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A New Reliability Allocation Method for Machine Tools Using the Intuitionistic Trapezoidal Fuzzy Numbers and TOPSIS

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A new reliability allocation method for machine tools using the intuitionistic trapezoidal fuzzy numbers and TOPSIS

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Abstract: The reliability allocation of machine tools which is a multi-attribute decision making problem has great significance in designment of machine tools. This paper integrates the intuitionistic trapezoidal fuzzy numbers and performance sorting technique based on similar ideal solutions to achieve the flexible allocation of machine tools reliability. Firstly, intuitionistic trapezoidal fuzzy numbers are employed to integrate decisions made by multiple decision makers and fuzzy information of their preferences. Then, intuitionistic trapezoid fuzzy numbers' expectations are treated as the weights of criteria. Finally, performance sorting technique based on similar ideal solutions is used to obtain the reliability allocation weight of every subsystem. To investigate the efficacy and simplicity of the provided approach, reliability distribution in a CNC machine tool is introduced as an example to explain its specific contents. It can be concluded that the provided method is more precise and convenient by comparing the results of this approach with those obtained by analytic hierarchy process method.

Keywords: Reliability allocation, Intuitionistic trapezoidal fuzzy numbers, Performance sorting technique based on similar ideal solutions, Machine tools.

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1. Introduction

As a crucial facility of other industrial components, machine tools play an important role in manufacturing industry[1]. They are widely used in the manufacturing industry, so the investigation of machine tools' reliability is very important[2]. The reliability of machine tool is very important to improve the processing quality of products and increase the benefit of enterprises [1]. The importance of reliability in the whole process of production is self-evident, it also has a certain influence on the accuracy of machine tools, which is also concerned by the relevant personnel[3]. The reliability of machine tools includes reliability prediction, reliability growth, reliability analysis[4] and assessment, reliability allocation and so on. In the process of product design and optimization, reliability distribution is an essential step, and its main content is to distribute product reliability to each key subsystem [5]. Many researchers and users have conducted a lot of in-depth research and experiments on reliability distribution [6]. There are also a series of achievements in the reliability distribution of CNC machine tools [7,8]. In recent years, the reliability distribution method has been constantly optimized[9].

Reliability distribution is to assign the determined system reliability to each subsystem according to the different attributes and then each reliability will be distributed to the parts in the subsystem [7]. Traditional reliability allocation methods include analytic hierarchy process (AHP) method[10], integration of maximal entropy ordered weighted averaging (ME-OWA) [11], FOO method[12] and maximal entropy ordered weighted averaging method[13] But these methods do not take into account the fuzziness of reliability allocation[14], so the research on reliability assignment based on fuzzy mathematical model is also increasing [5]. 0Zadeh[15]put forward the concept of fuzzy set in 1965. Fuzzy sets[15]is a tool dealing with the fuzzy and imprecise information, but there are always various problems when the fuzzy set was used in practice due to various uncertainties. Considering the personal preference information of the decision makers often be affected by the different levels of hesitation or the lack of relevant knowledge, Atanassov[16] came up with the intuitionistic fuzzy sets which contains information on membership, non-membership and hesitation based on the fuzzy set, it can express the preference information easily and effectively. However, intuitionistic fuzzy sets can only roughly represent membership or membership of a property for a scheme, or the degree of "good" and "bad" of a fuzzy concept[17]. Sriramdas V[18] put forward trapezoidal fuzzy numbers to represent the distribution factors in the early days of design optimization and proposed the trapezoidal fuzzy number approximation method according to linear programming. The literature [19] introduced a ITrFN and the concept of continuous set is put forward, which is a generalization of discrete set and fuzzy Numbers[20] extending the intuitionistic fuzzy sets from another direction. Intuitionistic trapezoidal fuzzy number is able to not only express the extent of "good" and "bad", but also express different dimensional decision-making information[17].

Currently, intuitionistic trapezoidal fuzzy numbers (ITrFNs) are widely applied to illustrate the fuzziness and one-sidedness of data. It can express different dimension decision information and contains non-membership degree and hesitation information, so the intuitionistic trapezoidal fuzzy number is more precise and accurate to cope with the vague problems. It has more great flexibility. Ranking intuitionistic trapezoidal fuzzy numbers has great significance in coping with problems containing partial and vague information [21].

In the process of machine tools reliability allocating, many influencing factors and alternatives must be taken into account. Therefore, it is a process in which various factors and alternatives are comprehensively weighed[22]. There are many factors that need to consider during the conducting of reliability allocation. The reliability allocation for machine tools hence to be deemed as a multi-attribute decision making (MADM) problem[8]. Li ZK[23] proposed a reliability allocation method that integrates fuzzy arithmetic with decision making trial and evaluation laboratory (DEMATEL). It attached importance to the correlations among different subsystems and improved the accuracy of reliability allocation. Kuei-Hu Chang[5] proposed a synthesis of the hesitant fuzzy linguistic term sets and minimal variance OWGA weights to allocate the system reliability flexibly. Gu Y K and Wu L H[10] proposed a new method of reliability allocation according to the fuzzy theory and AHP method. In this method, the weight of factors is computed by the AHP method, and the fuzzy evaluation allocation method is used for comprehensive evaluation. This paper propsed a modified method based on fuzzy theory and TOPSIS, which will mirror subjective judgement and objective information truly[24].

The MADM proposes an effective method to compare alternatives according to the assessment of different criteria and the aim of the MADM is to seek out the most satisfactory alternative(s) rather than the criteria from a set of available options[25]. A multiple attribute decision making problem is to seek out a satisfactory solution from limited number of viable options which is evaluated on series of attributes including quantitative and qualitative[26]. But the reliability distribution problem of the machine tool is to seek out all desirable solutions and order them by weight. The technique for order preference by similarity to ideal solution (TOPSIS)[27] is a helpful and effective method to select the desirable solution and also to sort the alternatives exactly[28]. The concepts of TOPSIS is that the chosen solution is supposed to have the shortest distance from the positive ideal point and the farthest distance from the negative ideal point simultaneously[25].

In summary, related researchers have studied AHP method and the intuitionistic fuzzy sets. However, these methods do not take into account the fuzziness of reliability allocation. In addition, the traditional TOPSIS method deals with accurate information, and the machine reliability distribution contains a lot of fuzzy information.In order to solve these problems, an approach to allocationg reliability of machine tools based on TOPSIS is propsed. In this paper, the intuitionistic trapezoidal fuzzy numbers (ITrFNs) and the TOPSIS method[29] is applied to deal with the machine tools' problems of reliability allocation. By using this method, we have developed simple and precise formulas to achieve reliability allocation of machine tools.

This article is arranged as follows: In Section 2, definition of TOPSIS method, ITrFNs and AHP method is reviewed. In Section 3, to allocate the system reliability flexibly, a more integrated reliability allocation method is presented, which proposed a synthesis of the ITrFNs and TOPSIS method. In Section 4, an example of a CNC machine tool is illustrated, and comparison with the AHP method is made to verify the effectiveness and superiority of the solution. Summaries are given in the last part. Some conclusions or results are summarized in the text.

2. Preliminaries

In this section, reliability allocation of machine tools[12] is reviewed and some definitions and basic concepts related to analytic hierarchy process (AHP) method[30], intuitionistic trapezoidal fuzzy numbers (ITrFNs)[29,31,32] and TOPSIS method[26] is introduced.

2.1 Reliability allocation of machine tools

After the overall reliability is determined, it needs to be assigned to subsystems and their parts properly according to certain principles [8]. A machine tools is often deemed to consist of n subsystems in the conventional reliability allocation methods. Let $R^*(t)$ be the reliability goal of a machine tool which is in operation for some time $t \geq 0$. Let $R_i(t)$ be the reliability distributed to the ith subsystem. The reliability of machine tool is indicateded by the reliability of subsystem as follows:

$$\prod_{n}^{i=1} R_i(t) \geq R^*(t)$$

The division of the machine tool subsystems by different experts is different, but this does not have a big impact on the reliability of the machine tool as a whole. Conventional reliability allocation methods of machine tools are complicated, so this paper presents an easy calculation method of reliability allocation.

2.2 AHP method

Analytic hierarchy process （AHP) method is to layer the problem. First, according to the nature of the problem, the overall goal to be achieved and influencing factors, the problem is decomposed into different levels. Next, judgment matrices are established by analyzing the relationship between the elements of each layer, and we should conduct consistency tests. These matrices should be reconstructed if they are not consistent. Then, weights of all elements at different levels which will be used to complete the reliability allocation problems at last are obtained by calculation. Hierarchical model of reliability allocation for CNC machine tools is shown in Figure 1.

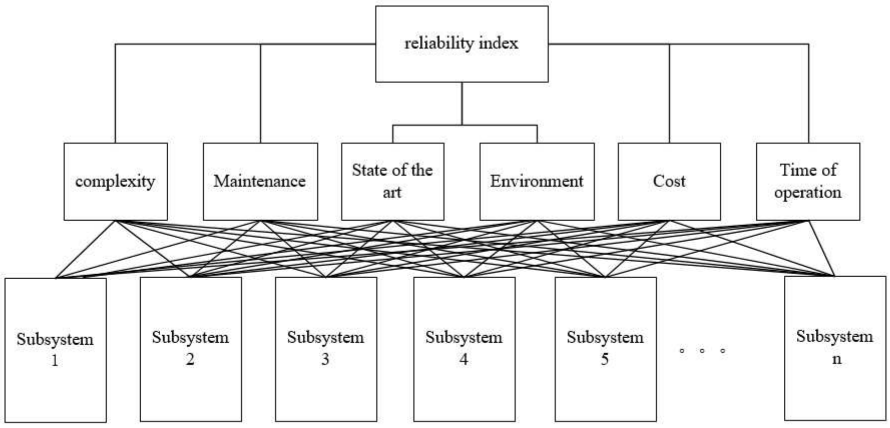

Figure 1 Hierarchical model of reliability allocation for CNC machine tools

After the hierarchical model is established, the judgment matrix $A = (a_{ij})_{n \times n}$ is constructed by pairwise comparison of all the elements in the middle layer with the target elements at the highest level as the criterion，where a_{ij} represents the importance scale of element i and j relative to the top-level target element, and n represents the number of elements in the middle layer, that is, the number of influencing factors to be considered.Set Wj as the weight value of the importance of the JTH influencing factor in reliability allocation. Set the weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_j), j = 1,2, \dots, n$,j=1,2... n to determine the weight value of the influencing factor.After verifying the consistency of the judgment matrix, the weight value of each influencing factor in the process of machine tool reliability distribution is finally obtained.

But, when the number of components or experts is large, its calculation process becomes very complicated and difficult. Besides, it cannot effectively integrate the experience of multiple experts. However, the approach provided in the paper is supposed to cope with these problems very well[33].

2.3 Intuitionistic trapezoidal fuzzy numbers

Definition 1. An intuitionistic trapezoidal fuzzy number $\boldsymbol{A}$ with parameters $b_1 \le a_1 \le b_2 \le a_2 \le a_3 \le b_3 \le a_4 \le b_4$ is represented as $A = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \rangle$ in the set of real numbers $\boldsymbol{R}$. In this case, its membership function and non-membership function can be expressed as

$$\mu_A(x) = \begin{cases} 0, x < a_1; \\ \frac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2; \\ 1, a_2 \le x \le a_3; \\ \frac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4; \\ 0, x > a_4; \end{cases} \tag{1}$$

$$\nu_A(x) = \begin{cases} 1, x < b_1; \\ \frac{x-b_2}{b_1-b_2}, b_1 \le x \le b_2; \\ 0, b_2 \le x \le b_3; \\ \frac{x-b_3}{b_4-b_3}, b_3 \le x \le b_4; \\ 1, x > b_4; \end{cases} \tag{2}$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote the membership degree and non-membership degree of the element x to the set $\boldsymbol{A}$ severally.

Definition 2. Let $A_i = \langle (a_{i1}, a_{i2}, a_{i3}, a_{i4}), (b_{i1}, b_{i2}, b_{i3}, b_{i4}) \rangle (i = 1,2)$ be two ITrFNs and μ be a positive scalar number. Then,

$$A_1 + A_2 = \langle (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}), (b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}) \rangle \tag{3}$$

$$\mu A_1 = \langle (\mu a_{11}, \mu a_{12}, \mu a_{13}, \mu a_{14}), (\mu b_{11}, \mu b_{12}, \mu b_{13}, \mu b_{14}) \rangle, \mu > 0 \tag{4}$$

Definition 3. Let $A_i (i = 1,2, \cdots, n)$ be a set of intuitionistic trapezoidal fuzzy numbers. $w = (w_1, w_2, \cdots, w_n)^T$ is the weight vector of A_i. The intuitionistic trapezoid fuzzy numbers weighted average (ITrFNWA) operator is defined by

$$ITrFNWA(A_1, A_2, \cdots, A_n) = \sum_{i=1}^{n} w_i A_i \tag{5}$$

The ITrFNWA operator is a ITrFNs. When $w_i = \frac{1}{n}$, the formula is as follows:

$$ITrFNWA(A_1, A_2, \cdots, A_n) = \frac{1}{n} \sum_{i=1}^{n} A_i \tag{6}$$

Another crucial definition is the expectation of the ItrFN.

Definition 4. Let $A = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4) \rangle$ be an ITrFN. Its expected value is obtained by

$$EV(A) = \frac{1}{8} \left(\sum_{i=1}^{4} a_i + \sum_{j=1}^{4} b_j \right) \tag{7}$$

Definition 5. A helpful way to cope with vague numbers is their α-cuts. Let $\boldsymbol{A}$ be an ITrFN in the set of real numbers $\boldsymbol{R}$. In the context of intuitionistic fuzzy numbers it is easy to differentiate following α-cuts: $(A^+)_\alpha$ and $(A^-)_\alpha$. It is obvious that

$$(A^+)_\alpha = \{x \in R | \mu_A(x) \ge \alpha\} \tag{8}$$

$$(A^-)_\alpha = \{x \in R | 1 - \nu_A(x) \ge \alpha\} \tag{9}$$

Based on the concept it is observed that every α-cut$(A^+)_\alpha$or$(A^-)_\alpha$is a closed interval. Thus, we can get $(A^+)_\alpha = [A_L^+(\alpha), A_U^+(\alpha)]and(A^-)_\alpha = [A_L^-(\alpha), A_U^-(\alpha)]$, severally, where

$$A_L^+(\alpha) = inf\{x \in R | \mu_A(x) \ge \alpha\} \tag{10}$$

$$A_U^+(\alpha) = sup\{x \in R | \mu_A(x) \ge \alpha\} \tag{11}$$

$$(A_L^-)_\alpha = inf\{x \in R|1 - \nu_A(x) \geq \alpha\} \quad (12)$$

$$(A_U^-)_\alpha = sup\{x \in R|1 - \nu_A(x) \geq \alpha\} \quad (13)$$

According to the concept of α-cut, a distance formula for ITrFNs is provided.

Definition 6. Let ***A*** and ***B*** be two intuitionistic fuzzy numbers. The distance from A to B can be expressed as:

$$D(A,B) =$$

$$\left(\frac{1}{4}\int_0^1\left(\left(A_L^+(\alpha) - B_L^+(\alpha)\right)^2 + \left(A_R^+(\alpha) - B_R^+(\alpha)\right)^2 + \left(A_L^-(\alpha) - B_L^-(\alpha)\right)^2 + \left(A_R^-(\alpha) - B_R^-(\alpha)\right)^2\right)d\alpha\right)^{\frac{1}{2}} \quad (14)$$

Theorem 1. Let $A_i = \langle(a_{i1}, a_{i2}, a_{i3}, a_{i4}), (b_{i1}, b_{i2}, b_{i3}, b_{i4})\rangle (i = 1,2)$ be two intuitionistic fuzzy numbers. The distance from A_1 to A_2 is as follows:

$$D(A_1, A_2) =$$

$$\left(\frac{1}{12}\left(\begin{matrix}\sum_{i=1}^4(a_{2i} - a_{1i})^2 + \sum_{i=1}^4(b_{2i} - b_{1i})^2 + (a_{21} - a_{11})(a_{22} - a_{12}) + \\ (b_{21} - b_{11})(b_{22} - b_{12}) + (a_{23} - a_{13})(a_{24} - a_{14}) + (b_{23} - b_{13})(b_{24} - b_{14})\end{matrix}\right)\right)^{\frac{1}{2}} \quad (15)$$

2.4 TOPSIS method

TOPSIS method is an approach among the common MADM methods. The rationale is that the chosen solution is supposed to have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. TOPSIS's process can be represented by a sequence of procedures:

Step 1. List decision matrices C^k.

$$C^k = \{c_{ij}^k | i = 1 \cdots m, \quad j = 1 \cdots n, \quad k = 1 \cdots l\}$$

Step 2. Determine the positive ideal and negative ideal solution

$$C^+ = \{c_{ij}^+ | i = 1, \cdots m, \quad j = 1, \cdots n\}$$

$$C^- = \{c_{ij}^- | i = 1, \cdots m, \quad j = 1, \cdots n\}$$

$$c_{ij}^+ = max\{c_{ij}^1, c_{ij}^2, \cdots c_{ij}^l | i = 1, \cdots m, \quad j = 1, \cdots n\}$$

$$c_{ij}^- = min\{c_{ij}^1, c_{ij}^2, \cdots c_{ij}^l | i = 1, \cdots m, \quad j = 1, \cdots n\}$$

Step 3. Calculate the distance measures.

$$D_k^+ = D(C^k, C^+), \quad k = 1, \cdots l$$

$$D_k^- = D(C^k, C^-), \quad k = 1, \cdots l$$

There are many common formulas for the distance measure. In this paper, the Eq. (15) was used to compute the distance.

Step 4. Calculate the relative closeness degree λ_k.

$$\lambda_k = \frac{D_k^-}{D_k^+ + D_k^-}$$

According to the relative closeness degree di, the smaller the value of the λ_k is, the closer it is to the positive ideal solution. On the other hand, the closer it get to the negative ideal solution.

3. Proposed method of the machine tool reliability allocation according to intuitionistic trapezoidal fuzzy TOPSIS.

AHP method and the conventional TOPSIS method do not take into account the fuzziness of reliability allocation. In addition, the intuitional trapezoid fuzzy number is more precise and accurate in describing the fuzziness of the objective information and has further flexibility, it can effectively decrease the reduction of decision information. The multi-attribute group decision making method according to intuitionistic trapezoidal fuzzy TOPSIS is more in line with the evaluation and perception of the decision-maker.

The reliability allocation of machine tools can be regarded as a MADM problem. Let $O = \{o_1, o_2, \cdots, o_m\}$be a series of alternatives and let $C = \{c_1, c_2, \cdots, c_n\}$be a set of criteria. Let $D = \{d_1, d_2, \cdots, d_l\}$ be a set of decision makers and let $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ be decision matrices. The preference value of an option on a standard $c_j (j = 1,2, \cdots, n)$ is a ITrFN $r_{ij}^{(k)} (i = 1,2, \cdots, m; j = 1,2, \cdots, n)$, it expresses the degree to which the alternative o_i meets or not meets the standard c_j established by decision makers based on linguistic values of ITrFNs for linguistic terms.

Step 1. List configurations of the machine tool under study.

Step 2. Ascertain the system reliability requirement and factors affecting the reliability distribution of the CNC machine tool.

Step 3. List decision matrices$R^{(k)}$.

Step 4. Construct integration intuitionistic trapezoidal fuzzy decision matrix$R$.

An intuitionistic trapezoidal fuzzy decision matrix$R$ is constructed by using ITrFWA operator to integrate the decision matrix of each decision maker.

The formula is as follows:

$$r_{ij} = \langle (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}), (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}) \rangle = ITrFWA\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \cdots, r_{ij}^{(l)}\right) = \sum_{k=1}^{l} \frac{1}{l} r_{ij}^{(k)} \tag{16}$$

Step 5. Use the expected values of intuitive trapezoid fuzzy numbers as criterion weights.

Compute the expected weight value for a criterion by using the Eq. (7). By using these weight values, the weight matrix of criteria ***U*** is obtained.

Step 6. Ascertain the positive ideal and negative ideal solution.

$$R^+ = (r_1^+, r_2^+, \cdots, r_n^+) \tag{17}$$

$$R^- = (r_1^-, r_2^-, \cdots, r_n^-) \tag{18}$$

where

$$r_j^+ = \left\langle \left(\max_i a_{ij1}, \max_i a_{ij2}, \max_i a_{ij3}, \max_i a_{ij4} \right), \left(\max_i b_{ij1}, \max_i b_{ij2}, \max_i b_{ij3}, \max_i b_{ij4} \right) \right\rangle$$

$$r_j^- = \left\langle \left(\min_i a_{ij1}, \min_i a_{ij2}, \min_i a_{ij3}, \min_i a_{ij4} \right), \left(\min_i b_{ij1}, \min_i b_{ij2}, \min_i b_{ij3}, \min_i b_{ij4} \right) \right\rangle$$

Step 7. Compute the weighted positive and negative distance measure.

The weighted positive and negative distance measure ($D_i^+$and D_i^-) are calculated as

$$D_i^+ = \sum_{j=1}^{n} U_{ij}\, D\left(r_{ij}, r_j^+\right) \tag{19}$$

$$D_i^- = \sum_{j=1}^{n} U_{ij}\, D\left(r_{ij}, r_j^-\right) \tag{20}$$

where U_{ij} is the element of the weight matrix of criteria ***U***.

Step 8. Compute the relative closeness degree λ. The relative closeness of the alternative o_i is defined as

$$\lambda_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{21}$$

Step 9. Calculate the reliability allocation coefficient k.

According to the reliability distribution standard of the machine tool, the bigger λ_i is, the more difficult it is to ameliorate the reliability of the alternative o_i. Thus this paper makes the following decision of the reliability allocation coefficient k_i:

$$k_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i} \tag{22}$$

Step 10. The reliability allocation of the machine tool is completed according to the reliability distribution coefficient and the configuration of the machine tool. The formula is as follows:

$$R_i = R_s^{k_i} \quad i = 1,2 \cdots, n \tag{23}$$

where, R_s is the system reliability of the machine tool.

The flow chart of the method is shown in Figure 2.

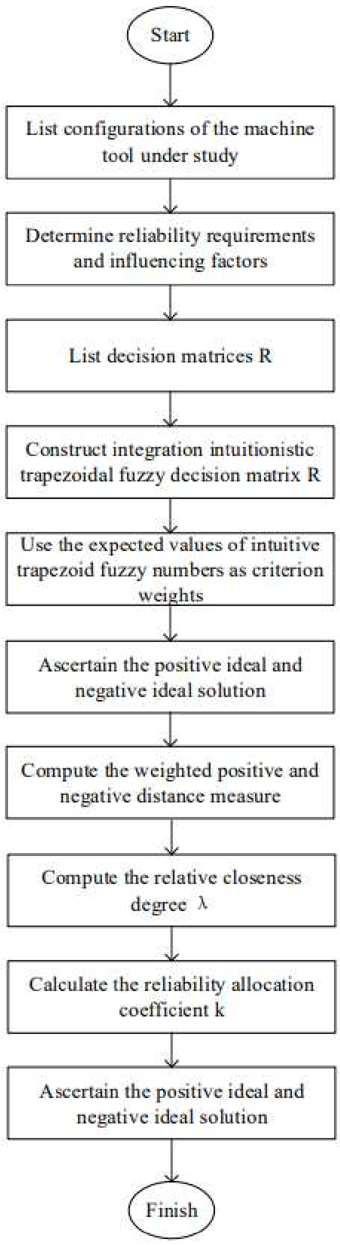

Figure 2 Flowchart of the fuzzy TOPSIS process

4. An illustrative example

In this part, to demonstrate the validity of the reliability distribution method, a CNC machine tool is introduced as an instance. Table 1 shows the common subsystem division of a typical machine tool. The pictures of CNC machine tools and subsystems are shown in Figure 3.

Table1 The common subsystem division of a typical machine tool

	Subsystems	The main components
The machine tool	Hydraulics and pneumatics system	Static pressure ram Static pressure slide carriage Static pressure rotary table
	Feeding system	Ball screw, Guide, Worktable
	Spindle system	Bearing, Gear, Spindle
	Servo system	Servomotor
	Lubricant system	Oil pump, Pipeline Lubricating oil and grease
	Cooling system	Cooling pump, Coolant Pipeline, Filter device
	Automatic tool changing system	Tools, Tool magazine
	CNC system	CNC, PLC
	Main transmission	
	Auxiliary system	Lamp, Bed
	Guard	Safety devices
	Swarf conveyors	Conveyor

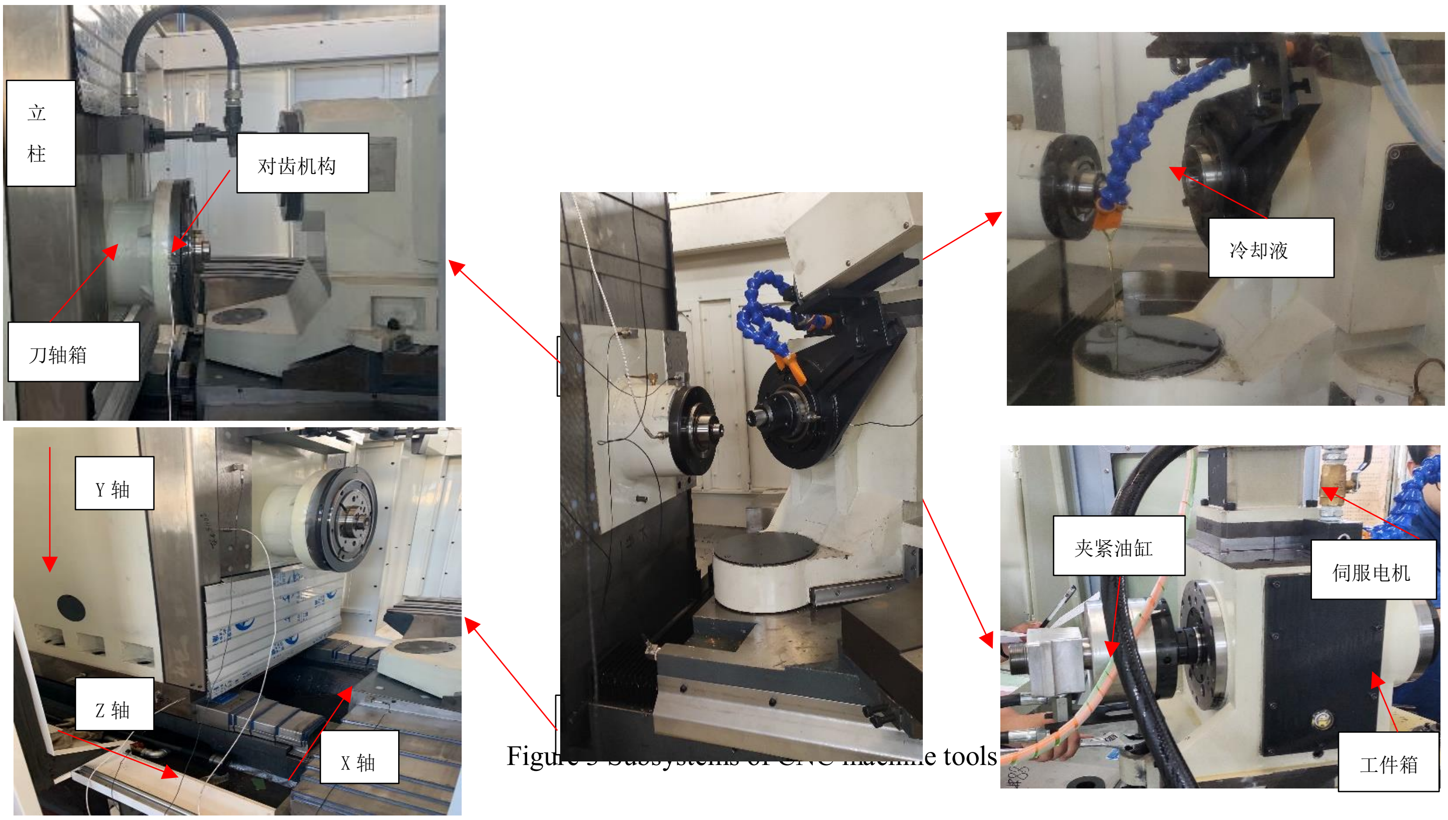

Figure 3 Subsystems of CNC machine tools

Step 1. List configurations of the CNC machine tool.

The CNC machine tool was divided into 8 main subsystems according to its structure feature. The 8 main subsystems are shown in Table 2.

Table 2 Subsystem division of the CNC machine tool

No.	Subsystem
1	Hydraulics and pneumatics system
2	Feeding system
3	Spindle system
4	Servo system
5	Lubricant system
6	Cooling system
7	Automatic tool changing system
8	CNC system

These 8 subsystems constitute a set of alternatives $O = \{o_1, o_2, \cdots, o_8\}$.

Step 2. Ascertain the system reliability requirement and factors affecting the machine tool reliability distribution.

Based on the design specification, the system reliability of the CNC machine tool is set to 0.85, and the mission time is 1200 h.

There are 6 influencing factors of reliability allocation: complexity, maintenance, the technical status, environment, cost and time of operation. These 6 reliability allocation factors constitute a set of criteria $C = \{c_1, c_2, \cdots, c_6\}$. From the qualitative perspective, the reliability allocation of machine tools can be enforced based on the following standards, as shown in Table3.

Table 3 The reliability allocation principle

Criterion	Influencing factor	Reliability allocation principle
c_1	Complexity	The more complex the structure, the lower the reliability allocated to the subsystem.
c_2	Maintenance	The easier the subsystem is to maintain, the lower the reliability of the allocation.
c_3	State of the art	The higher the technology maturity, the higher the reliability of the allocation.
c_4	Environment	The better the environment, the higher the reliability of the allocation.
c_5	Cost	The higher the cost, the lower the reliability of the allocation.
c_6	Time of operation	The longer the work, the lower the reliability of the allocation.

In order to be consistent with other criteria, for the influencing factor c_3 and c_4, experts evaluate according to technology immaturity and environmental severity, thus the higher their linguistic values, the lower the reliability of the allocation.

Step 3. List decision matrices $R^{(k)}$.

The 8 alternatives are computed by three professional decision makers using the linguistic terms in accordance with the above 6 principles, and linguistic values of ITrFNs for linguistic terms is listed in Table 4 [32].

Table 4 Linguistic values of ITrFNs for linguistic terms

Linguistic terms	Abbreviation	Linguistic values of intuitionistic trapezoidal fuzzy numbers
Absolutely low	AL	$\langle(0.0,0.0,0.0,0.0),\ (0.0,0.0,0.0,0.0)\rangle$
Low	L	$\langle(0.0,0.1,0.2,0.3),\ (0.0,0.1,0.2,0.3)\rangle$
Fairly low	FL	$\langle(0.1,0.2,0.3,0.4),\ (0.0,0.2,0.3,0.5)\rangle$
Medium	M	$\langle(0.3,0.4,0.5,0.6),\ (0.2,0.4,0.5,0.7)\rangle$
Fairly high	FH	$\langle(0.5,0.6,0.7,0.8),\ (0.4,0.6,0.7,0.9)\rangle$
High	H	$\langle(0.7,0.8,0.9,1.0),\ (0.7,0.8,0.9,1.0)\rangle$
Absolutely high	AH	$\langle(1.0,1.0,1.0,1.0),\ (1.0,1.0,1.0,1.0)\rangle$

The decision matrices $R^{(k)}(k=1,2,3)$as follows:

$$R^{(1)}=\begin{pmatrix} AH & FL & H & AH & H & AH \\ M & FH & M & FL & M & H \\ AH & H & FH & M & H & AH \\ H & FH & AH & L & H & H \\ M & H & FL & FH & FL & H \\ FL & H & L & M & L & H \\ AH & FH & FH & M & H & M \\ AH & FH & FL & L & H & AH \end{pmatrix}$$

$$R^{(2)}=\begin{pmatrix} FH & L & M & FH & FL & H \\ H & H & H & L & H & FH \\ H & FH & M & M & H & AH \\ AH & H & H & FH & FH & AH \\ FL & M & FL & L & FL & FH \\ L & FL & L & FH & FL & M \\ H & H & H & FH & M & FL \\ AH & FL & M & FH & H & AH \end{pmatrix}$$

$$R^{(3)}=\begin{pmatrix} FH & FL & L & FL & M & H \\ FH & FH & M & L & FH & AH \\ H & M & M & FH & H & AH \\ FH & H & FH & M & FH & AH \\ FL & FL & L & FL & FL & H \\ FL & M & L & FH & FL & FH \\ H & FH & FH & M & M & FL \\ AH & M & FL & FL & H & AH \end{pmatrix}$$

Step 4. Construct integration intuitionistic trapezoidal fuzzy decision matrix$R$.

Combined with the formula (16) and Table 4, an intuitive trapezoidal fuzzy decision matrix R is constructed, as listed in Table 5.

Table 5 Preference values of options provided by three experts by linguistic values of ITrFNs.

c_1	c_2

o_1	⟨(0.67,0.73,0.8,0.87)(0.6,0.73,0.8,0.93)⟩	⟨(0.07,0.17,0.27,0.37)(0,0.17,0.27,0.43)⟩
o_2	⟨(0.5,0.6,0.7,0.8)，(0.43,0.6,0.7,0.87)⟩	⟨(0.57,0.67,0.77,0.87)(0.5,0.67,0.77,0.93)
o_3	⟨(0.8,0.87,0.93,1)，(0.8,0.87,0.93,1)⟩	⟨(0.5,0.6,0.7,0.8)，(0.43,0.6,0.7,0.87)⟩
o_4	⟨(0.73,0.8,0.87,0.93)(0.7,0.8,0.87,0.97)⟩	⟨(0.63,0.73,0.83,0.93)(0.6,0.73,0.83,0.97)
o_5	⟨(0.17,0.27,0.37,0.47)(0.07,0.27,0.37,0.57	⟨(0.37,0.47,0.57,0.67)(0.3,0.47,0.57,0.73)
o_6	⟨(0.07,0.17,0.27,0.37)(0,0.17,0.27,0.43)⟩	⟨(0.37,0.47,0.57,0.67)(0.3,0.47,0.57,0.73)
o_7	⟨(0.8,0.87,0.93,1)，(0.8,0.87,0.93,1)⟩	⟨(0.57,0.67,0.77,0.87)(0.5,0.67,0.77,0.93)
o_8	⟨(1,1,1,1)，(1,1,1,1)⟩	⟨(0.3,0.4,0.5,0.6)，(0.2,0.4,0.5,0.7)⟩

	c_3	c_4
o_1	⟨(0.33,0.43,0.53,0.63)(0.3,0.43,0.53,0.67)	⟨(0.53,0.6,0.67,0.73)(0.47,0.6,0.67,0.8)⟩
o_2	⟨(0.43,0.53,0.63,0.73)(0.37,0.53,0.63,0.8)	⟨(0.03,0.13,0.23,0.33)(0,0.13,0.23,0.37)⟩
o_3	⟨(0.37,0.47,0.57,0.67)(0.27,0.47,0.57,0.77	⟨(0.37,0.47,0.57,0.67)(0.27,0.47,0.57,0.77
o_4	⟨(0.73,0.8,0.87,0.93)(0.7,0.8,0.87,0.97)⟩	⟨(0.27,0.37,0.47,0.57)(0.2,0.37,0.47,0.67)
o_5	⟨(0.07,0.17,0.27,0.37)(0,0.17,0.27,0.43)⟩	⟨(0.2,0.3,0.4,0.5)，(0.13,0.3,0.4,0.57)⟩
o_6	⟨(0,0.1,0.2,0.3)，(0,0.1,0.2,0.3)⟩	⟨(0.43,0.53,0.63,0.73)(0.33,0.53,0.63,0.83
o_7	⟨(0.57,0.67,0.77,0.87)(0.5,0.67,0.77,0.93)	⟨(0.37,0.47,0.57,0.67)(0.27,0.47,0.57,0.77
o_8	⟨(0.17,0.27,0.37,0.47)(0.07,0.27,0.37,0.57	⟨(0.2,0.3,0.4,0.5)，(0.13,0.3,0.4,0.57)⟩

	c_5	c_6
o_1	⟨(0.37,0.47,0.57,0.67)(0.3,0.47,0.57,0.73)	⟨(0.8,0.87,0.93,1)，(0.8,0.87,0.93,1)⟩
o_2	⟨(0.5,0.6,0.7,0.8)，(0.43,0.6,0.7,0.87)⟩	⟨(0.73,0.8,0.87,0.93)(0.7,0.8,0.87,0.97)⟩
o_3	⟨(0.7,0.8,0.9,1)，(0.7,0.8,0.9,1)⟩	⟨(1,1,1,1)，(1,1,1,1)⟩
o_4	⟨(0.57,0.67,0.77,0.87)(0.5,0.67,0.77,0.93)	⟨(0.9,0.93,0.97,1)，(0.9,0.93,0.97,1)⟩

o_5	$\langle(0.1,0.2,0.3,0.4),(0,0.2,0.3,0.5)\rangle$	$\langle(0.63,0.73,0.83,0.93)(0.6,0.73,0.83,0.97)$
o_6	$\langle(0.07,0.17,0.27,0.37)(0,0.17,0.27,0.43)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.43,0.6,0.7,0.87)\rangle$
o_7	$\langle(0.43,0.53,0.63,0.73)(0.37,0.53,0.63,0.8)$	$\langle(0.17,0.27,0.37,0.47)(0.07,0.27,0.37,0.57$
o_8	$\langle(0.7,0.8,0.9,1),(0.7,0.8,0.9,1)\rangle$	$\langle(1,1,1,1),(1,1,1,1)\rangle$

Step 5. Compute the expected values of ITrFNs and take them as weights of criteria.

By using the Eq. (7) to calculate weight values of standard and the weight matrix of criteria ***U*** is

$$U=(U_{ij})_{8\times 6}=\begin{pmatrix}0.77 & 0.22 & 0.48 & 0.63 & 0.52 & 0.9\\ 0.65 & 0.72 & 0.58 & 0.18 & 0.65 & 0.83\\ 0.9 & 0.65 & 0.52 & 0.52 & 0.85 & 1.0\\ 0.83 & 0.78 & 0.83 & 0.42 & 0.72 & 0.95\\ 0.32 & 0.52 & 0.22 & 0.35 & 0.25 & 0.78\\ 0.22 & 0.52 & 0.15 & 0.58 & 0.22 & 0.65\\ 0.9 & 0.72 & 0.72 & 0.52 & 0.58 & 0.32\\ 1.0 & 0.45 & 0.32 & 0.35 & 0.85 & 1.0\end{pmatrix}$$

Step 6. Determine the positive ideal and negative ideal solution by using Eqs. (17) and (18) and calculating the weighted positive and negative distance measures by using Eqs. (19) and (20). The results are shown in Table 6 and Figure 4.

Table 6 The weighted positive and negative distance measures

D_1^+=0.8970	D_1^-=0.7102
D_2^+=0.6822	D_2^-=0.5532
D_3^+=0.3754	D_3^-=0.2585
D_4^+=0.6114	D_4^-=0.7046
D_5^+=0.3818	D_5^-=0.3537
D_6^+=0.3598	D_6^-=0.3780
D_7^+=0.5676	D_7^-=0.2919
D_8^+=0.2374	D_8^-=0.1831

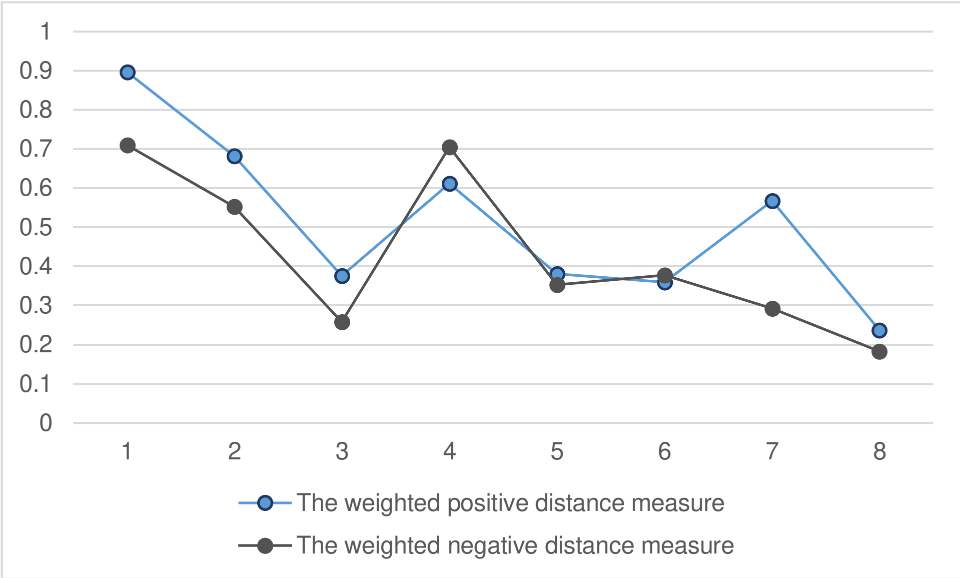

Figure 4 The weighted positive and negative distance measures

Step 7. Calculate the relative closeness degree and the reliability allocation coefficient by using Eqs. (21) and (22), as shown in Table 7. According to the reliability distribution principle of the machine tool, the higher λ is, the lower the reliability distribution is.

Table 7 The relative closeness degree and the reliability allocation coefficient

Subsystems	λ	k
1	0.4419	0.1227
2	0.4478	0.1244
3	0.4078	0.1132
4	0.5354	0.1487
5	0.4809	0.1335
6	0.5123	0.1423
7	0.3396	0.0943
8	0.4354	0.1210

Step 8 According to the result of Table 7, the Eq. (23) is applied to compute the reliability of each subsystem and the results are listed in Table 8. In order to compare with the provided method, the reliability allocation result for the CNC machine tool by the AHP method is also shown in Table 8. The distributed reliability of every subsystem by two allocation approach are listed in Figure 5.

Table 8 Reliability allocation results of two allocation methods

Subsystems	ITrFNs	AHP
1	0.9803	0.9976
2	0.9800	0.9729
3	0.9818	0.9536
4	0.9761	0.9571
5	0.9785	0.9955
6	0.9771	0.9961
7	0.9848	0.9870
8	0.9805	0.9804

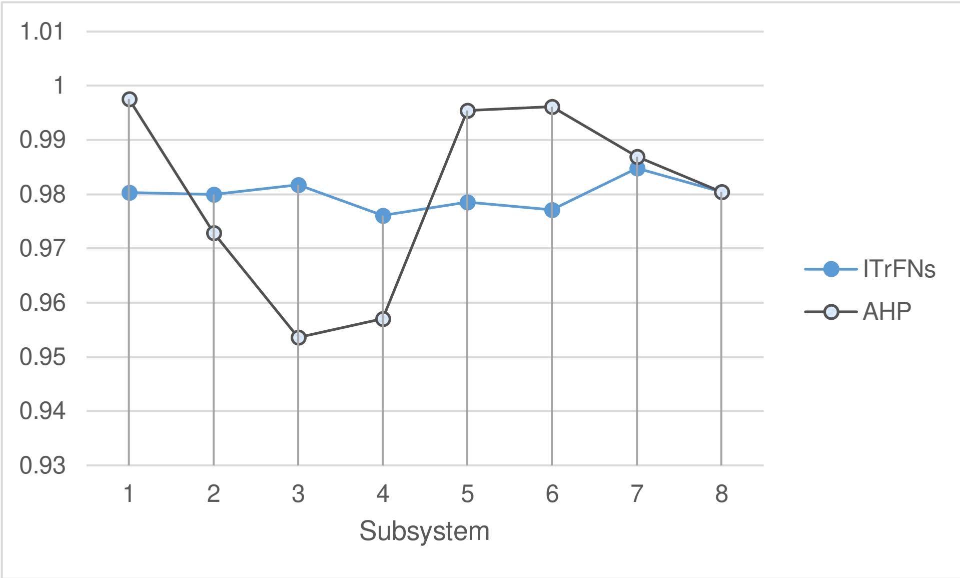

Figure 5 Comparison of reliability allocation between the two methods

By comparing ITrFNs with AHP methods, it is evident that the calculation of the former is easier than the latter. In the two methods, the system reliability of the CNC machine tool is set to 0.85, but the latter has higher requirements on the reliability of subsystems. The purpose of reliability allocation is to achieve the reliability target set according to the design requirements at the lowest cost and time [5], so the result of ITrFNs method is easier to accept. In addition, the ITrFNs method can better cope with fuzzy information. In summary, the ITrFNs method is more reasonable and more effective.

Conclusion

Reliability distribution has great significance in reliability design of the machine tool. A reliability distribution method for the machine tool was provided. The affecting factors and steps of the proposed method were researched, and a CNC machine tool was brought out to study as one example.

The advantage of the modified method in the paper are listed as follows:

1) Traditional reliability allocation methods are difficult to calculate, but the reliability distribution method in the paper is easy to calculate. The method in consideration of the weights of the factors that are taken into account, and it can cope with fuzzy information. Moreover, the decision makers can provide their decisions factually based on linguistic terms.
2) The method provided in this paper is easier to automate by programming, and the more decision makers, the more accurate the reliability allocation result. Fuzzy numbers are applied to denote inaccurate, fuzzy, and subjective information of a series of experts which makes the reliability allocation more precise. In addition, it is easy to integrate the experiences of multiple experts effectively.

The method proposed does not consider the weight of experts, because there is no uniform evaluation standard at present. In the future we will study this issue. The influencing factors that are considered is not limited to only these six and subsystems of other machine tools are not just these eight in this paper. The proposed approach is desirable to not only the machine tool but any large and complex system for which reliability distribution is essential.

Author contribution Qiang Cheng：methodology, validation, investigation, formal analysis, writing–review and editing. Chang Wang：Writing, methodology, and investigation. Dongyang Sun： resources, overseeing of analysis. Hongyan Chu：Supervision, review of experimental setup. Jigui Zheng： investigation.

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Data availability The datasets used or analyzed during the current studyare available from the corresponding author on reasonable request.

Code availability Not applicable

Declarations

Ethics approval Not applicable

Consent to participate All authors voluntarily agree to participate in this research study.

Consent for publication All authors voluntarily agree to the publication of the paper.

Conflict of interest The authors declare no conflict of interest.

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Figures

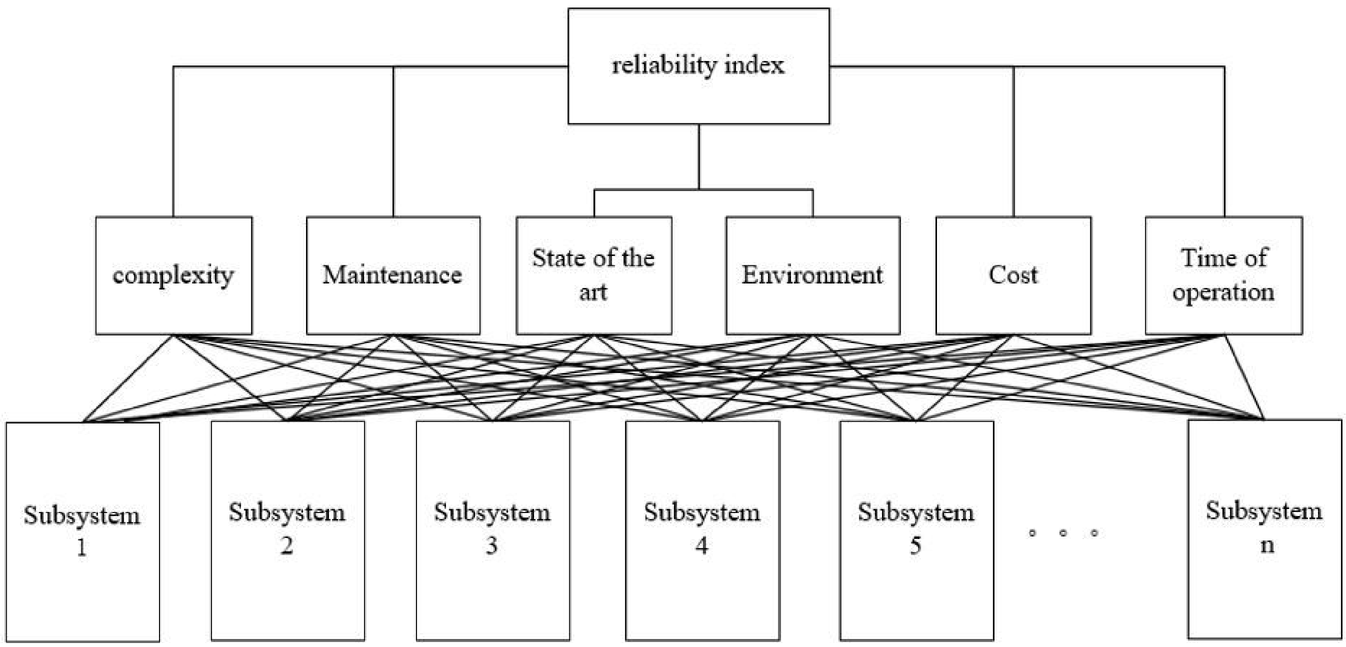

Figure 1

Hierarchical model of reliability allocation for CNC machine tools

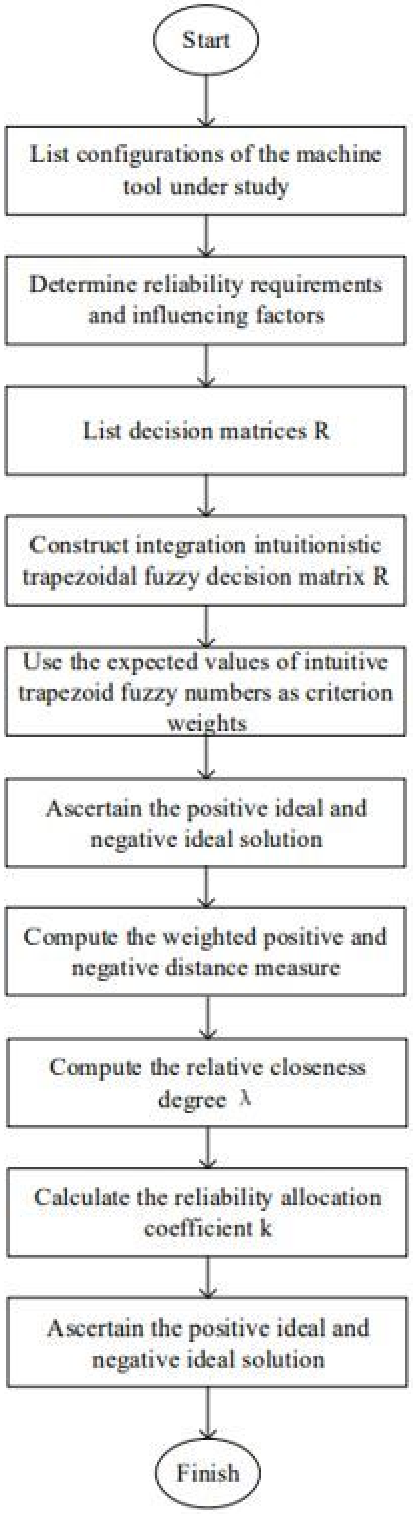

Figure 2

Flowchart of the fuzzy TOPSIS process

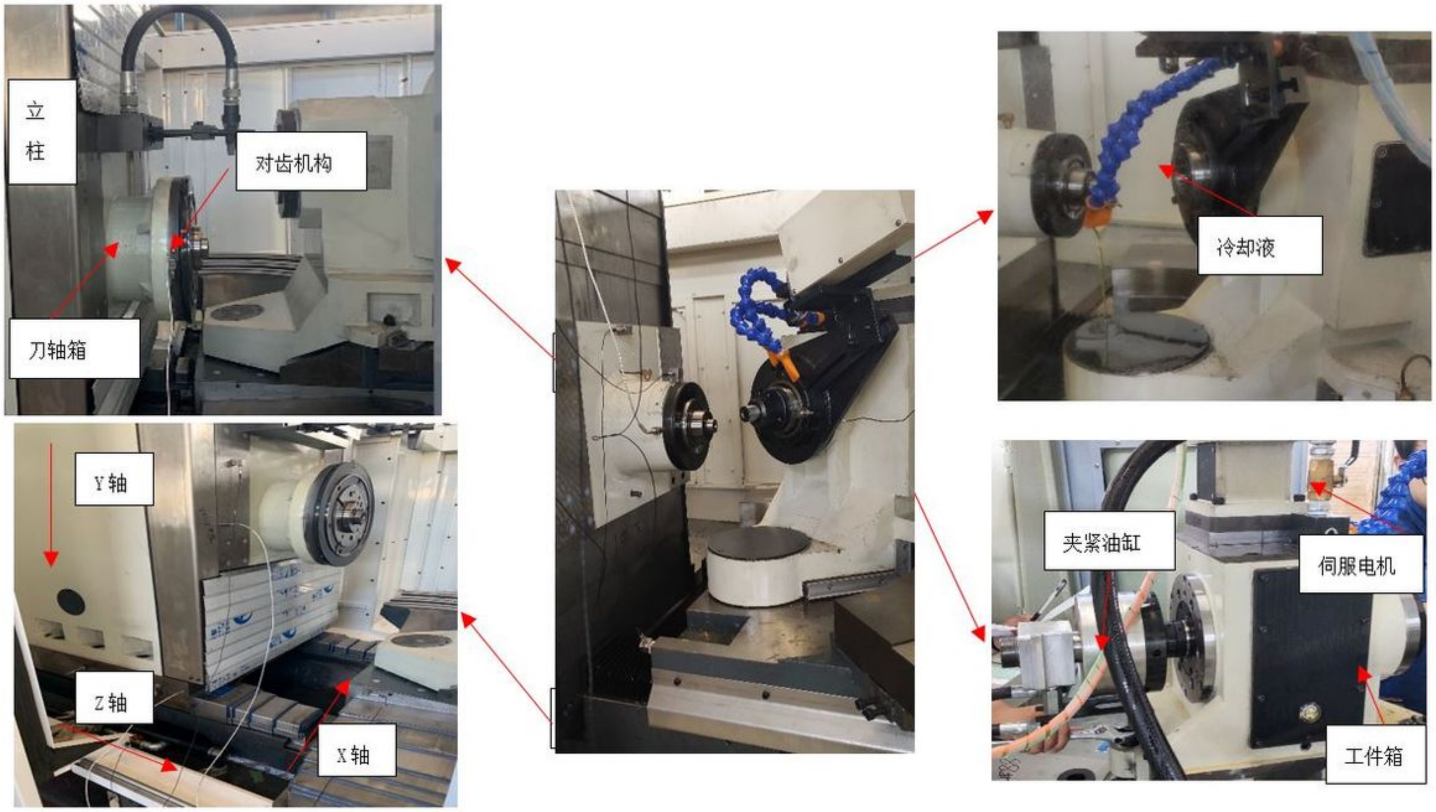

Figure 3

Subsystems of CNC machine tools

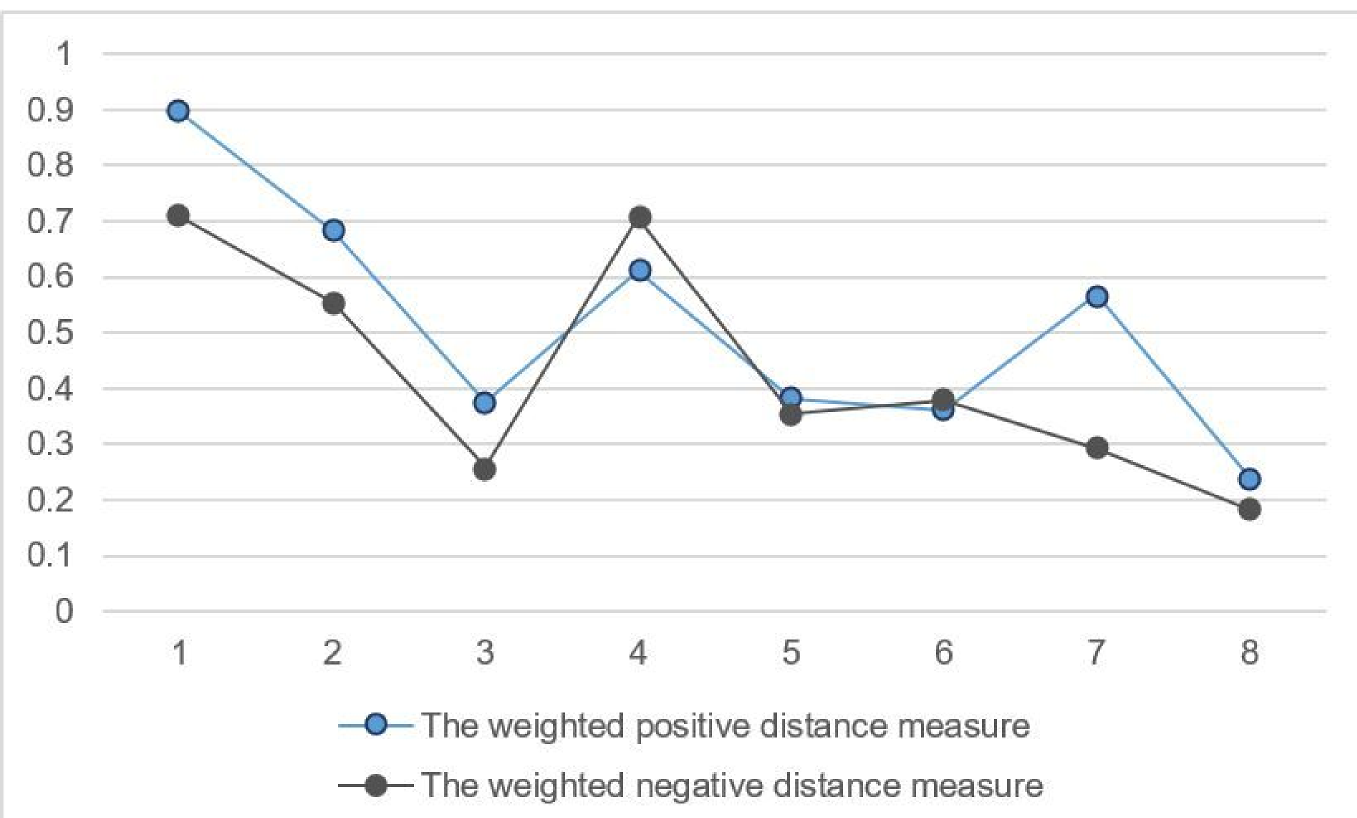

Figure 4

The weighted positive and negative distance measures

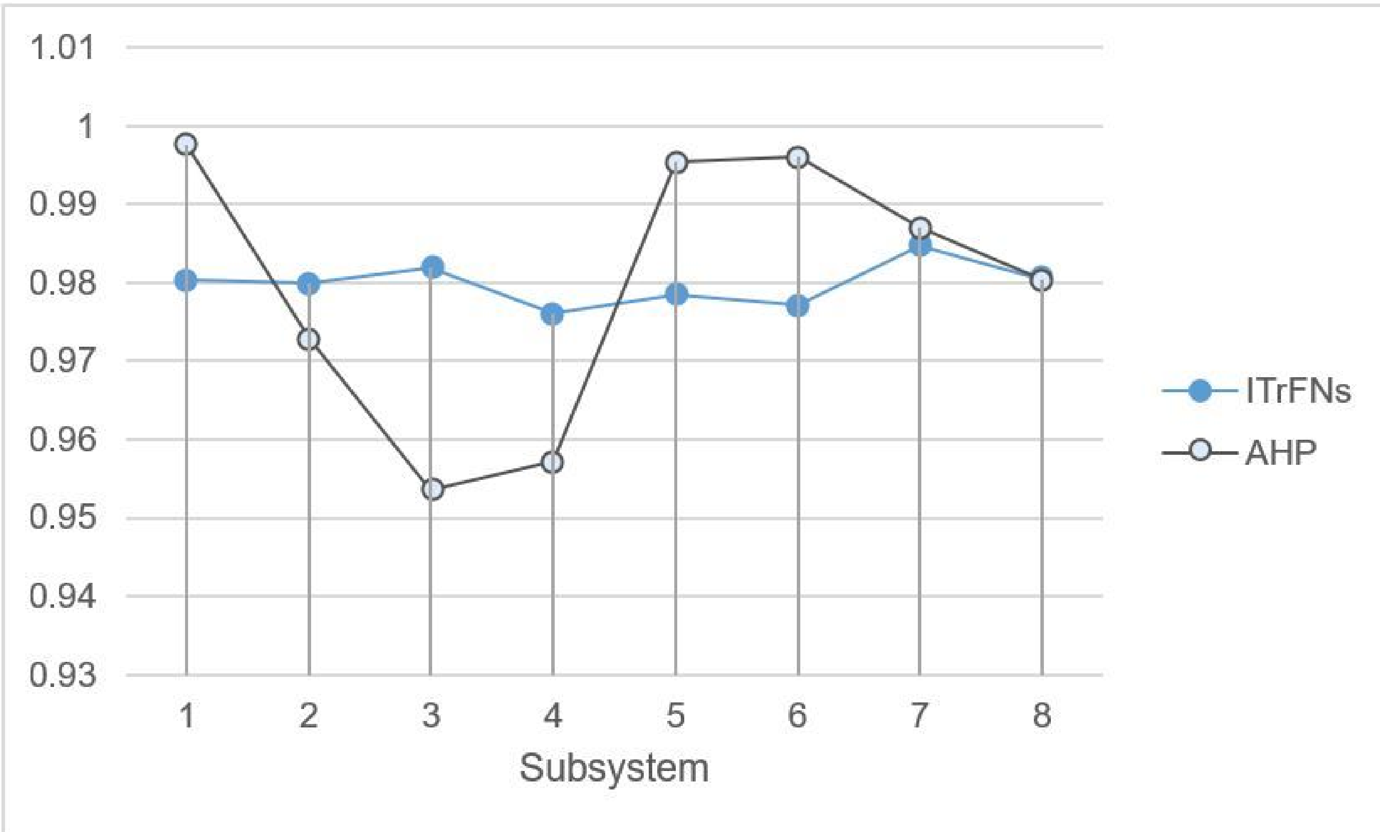

Figure 5

Comparison of reliability allocation between the two methods