Counting algorithms with phase sensitive detection for hover and S turn of aircraft flight

Moli Chen (chenmoli160@nuaa.edu.cn)  
Nanjing University of Aeronautics and Astronautics

Xunkai Wei  
Beijing Aeronautical Engineering Technology Research Center

Hao Wang  
Beijing Aeronautical Engineering Technology Research Center

Zhenhe Jiang  
Nanjing University of Aeronautics and Astronautics

Research Article

Keywords: Aircraft, Intelligence, Phase sensitive detection, Redundancy, Turn

Posted Date: June 26th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3084799/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License. Read Full License
Abstract

Hover and S turn of an aircraft can be characterized by flight yaw angle, but the yaw angle signal in actual fly parameters is not only non-stationary and non-ergodic, but also often appears violent mutations and weak fluctuations, which makes it difficult to work out service environment spectra related to the numbers of hovering and S-turning training times. This article develops intelligent counting algorithms for the numbers of hovers and S turns based on yaw angle data that from fly parameters. Using the median-crossing de-redundant function of Phase Sensitive Detection (PSD) and analyzing the characteristics of 360° hovering fly parameters, a counting algorithm for the number of hovers during a flight profile is presented. Using the split-half function of PSD, a triangle layering algorithm about yaw angle signal is developed to count the number of S turns during a flight profile, where the signal of each sublayer is segmented into the median-crossing intervals so as to eliminate the redundant median-crossing marks from the previous layer. Compared with the artificial means, the counting results of flight example show that the intelligent algorithms presented are effective.

1. Introduction

Both hover and S turn are maneuvers related to changes of flight yaw angle in an aircraft cruising, maneuvering flights with a yaw angle change of 360° or above are called hover, while below 360° and a shape similar to the letter "S" or lazy "8" are called S turn. Hover and S turn are two fundamental movements in flight training tasks, by manipulating rudder and accelerator, yaw angle of an aircraft can be changed to implement hover and S turn at different radii, slopes, and speeds. Many difficult maneuvers need hover or S turn as auxiliary movement, such as fancy somersault, barrel roll, etc., and the ultimate radius and speed of aircraft hover and S turn are closely related to the performance of aircraft engine. Hover and S turn movements effect the loads of an aircraft [1], the fatigue reliability design of structures and components in aircraft engine must meet the requirements of hovering and S-turning maneuverability. When compiling service environment spectra for structural design of aircraft engine rotors and main bearings based on flight training tasks, hover and S turn related to flight yaw angle are also two commonly involved maneuvers [2, 3]. However, although the fly parameter data recorded by aircraft onboard instruments includes yaw angle and yaw angle rate, it generally does not record the numbers of hovers and S turns, as well as the curvature radius of the cruising. This makes the compilation of aircraft service environment spectra based on hover or S turn highly dependent on the extraction of yaw angle signal features, especially when lots of samples with a big data from various flight subjects.

According to different flight training tasks and the subjects in them, the yaw angle data in different profiles of the same aircraft has obvious non-stationary and non-ergodic characteristics, while the characteristics of just one flight profile are difficult to represent the other flight profile samples. The hovers and S turns during an aircraft cruising are not ideal actions due to weather, geographical conditions, etc., an aircraft often experiences occasional changes in direction during hover and S turn, and the yaw angle data often appears violent mutations and weak fluctuations. This provokes great
troubles into compiling the service environment spectra for hovering and S-turning maneuvers, with the first being the statistics of the numbers of hovering and S-turning maneuvers. The actual fly parameter data has many weak fluctuations so that the peak-valley value method that suitable to calculate the number of inflection points, cannot be used for counting the numbers of hovers and S turns. Due to the instability of frequency, the methods such as rain flow filtering [4] and cyclic counting that are often used in the compilation of aircraft service environment spectra, are difficult to accurately calculate the numbers of hovers and S turns in flight profile, while the other methods such as time-delay correlation [5] and empirical mode decomposition [6] used in the statistical analysis field of non-ergodic data, are highly dependent on frequency information and are not suitable to calculate the numbers of hovers and S turns. Due to the uncertainty of flight time history characteristics, the methods that with the theoretical fundament on solving distance between two maneuvering profiles, such as principal component analysis [7], neural network [8, 9] or other types of machine learning algorithm [10, 11], are unstable when applying to the classification and statistics of such yaw angle data.

This article presents Phase Sensitive Detection (PSD) counting algorithms for the numbers of 360° hovers and S turns in a flight profile intelligently. By removing redundancy in the median-crossing marks of yaw angle data, the number of hovers can be worked out, while by means of the layering function of PSD and layer-by-layer decomposition of the yaw angle signal, the number of S turns can also be obtained. Finally, the effectiveness of the counting algorithms presented in this article is verified through a practical flight example.

2. History of flight yaw angle

Figure 1 shows the time histories of the yaw angle obtained from two flight profiles. The yaw angle histories during aircraft cruising have obvious non-stationary and non-ergodic characteristics. There is a significant difference in hovering or turning time in the figure. One hover may take 3.7s, while the other 74s, with a difference of 20 times. The angle rate of a hover is not constant, but oscillates quickly or slowly in the whole process, and the frequency of yaw angles changes unstably. Each maneuver is not ideal either, and many actions changing yaw angle or angle rate are found during one hovering or turning process. For example, an aircraft enters a hover from level flight state with constant yaw angle, or from an S-turn action with variable yaw angle and angle rate, while completes a hover to a turn flight or other actions. All the above show that the profiles about flight yaw angle have not obvious time history characteristics. It can be said that, it is difficult to intelligently extract the numbers of 360° hovers and S turns when aiding by the signal analysis, but easy to manually count them from the time history when according to the concepts of hovers and S turns. However, the latter is not suitable for compiling flight service environment spectra when there are a large number of flight profile samples, thus signal processing algorithms need to be investigated further to intelligently count the numbers of hovers and S turns.

3. Phase sensitive detection
After clarifying the characteristics of yaw angle time history, this section designs counting algorithms for hover and S turn based on yaw angle data of flight profile.

PSD is a signal demodulation method that uses equilibrium positions processing the positive and negative phases of vibration signals. This method can be easily implemented with diode circuits and is commonly used in hardware devices such as dynamic strain gauges, wireless receiving devices. Considering the angle time history vibrates near its median during hover or S turn maneuver, this section uses the PSD with median theory to decompose the time history signal, and provides some PSD algorithms for processing yaw angle time history data as well as forms a set of algorithms for counting of the hovering and S-turning numbers.

Let yaw angle signal in a certain profile be as follow:

\[ \Psi_0 = \{\psi_0(1), \psi_0(2), \cdots \psi_0(i), \cdots \psi_0(N_0)\} \]

where \( \psi_0(i) \) is the \( i \)th sample point in signal \( \Psi_0 \), \( N_0 \) is the length of \( \Psi_0 \).

The following provides PSD algorithms of yaw angle signal, including median retrieving, median-crossing de redundancy, and signal splitting.

### 3.1. Median retrieving algorithm

Calculate median \( \bar{\Psi}_0 \) of the yaw angle sequence \( \Psi_0 \) with \( \bar{\Psi}_0 = [\max(\Psi_0) + \min(\Psi_0)]/2 \), the sequence \( \Psi' \) is defined by:

\[ \Psi' = \Psi_0 - \bar{\Psi}_0 I_{1 \times N_0} = \{ \psi'(1), \psi'(2), \cdots, \psi'(i), \cdots, \psi'(N_0) \} \]

where \( I_{1 \times N_0} \) denotes a \( 1 \times N_0 \) row vector. The median of sequence \( \Psi' \) is zero.

Let \( T_0 \) be the set of zero-crossing marks. To \( \forall \psi'(i) \in \Psi' (i = 2, 3, \cdots, N_0) \), Criterion 1 given by:

If \( \psi'(i - 1)\psi'(i) < 0 \) or \( \psi'(i) = 0 \), then the yaw angle signal \( \Psi' \) passes through zero value one time and \( i \in T_0 \).

That is, if the sign of the yaw angle \( \psi'(i - 1) \) and \( \psi'(i) \) is opposite, it can be inferred \( \Psi' \) passing through a zero point, and the number \( i \) that as the time mark in this case need be put into set \( T_0 \).

The number set \( T_0 \) yielded by Criterion 1 is notated as \( T_0 = \{ t(1), t(2), \cdots, t(i), \cdots, t(n_0) \} \), where \( t(i) \) being the \( i \)th term in the sequence, \( n_0 \) being the length of \( T_0 \) and the number of zero-crossing points
in $\Psi'$. Due to the number of zero-crossing points must be less than the length of yaw angle time history, thus $n_0 < N_0$, we can know that $T_0$ belongs to the natural number field with $T_0 \subset \{1, 2, 3, \cdots, N_0\}$.

By the above, PSD retrieves the median terms and can count the median-crossing marks of the original signal $\Psi_0$.

### 3.2. Median de redundancy algorithm

Mathematically, the time points and number of median-crossing marks from Criterion 1 are accurate, but for an actual yaw angle time history, it is necessary to remove the redundancy in the zero-crossing mark set $T_0$. The reason of this operation is that, during an aircraft flight, there are always some deviations in the angle history due to factors such as weather conditions, geographic environment, etc., however, these small deviations or oscillations near the median cannot denote hovering or S-turning maneuvers. A threshold for such minor deviations will be set based on actual flight conditions so as to eliminate redundancy of such time points in $T_0$.

Let the deviation threshold of yaw angles in a profile be $c$, the zero-crossing set of the yaw angle signal $\Psi'$ be:

$$T_0 = \{t(1), t(2), \cdots, t(i), \cdots, t(n_0)\} = \{t_1, t_2, \cdots, t_i, \cdots, t_n\}, t_i = t(i), i = 1, 2, \cdots, n_0.$$ (3)

Then we develop a de redundant algorithm which only remains the zero-crossing marks with the yaw angle variation larger than $c$ in a small interval.

As shown in Fig. 2, for segmental sequence $\{t_{i-1}, t_i, t_{i+1}, t_{i+2}\} \in T_0 (i = 2, 3, \cdots, n_0 - 2)$, define a semi open interval $[t_{i-1}, t_{i+2}] \subset \mathbb{N}$, where $\mathbb{N}$ being the positive natural number field, then elements of the yaw angle within this argument interval partition are as follows:

$$\Psi_{si} = \{\psi'(t_{i-1}), \psi'(t_{i-1} + 1), \cdots, \psi'(t_{i+2} - 1)\}$$

Let $A_{si} = \max(\Psi_{si}) - \min(\Psi_{si})$ and the set of redundant zero-crossing points be $\tilde{T}_0$. The criterion to judge whether zero-crossing points $t_i$ and $t_{i+1}$ redundant is given by:

Criterion 2: if $A_{si} < c$, then $t_i$ and $t_{i+1}$ are redundant and $\{t_i, t_{i+1}\} \subseteq \tilde{T}_0$.

Similar to obtaining set $T_0$ through Criterion 1, set $\tilde{T}_0$ can be yielded through Criterion 2. The meaning of Criterion 2 is that, for the zero-crossing points $t_i$ and $t_{i+1}$, if the maximum fluctuation value of the yaw angles near them is less than the threshold $c$, it is determined that both of them are caused by small deviations of the yaw angles, but not the zero-crossing points that can represent a hover or S-turn.
Calculate the difference set \( \hat{T}_0 = T_0 \setminus \text{backslash} \setminus \text{backslash} \hat{T}_0 \), where \( T_0 \) and \( \hat{T}_0 \) are the zero-crossing set and the zero-crossing redundant set respectively, the symbol "\( \setminus \)" denotes the difference operator, that is, removing the elements belonging to \( \hat{T}_0 \) from \( T_0 \). Write:

\[
\hat{T}_0 = \{ \hat{t}_1, \hat{t}_2, \cdots, \hat{t}_i, \cdots, \hat{t}_{n_0} \} = \{ \hat{t}(1), \hat{t}(2), \cdots, \hat{t}(i), \cdots, \hat{t}(n_0) \}, \hat{t}_i = \hat{t}(i),
\]

where \( n_0 \) is the length of \( \hat{T}_0 \). It can be known that \( n_0 \) is the number of effective zero-crossing or median-crossing marks in the yaw angle signal \( \Psi' \), as shown in Fig. 3.

### 3.3. Signal splitting algorithm

Use PSD to split the yaw angle signal in half, the calculation formula is:

\[
\psi_D(i) = \begin{cases} 
\psi'(i)\psi'(i) < 0 & \psi_U(i) = \begin{cases} 
\psi'(i)\psi'(i) > 0 \quad \psi_U(i) = \begin{cases} 
\psi'(i)\psi'(i) \leq 0, i = 1, 2, 3, \cdots, N_0, (5)
\end{cases}
\end{cases}
\end{cases}
\]

\( \Psi_D = \{ \psi_D(1), \psi_D(2), \cdots, \psi_D(i), \cdots, \psi_D(N_0) \} \) denotes the lower part after yaw angle signal decomposed by PSD as well as \( \Psi_U = \{ \psi_U(1), \psi_U(2), \cdots, \psi_U(i), \cdots, \psi_U(N_0) \} \) the upper part.

PSD decompose \( \Psi_0 \) into the upper and lower halves by its median, with the upper retaining the terms which \( \Psi' > 0 \) in their amplitudes and filling the others with the median, while the lower half conversely retaining which \( \Psi' < 0 \) and filling the others with the median too. As will be seen later in this article, this signal processing method can be applied to the layer-by-layer counting of S turns.

The functions of PSD are shown in Fig. 4.

### 4. Counting algorithm for hovers

After understanding the time history of yaw angle and the PSD algorithm, this section will introduce a counting algorithm for 360° hover. Using the median-crossing retrieval and de redundancy algorithms in sections 3.1 and 3.2, we can obtain the zero-crossing mark set \( \hat{T}_0 = \{ \hat{t}_1, \hat{t}_2, \cdots, \hat{t}_i, \cdots, \hat{t}_{n_0} \} \) that removed the redundant marks of \( \Psi_0 \), for the next step, we need to further count the number of hovers by the aid of the sequence \( \hat{T}_0 \).

Firstly, based on the elements in \( \hat{T}_0, \Psi_0 \) is segmented to \( \lfloor n_0/2 \rfloor \) semi open intervals, where the symbol \( \lfloor \cdot \rfloor \) denotes a downward rounding operator. To \( \forall \{ \hat{t}_{i-1}, \hat{t}_{i+1} \} \subseteq \hat{T}_0 \), the elements of the yaw angle within the segmental interval \( [\hat{t}_{i-1}, \hat{t}_{i+1}) \subset \mathbb{N}^* \) are as follows:

\[
\Psi_{s_i}^0 = \{ \psi_0(t_{i-1}), \psi_0(t_{i-1} + 1), \cdots, \psi_0(t_{i+1} - 1) \} \in \Psi_0, i = 2, 4, \cdots, 2 \lfloor n_0/2 \rfloor. (6)
\]

Let \( A_{s_i} = \max(\Psi_{s_i}^0) - \min(\Psi_{s_i}^0) \), and define the set \( H \) for 360° hover maneuver while \( S \) for non-hovering maneuver. Note that the angle change for hover once is 360°, and a criterion for hover can be expressed as follow based on this characteristic,
Criterion 3: If $A_{si}^0 > 360 - c$, then the aircraft does one hovering action, $i \in H$; otherwise, $i \in S$.

In Criterion 3, 360 represents a hovering angle of 360°, and $c$ represents the threshold for deviation of the yaw angle, while the purpose of this threshold is to avoid Criterion 3 failure due to sampling errors of the yaw angle recorded in a discrete manner. The meaning of Criterion 3 is that, for the zero-crossing mark $\hat{t}_i$, if the amplitude fluctuation of the yaw angles near it is greater than the threshold $(360 - c)^\circ$, it is inferred that the aircraft has taken hover once; otherwise, the action taken is not a hover, although the aircraft crosses the median.

As mentioned above, if the length of set $H$ is $N_H$, then $N_H$ is the number of the hovers in a flight profile.

5. Counting algorithm for S turns

From Criterion 3 and the concept of S turn, it can be seen that $S$ is actually a mark set for S turn maneuvers which the change in yaw angle is less than 360° (or $(360 - c)^\circ$ when considering the threshold). Similar to set $H$, if the number of elements in set $S$ is $N_S^0$, then the aircraft takes S turns $N_S^0$ times near the median $\bar{\Psi}_0$ of $\Psi_0$, where the superscript or subscript "0" represents the S turn number obtained from the original signal $\Psi_0$.

However, in addition to S turns nearby $\bar{\Psi}_0$, an aircraft also turns in S shape near other yaw angles, as shown in Fig. 1(a), where the aircraft experiences S turn near a yaw angle of 50°. Thus, counting algorithms for S turns of an aircraft near other angles rather than $\bar{\Psi}_0$ will be studied in this section. The general thinking is that, with the help of PSD splitting the signal into halves, the original signal $\Psi_0$ is divided into several layers, then the S turns of each layer are cyclically counted, at the end, counting algorithms for S turns of an aircraft in a flight profile are presented.

5.1. Layering algorithm

As shown in Fig. 5, the signal splitting algorithm in Section 3.3 is used to decompose $\Psi_0$ to triangle sublayers. The first layer of decomposition obtains $2^1 = 2$ yaw angle signals, which are notated by $\Psi_{11}$ and $\Psi_{12}$; the second layer obtains $2^2 = 4$ yaw angle signals, which are notated by $\Psi_{21}, \Psi_{22}, \Psi_{23},$ and $\Psi_{24}$; if a total of $l$ layers of decomposition are carried out, the number of yaw angle signals in the last layer is $2^l$, notated in sequence by $\Psi_{11}, \Psi_{12}, \Psi_{13}, ..., \Psi_{l,2^l}$.

5.2. Counting algorithm

Using the first layer as an example to illustrate the S-turning counting algorithm for each sublayer signal.

For the lower half signal $\Psi_{11}$, based on the theory in section 3.2, solve the set $T_{0}^{11} = \{\hat{t}_1, \hat{t}_2, ..., \hat{t}_i, ..., \hat{t}_{\hat{n}_0}\}$, which is the set of de redundant median-crossing marks. The subscript or superscript "11" represents the number of the solved sublayer, and $\hat{n}_0$ is the length of $T_{0}^{11}$. It should be noted that for the lower $\Psi_{11}$ of $\Psi_0$, there are hovering and S-turning marks in set $T_{0}^{11}$ which
have already been counted by set $H$ and set $S$. That is to say, even $\hat{T}_{0}^{11}$ after de redundant median-crossing points of yaw angle deviation below threshold $c$, for the sublayer sequence, there are still redundant points which have been counted by the previous layers, as shown in Fig. 6. Therefore, further redundancy removal algorithms are needed for $\hat{T}_{0}^{11}$.

Before designing the algorithm, we note that an S turn have paired rising and falling edges, therefore, one can mark an S turn just by the median-crossing point of its rising edge, as shown in Fig. 7.

To $\forall \hat{t}_{i} \in \hat{T}_{0}^{11}$, if the yaw angle at $\hat{t}_{i}$ is in the rising edge, $\hat{t}_{i}$ needs satisfy the following inequality:

$$\Psi_{11}(\hat{t}_{i} + 1) - \Psi_{11}(\hat{t}_{i} - 1) > 0$$

Then determine whether median-crossing mark $\hat{t}_{i}$ of the rising edge is a redundant point. From Fig. 6, it can be seen that if $\hat{t}_{i}$ is a redundant point, then, in the open interval $(\hat{t}_{i}, \hat{t}_{i+1})$, there exists one or more points of which yaw angle equals the median of the previous layer, and because $\Psi_{11}$ is the lower half signal of the previous layer, for the angle segment

$$\Psi_{si}^{11} = \{\Psi_{11}(\hat{t}_{i}), \Psi_{11}(\hat{t}_{i} + 1), \Psi_{11}(\hat{t}_{i} + 2), \cdots, \Psi_{11}(\hat{t}_{i+1})\},$$

it must be satisfied:

$$\max(\Psi_{si}^{11}) = \bar{\Psi}_{0}$$

By redundancy of the elements in set $\hat{T}_{0}^{11}$, and just retaining the elements that satisfy Eq. (7) but not Eq. (8), a set of median-crossing points can be obtained by removing the marks related to the previous layer. Notate the new set as $S_{11}$, if the length of $S_{11}$ is $N_{S}^{11}$, then $N_{S}^{11}$ is the number of S turns in current sublayer.

For the upper half signal $\Psi_{12}$, in order to use the above algorithm of the lower $\Psi_{11}$, let $\Psi_{12}^{'} = -\Psi_{12}$, similar to $\Psi_{11}$, the number $N_{S}^{12}$ of S turns on $\Psi_{12}^{'}$ can be obtained as well. Because of $\Psi_{12}^{'} = -\Psi_{12}$, $N_{S}^{12}$ is the number of S turns in the upper half signal $\Psi_{12}$.

After obtaining the S-turning numbers of the lower $\Psi_{11}$ and the upper $\Psi_{12}$, the total number of S turns on the first layer can be expressed by $N_{S}^{1} = N_{S}^{11} + N_{S}^{12}$.

All the above, if $\Psi_{0}$ is divided into $l$ layers, the number of S turns during a whole takeoff-to-landing profile of an aircraft is the sum of the numbers from the original signal $\Psi_{0}$ and all sublayers, that is, the total number of S turns on a profile is:
It should be noted that, the number of signal layers should not be more than \( \sqrt{\frac{360}{c}} \), otherwise, when the median-crossing de redundant algorithm is applied to layer of which the number larger than \( \sqrt{\frac{360}{c}} \), 0 point passing through the median position will be made out, that is, the length of \( \hat{T}_0 \) will be \( \hat{n}_0 = 0 \), thus the counting algorithm become invalid.

6 Flowchart of counting algorithms

Fig. 8 shows the general flowchart of counting the numbers of hovers and S turns from the yaw angle time series during a takeoff-to-landing flight profile.

7. Flight example

For the yaw angle signal shown in Fig. 1 (a), determine the numbers of hovers and S turns. Let the yaw angle deviation threshold \( c = 22.5^\circ \) and the number of S-turning decomposed layers \( l = 2 \), the results are shown in Figs. 9–12.

Figure 9 shows the original yaw angle signal, from which it can be inferred that the number of 360° hovers of the aircraft during this profile is 10, and the number of S turns in this layer is 6.

Figure 10 is the lower part from the original layer, which uses the median-crossing de redundant algorithm in Section 3.2 to eliminate the invalid median-crossing points.

Figure 11 shows the two sublayers of the first layer, with 4 S-turning marks obtained in both the upper and lower halves, resulting in a total of 8 S-turning maneuvers in this layer.

Figure 12 shows the four sublayers of the second layer, with 2 and 1 S turns appearing in Fig. 12 (a) and (c) respectively. There is no S-turning mark in Fig. 12 (b) and (d), indicating that there are no S-turning actions on these two sublayers. Therefore, 3 S turns can be obtained from Fig. 12.

Sum up the numbers of S turns in all layers, then we can know a total of 17 S turns in this flight profile. By observing the yaw angle history in each sublayer, the numbers of hovers and S turns can also be worked out artificially. The same results about the numbers of hovers and turns by both the PSD and artificial methods, verify the effectiveness and intelligence of the PSD algorithm presented in this article.
8. Conclusions

Based on the median-crossing de redundancy and signal splitting functions of PSD, this article presents algorithms for counting the numbers of 360° hovers and S turns in a flight profile. By means of median-crossing de redundancy of the yaw angle data, the number of hovers in a flight profile is obtained. With the help of the PSD layer-by-layer splitting function, the yaw angle signal is decomposed by triangular form into sublayers with the median of each layer as the boundary, and the number of S turns is calculated. Applying the presented algorithm to an example of flight yaw angle profile, the effectiveness of the counting algorithms is verified.

Declarations

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

The research is supported by “National Science and Technology Major Project (J2019-IV-004-0071)“.

Data Availability Statements

The data that support the findings of this study are available on request to the corresponding author.

References


**Figures**

(a) Profile 1  
(b) Profile 2

**Figure 1**

Time history of yaw angles.
Figure 2

The interval partition from zero-crossing marks.
Figure 3

The redundant median-crossing points.

(a) Marking the median-crossing points

(b) Splitting signal to halves with the median
Figure 4
The functions of PSD.

Layer 0

Layer 1

Layer 2

Layer \( l \)

\[ \Psi_0 \rightarrow \Psi_{11} \rightarrow \Psi_{21} \rightarrow \Psi_{i_1} \rightarrow \cdots \rightarrow \Psi_{i_2} \rightarrow \cdots \rightarrow \Psi_{1,2} \]

Figure 5
Layering algorithm.
Figure 6

The redundant median-crossing points in sublayer.
Figure 7

The rising edge on a sublayer.
Figure 8

The general flowchart.
Figure 9

The numbers of hovers and S turns from the original signal.
**Figure 10**

Median-crossing de redundancy for sublayer.

**Figure 11**

The first layer of S turn.

**Figure 12**

The 1st sublayer

The 2nd sublayer

The 3rd sublayer

The 4th sublayer
The second layer of S turn.