, \tilde{J}_m\right) \) where \( k = 1, 2, 3, 4; l, m = 1, 2, 3, 4, 5 \) by using Eq.(35).

Now calculate the \( T\left(\tilde{J}_k\right) \) by using Eq.(36). To simplify \( T\left(\tilde{J}_k\right) \) denoted as \( T \) and obtain,

\[
\]

**Step 6:** Calculate the Power weight vector of alternative \( \tilde{J}_k \) for \( k = 1, 2, 3, 4 \) with respect to the criteria \( \tilde{J}_i \) \( i = 1, 2, 3, 4, 5 \) by using Eq. (37) and obtain.

\[
W_{ki} = \begin{bmatrix} 0.1546 & 0.3167 & 0.1489 & 0.2935 & 0.08623 \\ 0.1621 & 0.2998 & 0.1571 & 0.2927 & 0.08826 \\ 0.1609 & 0.2924 & 0.1674 & 0.2907 & 0.08859 \\ 0.1580 & 0.3081 & 0.1615 & 0.2838 & 0.08858 \end{bmatrix}.
\]
Step 7: Compute the overall performance for alternative $\mathcal{E}_m \ (m = 1, 2, 3, 4)$ by applying Eq. (30) to all criterias.

$$
\mathcal{E}_1 = \left( 0.6694.e^{i2\pi(0.7249)}, 0.5024.e^{i2\pi(0.6082)} \right) ; 
\mathcal{E}_2 = \left( 0.4870.e^{i2\pi(0.5334)}, 0.7096.e^{i2\pi(0.7539)} \right) ;
\mathcal{E}_3 = \left( 0.7397.e^{i2\pi(0.4766)}, 0.4537.e^{i2\pi(0.5538)} \right) ;
\mathcal{E}_4 = \left( 0.6684.e^{i2\pi(0.5282)}, 0.7263.e^{i2\pi(0.5617)} \right).
$$

Step 8: Compute the score value alternative $\mathcal{E}_m \ (m = 1, 2, 3, 4)$ by using Eq. (3).

$$
S(\mathcal{E}_1) = 0.1645 ; 
S(\mathcal{E}_2) = 0.2591 ; 
S(\mathcal{E}_3) = 0.1229 ; 
S(\mathcal{E}_4) = 0.05720.
$$

Step 9: Based on the calculated values, the final ranking is determined as $\mathcal{E}_1 \succ \mathcal{E}_3 \succ \mathcal{E}_4 \succ \mathcal{E}_2$. Consequently, $\mathcal{E}_1$ emerges as the optimal location for hostel.

8 Comparative study

In the subsequent section, a comparative analysis is conducted, contrasting the current aggregation operators with other existing ones including q-Rung orthopair fuzzy weighted power partitioned maclaurin symmetric mean (q-ROFWPPMSM) operator [43], q-Rung orthopair fuzzy weighted partitioned maclaurin symmetric mean (q-ROFWPMSM) operator [43], complex q-rung orthopair fuzzy weighted Madclaurin symmetric mean (Cq-ROFWMSM) [44] operator, complex q-rung orthopair fuzzy weighted dual Macclaurin symmetric mean (Cq-ROFWDMSM) [44] operator, Fermatean fuzzy weighted power average (FFWPA) [11] operator, Fermatean fuzzy weighted power geometric (FFWPG) [11] operator, Fermatean fuzzy Dombi weighted averaging (FFDWA) [45] operator are undertaken to illustrate the benefits of the presented method. The final results obtained from employing all of these aggregation operators to the preceding example are displayed in Table 3.
Table 3: Comparison with the existing aggregation operators

<table>
<thead>
<tr>
<th>Aggregation operator</th>
<th>$S(\mathbf{E}_1)$</th>
<th>$S(\mathbf{E}_2)$</th>
<th>$S(\mathbf{E}_3)$</th>
<th>$S(\mathbf{E}_4)$</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed CFWPPMSM</td>
<td>0.1645</td>
<td>-0.2391</td>
<td>0.1220</td>
<td>-0.05720</td>
<td>$\mathbf{e}_1 &gt; \mathbf{e}_3 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>q-ROFWPMSM [43]</td>
<td>0.1732</td>
<td>-0.2418</td>
<td>0.3113</td>
<td>-0.0845</td>
<td>$\mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>q-ROFWPMSM [43]</td>
<td>-0.5630</td>
<td>-0.7835</td>
<td>-0.5147</td>
<td>-0.7350</td>
<td>$\mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>Cq-ROFWMSM [44]</td>
<td>0.7800</td>
<td>0.5727</td>
<td>0.6993</td>
<td>0.7132</td>
<td>$\mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_3 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>Cq-ROFWDMMSM [44]</td>
<td>-0.6706</td>
<td>-0.7878</td>
<td>-0.5851</td>
<td>-0.5108</td>
<td>$\mathbf{e}_4 &gt; \mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>FFWPA [11]</td>
<td>0.2378</td>
<td>-0.2380</td>
<td>0.2875</td>
<td>0.0232</td>
<td>$\mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>FFWPG [11]</td>
<td>0.2727</td>
<td>-0.2662</td>
<td>0.3414</td>
<td>0.0151</td>
<td>$\mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
<tr>
<td>FFDAW [45]</td>
<td>0.5005</td>
<td>-0.0900</td>
<td>0.5481</td>
<td>0.2481</td>
<td>$\mathbf{e}_3 &gt; \mathbf{e}_1 &gt; \mathbf{e}_4 &gt; \mathbf{e}_2$</td>
</tr>
</tbody>
</table>

The approach provided in the present article has some obvious benefits. In this approach, we consider criteria weights to be unknown, which enhances the robustness of our method. Another reason that makes our approach more reliable than others is its use of the power aggregation operator which calculate the support degree between arguments, whereas the latter not only includes the power aggregation operation but also considers the interrelationship among arguments. According to the conducted analysis, the proposed operators have the following merits over the existing ones: i). The proposed approach and the methodologies of Bai et al. [43] (q-ROFWPMSM and q-ROFWPMSM) yield different rankings. A key distinction is that the proposed operator assumes the weights of the criteria to be unknown. Therefore, the existing operators q-ROFWPMSM and q-ROFWPMSM are applicable when the data consists of real numbers. In contrast, our approach exhibits superiority as it can be applied to complex fuzzy numbers.

ii). Our devised MCDM method and Ali’s MCDM approach [44] both work with complex fuzzy sets. However, Ali’s MCDM approach does not capture the interrelationship among the criteria, nor does it incorporate power aggregation operators, which calculate the support degree between arguments. This omission reduces the impact of high or low values.

iii). Aydemir’s MCDM approach [45] and our proposed method both capture the power operator which helpfull to control the effect of low or high values but Aydemir’s approach does not capture the interrelationship among the criteria. FFWPA and FFWPG are primarily tailored for real-valued data, and adapting them for complex data may present challenges related to their interpretability and sensitivity to complex numbers. The inclusion of both real and imaginary components in complex data can lead to less intuitive outcomes and complexities in controlling power parameters’ effects. So CFWPPMSM is often a more appropriate choice, as it is explicitly designed to handle complex fuzzy data with precision and tailored control. While FFDAW offers flexibility, it introduces complexity in parameter selection and may compromise interpretation and robustness in complex data scenarios.

iv). The introduced operators come with two parameters, denoted as $t$ and $\rho$, offering decision makers enhanced flexibility and resilience when it comes to choosing the optimal $\rho$ in line with their risk preferences. Furthermore, it’s noteworthy that numerous established operators can be considered as specific instances of the newly created operator, characterized by particular parameter values. This comprehensiveness renders the newly developed operators even more adept at effectively addressing a wide array of Multi-Criteria Decision Making (MCDM) challenges.
9 Conclusions

This paper introduces a novel methodology aimed at addressing challenges associated with complex fermatean fuzzy MCDM problems. This contribution can be delineated into three distinct phases. Firstly, to aggregate complex fermatean fuzzy information effectively, the paper extends the Partitioned Maclaurin Symmetric Mean (PMSM) operator to a complex fermatean fuzzy partitioned Maclaurin symmetric mean (CFFPMSM) and complex fermatean fuzzy weighted partitioned Maclaurin symmetric mean (CFFWPMSM) operators. These operators are designed to address scenarios where attributes are partitioned into groups with interrelationships within each group but not across groups. Secondly, to mitigate the potential adverse effects of arbitrary assessment values provided by decision makers on the final decision outcomes, the paper leverages the Power Average (PA) operator. This leads to the development of the complex fermatean fuzzy power partitioned Maclaurin symmetric mean (CFFPPMSM) and complex fermatean fuzzy weighted power partitioned Maclaurin symmetric mean (CFFWPMPMSM) operators, which combine the strengths of the PMSM and PA operators. Finally, we handle scenarios where the criteria weights are initially unknown, employing the statistical variance method to determine these weights and enhance the robustness of our approach. A new approach, based on the CFFWPMPMSM operator, is proposed for selecting the optimal site for a hostel at a renowned university under complex fermatean fuzzy MCDM scenarios. To contrast our developed MCDM approach with existing ones, we conducted a case study implementation [11,43-45]. The comparison results indicate that our developed MCDM approach exhibits greater efficiency than Bai et al. methodology [43], Ali and Mahmood method [44], Aydemir and Gunduz MCDM approach [45], and Senapati and Yger [11] which fails to capture the interdependence among criteria to effectively address MCDM problems in a Complex fermatean fuzzy environment.

In future research, we will apply the proposed approach in other practical decision making problems, such as urban planning, healthcare resource allocation, financial risk assessment, and environmental impact analysis, etc.

References


