Does e-commerce ease or intensify tax competition? Destination principle vs. origin principle

Hiroshi Aiura
Nanzan University

Hikaru Ogawa (✉ ogawa@e.u-tokyo.ac.jp)
University of Tokyo

Research Article

Keywords: Tax competition, product differentiation, E-commerce, origin principle, and destination principle

Posted Date: July 4th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3126658/v1

License: © This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Version of Record: A version of this preprint was published at International Tax and Public Finance on September 14th, 2023. See the published version at https://doi.org/10.1007/s10797-023-09796-8.
Does e-commerce ease or intensify tax competition?
Destination principle vs. origin principle

Hiroshi Aiura∗
Nanzan University

Hikaru Ogawa†
University of Tokyo

June 30, 2023

Abstract

This study examines the relationship between the development of e-commerce and the intensity of commodity tax competition under two different tax principles on goods purchased online: the destination principle and the origin principle. Our main findings are as follows: Given that origin-based tax is applied to purchases made in brick-and-mortar stores, (i) tax competition under destination-based taxation on e-commerce is more intense than tax competition under origin-based taxation; (ii) tax competition under origin-based tax on e-commerce generates higher tax revenues; and (iii) the development of e-commerce raises the tax rate and tax revenues under origin-based taxation but lowers them under destination-based taxation. The main factor leading to these results is that replacing the choice of “where to purchase” goods, consumers will have a new choice of “how to purchase” when online purchasing becomes available, and the destination-based taxation distorts the latter choice, while the origin-based taxation is neutral to it. Tax competition between asymmetric regions has also been analyzed, showing that both principles result in higher tax rates and greater tax revenues for large regions than for small ones.

Keywords: Tax competition; product differentiation; E-commerce, origin principle, and destination principle

JEL Classification Codes: H21; H71; H87

Statements and Declarations (Competing Interests). The authors have no conflicts of interest to declare that are relevant to the content of this article.
1 Introduction

The development of e-commerce and the associated change in the role of consumption tax have recently led researchers to study taxation on e-commerce (Bacache-Beauvallet 2018; Agrawal and Wildasin 2020). Underlying this is the policy concern that failing to design an adequate tax system for expanding e-commerce will create significant tax revenue losses, as e-commerce could trigger increased tax competition, as well as more channels for tax avoidance.\(^1\) Specifically, the ability to purchase goods and services via the Internet allows consumers to choose the region from which to purchase, not only from neighboring regions but also from more distant regions and, therefore, from a greater number of regions. This would accelerate the interregional commodity tax competition for cross-border consumption. Our study intends to present one view in this new field of research that search for an appropriate way to tax e-commerce transactions. Specifically, the purpose of our study is to provide one possible idea to the question of whether the imposition of taxes on e-commerce should be based on the origin principle or the destination principle. In the case of taxation based on the former, tax is levied on transactions of goods and services at the supplier’s location, and in the case of the latter, at the recipient of the goods or services. VAT in the EU has experienced both of these tax principles: it was crafted over two decades ago, when e-commerce was almost non-existent, with the aim of arriving at a definitive VAT system based on the origin principle, but has now shifted toward destination principle taxation (European Commission, 2016, p.11). A result derived in this study is that taxing e-commerce under the origin principle rather than the destination principle will be superior from the standpoint of revenue-maximizing government if, as we still observe in a number of cases, origin-based taxes are applied to purchases of goods in brick-and-mortar stores.

Many classic studies on cross-border shopping that do not address e-commerce have analyzed the choice between origin and destination principles. There is general consensus that the destination principle of taxation is superior to the origin principle of taxation when assuming a competitive market (Keen and Lahiri, 1998, p.325). In particular, economists have recognized the importance of enforcing a destination-based tax from the perspective of avoiding harmful competition to lower tax rates (Lockwood et al., 1994a, p.5; Agrawal and Fox, 2017, p.917). In the case of origin-based taxation, a country with a lower tax rate has an advantage in terms of the tax burden for mobile firms and consumers, which causes a race to lower taxes. However, in the case of destination-based taxation, even if tax rates differ among countries, the tax burden on them is the same regardless of where the production and consumption activities take place, so competition to lower taxes is avoided, and efficiency is not impaired.\(^2\) Subsequent studies have broadened the scope of the analysis to include imperfectly competitive markets and markets with factors such as trade costs, spillovers, and unemployment. Some of these studies have confirmed that the superiority of the destination principle still holds, whereas others have indicated the possibility of a counterview (Lockwood et al., 1995; Lockwood, 2001; Haufler and Pfüger, 2004; Haufler et al., 2005; Hashimzade et al., 2011; Antoniou et al., 2019, 2022; Agrawal and Mardan, 2019). Our study’s contribution is to present a model that includes e-commerce, which was not included in the analysis in any of the various types of models mentioned above, but such an analysis will inevitably be complicated by the addition of new purchase options to these models in which consumers only purchase goods in brick-and-mortar stores. To avoid complexity and present our main findings with analytical solutions, we focus on the symmetric equilibrium in the majority of the paper. The findings when asymmetry is included are presented in the Discussion section.

As mentioned above, two studies are closely related to our analysis of taxation principles applicable to e-commerce. The first is Bacache-Beauvallet (2018), who developed a model of taxation on e-commerce in which firms are competitive and a certain percentage of consumers use the Internet to buy one unit of goods, while the remainder buy goods in brick-and-mortar stores. She compared tax revenue under the origin and destination principles to show that large regions always prefer the destination principle for e-commerce, whereas small regions prefer the origin principle. The second study is Agrawal and Wildasin’s

\(^1\) There have been numerous estimates of the magnitude of tax revenue lost due to the growth of e-commerce, which poses a new policy challenge to the government. In one of the earliest studies, Bruce and Fox (2000) and Bruce et al. (2009) found that as of 1999, the U.S. had lost $7 billion in annual state and local tax revenue due to e-commerce, and by 2012, that amount is estimated to increase to about $11.4 billion to $12.7 billion. Beem and Bruce (2021) show that an increase in online firms could affect (and slightly increases) tax revenues by changing the number of firms with sales tax liability.

\(^2\) See also Lockwood et al. (1994a, 1994b) and Genser (1996) for studies that identify the conditions under which the two tax principles could be equivalent. They point to the importance of adjustments in wages and exchange rates, and show that the two tax principles are equivalent when wages and exchange rates are freely adjusted; a shift from the destination principle to the origin principle could cause real prices to adjust so that changes in wages and exchange rates offset the effects of the change in tax principle.
(2020), which extends commodity tax competition models to include e-commerce, assuming that it is taxed at the destination, whereas purchases of goods in brick-and-mortar stores are subject to tax based on the origin principle. They show that as the cost of e-commerce declines, tax rates decrease in the core region where essential goods that can be purchased online are produced but increase in peripheral ones, thus reducing tax differentials. On the one hand, these studies share the same objective as our study, which is to elucidate equilibrium tax rates for this new means of conducting transactions through the Internet. However, there are two important differences between these studies and ours, which allow us to present different theoretical results. First, we relax Bacache-Beauvallet’s (2018) assumption that the number of consumers purchasing goods over the Internet is always constant. Instead, we develop a model in which the number of consumers participating in e-commerce differs if the tax rates differ between the two principles. Given the current situation in which the number of e-commerce users is increasing, it would be appropriate to adopt a model in which the proportion of consumers who purchase goods online changes, leading to our results. Second, we extend Agrawal and Wildasin’s (2020) model, which analyzes the case of a destination-based tax on e-commerce, to allow the government to impose an origin-based tax on purchases of goods and services via the Internet. By adding a technically feasible origin-based tax to the analysis, it is possible to determine which of the two tax principles is superior from the government’s perspective. Furthermore, what is common to Bacache-Beauvallet (2018) and Agrawal and Wildasin (2020) is that they both impose the constraint that consumers purchase one unit of a good inelastically, regardless of how the tax rate changes. While this might be an acceptable assumption for certain goods, it is difficult to apply to many other goods and services in online retail markets.

We generalize the demand structure using a model in which demand is elastic in response to changes in prices and tax rates, which gives consumers the choice of “how much to buy” goods in addition to “how to buy” them. By considering these elements under the assumption, common to prior studies, that origin-based taxes are imposed on purchases of goods in brick-and-mortar stores, we reach the following results. First, the expansion of the online market, expressed in lower online purchase costs, increases the equilibrium tax rates and tax revenues under origin-based tax competition and decreases them under destination-based tax competition. This suggests that the expansion of the online market intensifies destination-based tax competition while easing origin-based tax competition. Second, for any given online purchase cost, the tax rates and tax revenues under origin-based tax competition are higher than those under destination-based tax competition. In other words, tax competition is less intense if an origin-based tax is applied to goods purchased online than if a destination-based tax is applied. These results are derived under a model in which consumers purchase two different goods and have elastic demands, which is a more generalized version of the two studies mentioned above. In our model, with each region having a different history, culture, climate, resources, etc., competitive firms operating in region $x$ produce good $x$, while competitive firms in the other region $y$ produce good $y$, which is differentiated from good $x$. Since the goods produced in the two regions are differentiated, consumers in region $x$ with a preference for variety will buy not only the good $x$ produced in their own region but also good $y$ produced in region $y$. In this case, consumers have no choice of where to purchase the goods from. Instead, consumers in region $x$, for example, will have a new choice of how to buy it, that is, whether to travel to a brick-and-mortar store in region $y$ or buy it from stores in region $y$ online. Excluding the choice of where to purchase is a device in the model to extract the effect of a new choice of how to purchase for the consumers we focus on.

Our finding that origin-based tax is superior to destination-based tax in terms of loosening tax competition may be somewhat unexpected. However, given that we included e-commerce, a new way of making purchases, the mechanism leading to this result is simple. Destination-based taxation is considered superior because it is neutral to the consumer’s choice of “where to buy” goods. However, when a new means of purchasing goods online is added, in addition to purchasing goods in brick-and-mortar stores...
stores, consumers are given the choice of “how to buy” goods. Because the origin-based tax is applied to purchases made in brick-and-mortar stores, when consumers in region \( x \) purchase goods \( y \) in stores in region \( y \), the tax they pay is paid to region \( y \) and not received by the government in region \( x \). However, if they purchase the same goods online from region \( y \), the government in region \( x \) will receive the tax they pay if destination-based tax is applied to online purchases. This means that governments have the incentive to shift consumers residing in their region from purchasing goods at brick-and-mortar stores to online purchases when destination-based taxes are imposed on online purchases, thus causing competition among governments for a tax base and creating fiscal externalities. If the origin principle of taxation is applied to online purchases in the same way as it is applied to purchases of goods in brick-and-mortar stores, the government loses the incentive to change the way consumers buy, and the tax becomes neutral to the consumer’s choice of “how to buy”.

As Agrawal and Fox (2017, 2021) have noted, there is an international trend toward destination-based taxation of goods purchased online. This trend can also be seen in the legal changes in the taxation rules for e-commerce across states in the United States. In 1992, the U.S. Supreme Court’s decision in Quill Corp. v. North Dakota prohibited state governments from collecting taxes from retail purchases made via the Internet or other e-commerce channels unless the seller had physical establishments in the state. This caused the inability of state governments to proper tax consumption in the state. However, the recent case of South Dakota v. Wayfair Inc. in 2018 overturned the decision of the Quill on the grounds that it is “unsound and incorrect” in the current age of Internet services, providing a legal basis for taxation under the destination principle.\(^6\) This is an example of e-commerce taxation moving in a direction based on the destination principle. However, it would not be appropriate to leave the applicability of origin-based taxation to e-commerce out of the analysis altogether, as origin-based taxation is less costly from a practical point of view (OECD, 2001, p.16). For this reason, some emerging economies are exploring the implementation of origin-based taxation of e-commerce.\(^7\) Examples of the origin principle taxation of e-commerce transactions in developed countries are also given by Bacache-Beauvallet (2018, p.101): since 2015, the tax rate of the country in which the consumer resides has been applied to e-commerce services in Europe, but the application of such a destination principle tax involves accounting checks and controls compliance costs. Therefore, only sellers with sales above a certain level are subject to destination tax, while the origin principle tax is applied to electronic transaction services sold by many other small businesses. Such cases demonstrate that both taxation principles can be applied to e-commerce transactions, and thus, the question can be set as to which of them to apply.

The remainder of this paper is organized as follows. Section 2 introduces the model setting. In Sections 3 and 4, we derive the symmetric equilibrium of tax competition when taxes are imposed under the origin and destination principles, respectively. By comparing the results of these two sections, we show the superiority of origin-based taxation in e-commerce. In Section 5, we present the additional results obtained under the assumption of two regions with different population sizes. We also discuss the equilibrium when consumers have no choice in how they purchase, based on the special case in which they purchase exclusively online. Finally, Section 6 presents the conclusions of the study.

2 Model

2.1 Basic setup

We assume a model analogous to Hotelling (1929)’s line economy with two regions named region 0 and region 1 (Kanbur and Koen, 1993; Nielsen, 2001). Consumers and competitive firms are uniformly distributed in the line economy. The populations of the two regions are denoted by \( N_0 \) and \( N_1 \), respectively. We assume \( N_0 = 1 + b \) and \( N_1 = 1 - b \) where \( b \in [0, 1) \) is a constant. Competitive firms exist in each region, and firms operating in region 0(1) produce good 0(1). Each firm has a brick-and-mortar store that sells goods to consumers. It also ships goods to consumers who order over the Internet without

---

\(^6\)Fox et al. (2022) show that the 2018 ruling increased sales tax revenues by 7.9% at the state level data.

\(^7\)In 2014, there was a case in Brazil where the Supreme Court ruled on the application of the origin principle taxation by the government on the Brazilian value-added tax called ICMS (Imposto sobre Circulação de Mercadorias e Serviços de Transporte Intermunicipal, Interestadual e de Comunicação) on interstate electronic transactions. Gutuza (2010) studies how the source, or origin, principle can be applied to e-commerce firms operated through the use of a server in which physical presence is not required and argues that origin-based taxation still remains an important consideration even though South Africa is moving to a residence basis of taxation.
coming to a store. Although one region can produce two goods, we consider a situation in which one region specializes in the production of a particular good to present a clear result. This setting allows the analysis to focus on the consumer’s choice of how to buy because it excludes the consumer’s choice of where to buy a good from. Although the model is set up to simplify the analysis, this situation applies to the consumption of regional specialties such as wine, which has different characteristics depending on where it is produced. Therefore, we assume that the goods produced in each region are horizontally differentiated (not vertically differentiated), and consumers buy both.

All competitive firms operating in the same region are homogeneous, and their profit in equilibrium will be zero.\(^8\) When consumers purchase two goods, they can choose to purchase each good either by visiting a brick-and-mortar store or purchasing it online, along with the quantity purchased. When consumers purchase good \(i\), they face the same price as \(p_i\) regardless of how they purchase it. The price of the good supplied by competitive firms in region \(i\), \(p_i\), is at a level equal to its marginal cost, \(c_i\), in equilibrium. While prices may differ depending on the method of purchase, Cavallo (2017)’s survey of ten countries shows that for 72% of goods (69% in the U.S.), the price of goods is the same for online and in-store purchases. In particular, clothing and electronics are more likely to have the same price regardless of the method of purchase. In our analysis, we assume goods for which there is no difference in price by method of purchase, or if there is, the price gap remains reasonably small.

Following the assumptions of Dixit and Stiglitz (1977), the goods are differentiated, but the marginal cost of producing each good does not differ across firms since the production technology is the same for all goods and the inputs needed to produce them are supplied by the same factor market. In the following symmetric equilibrium analysis, we assume that marginal costs are the same in the two regions, \(c_i = c\). The government in each region imposes a unit tax on goods, denoted by \(t_i\). Although an ad valorem (ADV) tax could be used, as in Haufler and Pfüger (2004), a unit tax is used here because ADV and unit taxes are equivalent when assuming the supply of goods by perfectly competitive firms, as in this study (Suits and Musgrave, 1953).

We follow Bacache-Beauvallet’s (2018) setup and analyze the situation in which the origin principle tax is applied when goods are purchased in brick-and-mortar stores, whereas either the origin or destination principle taxes are applied when goods are purchased online. The application of origin-based taxes to purchases of goods in brick-and-mortar stores has been assumed in the literature, e.g., Kanbur and Keen (1993), Nielsen (2001), and Agrawal and Wildasin (2020), as governments typically have no feasible methods of monitoring consumer purchases of goods across cities or states and taxing consumption in stores outside their jurisdictions unless there is a tax adjustment mechanism at the regional boundary. The purchase of goods via the Internet, however, allows firms to directly (and indirectly for the government) identify the place of consumption and the place of purchase, which raises the issue of choice of tax principles for e-commerce.

Let \(s_{ij}\) denote the cost, other than the price of the good, that a consumer in region \(i\) incurs when purchasing goods produced in region \(j\). The size of \(s_{ij}\) depends on whether the consumer purchases goods in a brick-and-mortar store or online, as explained below.

**Purchasing goods in brick-and-mortar stores.** When residents of region \(i\) purchase goods produced in region \(i\) (their own region) from stores, the cost to purchase per unit of goods is \(p_i + s_i\), where \(s_i = t_i\). Because goods produced in region \(i\) are sold everywhere in region \(i\), consumers can purchase goods without traveling. Consumers in region \(i\) also purchase good \(j(\neq i)\) produced in region \(j\). When they go to a neighboring region to buy goods \(j\) from brick-and-mortar stores, the per-unit purchase cost is \(p_j + s_{ij}\), where \(s_{ij} = t_j + \delta x_{ij}\), \(\delta\) and \(x_{ij}\) are the travel expense per distance and the travel distance from region \(i\) to region \(j\). We assume that \(x_{ij}\) is different depending on where consumers reside and are uniformly distributed in the interval \([0, N]\). Because the in-store purchase of goods is taxed based on the origin principle, the consumer pays the tax imposed by the region where the goods are supplied. We assume that the goods cannot be resold to exclude from analysis the situation in which a consumer in region \(i\) living near the border buys goods in region \(j\) and resells them to a consumer in region \(i\) far from the regional boundary.

**Purchasing goods online.** When goods are purchased online, governments impose a tax on goods based on

\(^8\)While we have included the behavior of oligopolistic firms in our discussion paper, that is, Aiura and Ogawa (2021), we develop a model here that discards firm behavior because similar results can be presented in a simpler way. Thus, we analyze situations where goods are purchased through the Internet, either by ordering from competitive stores or through mail orders, rather than situations where goods are purchased from a monopolistic multinational digital company such as Amazon.
either the origin or the destination principle. First, if a consumer in region \(i\) purchases a good in region \(j(\neq i)\) online, the per-unit cost of purchasing the good is \(p_j + s_{ij}\), where \(s_{ij} = t_i + e\) if it is taxed under the origin principle. Here, \(e(>0)\) is the common cost in any region associated with purchasing the good online. When purchasing goods online, the cost of traveling to a store is not a factor. Instead, consumers bear certain costs for shipping and costs associated with the risk of purchasing goods that are different from those they had imagined because they do not see the actual goods in stores before purchasing them online. These are represented by the cost \(e\), which is independent of the place of residence.\(^9\) A situation where goods could not be purchased through e-commerce, as was the case a few decades ago, corresponds to a sufficiently large value of \(e\). Second, when goods are taxed under the destination principle, regardless of where the goods are supplied, the tax rate in the place of residence applies, so consumers in region \(i\) bear the cost of \(p_j + s_{ij}\), where \(s_{ij} = t_i + e\). Finally, the cost of online purchase of a domestically produced good by a consumer in region \(i\) is \(p_i + s_{ii}\), where \(s_{ii} = t_i + e\), regardless of the taxation principle.

**Selecting a purchase method.** Consumers purchase goods either in brick-and-mortar stores or online, whichever has a lower purchase cost. Regardless of taxation principles, consumers residing in region \(i\) always buy good \(i\) in brick-and-mortar stores in region \(i\), not online, and pay taxes to region \(i\); thus, \(s_{ii} = t_i\). This is because consumers in region \(i\) pay the tax of \(t_i\) regardless of the method of purchase, and the cost of buying online, \(p_i + t_i + e\), is higher than the cost of buying in a brick-and-mortar store, \(p_i + t_i\). Contrarily, when consumers residing in region \(i\) buy good \(j(\neq i)\), there are two ways to purchase it: first, they go to region \(j\) to buy it at the brick-and-mortar store. The second option is to buy the product online. In the former case, the cost incurred is \(p_j + t_j + \delta x_{ij}\). The second term is the origin-based tax, and the third term is the travel expense to go to region \(j\). In the latter case, that is, online purchases, the cost incurred depends on the taxation principle applicable to the good he buys online. If the origin principle is applied, the cost incurred by the user is \(p_j + t_j + e\). If the destination principle is applied, the cost incurred will be \(p_j + t_j + e\). The applied tax rate depends on the tax principle. The travel distance from the place of residence to region \(j\), \(x_{ij}\), is different for consumers, and two purchase methods are indifferent in distance at \(x^{O}_{ij}\) satisfying

\[
p_j + t_j + \delta x^{O}_{ij} = p_j + t_j + e \quad \Rightarrow \quad x^{O}_{ij} = \frac{e}{\delta},
\]

if the origin principle of taxation applies to e-commerce, where \(O\) in the superscript represents the value when taxed, according to the origin principle. A consumer with \(x_{ij}\) satisfying \(x_{ij} > x^{O}_{ij}\) purchases good \(j\) online. Conversely, a consumer with \(x_{ij}\) satisfying \(x_{ij} < x^{O}_{ij}\) travels to region \(j\) and purchases good \(j\) at the brick-and-mortar stores. Hence, we obtain

\[
s^{O}_{ij}(x_{ij}) = \begin{cases} \ t_j + \delta x_{ij} & \text{if } 0 \leq x_{ij} \leq x^{O}_{ij} \\ t_j + e & \text{if } x^{O}_{ij} < x_{ij} \end{cases},
\]

indicating that costs other than prices vary according to the distance from the border. If the destination principle of taxation is applied, \(x^{D}_{ij}\) where the choice between the two purchase methods is indifferent, satisfies

\[
p_j + t_j + \delta x^{D}_{ij} = p_j + t_i + e \quad \Rightarrow \quad x^{D}_{ij} = \frac{e}{\delta} + \frac{t_i - t_j}{\delta}.
\]

As in the previous case, a consumer with \(x_{ij}\) satisfying \(x_{ij} > x^{D}_{ij}\) purchases good \(j\) online, while a consumer with \(x_{ij}\) satisfying \(x_{ij} < x^{D}_{ij}\) purchases it in a brick-and-mortar store. In this case, costs other than price are

\[
s^{D}_{ij}(x_{ij}) = \begin{cases} \ t_j + \delta x_{ij} & \text{if } 0 \leq x_{ij} \leq x^{D}_{ij} \\ t_i + e & \text{if } x^{D}_{ij} < x_{ij} \end{cases}.
\]

What makes (4) different from (2) is that different taxes apply when buying goods online. Assuming \(t_i > t_j\), Figure 1 depicts \(s_{ij}\) for each of the two taxation principles. In both Figures 1(a) and 1(b), consumers residing in region \(i\) who compare the costs of online and in-store purchases are more likely to purchase goods \(j\) in stores if the distance from their residence to the border, \(x_{ij}\), is shorter, and conversely.

\(^9\)When goods are purchased online, it takes a certain amount of time for the goods to reach the consumer via delivery. With this view, Miyatake et al. (2016) estimate the cost of purchasing goods online, or \(e\) in our model, to be \$7.5 per item.
more likely to purchase online if the distance is farther. The two figures look similar, but there is a key difference that arises under the two taxation principles: the critical point that divides online and offline purchases is affected by tax rates when taxed under the destination principle (Figure 1(b) and Eq. 3), but not when taxed under the origin principle (Figure 1(a) and Eq. 1). Under origin-based tax, consumers pay taxes to the region where the goods are supplied, regardless of the purchase method, so the distance for determining the purchase method is not affected by the tax rate. In contrast, under destination-based tax, the tax rate will affect consumers’ choice of purchase method, since the tax rate will be different for the same good when purchased online versus in a brick-and-mortar store.

Based on the classification according to the two taxation principles shown in Figures 1(a) and 1(b), it can be intuitively obvious that under the destination principle of taxation, there is an incentive for the government to manipulate tax rates because $\hat{x}^D_{ij}$ depends on the tax rates; thus, consumers in the home region purchase goods from other regions online. There is no such incentive when taxing under the origin principle of taxation, since changing the tax rate will not influence consumers’ choice of “how to buy”.

2.2 Consumers

Consumers in region $i$ have a utility function given by:

$$U_i(q_{ii}, q_{ij}, X_i) = \alpha - \gamma \frac{1}{2(1+\gamma)}(q_{ii}^2 + q_{ij}^2) - \frac{\gamma}{2(1-\gamma^2)}(q_{ii} + q_{ij})^2 + X_i,$$

where $q_{ii}$ and $q_{ij}$ are the individual consumption of the consumer living in region $i$ for differentiated goods produced in regions $i$ and $j$, and $X_i$ is the individual consumption of a numeraire good in region $i$. $\alpha$ and $\gamma$ are parameters, and the former is assumed to be sufficiently large to ensure $q_{ii} > 0$ and $q_{ij} > 0$, as described below. $\gamma$ represents the substitutability between the two goods and is assumed to take values between zero and one to ensure that the utility function is concave. Two goods are independent if $\gamma \to 0$ and perfect substitutes if $\gamma \to 1$. This quasi-linear utility function has been widely used because it provides a linear demand function for differentiated products in its simplest form.\(^{10}\) If the price of each good is $p_i$ and the income is $y_i$, the budget constraint faced by a consumer with $x_{ij}$ in region $i$ is given by:

$$y_i = [p_i + s_{ii}]q_{ii} + [p_j + s_{ij}(x_{ij})]q_{ij} + X_i.$$

\(^{10}\)The assumption for a linear demand function is particularly essential to solving the tax competition model analytically, but it does not significantly alter the results if the analysis is restricted to the neighborhood of equilibrium (Devereux et al., 2008). For the microfoundations of (5), see Amir et al. (2017), which presents the historical background on this useful function, dating back to Bowley (1924).
The consumer chooses \( q_{it} \) and \( q_{tj} \) that maximizes (5), with (6) as the constraint. The individual demand functions of consumers residing in region \( i \) for goods \( i \) and \( j \) are as follows.

\[
q_{it}(x_{ij}) = \alpha - [p_t + s_{it}] + \gamma[p_j + s_{ij}(x_{ij})], \tag{7}
\]

\[
q_{tj}(x_{ij}) = \alpha - [p_j + s_{ij}(x_{ij})] + \gamma[p_t + s_{it}], \tag{8}
\]

### 2.2.1 Individual demand function under the origin principle tax

Individual demand under the origin principle tax can be explicitly obtained by substituting \( s_{it} = t_i \) and (2) into (7) and (8). All consumers in region \( i \) purchase good \( i \) in the brick-and-mortar store, while some purchase good \( j \) in stores in region \( j \) and the rest purchase it online. Let Case A (Non-online) be the pattern in which consumers buy both goods in brick-and-mortar stores, and Case B (partially online) be the pattern in which consumers in region \( i \) buy good \( i \) in their home region in stores but buy good \( j \) online. The demand function of the consumer with \( x_{ij} \) in region \( i \) purchasing goods \( i \) and \( j \) is shown for each case as follows:

**Case A (Non-online):** For consumers with \( x_{ij} \) satisfying \( 0 \leq x_{ij} \leq \hat{x}_{ij} \),

\[
q_{it}^O(x_{ij}) = q_{it}^{OA} = \alpha - (p_t + t_i) + \gamma(p_j + t_j + \delta x_{ij}), \tag{9}
\]

\[
q_{tj}^O(x_{ij}) = q_{tj}^{OA} = \alpha - (p_j + t_j + \delta x_{ij}) + \gamma(p_t + t_i). \tag{10}
\]

**Case B (Partially online):** For consumers with \( x_{ij} \) satisfying \( x_{ij} \leq \hat{x}_{ij} \),

\[
q_{it}^O(x_{ij}) = q_{it}^{OB} = \alpha - (p_t + t_i) + \gamma(p_j + t_j + e), \tag{11}
\]

\[
q_{tj}^O(x_{ij}) = q_{tj}^{OB} = \alpha - (p_j + t_j + e) + \gamma(p_t + t_i). \tag{12}
\]

In (9)-(12), \( q_{it}^O(x_{ij}) \) and \( q_{tj}^O(x_{ij}) \) are, respectively, the demand functions for goods \( i \) and \( j \) of consumers with \( x_{ij} \) living in region \( i \) when the origin principle tax is applied. Case A represents the demand function for consumers living near the border; that is, consumers with \( x_{ij} \) satisfying \( 0 \leq x_{ij} \leq \hat{x}_{ij}^O \). They purchase goods \( i \) and \( j \) in a brick-and-mortar store. Case B is the demand function for consumers living far from the border, that is, with \( x_{ij} \) satisfying \( \hat{x}_{ij}^O \leq x_{ij} \). This consumer purchases good \( i \) in a brick-and-mortar store and good \( j \) online.

Figure 2(a), with the consumption of goods \( q^O \) under the origin principle on the vertical axis, visually illustrates (9)-(12). Consider a consumer who lives at point \( N_0 \). This consumer purchases \( q_{00}^{OB} \) for good 0 at a brick-and-mortar store and \( q_{01}^{OB} \) for good 1 online. Since consumers in region \( i \) residing away from the border purchase good \( j \) online, their costs are independent of the distance to the border. Hence, \( q_{0i}^{OB} \); hence, \( q_{00}^{OB} \) is constant, regardless of where the consumer resides. Consumers in region 0 whose distance to the border is shorter than \( e/\delta \) purchase \( q_{00}^{OA} \) of good 0 and \( q_{01}^{OA} \) of good 1 at brick and mortar stores. The shorter the distance to the border, the lower the travel cost required to buy good 1; thus, the consumer purchases more of good 1. Correspondingly, this consumer’s purchase of good 0 is lower because good 0 is substituted for good 1. Next, we consider a consumer in region 1 who lives at point \( N_1 \). This consumer purchases \( q_{10}^{OB} \) of good 0 online and \( q_{11}^{OB} \) of good 1 at a brick-and-mortar store. Furthermore, consumers in region 1 whose distance to the border is shorter than \( e/\delta \) purchase \( q_{10}^{OA} \) of good 0 and \( q_{11}^{OA} \) of good 1 at the stores. Again, consumers who travel shorter distances purchase more of good 0 but consume less of good 1.

### 2.2.2 Individual demand function under the destination principle tax

Similarly, consumers’ individual demand under the destination principle tax can be obtained by substituting \( s_{it} = t_i \) and (4) into (7) and (8). Case A is the demand function for a consumer in region \( i \) who purchases both goods in the brick-and-mortar store, and Case B is the demand function for a consumer who purchases goods \( i \) in a store and goods \( j \) online.

**Case A (Non-online):** For consumers with \( x_{ij} \) satisfying \( 0 \leq x_{ij} \leq \hat{x}_{ij}^D \),

\[
q_{it}^D(x_{ij}) = q_{it}^{DA} = q_{it}^{OA}, \tag{13}
\]

\[
q_{tj}^D(x_{ij}) = q_{tj}^{DA} = q_{tj}^{OA}. \tag{14}
\]

8
As follows:

The second (fourth) term is the aggregate consumption of good \( i \) in 
brick-and-mortar stores, whereas consumers away from the border 
purchase goods \( i \) in the store and \( j \) online. Individual consumption under destination-based tax is shown 
in Figure 2(b).

The difference between the demand functions in (15) and (16) from those presented in (11) and (12) lies in the tax rate applied when goods are purchased online; in the former, consumers in region \( i \) pay \( t_i \) when they purchase goods \( j \) online, whereas, in the latter, they pay \( t_j \). This leads to an important difference between Figures 2(a) and 2(b). In Figure 2(a), the point at which the purchase method is indifferent is uniquely determined at \( e/\delta \) independent of the tax rate, while in Figure 2(b), it depends on the difference in tax rates between the two regions.

**2.3 Governments**

Following a series of studies since Kanbur and Keen (1993), we assume that each government aims to 
maximize its tax revenue, given that the origin-based tax applies to purchases made in brick-and-mortar 
stores. When origin-based tax is also applied to goods purchased online, the tax revenues in region \( i \) are 
as follows:

\[
R_i^O = t_i^O Q_i^O, \quad 
Q_i^O \equiv \int_0^{\min[p_{ij}, N_i]} q_{ii}^{O_A} dx_{ij} + \int_{\min[p_{ij}, N_i]}^{N_i} q_{ii}^{O_B} dx_{ij} + \int_0^{\min[p_{ij}, N_i]} q_{ij}^{O_A} dx_{ji} + \int_{\min[p_{ij}, N_i]}^{N_i} q_{ij}^{O_B} dx_{ji},
\]

which is equal to the tax rate times the tax base (denoted by \( Q_i^O \)). The first (third) term in (17) is the 
aggregate consumption of good \( i \) by consumers in region \( i \) (\( j \)) who purchase all goods in brick-and-mortar 
stores. The second (fourth) term is the aggregate consumption of good \( i \) by consumers in region \( i \) (\( j \)) who 
purchase good \( i \) (\( j \)) in stores and good \( j \) (\( i \)) online. In Figure 2(a), for example, the gray area corresponds 
to the tax base of region 0 under the origin principle of taxation.

If the origin-based tax is applied to purchases of goods in brick-and-mortar stores but the destination-
based tax is applied to e-commerce, the tax revenue in region \( i \) is expressed as
\[ R_i^D = t_i^D Q_i^D, \]  
\[ Q_i^D = \int_{0}^{\min[x^D_{ij}, N_i]} q_{ii}^{DA} dx_{ij} + \int_{0}^{N_i} q_{ij}^{DB} dx_{ij} + \int_{0}^{\min[x^D_{ij}, N_i]} q_{ij}^{DA} dx_{ij} + \int_{0}^{N_i} q_{ij}^{DB} dx_{ij}. \]  

\( Q_i^D \) denotes the tax base under destination principle of taxation. The first (third) term is the aggregate consumption of good \( i \) (\( j \)) by consumers who purchase all goods in the stores, and the second and fourth terms are the aggregate consumption of good \( i \) by consumers who buy goods from their own region in brick-and-mortar stores while buying goods from other countries online. In Figure 2(b), for example, the gray area corresponds to the tax base of region 0 under the destination principle.\(^{11}\) The key difference between \( Q_i^D \) and \( Q_i^O \) is that the subscript of \( q \) is \( ij \) in the fourth term of (17), but is replaced by \( ij \) in (18), and the tax in region \( i \) is applied under the origin (destination) principle tax on the consumption of good \( i \) (\( j \)) purchased online by consumers living in region \( j \) (\( i \)), that is, \( q_{ij} (q_{ji}) \).

### 2.4 Equilibrium without online purchases

First, we show the equilibrium of the symmetric model, that is, \( N_0 = N_i = 1 \), with no online purchases.\(^{12}\) For this, we assume a situation in which the online purchase cost \( e \) is sufficiently high such that consumers do not have the option to purchase goods online. By comparing the equilibrium obtained here with that of the model that allows online purchases presented in Section 3, we can observe the impact of the emergence of e-commerce.

When the online purchase cost \( e \) is sufficiently high, \( \hat{x}^O_{ij} \) and \( \hat{x}^O_i \) are larger than \( N_i (= 1) \), and (17) and (18) can be expressed simply as follows:

\[ R_i^N = t_i^N Q_i^N, \]  
\[ Q_i^N = \int_{0}^{1} q_{ii}^N dx_{ij} + \int_{0}^{1} q_{ij}^N dx_{ji} = \int_{0}^{1} q_{ii}^{OA} dx_{ij} + \int_{0}^{1} q_{ij}^{OA} dx_{ji}, \]  

where the superscript \( N \) indicates that it is a variable in a model in which there is no e-commerce. As all consumers purchase both goods at the brick-and-mortar store, \( q_{ii}^N = q_{ii}^{OA} \) and \( q_{ij}^N = q_{ij}^{OA} \) hold. The first-order condition for maximizing \( R_i^N \) with respect to \( t_i^N \) is:

\[ \frac{\partial R_i^N}{\partial t_i^N} = Q_i^N + t_i^N \frac{\partial Q_i^N}{\partial t_i}, \]  

which includes the two conflicting effects of increasing tax rates on tax revenue. The first term in (20) represents an increase in tax revenue per tax base and the second term represents a decrease in tax revenue due to a contraction of the tax base caused by a decrease in demand. Substituting \( p_i = p_j = c \), \( N_i = N_j = 1 \), (9) and (10) into \( Q_i^N \) in (19), we obtain:

\[ Q_i^N = 2 \left\{ A - \left[ t_i^N + \frac{\delta}{4} \right] + \gamma \left[ t_j^N + \frac{\delta}{4} \right] \right\}, \]

where \( A \equiv \alpha - c(1 - \gamma) \) and \( \frac{\partial Q_i^N}{\partial t_i} = -2 \). Using these equations, (20) can be rewritten as

\[ \frac{\partial R_i^N}{\partial t_i^N} = 2 \left[ A - t_i^N + \gamma t_j^N - \left( \frac{1 - \gamma}{4} \delta \right) \right] - 2t_i^N = 0. \]  

(21) shows that the tax response curve has a positive slope and that the slope varies with the degree of substitutability (\( \gamma \)) of the two goods. If \( \gamma = 0 \), there is no tax competition for consumption as a tax base among the regions.

Using (21) for \( i \) and \( j \), we can solve for the tax rate and tax revenues of the Nash equilibrium (NE) when consumers do not have a channel for online purchases, as follows:

---

\(^{11}\)The area in darker gray is counted twice in the calculation of the tax base.

\(^{12}\)Contrarily, in Section 5.2, we show the equilibrium with only online purchases without purchase at brick-and-mortar store.
\[ t_i^{N^*} = \frac{A}{2-\gamma} - \frac{\delta}{4} \cdot \frac{1-\gamma}{2-\gamma}, \]  
(22)

\[ R_i^{N^*} = 2(t_i^{N^*})^2 = 2 \left( \frac{A}{2-\gamma} - \frac{\delta}{4} \cdot \frac{1-\gamma}{2-\gamma} \right)^2. \]  
(23)

3 Symmetric equilibrium under the origin principle of taxation

Here, we present a symmetric model of online purchases. In this section, the origin-based tax applies when the purchase of goods is made in a brick-and-mortar store or through e-commerce, deferring a discussion of equilibrium under destination-based taxation until later. If the cost of purchasing goods online is too high, and consumers who purchase goods online disappear from the economy, then the equilibrium would be that presented in section 2.4. Therefore, we make the following assumption to validate online purchase:

**Assumption 1.** \( e < \delta. \)

The tax base for region \( i \) is obtained by using \( p_i = p_j = c, N_i = N_j = 1, (1), (9), (10), (11), (12), \) and (17) as

\[ Q_O^i = 2 \left\{ A - t_O^i + 2\delta - e \frac{1-\gamma}{4\delta} \gamma t_O^j + 2\delta - e \frac{1-\gamma}{4\delta} \right\}. \]  
(24)

Using (24) with \( \partial Q_O^i / \partial t_i = -2 \), the first-order condition for maximizing \( R_O^i = t_i^O Q_O^i \) is:

\[ \frac{\partial R_O^i}{\partial t_i} = Q_O^i + t_i^O \frac{\partial Q_O^i}{\partial t_i} \]
\[ = 2 \left[ A - t_i^O + \gamma t_j^O - (1-\gamma) \frac{2\delta - e}{4\delta} \right] - 2t_i^O = 0. \]  
(25)

In the e-commerce model, the response curve still has a positive slope, indicating that taxes are a strategic complement. The first term on the right-hand side of (25) represents the effect of increased revenue associated with a tax increase when the tax base remains unchanged, and the second term represents the effect of a smaller tax base associated with a tax increase. The latter effect is depicted in Figure 3, which shows that individual consumption subject to tax in region \( i \) varies with distance from the border. Under the origin-based tax, the tax base of country \( i \) is the amount of good \( i \) purchased by the consumer, regardless of the method of purchase and their place of residence, which are indicated by the bold line in the figure. Then, as can be seen from (9) to (12), when \( t_i \) increases by \( \Delta t_i \), the purchases of good \( i \) are reduced by \( \Delta t_i \) equally for all consumers and the tax base shrinks accordingly, which is captured in the second term of (25).

Using (25) for \( i \) and \( j \), we solve for the tax rate and tax revenues of the NE under the origin principle of taxation, as follows:

\[ t_i^{O^*} = \frac{A}{2-\gamma} - \frac{1-\gamma}{2-\gamma} \cdot \frac{2\delta - e}{4\delta} , \]  
(26)

\[ R_i^{O^*} = 2(t_i^{O^*})^2 = 2 \left( \frac{A}{2-\gamma} - \frac{1-\gamma}{2-\gamma} \cdot \frac{2\delta - e}{4\delta} \right)^2. \]  
(27)

Now, let us express the progress of e-commerce as a decline in \( e \) (Agrawal & Wildasin, 2020). Differentiating \( t_i^{O^*} \) with respect to \( e \) in (26), we can formally show the impact of the development of online purchasing on the equilibrium tax rate under the origin principle of taxation as follows:

**Proposition 1.** Under the origin principle of taxation, as the cost of online consumption decreases, taxes increase, \( \partial t_i^{O^*} / \partial e < 0. \)

\[ \text{It can be easily shown by } \partial R_i^{O^*} / \partial e = 2t_i^{O^*} (\partial t_i^{O^*} / \partial e) < 0 \text{ that the increase in } t_i^{O^*} \text{ associated with the decrease in } e \text{ leads to an increase in } R_i^{O^*}. \]  

11
Figure 3: Change in individual consumption taxed by region $i$ when $t_i$ increases by $\Delta t$: Origin principle of taxation

Note. The bold and thin lines represent individual consumption before and after the tax rate in country $i$ is raised by $\Delta t$, respectively. The change in the tax base resulting from the tax increase is shown in the gray area, which applies to subsequent figures.

**Proof.** Using (26), we obtain

$$\frac{\partial Q_{O_i}^O}{\partial t_i} = -\frac{(1 - \gamma)(\delta - e)}{2\delta(2 - \gamma)} < 0. \tag{28}$$

The sign is based on Assumption 1. $\square$

The tax increases as online purchase cost decreases because the decrease in $e$ increases the marginal revenue from a higher tax rate:

$$\frac{\partial^2 R_{O_i}^O}{\partial t_i \partial e} \bigg|_{t_i} = \frac{\partial Q_{O_i}^O}{\partial e} + \frac{\partial^2 Q_{O_i}^O}{\partial t_i \partial e} = -\frac{(1 - \gamma)(1 - e)}{2\delta} < 0. \tag{29}$$

In (29), there are two possible impacts of the decline in $e$ on marginal revenue. The first is the impact of the decrease in $e$ directly changing the tax base through changes in how and how many purchases are made. This is captured by the first term on the right side of (29). The second is the indirect effect of a decrease in $e$, which changes the sensitivity of the tax base to tax rates, as indicated by the second term. However, because $\frac{\partial Q_{O_i}^O}{\partial t_i} = -2$ in (24), no indirect effects occur in our model, and thus the second term is zero.

Direct effect of the decline in $e$ on the tax base, that is, the first term, can be broken down into two parts: (i) a decrease in the cost of online purchases increases the number of consumers who purchase goods online, and (ii) lower online purchase costs increase the quantity of goods purchased online. Figure 4 illustrates these two effects, where the bold line shows the individual consumption of good $i$ in region $i$ and region $j$. The change in the number of online users with a marginal decrease in $e$ is indicated by the two horizontal arrows, and the decrease in $e$ increases the number of online users by $\Delta e/\delta$ and reduces the number of offline users by the same amount in both regions. When consumers switch from online to in-store purchases, consumption before and after the switch is the same for marginal consumers whose purchase method is indifferent, and the expansion of the tax base through an increase in online users is offset by a contraction of the tax base through a decrease in in-store users at the margin.\(^{14}\)

\(^{14}\)In fact, an increase in tax base through the path of (i) is $(1 - \gamma)(\Delta e)^2/(2\delta)$ in Figure 4, which converges to zero when $\Delta e \to 0$. 

12
Figure 4: Change in individual consumption taxed by region $i$ when $e$ declines: Origin principle of taxation

the path of (i) does not affect the tax base; hence, the tax rate does not change through this path. The sign in (29) is due to path (ii), which is indicated by the two vertical arrows. A decrease in online costs increases the purchase of good $i$ by consumers in region $j$ by $\Delta e$. It also increases the purchase of good $j$ by consumers in region $i$ by $\Delta e$, which, in turn, decreases the purchase of good $i$ by $\gamma \Delta e$. Thus, as $e$ decreases, the tax base in region $i$ shows a net increase. The larger the tax base, the greater the marginal revenue when taxes are raised; thus, a larger tax base due to a lower $e$ increases the incentive to set higher tax rates. However, it is also true that when the two goods are perfectly substitutable ($\gamma = 1$), an increase in demand for good $i$ by consumers in region $j$ is completely offset by a decrease in demand for good $i$ by consumers in region $i$, meaning that the tax base does not change even if $e$ decreases. Therefore, when $\gamma = 1$, the tax rate does not change even if $e$ decreases.

Thus, the policy implications of Proposition 1 are clear. The expansion of online purchases due to lower e-commerce costs will ease competition for lower tax rates, thereby reducing the decline in tax revenues under the origin principle of taxation, except in the special case of the perfect substitution of two goods, $\gamma = 1$.

In our analysis, as detailed above, we have made the following assumption, which ensures a normal situation in which the consumption of goods in equilibrium is positive, $q_{ii}(x_{ij}) > 0$ and $q_{ij}(x_{ij}) > 0$, for any $x_{ij}$ and $e$ under Assumption 1 (see Appendix A).

Assumption 2.

$$\alpha > \bar{\alpha} \equiv (1 - \gamma) c + \frac{7 - 2\gamma - \gamma^2}{4} \delta \quad \leftrightarrow \quad A > \frac{7 - 2\gamma - \gamma^2}{4} \delta.$$  

Finally, we formally derive the impact of the emergence of a new means of purchasing online tax rates and revenue. Comparing (22) and (23) with (26) and (27), we obtain the following results:

**Corollary 1.** If an origin-based tax is imposed, regardless of whether goods are purchased from a brick-and-mortar store or via the Internet, the tax rate and revenue will be higher when consumers can purchase goods online than when they cannot: $t^N_1 < t^O_1$ and $R^N_1 < R^O_1$.

**Proof.** The comparison gives

$$t^O_1 - t^N_1 = \frac{1 - \gamma}{2 - \gamma} \frac{(\delta - e)^2}{4\delta} > 0,$$

$$R^O_1 - R^N_1 = 2(t^O_1)^2 - 2(t^N_1)^2 = 2(t^O_1 - t^N_1)(t^O_1 + t^N_1) > 0. \quad \square$$

The demand for goods produced in other regions will increase when consumers can purchase goods online because the cost is lower than when they can only purchase goods in brick-and-mortar stores. This means an increase in the tax base in the presence of e-commerce, which allows governments to impose higher taxes and generate higher tax revenues.

15 The effect of lower $e$ on changing individual consumption does not occur in models of Bacache-Beauvallet (2018) and Agrawal and Wildasin (2020) since they assume that consumers purchase one unit of goods inelastically.
4 Symmetric equilibrium under the destination principle of taxation

This section derives an equilibrium in which destination-based taxation applies to purchases of goods online, whereas origin-based taxation applies to purchases of goods in brick-and-mortar stores.

By inserting $p_i = p_j = c$, $N_i = N_j = 1$, (3), (13), (14), (15), and (16) into (18), we obtain the tax base of region $i$:

$$Q^D_i = \left(1 - \frac{E_i}{\delta}\right) [2A - (1 - \gamma)(2t^D_i + e)] + \frac{2e}{\delta} (A - t^D_i + \gamma t^D_j) - \frac{E^2_i}{2\delta} + \gamma \frac{E^2_j}{2\delta},$$

where $E_i = e - (t^D_i - t^D_j)$. The maximization of $R^D_i = t^D_i Q^D_i$ yields the first-order condition

$$\frac{\partial R^D_i}{\partial t^D_i} = Q^D_i + t^D_i \frac{\partial Q^D_i}{\partial t^D_i}$$

$$= 2 \left[A - (1 - \gamma)t^D_i - (1 - \gamma)\frac{2\delta - e}{4\delta} e\right] - \left\{2t^D_i - 2\gamma \left(1 - \frac{e}{\delta}\right) t^D_j + 2\gamma \left(1 - \frac{e}{\delta}\right) t^D_j\right\} = 0,$$

which yields the reaction function $t^D_i = t(t^D_j)$. To evaluate the first-order condition at the symmetric equilibrium, we set $t^D_i = t^D_j$, which gives

$$\left.\frac{\partial R^D_i}{\partial t^D_i}\right|_{t^D_i = t^D_j} = 2 \left[A - (1 - \gamma)t^D_i - (1 - \gamma)\frac{2\delta - e}{4\delta} e\right] - 2t^D_i \left[1 - \gamma \left(1 - \frac{e}{\delta}\right)\right] - 2t^D_i \left[1 - \gamma \left(1 - \frac{e}{\delta}\right)\right] t^D_i = 0. \quad (31)$$

The first-order condition has two solutions, and only the smaller one satisfies the second-order condition (see Appendix B). Figure 5 shows the individual consumption subject to the tax for region $i$ under the destination principle tax. The first term on the right-hand side of (31), which corresponds to the tax base $Q^D_i$, is represented by the integral along the bold line in the figure.\(^{16}\) The second and third terms in (31) represent $t^D_i(\partial Q^D_i / \partial t^D_i)$, which shows the impact of the tax base reduction due the tax increase on tax revenues. $\partial Q^D_i / \partial t^D_i$ is visually captured by the total change in consumption, indicated by the six arrows in Figure 5. It has the following two properties that are different from those in Figure 3 under the origin principle tax. First, the decline in individual consumption due to the tax increase, as captured by the downward arrows, is smaller in Figure 5 than in Figure 3 by $\gamma \Delta t_i$. Second, as captured by the two left arrows, the increase in $t_i$ reduces the number of consumers paying taxes to region $i$ in Figure 5, but there is no such change in Figure 3.

These two differences arise because, under the origin principle, taxes are imposed where goods are supplied, whereas, under the destination principle, taxes are imposed where goods are consumed. This brings us to the difference between the segment $q^{OB}_{ii}$ in Figure 3 and $q^{OB}_{ij}$ segment in Figure 5, which provides the following two implications. First, under the destination principle, for consumers purchasing goods online, regions impose a tax on the consumption of both goods $i$ and $j$ regardless of where they are supplied. As the tax rates imposed on the two goods are simultaneously increased, consumers cannot substitute their purchases of the goods. In this case, consumers reduce their consumption of both goods proportionately. This leads to a smaller decrease in the consumption of good $i$ by $\gamma \Delta t_i$ than when, for example, the origin principle tax is applied, in which only the tax on good $i$ is raised and a shift in demand from good $i$ to good $j$ occurs.\(^{17}\) Under the origin principle tax, the number of consumers

\(^{16}\)This is consistent with the first term of (25). Since $t_i = t_j$, $p_i = p_j = c$, $x^D_{ij} = x^D_{ji} = e/\delta$, $q^{OB}_{ii} = q^{OB}_{ij} = A - (1 - \gamma)t_i + \gamma e$, and $q^{OB}_{ij} = q^{OB}_{ij} = A - (1 - \gamma)t_i - e$ hold in the symmetric equilibrium, we have $Q^D_i = Q^D_j$ from (17) and (18).

\(^{17}\)A higher $t_i$ reduces the consumption of good $i$, $q^{OB}_{ii}$, by $\Delta t_i$, and increases the consumption of the good $j$, $q^{OB}_{ij}$, that replaces it by $\gamma \Delta t_i$. Simultaneously, increasing $t_i$ reduces the consumption of online-purchased good $j$, $q^{OB}_{ij}$, by $\Delta t_i$, instead increasing the consumption of the good $i$, $q^{OB}_{ij}$, which replaces it with $\gamma \Delta t_i$. Through these two paths, when $t_i$ is higher, the purchases of goods $i$ and $j$ decrease by $(1 - \gamma)\Delta t_i$. 


14
purchasing goods online is fixed at $1 - e/\delta$; therefore, if the tax rate in region $i$ is increased by $\Delta t_i$, the tax revenue reduced by the drop in individual consumption is smaller than the revenue reduction under the destination principle tax by $2(1 - e/\delta)$ times $\gamma \Delta t_i$. This is expressed as the difference between the second terms in (25) and (31). Second, under the destination principle, changing the tax rate in region $i$ changes the number of consumers paying tax to region $i$. For example, increasing $t_i$ by $\Delta t_i$ makes $\Delta t_i/\delta$ consumers in region $j$ switch from in-store to online purchases of good $i$ and $\Delta t_i/\delta$ consumers in region $i$ switch from online to in-store purchases of good $j$. Such a change in purchasing method would reduce the number of consumers paying taxes to region $i$ by $2\Delta t_i/\delta$. Because the consumption of consumers who switch purchase methods is equal to $q_{ij}^{DB}$ in Figure 5, the tax revenue reduced by the reduction in the number of consumers paying taxes in region $i$ when $t_i$ is raised is equal to $2\Delta t_i/\delta$ multiplied by $q_{ij}^{DB}$. This is captured in the third term of (31). These effects, which were not present when the origin principle tax was applied, are added in the case of the destination principle so that the two tax principles lead to different equilibrium tax rates.

Next, we evaluate (31) at $t_i^{O*}$ to determine how equilibrium tax rates differ between the two tax principles. Using (26) and (31), we obtain:

$$\left. \frac{\partial R_i^D}{\partial t_i^{O*}} \right|_{t_i^{p}=t_i^{p}=t_i^{O*}} = t_i^{O*} H(e), \quad \text{where } H(e) \equiv 2\gamma \left(1 - \frac{e}{\delta}\right) - \frac{2}{\delta} \left[ A - \frac{3 - \gamma^2}{2(2 - \gamma)\delta} e - \frac{(1 - \gamma)^2}{4(2 - \gamma)\delta} e^2 \right]. \quad (32)$$

The bracket in the second term of $H(e)$ is identical to $q_{ij}^{OB}$ in the symmetric equilibrium under the origin principle of taxation, and its sign is positive; however, the sign of (32), consisting of the two terms, is nontrivial. However, it is immediately clear that if $\gamma$ is small, for example, $\gamma \to 0$, or if $e/\delta$ is large, for example, $e/\delta \to 1$, then the first term of $H(e)$ is negligible, and thus, $\partial R_i^D/\partial t_i^{O*} \big|_{t_i^{p}=t_i^{p}=t_i^{O*}} < 0 \to t_i^{O*} > t_i^{D*}$. Moreover, we can formally determine the sign of (32) as in Proposition 2 below:

**Proposition 2.** The tax rate is lower under the destination principle than under the origin principle. $t_i^{O*} < t_i^{D*}$.

**Proof.** $\frac{\partial R_i^D}{\partial t_i^{O*}} \big|_{t_i^{p}=t_i^{p}=t_i^{O*}} = t_i^{O*} H(e) < 0$ for $0 < e < \delta$ because $H(\delta) < 0$ and

$$\frac{\partial H(e)}{\partial e} = \frac{1 - \gamma}{(2 - \gamma)\delta} \left[ (3 - \gamma) + \frac{(1 - \gamma)e}{\delta} \right] > 0. \square$$

The intuitive mechanism for this result is as follows: In the case of taxing online purchases of goods under the destination principle, the government in region $i$ can induce consumers in region $j$ who purchase goods online from region $i$ to purchase in stores. Simultaneously, a reduction in the tax rate in region $i$ can induce consumers in region $i$ who used to buy goods in brick-and-mortar stores in region $j$ to buy...
them online. This leads to an increase in the tax base and provides an incentive for region $i$ to lower its tax rate. This effect does not occur under the origin principle of taxation; therefore, the equilibrium tax rate under the destination principle will be lower than the tax rate under the origin principle.

Proposition 2 immediately yields the following results for the comparison of tax revenue in a symmetric equilibrium under different tax principles.

**Proposition 3.** The tax revenue is lower under the destination principle than under the origin principle.

\[ R^D_i | t_i = t^D_i < R^O_i | t_i = t^O_i. \]

**Proof.** At symmetric equilibrium,

\[ R^D_i | t_i = t^D_i = R^D_i | t_i = t^O_i = 2 \left\{ A - (1 - \gamma)\hat{t} - (1 - \gamma) \frac{2\delta - e}{4\delta} e \right\} \hat{t}, \]

which is inverted U-shaped with respect to $\hat{t}$ and is maximized at

\[ \hat{t}^* = \frac{A}{2(1 - \gamma)} - \frac{2\delta - e}{8\delta} e. \]

Here, $t^D_i < t^O_i < \hat{t}^*$ holds, because

\[ \hat{t}^* - t^O_i = \frac{\gamma}{2(1 - \gamma )(2 - \gamma )} A + \frac{\gamma}{2(2 - \gamma)} \cdot \frac{2\delta - e}{4\delta} e > 0. \]

Therefore, $R^D_i | t_i = t^D_i < R^O_i | t_i = t^O_i$. □

Next, we consider how the tax rate changes when the cost of online purchases declines. The equilibrium tax rate under the destination principle is determined at the level at which (31) is set to zero.\(^{18}\) To see how $t^D_i$ changes as $e$ decreases, we differentiate (31) by $e$:

\[ \frac{\partial^2 R^D_i}{\partial t^D_i \partial e} \bigg|_{t^D_i = t^O_i} = -(1 - \gamma) \left( 1 - \frac{e}{\delta} \right) + t^D_i \cdot \frac{2}{\delta} (1 - \gamma) \cdot \left( \frac{\partial^2 Q^D_i}{\partial \gamma \partial e} + \frac{\partial^2 Q^D_i}{\partial t^D_i \partial e} \right). \] (33)

The first term in (33) is the same as that derived in (29) and captures the direct impact of lower $e$ on the tax base. It is immediately clear from (13)-(16) that the amount of change in individual consumption in response to a change in $e$ is independent of the tax rate. Furthermore, as Figure 1(b) shows, changes in the number of online users in response to changes in $e$ are also independent of the tax rate. Because the tax base is determined by individual consumption and the number of consumers, the amount of change in the tax base due to a change in $e$ is equal at any tax rate and tax principle. The second term in (33) captures the change in the sensitivity of the tax base to taxes as $e$ declines. This term is not zero but positive under destination principle taxation, suggesting that a decrease in $e$ reduces the sensitivity of the tax base to tax rates. This is because the second and third terms in (31) representing $t^D_i (\partial Q^D_i / \partial e)$ include the number of online users and individual consumption, both of which increase with a lower $e$ and strengthen tax competition. Because of this term, unlike in the case of origin principle tax, a decrease in $e$ may result in a decrease in the tax rate for the destination principle tax.

In fact, if the negative effect of the first term in (33) exceeds the positive effect of the second term, a decrease in $e$ increases $t^D_i$. Conversely, if the negative effect is less than the positive effect, a decrease in $e$ decreases $t^D_i$. The former is the same as that in Proposition 1: when taxing under the origin principle on e-commerce, a decrease in $e$ causes an increase in the tax rate. Interestingly, contrary to the original principle of taxation, a decline in $e$ may cause a decline in tax rates, exacerbating the race to lower taxes. To see how likely it is that a decrease in $e$ will lower the tax rate, we check the change in marginal revenue associated with a tax increase due to a decrease in $e$. Noting that the right-hand side of (33) is an increasing function of $e/\delta$, by substituting $e/\delta = 0$, we obtain

\(^{18}\)The explicit solution is given in Appendix C.
\[
\frac{\partial^2 P^D}{\partial t^D \partial e} \bigg| _{t^D = t^D} > 2(1 - \gamma) \frac{t^D}{\delta} - (1 - \gamma) = \frac{2(1 - \gamma)}{\delta} \left( t^D - \frac{\delta}{2} \right). \tag{34}
\]

From (34), if \( t^D_i > \delta/2 \), which is proved to hold in the following proposition under Assumptions 1 and 2, the positive effect of the second term in (33) outweighs the negative effect of the first term and a decline in the cost of online purchases accelerates the race to lower tax rates under the destination principle of taxation. The formal results are shown in the following proposition.

**Proposition 4.** Under the destination principle of taxation, as the cost of online consumption decreases, taxes decrease, \( \partial t^D_i / \partial e > 0 \).

**Proof.** See Appendix D. □

Proposition 4 contrasts with Proposition 1: When goods purchased online are taxed under the origin principle, the increase in demand associated with the lower cost of online consumption reduces the incentive for regions to lower their tax rates, thus resulting in higher equilibrium tax rates. Under the destination principle tax, however, the result is the opposite. Basically, when goods purchased online are taxed under the destination principle, the government in region \( i \) has an incentive to lower the tax rate to induce consumers in region \( j \) to buy good \( i \) in brick-and-mortar stores and to make consumers in region \( i \) buy good \( j \) online. In this case, as the cost of online consumption declines and the attractiveness of online consumption increases, governments will have to lower tax rates to a greater extent to counteract the effect of the lower costs of online shopping and induce customers to visit brick-and-mortar stores. For this reason, the sign of \( \partial t^D_i / \partial e \) is positive; the smaller the value of \( e \), the more likely consumers are to buy goods online, so in order to induce consumers in region \( j \) to buy in brick-and-mortar stores in region \( i \), the tax rate in region \( i \) needs to be reduced. Contrary to the result under origin-based taxation, a decline in the cost of online purchases lowers tax rates and worsens tax competition under destination-based taxation.

We derive two useful policy implications from Propositions 1 to 4. First, given that origin-based tax is applied to the purchase of goods in brick-and-mortar stores, taxing online purchases of goods under the origin principle is superior to taxing them under the destination principle from a revenue-maximizing viewpoint. Although origin-based taxes are neutral to the consumer’s decision on how to purchase goods, destination-based taxes distort this decision, thus encouraging the latter to compete to lower the tax. Second, as the cost of online purchases declines and online consumption expands, the advantage of taxing goods purchased online under the origin principle is strengthened.

## 5 Extension

Among the several assumptions made in the analysis thus far, this section first discusses the properties of equilibrium when regions are asymmetric. So far, we assume that the two regions are symmetric, which is a necessary assumption to obtain analytical solutions. In Section 5.1, to help with numerical analysis, we examine the properties of equilibrium in the case of two regions of different sizes and clarify how the tax rate will change as the online market expands due to lower online costs. As the process of finding the NE is the same as that in the previous sections, we focus only on new findings that can be drawn owing to population differences between regions. Section 5.2 refers to the equilibrium at which all consumers purchase goods exclusively online. This may not be a realistic situation given the current situation, but it enables us to study an extreme case in which there is no longer a choice of how goods are purchased. This clarifies the critical role of the choice of “how to purchase” on the efficiency of the tax principle.

### 5.1 Asymmetric equilibrium

#### 5.1.1 Origin principle of taxation

The NE tax rates under the origin principle of taxation are obtained as follows (see Appendix E):
\[ t_{0}^{O^{*}} = \frac{A}{2 - \gamma} - \left[ \frac{1 - \gamma}{2(2 - \gamma)} - \frac{1 + \gamma}{2(2 + \gamma)} b \right] e + \frac{1 - \gamma}{4(2 - \gamma)} \cdot \frac{e^2}{\delta}. \] \tag{35}

\[ t_{1}^{O^{*}} = \frac{A}{2 - \gamma} - \left[ \frac{1 - \gamma}{2(2 - \gamma)} + \frac{1 + \gamma}{2(2 + \gamma)} b \right] e + \frac{1 - \gamma}{4(2 - \gamma)} \cdot \frac{e^2}{\delta}. \] \tag{36}

Note that (35) and (36) are reduced to (26) if \( b = 0 \). A comparison of the equilibrium tax rates and tax revenues derived using (35) and (36) between the two regions yields the following results.

**Proposition 5.** When region 0 has a population larger than region 1 \((b \geq 0)\), the tax rates and tax revenues in region 0 are higher than those in region 1, \( t_{0}^{O^{*}} \geq t_{1}^{O^{*}} \), and \( R_{0}^{O^{*}} \geq R_{1}^{O^{*}} \).

**Proof.** See Appendix F. \(\square\)

The conclusion that regions with larger populations set higher tax rates have been a well-known view since Kanbur and Keen (1993), Trandel (1994), and Nielsen (2001). Proposition 5 confirms that this feature of the equilibrium tax rate holds robustly when the model is extended to a situation where the consumption of goods is elastic and online purchases are possible.

The comparative statistics of (35) and (36) also provide the following results:

**Proposition 6.** In small regions, the origin-based tax increases monotonically as the cost of online consumption \((e)\) decreases, \( \frac{\partial t_{0}^{O^{*}}}{\partial e} < 0 \). In a large region, the changes in the cost of online consumption \(e\) and the tax rate are non-monotonic. When \(e\) is large, the origin-based tax decreases as \(e\) falls, \( \frac{\partial t_{0}^{O^{*}}}{\partial e} > 0 \), but when \(e\) is small, taxes rise as \(e\) falls, \( \frac{\partial t_{0}^{O^{*}}}{\partial e} < 0 \).

**Proof.** From (35) and (36), we have that

\[ \frac{\partial t_{0}^{O^{*}}}{\partial e} = -\frac{1}{2} \left[ \frac{1 - \gamma}{2 - \gamma} \left( 1 - \frac{e}{\delta} \right) - \frac{1 + \gamma}{2 + \gamma} b \right] \quad \text{and} \quad \frac{\partial t_{1}^{O^{*}}}{\partial e} = -\frac{1}{2} \left[ \frac{1 - \gamma}{2 - \gamma} \left( 1 - \frac{e}{\delta} \right) + \frac{1 + \gamma}{2 + \gamma} b \right]. \] \tag{37}

\( \frac{\partial t_{1}^{O^{*}}}{\partial e} < 0 \) is straightforward, from the second equation in (37). The first equation in (37) shows that the \( \frac{\partial t_{0}^{O^{*}}}{\partial e} < 0 \) if \(e\) is sufficiently small to satisfy:

\[ 0 < \frac{e}{\delta} < 1 - \frac{(1 + \gamma)(2 - \gamma)}{(1 - \gamma)(2 + \gamma)} b, \]

and \( \frac{\partial t_{0}^{O^{*}}}{\partial e} > 0 \) if \(e\) satisfies

\[ 1 - \frac{(1 + \gamma)(2 - \gamma)}{(1 - \gamma)(2 + \gamma)} b < \frac{e}{\delta} < 1 - b. \quad \square \]

This result can be intuitively explained as follows: The decline in \(e\) has two effects on demand. First, it increases the demand from consumers who buy goods online. Second, it reduces the purchase of goods at brick-and-mortar stores, which are substituted with goods purchased online. In a market with this feature, the demand from consumers in the larger region 0 online for good 1 produced in the smaller region 1 is relatively large. Therefore, the first effect outweighs the second effect in the small region and the tax base increases owing to the decrease in \(e\). This leads to higher tax rates in small regions owing to lower \(e\). Conversely, the demand from consumers in small region 1 online for good 0 supplied by large region 0 is relatively small. Therefore, the second effect tends to be larger than the first effect. In this case, the tax rate in the large region will decrease because the decrease in \(e\) will lead to a smaller tax base. However, as the size of the online market expands, owing to the decline in \(e\), the magnitudes of the two effects eventually reverse. Thus, if \(e\) declines further from a sufficiently low level, the large region’s tax rate will start to rise.
5.1.2 Destination principle of taxation

Even in our simple setting, it is difficult to analytically solve for the equilibrium when asymmetric regions apply destination-based taxation to e-commerce. Therefore, we rely on numerical calculations to demonstrate the relationship between tax rates and the decline in online purchase costs in each region. Figure 6(a) shows the tax rates in large and small regions under a set of plausible parameters within the range that satisfies the conditions corresponding to Assumptions 1 and 2: $\gamma = 0.25$, $\delta = 1$, $c = 1$, $\alpha = 4$, and $b = 0.2$. The blue lines are the large region’s tax rates, the red lines are the small region’s tax rates, and the dotted line indicates the tax rates in symmetric equilibrium. The lines at the high level, that is, above 1.6, represent the tax rate under the origin principle, and the lines at the low level, that is, below 1.0, represent the tax rates under the destination principle.

5.1.3 Comparison of the two tax principles

From Figure 6(a), we identify several features of the equilibrium tax rate under these two tax principles. First, under both tax principles, the tax rates in larger regions are higher than those in smaller regions, implying that the result in Proposition 5, derived under the origin principle of taxation, is also valid for taxation under the destination principle. Second, for all $\epsilon(< \delta = 1)$, the tax rate based on the destination principle is lower than that based on the origin principle, showing that what is shown in Proposition 2 holds even when there is a difference in the population of regions. Third, a lower $\epsilon$ lowers tax rates in both regions under the destination principle but raises rates in the smaller region under the origin principle and also raises rates in many areas in the larger region. These mean that the results shown in Propositions 1 and 4 are almost valid, even if we assume two asymmetric regions, clearly showing the difference in the change in the tax rate as the cost of online purchase declines under the origin and destination principles, respectively. Finally, a decrease in $\epsilon$ reduces regional differences in tax rates when taxed under the destination principle but increases them when taxed under the origin principle. These tendencies are maintained in almost all situations, even when the value of the parameter is changed.

19 In Agrawal and Wildasin (2020), under the destination-based tax on e-commerce, a decline in $\epsilon$ lowers tax rates in large regions and raises rates in small ones. By contrast, our results under the destination principle show that a decline in $\epsilon$ lowers the tax rates in both regions. This difference stems from the difference in the assumptions of the two studies: In the basic analysis of Agrawal and Wildasin (2020), goods purchased over the Internet are only sold in a core region, whereas in our model, goods can be purchased over the Internet in both regions. In their extended part of the analysis, they suggest that the results may vary depending on the export and import position if consumers can purchase goods over the Internet from the core as well as from peripheries.

---

**Figure 6: Tax rates and Tax revenues in two asymmetric regions ($b > 0$)**

Note. The blue line represents the tax rate and tax revenues for large region 0, and the red line represents that of small region 1. In (a), the lines where the tax rates are greater than 1.6 are the equilibrium tax rates under the origin principle, and the lines where the tax rates are less than 1.0 are the equilibrium tax rates under the destination principle. The same is true for (b): the lines at the high level are the tax revenues under the origin principle, and the lines at the low level are the tax revenues under the destination principle. The dotted line represents the tax rate and tax revenues for the case of two symmetric regions. The parameters are $\gamma = 0.25$, $\delta = 1$, $c = 1$, $\alpha = 4$, and $b = 0.2$. 

---

19 In Agrawal and Wildasin (2020), under the destination-based tax on e-commerce, a decline in $\epsilon$ lowers tax rates in large regions and raises rates in small ones. By contrast, our results under the destination principle show that a decline in $\epsilon$ lowers the tax rates in both regions. This difference stems from the difference in the assumptions of the two studies: In the basic analysis of Agrawal and Wildasin (2020), goods purchased over the Internet are only sold in a core region, whereas in our model, goods can be purchased over the Internet in both regions. In their extended part of the analysis, they suggest that the results may vary depending on the export and import position if consumers can purchase goods over the Internet from the core as well as from peripheries.
Figure 6(b) shows the tax revenue realized under the equilibrium tax rates shown in Figure 6(a). It shows the following two characteristics of tax revenue in regions with different population sizes: first, lower costs associated with online purchases tend to increase tax revenues under origin-based taxation but decrease tax revenues under destination-based taxation. Second, for any level of \( e \), origin-based taxation generates greater tax revenue than destination-based taxation does. The second feature numerically confirms the advantage of the origin-based taxation presented in Proposition 3 in the asymmetric regional framework. Specifically, assuming that origin-based taxes generate larger tax revenues than destination-based taxes since the latter distorts consumers’ choice of where to buy. In contrast, our model omits the consumer’s choice of where to buy but instead includes a choice of how to buy, and the destination-based tax distorts that choice, leading to the opposite result.

These results are only insights based on numerical calculations; they are not the results of analytical solutions. Thus, what can be said of the nature of equilibrium in an environment where there is tax competition between asymmetric regions under the destination principle on e-commerce is that in “many cases”, the expansion of e-commerce as online shopping costs fall intensifies competition to lower taxes and decreases regional tax revenue.

5.2 Equilibrium for online purchases only

Thus far, we have assumed \( e \geq 0 \). This allowed us to depict a situation in which consumers buy goods from their own region in brick-and-mortar stores. In this subsection, we analyze the situation where consumers in region \( i \) will purchase not only good \( j \) but also good \( i \) online. This situation is derived by simply assuming \( e < 0 \). In this case, consumers have no choice regarding how they purchase, and thus the number of online users does not change when the tax rate or the cost of online purchases changes. Then, the fourth term in (31) is zero, and (32) and (33) can be rewritten as follows:

\[
\frac{\partial R^D_i}{\partial t^D_i} \bigg|_{t^D_j = t^D_i = t^D_i \text{ and } e < 0} = 2\gamma \left(1 - \frac{\epsilon}{\delta}\right) t^O_i > 0,
\]

\[
\frac{\partial^2 R^D_i}{\partial t^D_i \partial e} \bigg|_{t^D_j = t^D_i \text{ and } e < 0} = -(1 - \gamma) \left(1 - \frac{\epsilon}{\delta}\right) - \frac{2\gamma t^D_i}{\delta (Q^D_i / (\partial e) - \partial^2 (t^D_i Q^D_i) / (\partial t^D_i \partial e))} < 0.
\]

The signs of these equations are exactly opposite to those obtained for (32) and (33). This means that the results of Propositions 2 through 4 are all reversed. If consumers have no choice about how they buy goods and all goods are purchased online, a destination-based tax avoids competition to lower the tax rate more than an origin-based tax. Conversely, as indicated in Propositions 2 through 4, the choice of how to purchase goods given to consumers owing to technological progress strongly influences the choice of efficient tax principles.

6 Conclusion

This study analyzes which of the two tax principles governments could apply to e-commerce to avoid a race to lower tax rates and achieve greater tax revenues. The distinctive feature of our approach compared to preceding tax competition studies is that we analyze the different implications that the two representative tax principles have on tax competition when consumers are given a new choice of how to purchase goods instead of where to purchase goods. Goods purchased in brick-and-mortar stores are taxed under the origin principle, which is a form of taxation observed especially in transactions that cross city or state borders or between regions where no customs or other border adjustment mechanisms are in place. By contrast, the government may apply the origin principle tax or destination principle tax to
e-commerce. This raises the question of whether the origin or destination principle should be applied to online purchases.

We compare the tax competition equilibrium under two different taxation principles and show that applying origin-based taxation to not only brick-and-mortar purchases but also online purchases mitigates the excessive tax reductions associated with tax competition and results in higher tax revenues. Furthermore, we show that the expansion of the online market through the offer of lower online purchase costs raises the equilibrium tax rate in an origin-based tax competition but lowers it in a destination-based tax competition. Thus, the main argument emerging from our study is that given that origin-based tax is applied to purchases of goods in brick-and-mortar stores, a move toward applying the destination principle when taxing online purchases may accelerate the race to lower the tax; applying the same taxation principle to both brick-and-mortar stores and online purchases would curb tax competition. The destination principle taxation creates distortions because it incentivizes governments to induce their own consumers to shop online. If a consumer in one region buys a good in a brick-and-mortar store in a neighboring region, a tax is paid to the neighboring region. However, if the consumer purchases goods online from the neighboring region, the tax can be paid to the home region. In addition, by encouraging consumers in neighboring regions to shift from online purchases to brick-and-mortar purchases of goods produced in their own regions, governments can increase their tax revenues. As a result, governments under the destination principle of taxation on e-commerce are in a race to steal other regions’ tax bases by inducing their own consumers to buy online, which leads to lower tax rates and revenues.

We derive key results from a symmetric tax competition model to simply present a new view of tax principles as they apply to e-commerce. In the second half of the study, the model was extended to two regions with different population sizes. This extension would be useful to confirm the robustness of the result that destination-based taxes are less distortive of the choice of “how to purchase” goods, as well as to provide insight into the possibility that large and small regions have different preferences for the tax principles they apply. As for the former, we confirm that the results under symmetric tax competition are generally valid, regardless of whether the origin or destination principle is applied; given the origin-based tax is applied to in-store purchases, applying the origin-based tax to e-commerce as well would avoid competition to lower the tax and generate greater tax revenue. For the latter, Figure 6(b) shows that, given that both regions apply the same tax principles, there is a preference for a destination-based tax that generates higher tax revenues for any region because the tax does not distort consumer choices about how to purchase goods. However, by shifting from the origin principle to the destination principle, government will be able to influence consumer choices by changing tax rates, thereby unilaterally expanding its tax base and increasing tax revenues. Any region, regardless of population size, has this incentive to shift to destination principle of taxation, but smaller regions would have a stronger incentive to deviate from the agreement to adopt the principle of origin since they have a stronger incentive to lower tax rates and broaden their tax base, i.e., Kanbur and Keen (1993). Although the current model is difficult to solve analytically, such inferences suggest that regions have incentives to deviate from the agreement to apply the origin-based tax to online purchases of goods, especially in regions with smaller populations, where the incentive to shift from the origin principle to the destination principle is stronger.

While our study provides some generalizations, such as assuming situations in which demand is elastic with respect to tax rates or where there is differentiation of goods, it retains some specific assumptions. The first assumption is that the objective function of the government is to maximize tax revenue. This setting is used in many tax competition models; however, for more generalization, it may be necessary to change the government’s objective. In such a case, we would need to adopt a different approach than analytically solving the model. Second, we assumed a competitive market. Although the case of one firm operating in each region has been analyzed by Aiura and Ogawa (2021), we may extend the analysis to more elaborate, imperfectly competitive e-commerce models. Specifically, while our model allows for the inclusion of consumer purchases of goods online in the analysis, it does not explicitly model platform companies that are dominant in the online market. Third, it is theoretically worthwhile to consider the case in which destination-based taxation is imposed on the purchase of goods in brick-and-mortar stores. Indeed, it is technically possible to apply the destination principle of taxation when making purchases at brick-and-mortar stores through inter-regional tax adjustments, and much research has been conducted in this direction (Haufler and Pflüger, 2004; Antoniou et al., 2019, 2022). If the destination-based tax applies to purchases of goods in brick-and-mortar stores, based on the mechanisms leading to our

20Haufler and Pflüger (2004) analyze the two tax principles under the names consumption-based tax and production-based tax.
results, applying the origin-based tax to the purchase of goods online would distort the choice of “how to purchase”. Thus, a tax method in line with the tax principle applicable to purchases in brick-and-mortar stores might be applied to online purchases, resulting in the adoption of a destination-based tax on e-commerce.

There are also a number of factors worth analyzing in the theory of optimal taxation with e-commerce. First, since the adoption of any tax principle implicitly assumes the coordination of tax systems across regions, there is room to analyze the incentives that make this coordination possible. Second, while the model in this study focuses on tangible assets, it is increasingly important to discuss taxation policy under a model in which semi-tangible assets such as data traded online. However, these issues are beyond the scope of our analysis and should be addressed in future research.

Appendices

Appendix A

Figure 2(a) shows that \( q_{ij}^OB \) is the smallest for individual consumption; therefore, if \( q_{ij}^OB \) is shown, then all consumption is positive. Substituting \( p_i = p_j = c \) and \( t_i = t_j = t_i^{O*} \) into (12) yields

\[
q_{ij}^OB = \frac{1}{2 - \gamma} \left( A - \frac{3 - \gamma^2}{2} e - \frac{(1 - \gamma)^2}{4} - \frac{\epsilon^2}{\delta} \right) > 0,
\]

where the first sign in the second line is from Assumption 1 and the last sign is from Assumption 2. This suggests that the equilibrium quantity under origin-based taxation is positive for any \( \epsilon \) and \( x_{ij} \) under two assumptions.

Appendix B

We express the first-order condition as \( \partial R_i^D / \partial t_i^D \big|_{t_i^D = t_i^D} \equiv f(t_i^D) = 0 \). Then, from (31), we have:

\[
f(t_i^D) = \frac{2(1 - \gamma)}{\delta} (t_i^D)^2 - \frac{2}{\delta} [A + (1 - \gamma)(2\delta - e)] t_i^D + 2 \left[ A - (1 - \gamma) \frac{2\delta - e}{4\delta} \right] = 0,
\]

where \( f(t_i^D) \) is a quadratic function that is convex downward with respect to \( t_i^D \). We also express the second-order condition as \( \partial^2 R_i^D / \partial (t_i^D)^2 \big|_{t_i^D = t_i^D} \equiv s(t_i^D) < 0 \), where

\[
s(t_i^D) = 7(1 - \gamma) \cdot \frac{t_i^D}{\delta} + 4(1 - \gamma) \left( \frac{e}{\delta} \right) - 4 \left[ \frac{A}{\delta} + (1 - \gamma) \right].
\]

\( s(t_i^D) \) is a monotonically increasing linear function. Now, the solution of \( s(t_i^D) = 0 \) is denoted by \( t_i^S \):

\[
t_i^S = \frac{4}{7} \left[ \frac{A}{1 - \gamma} + \left( 1 - \frac{e}{\delta} \right) \right] \cdot \frac{1 - \gamma}{\delta}.
\]

If the solution satisfying \( f(t_i^D) = 0 \) is greater than or equal to \( t_i^S \), then the second-order condition is not satisfied. Conversely, if the solution is less than \( t_i^S \), it satisfies the second-order condition:

\[
f(t_i^S) = -\frac{24}{49(1 - \gamma)} \left[ A - \frac{(8e - \delta)(1 - \gamma)}{8} \right]^2 - \frac{1 - \gamma}{8} \left[ 13 - 8 \left( \frac{e}{\delta} \right) - 4 \left( \frac{e}{\delta} \right)^2 \right] \delta < 0,
\]

indicating that \( t_i^S \) is between the two solutions satisfying \( f(t_i^D) = 0 \) because \( f(t_i^D) \) is a convex downward quadratic function. Therefore, \( f(t_i^D) = 0 \) has two solutions, and only the smaller solution satisfies the second-order condition.
Appendix C

Appendix B shows that among the two solutions satisfying (38), a larger value does not satisfy the second-order condition; however, a smaller value satisfies Assumptions 1 and 2. Therefore, the equilibrium tax rate is the smaller value of the two solutions in (38); that is,

\[ t_i^{D*} = \frac{2\delta - e}{2} + \frac{A - \sqrt{A^2 - 2(1 - \gamma) [A + (1 - \gamma) \delta] e + 4(1 - \gamma)^2 \delta^2}}{2(1 - \gamma)}. \]  \hspace{1cm} (39)

Appendix D

Because \( \partial t_i^{D*} / \partial e = -[\partial f(t_i^{D*})/\partial e]/[\partial f(t_i^{D*})/\partial t_i^{D*}] \), we derive the sign of \( \partial f(t_i^{D*})/\partial e \) and \( \partial f(t_i^{D*})/\partial t_i^{D*} \) respectively. From (38), we obtain:

\[ \frac{\partial f(t_i^{D*})}{\partial t_i^{D}} = - \frac{4(1 - \gamma)}{\delta} t_i^{D*} - \frac{2}{\delta} [A + (1 - \gamma)(2\delta - e)], \]

which can be rewritten by substituting (39) as follows:

\[ \frac{\partial f(t_i^{D*})}{\partial t_i^{D}} = - \frac{2}{\delta} \frac{A - (1 - \gamma)(2\delta - e)}{\delta} < 0. \]

Next, from (34), if \( t_i^{D*} > \delta/2 \), we have

\[ \frac{\partial f(t_i^{D})}{\partial t_i^{D}} = \frac{\partial^2 R_i^D}{\partial t_i^{D} \partial e} \bigg|_{t_i^{D} = t_i^{D*}} > 0. \]

Now, we check whether \( t_i^{D*} > \delta/2 \) holds. From (38), tax revenue is maximized at \( t_i^{D*} \); that is, \( f(t_i^{D*}) = 0 \), and thus \( t_i^{D} \) satisfying \( f(t_i^{D}) > (\gamma)0 \) is smaller (larger) than \( t_i^{D*} \). When we substitute \( t_i^{D} = \delta/2 \) into (38), we get

\[ f \left( \frac{\delta}{2} \right) = \frac{(1 - \gamma)e^2}{2\delta} + \left[ A - \frac{3}{2}(1 - \gamma) \delta \right]. \]

Because \( A > \delta(7 - 2\gamma - \gamma^2)/4 > 3\delta(1 - \gamma)/2 \) from Assumption 2, we have \( f(\delta/2) > 0 \). This means that \( t_i^{D*} > \delta/2 \) is valid, and thus \( \partial f(t_i^{D})/\partial e > 0 \). Therefore, \( \partial t_i^{D*}/\partial e > 0 \).

Appendix E

Substituting \( p_i = c, (1), (9), (10), (11), (12) \), with care to the subscript, into (17), we obtain the tax revenue of each government as follows:

\[ R_i^O = 2 \left\{ \frac{1}{2} a_i^O \left[ t_i^O + \frac{2(1 - b)\delta - e}{4\delta} \right] + \frac{1}{2} a_i^O \left[ t_i^O + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} t_i^O. \] \hspace{1cm} (40)

\[ R_i^F = 2 \left\{ \frac{1}{2} a_i^F \left[ t_i^F + \frac{2(1 - b)\delta - e}{4\delta} \right] + \frac{1}{2} a_i^F \left[ t_i^F + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} t_i^O. \] \hspace{1cm} (41)

The first-order conditions for tax revenue maximization are obtained as:

\[ \frac{\partial R_i^O}{\partial t_i^O} = 2 \left\{ a_i^O \left[ t_i^O + \frac{2(1 - b)\delta - e}{4\delta} \right] + \frac{1}{2} a_i^O \left[ t_i^O + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} = 0, \]

\[ \frac{\partial R_i^F}{\partial t_i^O} = 2 \left\{ a_i^F \left[ t_i^O + \frac{2(1 - b)\delta - e}{4\delta} \right] + \frac{1}{2} a_i^F \left[ t_i^O + \frac{2(1 + b)\delta - e}{4\delta} \right] \right\} = 0. \]

By solving these equations, we obtain (35) and (36).
Appendix F

Comparing (35) and (36) yields
\[ t^{O_0} - t^{O_1} = b(1 + \gamma)/(2 + \gamma) \geq 0 \]
because \( b \geq 0 \). Using \( p_i = c, (1), (9), (10), (11), (12), (17), (35), \) \text{and} \ (36), \text{and taking the difference in the equilibrium tax revenues, \text{we obtain}}
the following:
\[ R^{O_0} - R^{O_1} = \frac{2b(\gamma + 1)}{\delta(2 - \gamma)(2 + \gamma)} \left\{ 2\delta(\delta - e)(1 - \gamma) + \delta\Lambda + \delta^2 (5 - \gamma^2) + e^2 (1 - \gamma) \right\} \geq 0. \]

Given \( b \geq 0 \), the sign holds because \( \delta > e \) from Assumption 1 and \( \Lambda \equiv 4\Lambda - (7 - 2\gamma - \gamma^2)\delta > 0 \) from Assumption 2.

References


Miyatake, K., Nemoto, T., Nakahara, S., & Hayashi, K. (2016), Reduction in consumers’ purchasing cost by online shopping, Transportation Research Procedia, 12, 656-666.


