Descriptor and scaling relations for ion mobility in battery electrodes and solid electrolytes

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Abstract

Based on first-principles electronic structure calculations, we have derived an efficient physical descriptor for the ion mobility in battery electrodes and solid electrolytes which is a critical performance parameter in electrochemical energy storage and conversion. This descriptor is entirely composed of observables that are easily accessible: ionic radii, oxidation states and the difference in the Pauling electronegativities of the involved species. Within a particular class of materials, the activation barriers for migration are connected to this migration parameter through linear scaling relations both as far as the variation of the cation chemistry of the charge carriers and the anion chemistry of the host lattice are concerned. The validity of these scaling relations indicates that a purely ionic view of ion mobility in solids falls short of capturing all factors influencing this mobility. The identification of these scaling relations has the potential to significantly accelerate the discovery of materials with desired mobility properties.

Keywords: ion mobility, descriptor, scaling relations, mono- and multivalent charge carriers, density functional theory

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I. INTRODUCTION

Electrochemical energy storage devices play a central role in our attempts towards decarbonization through the storage of volatile renewable energy and the emission-free usage of vehicles and mobile devices. Significant progress has been made in this respect due to the development of advanced Li-ion battery technologies. In addition, recently so-called post Li-ion technologies have drawn a lot of attention in order to address, among others, sustainability issues associated with the materials typically used in Li-ion batteries. In post-Li ion batteries, other charge carriers such as monovalent Na and K cations or divalent Mg and Ca cations are used. These post-Li-ion batteries can compete with existing Li-ion batteries as far as efficiency and safety are concerned or even exhibit a better performance, in particular with respect to safety aspects. In addition, batteries based on divalent cations such as Mg can achieve higher volumetric energy densities compared to monovalent-based batteries. Furthermore, as liquid electrolytes are prone to corrosion processes and often represent fire hazards because of their flammability, all solid-state batteries with higher safety and better electrochemical stability based on materials such as inorganic oxides, hydrides, and chalcogenides have been intensively studied for all possible charge carriers.

A critical parameter that significantly influences the performance of batteries is the ion mobility both in the electrolyte and in the electrodes. Unfortunately, the efficiency of particularly batteries based on multivalent ions such as Mg is hampered by a low ion mobility due to the strong interaction of the multivalent ions with the host structures. Hence the identification and development of materials with improved ion mobility are essential for more efficient electrochemical energy storage devices.

A very useful concept in order to accelerate materials discovery is based on so-called descriptors. They represent fundamental materials properties or combinations thereof that are correlated with a desired or undesired functionality of the material. This concept has been very successfully used in heterogeneous catalysis, in particular in connection with so-called scaling relations, but also already in battery research. The identification of descriptors can significantly speed up the search for new materials with desired functional properties because once they are identified only the particular descriptors need to be optimized in a first step. Thus promising candidate materials can be proposed whose properties...
can then be scrutinized in detail.

In fact, also with respect to ion mobility in solids a number of possible descriptors have been proposed, based on the lattice volume and ionic size\textsuperscript{26,27}, the choice of the anion sub-lattice\textsuperscript{27,37}, the lattice dynamics\textsuperscript{27,38}, or the preferred crystal insertion site\textsuperscript{28}. However, many of the identified descriptors are restricted to some particular crystal structure. Furthermore, some are based on materials properties that are not easily accessible. Hence it is fair to say that so far no convenient descriptor has been established that is able to predict ion mobility across a set of different structures.

Based on the results of first-principles density functional theory (DFT) calculations and physico-chemical reasoning, here we propose such a convenient descriptor for the ion mobility, the so-called migration parameter or number, that is based on the product of Pauling’s electronegativity, ionic radii and oxidation states of the involved compounds, all properties that are easily accessible for any material. This particular descriptor, whose choice is also supported by a statistical analysis of our first-principles results, goes beyond current proposals by considering also deviations from a purely ionic interaction between the migrating ion and the host lattice. According to our calculations, the activation barrier for migration is connected to this migration number via linear scaling relations that are different for different classes of materials. This allows to predict the activation barriers both for the variation of the cation chemistry of the migrating ion as well as for the variation of the anion chemistry of the host lattice. Thus this descriptor will most probably significantly accelerate the discovery of materials with favorable mobility properties. As this migration number is based on basic physico-chemical quantities, it also enables a deeper fundamental understanding of the principles underlying ion mobility.

II. RESULTS AND DISCUSSION

From a microscopic viewpoint, migration or diffusion in solid crystalline materials occurs by atomic hops in a lattice. Such jump processes are typically thermally activated, and the corresponding tracer diffusion coefficient is given by

\[ D^{tr} = D_0^{tr} \exp \left( -\frac{E_a}{k_B T} \right) . \] (1)
FIG. 1. Illustration of a cation interstitial migration mechanism, using Ca diffusion in CaO as an example. A diffusion event corresponds to the migration of the Ca cation from the energetically most favorable octahedral site $A$ to the nearest equivalent site $A'$ through the transition state which corresponds to a saddle-point in the multi-dimensional potential energy surface and which can be derived by first-principles electronic structure calculations. The activation energy or diffusion barrier is denoted by $E_a$ which corresponds to the energy difference between the saddle-point and the initial configuration.

Here $D_0^{tr}$ is the pre-exponential factor, $k_B$ the Boltzmann constant, and $T$ the absolute temperature. $E_a$ is the activation barrier corresponding to the energy barrier along the minimum energy path connecting two equivalent intercalation sites, as illustrated in Fig. 1. Such a minimum energy path can be determined by automatic search routines $^{39}$. In the present work, we have used the nudged elastic band method (NEB) $^{40}$ in the DFT calculations to derive the activation barrier $E_a$. The electronic structure calculations were performed using the Vienna \textit{Ab-initio} Simulation Package (VASP) $^{41}$ employing the Projector Augmented Wave (PAW) $^{42}$ method with the exchange-correlation effects being described with the Perdew-Burke-Ernzerhof (PBE) functional $^{43}$. Further details are provided in the supporting information.

Motivated by the goal to identify the fundamental factors determining ion mobility in solids, in a previous study $^{26}$ we had derived the activation barriers for diffusion of a number
of ions of varying size and charge in the same host lattice, a chalcogenide spinel. We obtained the expected results, namely that the size and the charge of the diffusing ion matter. However, the ionic radius of the charge carrier alone could not explain the observed trends, but rather the distance between the ion in the tetrahedral site and the nearest chalcogenide atom. In order to further elucidate the mobility-determining factors, we decided to look at structurally simpler compounds, namely binary $A_nX_m$ materials with $A$ being the migrating ion. In total, we looked at 35 different compounds with Li, Mg, and Ca as the migrating ion $A$.

For these binary materials, we again found that size and charge of the propagating ions matter, but not in a very systematic way, as already observed by others\textsuperscript{27}. However, we could recently show that the stability of ions in chalcogenide spinels can only be understood if deviations from a purely ionic interaction are taken into account\textsuperscript{44}. It is essential to realize that the considered binary materials span the whole range of interaction characteristics between metallic and ionic bonding. Such bonding characteristics can in fact been classified in so-called Van Arkel-Ketelaar triangles\textsuperscript{45} in which compounds are placed according to the mean electronegativity $\chi_{\text{mean}}$ ($x$-axis) and the electronegativity difference $\Delta \chi$ ($y$-axis) of the constituting elements.

Fig. 2a shows the Van Arkel-Ketelaar triangle including the Mg binary compounds considered in this study. A large difference in electronegativity indicates ionic bonding characteristics (shown in yellow), as present in MgO and MgF$_2$. CsF (not shown) would lie at the apex of the triangle. At the bottom of the triangle corresponding to small electronegativity difference, an increasing mean electronegativity is associated with more directional bonding. Hence the lower left corner contains metallic systems whereas the lower right corner gathers covalent systems.

The $Mg_nX_m$ binaries considered in this study all fall along a line between metallic and ionic bonding. The green area in Fig. 2a corresponds to a region of bonding characteristic intermediate between ionic and metallic. In detail, MgF$_2$ has the highest electronegativity difference $\Delta \chi$ indicating a strong ionic bond. This is also true for MgO, whereas Mg$_3$Si is associated with the lowest value $\Delta \chi$ demonstrating metallic bonding. The rest of Mg-halides, Mg-chalcogenides, Mg-pnictides, and Mg-tetrels are located between strong ionic and metallic bonding. They are divided into three groups. MgCl$_2$, MgBr$_2$, and Mg$_3$N$_2$ are characterized by a large electronegativity difference of about 1.7 demonstrating a predomi-
FIG. 2. A$_n$X$_m$ binaries considered in this study. (a) Van Arkel-Ketelaar triangle with the considered Mg$_n$X$_m$ binaries plotted as a function of the mean electronegativity and the difference in the electronegativity of the two components. (b) Calculated activation energies for the migration of A = Li, Mg, and Ca in A$_n$X$_m$ binaries as a function of the migration number $N_{mig}^{AX}$ for various elements X according to Eq. (2). The solid lines correspond to linear regressions of these results.

nately ionic bonding (light yellow region). MgI$_2$, MgS, and MgSe have $\Delta \chi \approx 1.3$, the other Mg binaries have electronegativity differences below 1.

The fact that also non-ionic components of the interaction contribute to the bonding
in nominally ionic crystals suggests that also the interaction characteristics within the considered compounds represented by the electronegativity difference $\Delta \chi^2$ influences the ion mobility. Together with the well-known dependence of the diffusion barriers on the ionic radii ($r_i$) and oxidation states ($n_i$) of the involved compounds, this observation motivated us to define the migration parameter or number $N_{migr}$

$$N_{migr}^{AX} = (r_A + r_X)n_An_X\Delta \chi_{AX}^2/(N_A + N_X)$$

(2)
as the product of these three quantities where the ionic radii are given in Å, $n_A$ and $n_X$ are the absolute values of the formal integer oxidation states or numbers. In addition, also the number of atoms of the corresponding species in the unit cell of the crystal $N_A$ and $N_X$ enters. In Fig. 2b, we plot the dependence of the migration barriers as a function of the migration parameter for the three migrating ions Li, Mg and Ca in the low vacancy limit. In spite of some outliers, overall the migration barriers nicely follow separate scaling relations for each migrating ion

$$E_a^A(X) = E_0^A + C^A(r_A + r_X)n_An_X\Delta \chi_{AX}^2/(N_A + N_X) = E_0^A + C^AN_{migr}^{AX}.$$  

(3)

These universal scaling relations strongly suggest that the same factors govern the ion mobility in all considered binary compounds. The outliers indicate that other critical contributions to the activation energies such as Coulomb interactions beyond those represented by the oxidation states, quantum mechanical overlap effects and polarization can play a role.

In order to verify that we identified the crucial parameters governing ion mobility in these binary materials, we applied a statistical compressed-sensing approach using the sure-independence screening and sparsifying operator SISSO, as described in detail in the supporting information, to search for possible descriptors. We used the following input parameters or so-called primary features: number of atoms in the unit-cell ($N_{atom}$) and the atomic masses of the two elements in the binary compound ($m_A$, $m_X$), their formal oxidation numbers ($n_A$, $n_X$) and ionic radii ($r_A$, $r_X$), the Pauling electronegativity ($\chi_A$, $\chi_X$) of both elements, the A-X bond distances $d_{A-X}$, and the unit cell volume $V$. This approach allows to vary the dimensionality $\Omega$ of the descriptor space, and the descriptor is expressed as a linear combination of so-called features that are non-linear functions of the input parameter or primary features. For $\Omega = 1$, we obtained the descriptor

$$d = \left(\left(\frac{n_X}{n_A} - \cos(n_X)\right) / (\chi_X^6 \cdot \sin(m_X))\right),$$  

(4)
whereas for $\Omega = 2$ we found a two-dimensional descriptor consisting of the two features $d_1$ and $d_2$:

$$d_1 = (n_X)^2 \times (r_{\text{Mg}} + r_X),$$

$$d_2 = (\chi_X)^3/(N_{\text{atom}}).$$

Indeed these findings confirm that the oxidation states reflecting the charge of the atoms, the ion radii and the electronegativity differences are the determining factors for the migration barriers. Interestingly, the unit cell volume $V$, which has been shown to substantially influence the ionic mobility in some structural families\textsuperscript{26,27} does not show up in these statistically derived descriptors. However, note that the functional dependencies found by the SISSO operators do not allow for a straightforward interpretation of the physico-chemical factors underlying the migration process.

Therefore we decided to look for a verification whether the observed scaling relations as a function of the migration parameter (Eq. (2)) are also valid for other material types. As this study was originally motivated by the results for migration barriers of $A^{n+}$ in $AB_2X_4$ spinel structures, we reconsidered our previous results. For these structures, the NEB method was again applied in the low vacancy limit. In Fig. 3, we have plotted the migration barriers $E_a$(in eV) as a function of the migration parameter Eq. (2) for $A\text{Sc}_2\text{S}_4$ and $Mg\text{Sc}_2\text{X}_4$ spinels (panel a) and $A\text{Cr}_2\text{S}_4$ and $Mg\text{Cr}_2\text{X}_4$ spinels (panel b), respectively. Note that the factor $1/(N_A + N_X)$ has been omitted in the definition of the x-axis as this factor is constant for all considered materials. Again, as in Fig. 2, we find a linear scaling of the migration barriers upon variation of the anions $X^{n-}$ (blue symbols). Interestingly enough, we also find additional scaling relations upon variation of the cations $\text{Li}^+$, $\text{Na}^+$, $\text{K}^+$, $\text{Mg}^{2+}$, $\text{Ca}^{2+}$, and $\text{Sr}^{2+}$ (black symbols) (note that the $Mg\text{Sc}_2\text{S}_4$ and $Mg\text{Cr}_2\text{S}_4$ spinels, respectively, are part of both corresponding subsets). These results demonstrate that the scaling relations Eq. (3) are independently valid for the variation of both the cation chemistry of the migrating ions $A^{n+}$ as well as for the variation of the anion chemistry of the host lattice ions $X^{n-}$.

As Fig. 3 illustrates, upon variation of the host lattice cations $B^{n+}$ present in the sulfide spinels $AB_2X_4$, which are typically transition metal cations, the slope of the linear scaling relations represented by the parameter $C_A$ in Eq. 2 changes. We have determined the height of the migration barriers for the six additional transition metals $B=\text{Ti, V, Mn, Fe, Co and Ni}$ as a function of the migration number upon variation of the migrating cations $A^{n+}$ and
FIG. 3. Panel a: migration barriers (in eV) in ASc$_2$X$_4$ as a function of the migration parameter $(r_A + r_X)n_An_X\Delta\chi^2_{AX}$ (Eq. (2)) for ASc$_2$S$_4$ (black symbols) and MgSc$_2$X$_4$ spinels (blue symbols) for various mono- and multivalent cations A$^{n+}$ and anions X$^{n-}$. Panel b: the same as in panel a, but with Sc replaced by Cr.

collected the results in Fig. 4. We again find that the migration barriers follow linear scaling relations, but with different slopes. It is interesting to note that the difference $\Delta E_a^A(B)$ between the lowest and the highest migration barrier upon variation of the eight considered transition metals B increase with the size and the charge of the migrating cations A$^{n+}$, as
FIG. 4. Migration barriers (in eV) in AB$_2$S$_4$ spinels as a function of the migration number $N_{\text{migr}}^{\text{AS}}$ for eight different transition metal cations B= Sc, Ti, V, Cr, Mn, Fe, Ci and Ni upon variation of migrating cations A= Mg, Na, K, Mg, and Ca.

Illustrated in Tab. I. Apparently for increasing charge and size of the host lattice cations B, the specific nature of the interaction between the cations A and B becomes more prominent, as far as the migration barriers for A are concerned.

We have tried to derive some more trends in the height of the migration barrier as a function of the squared electronegativity difference between transition metal B and anion X, $\Delta \chi_{BX}^2$, and the ionic radius $r_B$ of the transition metal B (see supporting information, Fig. S1). Indeed, we find pronounced minima in $E_a$ as a function of these variables, but we did not manage to identify any linear scaling relations upon the variation of the cation B. However, based on the identification of these pronounced minima and the corresponding matching

<table>
<thead>
<tr>
<th>Migrating ion</th>
<th>Li$^+$</th>
<th>Na$^+$</th>
<th>K$^+$</th>
<th>Mg$^{2+}$</th>
<th>Ca$^{2+}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E_{a}^A(B)$ (eV)</td>
<td>0.08</td>
<td>0.19</td>
<td>0.42</td>
<td>0.44</td>
<td>0.61</td>
</tr>
</tbody>
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properties of Zr, we identified MgZr$_2$S$_4$ as a promising ion conductor with a high ion mobility, and indeed we found that MgZr$_2$S$_4$ has a rather low Mg migration barrier of only 0.3 eV.

We have applied the concept to yet another class of materials that are widely used as battery materials, namely olivines$^{47}$. Figure 5 shows the migration barriers in the olivine AFeSiO$_4$ as a function of the migration parameter $(r_A + r_X)n_An_X\Delta\chi^2_{AX}$ (Eq. (2)) for varying charge carriers A. Again a convincing linear scaling relation has been obtained.

The fact that the migration parameter $N_{\text{migr}}^{AX}$ captures the essence of the migration barrier height upon variation of the migrating cation A and the anion X of the host lattice calls for a critical assessment of this parameter. There are some obvious factors influencing the height of the migration barrier. For larger ions it will be harder to migrate through a given lattice, therefore it is no surprise that the ion radius $r_A$ enters the migration barrier. However, when also varying the size of the anion of the host lattice, it becomes apparent that it is both the size of the cation and of the anion represented by $r_A + r_X$ that is the critical length parameter, as already stressed in a previous study$^{26}$. Furthermore, note that in many cases the dependence of the mobility on the ionic radius is not monotonic$^{27}$, so any descriptor of the ion mobility taking into account the ionic radius needs to reflect this non-monotonic behavior.

It is also well-known that the charge of the migrating ion matters with respect to the ion mobility. The higher the charge of an ion, the stronger its interaction with the environment and thus the higher the migration barriers. This same argumentation of course also applies...
to the charge of the ions constituting the host lattice as the ionic interaction scales with
the product of the charges of interacting ions. These charges enter the migration parameter
through the product of the oxidation numbers \( n_A n_X \).

However, it is important to realize that in the migration of “ions” in a host lattice it is
not \textit{a priori} clear that the “ions” keep their ionic charge. Any crystal containing migrating
ions has to be overall charge neutral because macroscopically charged matter is unstable.
Hence any charge on the migrating ions has to be compensated by the host lattice. Of
course, the assumption that strong ions remain charged in a host lattice makes a lot of sense
and is the basis of the concept of formal oxidation numbers. Still, formal atomic charges in
a material are no good observables because it can not be uniquely defined which electrons
belong to the migration ion and which to the host lattice. This is also the reason why there
is a broad variety of different charge partition schemes used in quantum chemical codes in
order to derive atomic charge numbers which can give quite different quantitative results.
And furthermore, there are hardly any chemical systems in which the interaction is either
purely ionic or purely covalent or purely metallic. Therefore it is not surprising that trends
in the ion mobility can not be fully understood on the basis of formal oxidation states alone.

This deviation from the purely ionic interaction can be characterized by the difference in
the electronegativity \( \Delta \chi^2 \) of the interacting compounds which is also the basis for the Van
Arkel-Ketelaar triangle. In this context it should be noted that the Pauling electronegativity
in the form revised by Allred\textsuperscript{48} that has been used here is based on a quite accurate, semi-
empirical formula for dissociation energies, namely

\[
(\chi_A - \chi_B)^2 = E_d(AB) - \frac{E_d(AA) + E_d(BB)}{2}.
\]

This illustrates that the square of the difference in the electronegativities takes the deviation
from a purely ionic interaction in a compound crystal into account. It is in fact true that
the stronger polarizability of “soft” anions has already been used to explain the higher ion
mobility in chalcogenides containing sulfur and selenide compared to oxides\textsuperscript{11} with their
softness reflected in the lower electronegativity of sulfur and selenide\textsuperscript{49,50}. Still this notion
had not been transferred into any descriptor concept before.

The fact that the migration parameter including \( \Delta \chi^2 \) yields such a good descriptor for
the height of the migration barriers reconfirms that a purely ionic consideration of ion
mobility in crystals does not capture all factors determining this mobility. It also means
that this deviation from ionicity is the reason for the observed non-monotonic behavior of
the migration barriers as a function of the ionic radii which is correctly taken into account
by including the factor $\Delta \chi^2$ in the migration parameter. It is also important to stress the
fact that the parameters entering the migration number are basically independent of the
particular structure of the considered host lattice, as they correspond to general atomic and
ionic properties of the particular elements. The same parameters enter the scaling relations
for binaries, spinels and olivines, confirming the general fundamental nature of the scaling
relations.

Note that the linear scaling relations as a function of the migration parameter established
in our work do not allow the quantitative prediction of the height of migration barriers in
any particular system without any initially measured or calculated data. Thus they do not
correspond to a parameterization of the barrier height as a function of input parameter across
all families of possible structures. However, these scaling relations allow to make qualitative
predictions of the height of migration barriers, and once some migration barriers are known in
these structures, then even semi-quantitative predictions based on easily accessible materials
parameters can be made. This will be very beneficial for the identification of promising
candidate materials with improved mobility properties. Of course, this linear scaling is not
perfect, and we already identified some outliers. However, this descriptor is based on a
strict physico-chemical reasoning, so deviations from the scaling relations should point to
some interesting additional factors also influencing the ion mobility and thus to an enhanced
fundamental understanding of ion mobility.

III. CONCLUSIONS AND SUMMARY

In summary, we propose a descriptor called migration parameter for the ion mobility in
crystalline solids that is based on well-accessible materials parameters, namely ion sizes,
oxidation states and the Pauling electronegativity difference between anions and cations in
the compounds. Thus in contrast to previous attempts to derive descriptors for the ion
mobility we also take the deviation from ionic bonding in the compounds into account. For
a broad range of materials classes, we have shown that the height of the migration barrier
follows linear scaling relations as a function of this descriptor upon both the variation of the
cation chemistry of the migrating ion as well as upon variation of the anion chemistry of the
host lattice. This demonstrates the strong predictive power of the descriptor which should accelerate the discovery of materials with improved migration properties in electrochemical energy storage and conversion.

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AUTHOR CONTRIBUTIONS

AG designed and supervised the whole project. MS developed the conceptual framework of the project, performed all calculations and wrote the first draft of the manuscript. Both authors revised the manuscript and have approved the submitted version.


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